

# Visualization of Multi-variate Scientific Data

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## Abstract

*In this state-of-the-art report we discuss relevant research works related to the visualization of complex, multi-variate data. We focus on "non-classical" approaches, i.e. approaches which haven't been discussed in previous related reports, and we highlight techniques which potentially lead towards new directions in visualization research. We discuss how different techniques take effect at specific stages of the visualization pipeline and how they apply to multi-variate data sets being composed of scalars, vectors, and tensors. We also provide a categorization of these techniques in the aim for a better overview of related approaches. In the second part of this paper we take a look at recent techniques that are useful for the visualization of complex data sets either because they are general purpose or because they can be adapted to specific problems.*

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## 1. Introduction

In the last decade, there has been an enormous progress in scientific visualization [Max05], still the visualization of multi-variate continuous 3D data, especially also if the dataset is time-dependent, remains a great visualization challenge. Recent publications have stated a shift in visualization research from a classical approach dealing with visualization of small, isolated problems to a new kind of challenge: visualization of massive scale, dynamic data comprised of elements of varying levels of certainty and abstraction [TC06]. It is a well known fact that we are experiencing an increase in the amount and complexity of data generated that exceeds our ability to easily understand and make sense of it. To address this development, Lee et al. [LCG\*02] have indicated the work with multi-variate data sets as one important task for future visualization research that will require significant advances in visualization algorithms. Munzner et al. [MJM\*06] addressed top scientific research problems and identified multi-field visualization as one of the central questions for future research. Therefore, in this article, we give an overview of existing work in scientific visualization that points in this direction. Intentionally, we leave out a vast amount of work focusing on solving specific visualization tasks which repeatedly have been discussed in other related reports.

Even though there is a wealth of algorithms for automated

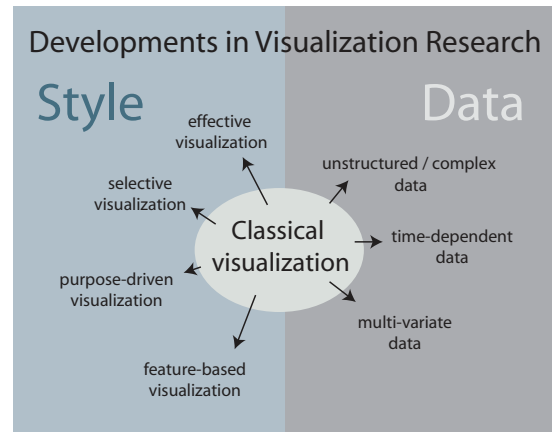
analysis to process data, these are only applicable when we know what we are looking for in the first place. Automated processing cannot generate new features and understanding beyond what is already known and might even remove important features of the data. Ward's Mantra [War02] "I'll Know it When I See it", stresses the fact that we often rely on our visual pattern recognition system to help us in gaining knowledge of the data. For this reason we need to discover new ways to visualize complex scientific data or adapt the existing ones to deal with the new situation of multi-field datasets.

Different tasks need specific qualities in a visualization method. For example when trying to gain overview of the data a continuous and smooth style might be most appropriate, whereas for information drill-down a researcher might want a specific feature to be shown in a way as true to the data as possible. We believe that the application of combined hybrid visualization techniques will increase the value of any visualization application, therefore most of the techniques discussed in this paper will be useful for other types of data as well.

By targeting the use of complex visualization techniques we do not speak in favor of a "more is better" approach, we would rather like to stress the importance of feature- and knowledge-driven visualization. The aim of this report is to give an overview of current techniques from various disci-

plines dealing with complex multi-dimensional scalar, vector and tensor data sets with the goal in mind to apply them to situations where these types of data are present at the same time in three dimensions. Kirby [KML99] defines the development of a visualization method as breaking the data into components, exploring the relationships among them, and visually expressing both the components and their relationships. To visualize the complexity of multiple data components and the relationships between them researchers have sought to find ways to combine the advantages of different types of visualization techniques. Hesselink et al. [HPvW94] give a short overview of research issues in visualization of vector and tensor fields that is still valid today. They declare four goals for future visualization research: feature-based representation of the data, comprising reduced visual complexity, increased information content and a visualization that matches the concepts of the application area. According to this, we see the three main advantages of multi-method visualization: first, improved effectiveness of visualization because each part of the data can be visualized by the most appropriate technique. Second, the ability to visualize multi-variate data sets while minimizing visual clutter at the same time. And third a separation between two questions that are too often intermingled: how to visualize vs. what to visualize. This intermingling is a drawback in the design of visualization systems featuring a single rendering technique. They very often induce a strong determination on the aspects of the data the user will be able to see. This does not mean that it is not possible to generate parameters that will show all the features but that a certain visualization tool might set the user on a track that hinders a successful investigation. An overview of current work on hybrid- and combined visualization algorithms can be a starting point for future visualization research in this direction of flexible and user-task driven visualization.

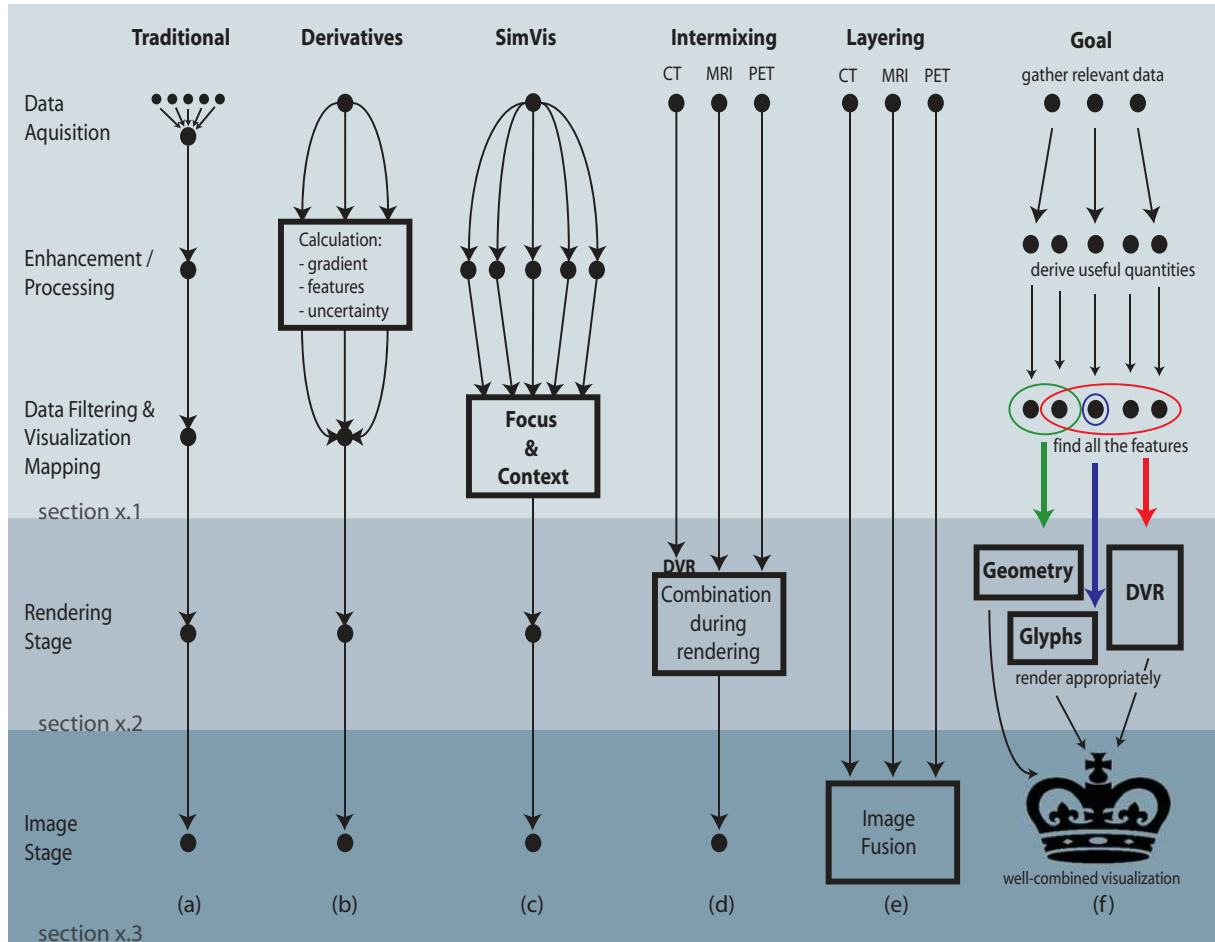
In the first part of this paper (sections 1 to 4) we will discuss existing techniques resulting from applications in various scientific fields such as meteorology, CFD simulation, medical imaging and geology. Some types of data are of distinguished importance and appear in so many applications that visualization of these has become a research area in its own respect. Among these are flow, tensor and (time-dependent) scalar data. We will take a closer look at techniques that have been developed in these fields to deal with multi-variate data. In this context we will structure the following discussion relating to the type of data a publication mainly deals with and focus on scalar data in the first chapter, vectorial data in the second and tensorial in the third. At the beginning of each chapter we will give references to survey articles that focus on the classical approaches concentrating on one or few data items of the respective data type at once. In the second part of this paper (sections 5 and 6) we will give a short overview of existing techniques that may not have been developed in the context of multi-variate



**Figure 1:** Visualization research is developing from classical 3D structured scalar visualization in many different directions. We believe that the use of techniques that allow the visualization of time-dependent, complex and multi-variate data in a useful way will be a key to successful visualization applications in the future.

visualization but that we consider as highly applicable for this purpose.

In each chapter we classify techniques dealing with multi-variate data sets according to the point of the visualization pipeline where the multi-variate nature of the data is being tackled (see Figure 2). The visualization pipeline is a model that describes how visual representations of data can be obtained following a procedure of well defined steps. (a) In traditional scientific visualization most of the acquired information is resampled onto a structured grid during an pre-processing and data analysis step. During data filtering the user selects the proportions of the data that will be visualized, e.g. by selecting values ranges of data attributes to be shown in focus. In the visualization mapping stage the data elements are mapped to visual representations. In the rendering stage the geometric information, data values and additional parameters such as viewing parameters and transfer functions contribute to the resulting image. The resulting image can undergo final modifications such as color enhancements or overdrawing (e.g. labels) to generate the final output of the visualization process. (b) Multi-variate visualization using derived quantities uses additionally computed values to improve the visualization, e.g. by using these values in the transfer function design [KKH01] or for color assignment [GGSC98]. Other important type of derived features are segmentation data, classification data and cluster information. These are very often generated in a (semi)automated fashion that also outputs uncertainty information for the generated information. These additional fields can be used to improve the visualization of the data [HBH03]. (c) The linking and brushing concept in SimVis [DGH03] uses different



**Figure 2:** Six examples of how to deal with multi-dimensional data-sets: (a) traditional (b) deriving additional quantities (c) linking brushing and other SimVis related techniques (d) data intermixing (e) layering rendered images (f) a well-combined visualization technique using all relevant data and derivations. Within the following sections on scalar, vectorial and tensorial data we will discuss techniques following the different stages of the visualization pipeline. In each section we will begin with techniques related to data acquisition, processing and visualization mapping (x.1), discuss techniques based on the rendering stage in the second subsection (x.2) and techniques working on the image stage in the third (x.3).

views and combines the selected features into a visualization mapping of the multi-variate data set. (d) The fourth type of multi-variate visualization combines the data coming from different modalities during the rendering step. The opacity is computed by using a combination operator to integrate the material properties into a single value [CS99]. (e) Layering renders different data items separately and combines them in an image fusion step [WFK\*02]. (f) A goal could be a visualization technique that would collect all relevant data, derive the inherent information, (interactively) detect all features of interest, match them to an appropriate rendering algorithm and combine the results cleverly to get the optimal visualization.

When talking of complex volumetric data we distinguish

between different types of data. We distinguish between multi-dimensional, multi-variate and multi-modal data. We will refer to multi-dimensional data in the scope of this article as data containing multiple independent physical dimensions (e.g., three spatial and the time dimension). Multi-variate data contains multiple variables or attributes, that do not have to be independent of each other. Multi-channel or multi-modal data is data that was acquired using multiple ways of measurement or operations. In many respects multi-modal and multi-variate data is considered equivalent in the context of visualization research. As an example we can think of a scan of the human chest using combined CT (computed tomography), MRI (magnetic resonance imaging) and ultrasound. The CT will capture the bones (e.g. ribs, spine)

best, resulting in a single scalar field. The MRI scan is more accurate in measuring soft tissue, resulting in a second scalar field and with the use of a contrast agent it is possible to obtain vector information about the blood flow. Using multiple scanning modalities, we have obtained a multi-variate data set where the data elements are indexed by three spacial dimensions. At each grid point we have two scalars which describe the same physical situation and one vector with three components describing the x,y and z direction of the flow. We see that scientific data has more structure than general multi-variate data sets (e.g. census data or questionnaire results).

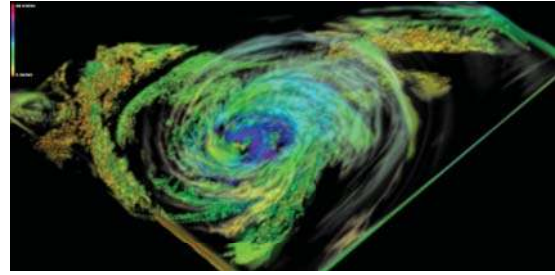
## 2. Dealing with scalar attributes

Scalar volume rendering is a central issue in medical data visualization. See [EHK\*06] for an extensive introduction. The acquisition devices used for medical imaging can be used for other purposes as well (e.g. industrial CT), but medical applications can be considered a large driving force of this field. Since medical data sets are often images obtained from different sources, the visualization of multi-valued data sets and their registration is a tightly coupled research area in medical image processing. Therefore medical imaging is a large source of multi-valued data sets. Also, medical images are very often segmented into regions containing different features which are of varying importance to the user - the location of a tumor might be of more interest than any other feature in the data set resulting in additional dimensions. An additional dimension for this kind of high dimensional data sets results from the uncertainty that comes with automated registration and segmentation algorithms [DKLP02]. Other types of scalar data result from sources such as simulations, marine sonar, meteorology radar/satellite scans and photographic volumes.

### 2.1. Techniques in the processing, filtering and visualization mapping stage

In this section we discuss visualization techniques that reduce the number of variables before rendering. Two important approaches are feature extraction methods and region of interest based methods. Feature extraction methods classify high dimensional data into features like iso surfaces, topological structures or other domain related features (such as vortices in flow data). They assign to each point in space a degree of membership to a feature of interest (e.g. a tumor) that can then be visualized using scalar rendering and color coding.

The SimVis technique uses linking and brushing for interactive feature selection [DMG\*05] (see Fig 3). The user can specify a degree of interest in a subset of data items based on their attributes. The degree of interest functions from several linked views (scatterplots, histograms, etc.) are then accumulated using fuzzy-logic operators [DGH03]. According



**Figure 3:** A DVR visualization of a hurricane dataset using interactive feature specification and focus+context visualization [DMG\*05]. (Image courtesy of H. Doleisch)

to this degree of interest function data samples are classified as focus or context and colored accordingly. This is an example of feature and knowledge driven visualization. Tzeng et al. [TLM05] suggested an intelligent systems approach to brushing. The user brushes portions of the volume that are of interest. A machine learning classifier (a neural network or support vector machine) is built from this training set. Based on the classifier the system then determines for each sample whether it belongs to the region of interest or not.

In the field of medical imaging the integration of different volumes into a single visual representation is called data intermixing (this compares to data fusion and volume fusion in other fields). The different modalities (e.g. Computed Tomography (CT), Magnetic Resonance Imaging (MRI) or Positron Emission Tomography (PET)) can show different, complementary and partially overlapping aspects of the situation. Therefore most algorithms are designed to allow flexible viewing of the original channels alone and more or less sophisticated combinations. A standard approach is to combine datasets based on segmentation information (e.g., the brain is visualized using MRI data, while the skull is shown based on data from the CT channel) combined with color coding (see Figure 4).

Illumination stage intermixing takes place in the visualization mapping stage: to combine the different attributes in the multi-valued volume voxel  $V$ , a combination function takes the attribute values  $a_1, \dots, a_n$  directly as input:

$$opacity(V) := opacity(combine(a_1, \dots, a_n))$$

This way only a single transfer function is necessary, but we have to define a combination function that deals with the different value ranges of the attributes (e.g., using a multi-dimensional transfer function). To handle the complexity of higher-dimensional transfer functions, Kniss suggests the use of (local support) gaussian transfer functions [KPI\*03]. Kniss and Hansen [KKH01] developed a technique for visualization of multi-variate data by applying multi-dimensional transfer functions and derived quantities. In a case study [KHGR02] they apply this approach to me-

teological simulation data using three-dimensional transfer functions (for instance two axes map data values and the third the gradient magnitude). A drawback of this method is that multi-dimensional transfer function design is a complicated task and the results are hard to predict.

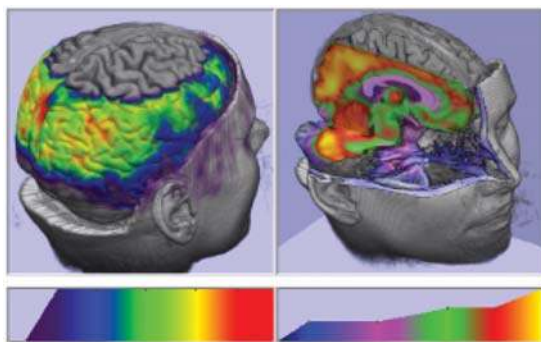
Kreeger and Kaufmann [KK99] present a hybrid rendering algorithm that combines volume rendering and translucent polygons embedded inside the volume. They apply their technique to combine an MRI-volume of a human head with an angiogram that visualizes blood vessels. Here the 'how' approach of the visualization (surfaces and volume) are matched to the 'what' context of the data (blood vessels and tissue).

Woodring and Shen [WS06] present a technique to visually compare different time steps of time-varying datasets using boolean and other operations. The operators over, in, out, atop and xor compare two timesteps A and B at each voxel to derive a new field.

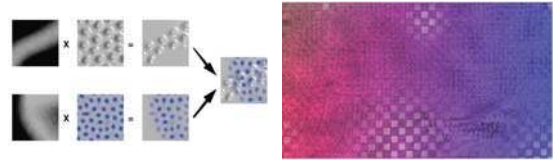
Another (rare) source of multi-modal data are photographic volumes. The visible human male data set contains vectorial (RGB) color information at each voxel taken by photographing each slice. Volume rendering is difficult in this context, because a high-dimensional transfer function from 3D to opacity is necessary. Ebert et al. [EMRY02] show how to use a perceptually appropriate colorspace for transfer function design. Ghosh et al. [GPKM03] render multi-channel color volumes consisting of CT, MRI and color information on the hardware. Muraki et al. [MNK00] have presented a method to assign color values to voxels from multi-modal datasets using a neuronal net trained on a photographic volume.

## 2.2. Rendering stage techniques

Cai and Sakas [CS99] present a ray casting technique that integrates the information of multiple volumes during rendering. Data intermixing is done in the rendering pipeline



**Figure 4:** Using multiple transfer functions, region selection and color coding to combine information from multiple channels [MFNF01]. (Image courtesy of I. H. Manssour)



**Figure 5:** A combination of different bump-mapping styles to visualize multiple fields on the same layer [Tay02]. (Image courtesy of R. M. Taylor)

during accumulation. On the accumulation stage the different modalities are already mapped to opacity and intensity values by their own transfer functions. This means they have the same intensity and opacity range  $([0, 1])$ . Intermixing on the accumulation stage can then be done by defining additional opacity and intensity evaluation function taking as input the opacities of the different attributes  $a_1, \dots, a_n$ :

$$\text{opacity}(V) := \text{combine}(\text{opacity}(a_1), \dots, \text{opacity}(a_n))$$

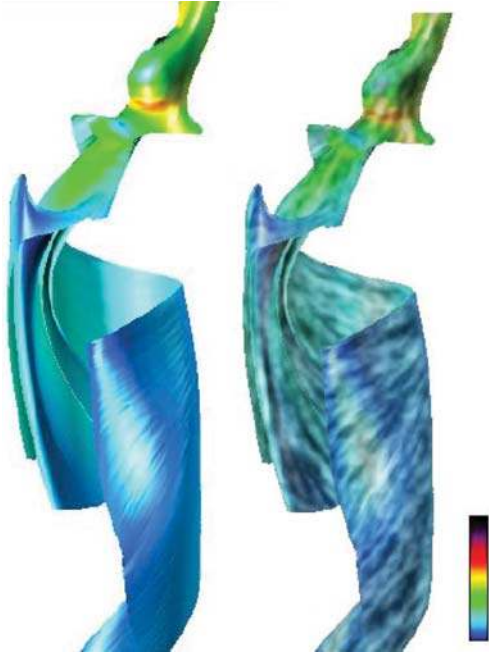
The authors suggest to use linear or boolean operators for combination. Ferre et al. [FPT04] discuss combination functions that take into account additional values, such as the gradient. Rössler et al. [RTF\*06] present a GPU-based implementation of the DVR intermixing technique working with 3D textures and shader programs. Each data volume is rendered separately using an individual shader program allowing for different render modes for the modalities. Then intermixing is done when volume slices are combined in back-to-front order.

The Spectral Volume Rendering Technique [NVS00] displays a multimodal volume using a physics based light interaction model: each material interacts with the light in its specific way. For different modalities, the interaction with several materials at one point in space is simulated. Spectral Volume Rendering is probably the physically most realistic technique to do illumination stage intermixing in terms of light propagation.

Grimm et al. [GBKG04] developed methods that allow efficient visualization of multiple intersecting volumetric objects that is applicable in the situation of multimodal volumes. They introduce the concept of V-Objects, which represent abstract properties like illumination, transfer functions, region of interest and transformations of an object connected to a volumetric data source.

## 2.3. Image stage techniques

Among the visual attributes that can represent data values are color, transparency, contour lines, surface albedo, texture and surface height. Textures are a versatile medium, that can be computed with approaches such as spot noise, texture advection, bump-mapping and reaction-diffusion techniques. Shenan and Interrante [SI05] discuss methods to combine



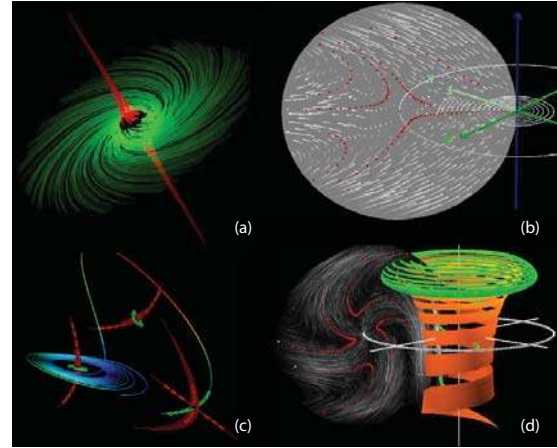
**Figure 6:** Combining texture advection and surface based flow visualization. Both the location of the iso-surface and its texture convey information about the flow [LGSH06].

color and texture to represent multiple values at a single location. Taylor [Tay02] describes a layering system for visualization of multiple fields on the same surface using data driven spots. He also discusses problems that arise due to the layering process. Their finding is that visualizing multiple data sets using a different technique for each layer is limited to four different fields, since the layers on top either mask or scramble the information below. Special care must be taken to keep the different layers distinguishable, for example by keeping the frequencies of the surface characteristics separated enough. In Figure 5 we see an example how bump-mapping and a reaction-diffusion texture are combined (left). On the right we see a resulting image using data driven spot textures. House et al. [HBW05] discuss optimal textures for information visualization, including a large user study design, that investigates layered textures for visualizing information located on a surface.

### 3. Vector Field and Flow Visualization

The velocity of a flow is represented by a vector field and a vector field can define a flow, therefore in most applications their visualizations can be considered equivalent [Max05].

Recent surveys and overview articles include: a classification of different flow visualization algorithms and a discussion on derived, second-order data by Hauser [Hau06] and the state of the art report on flow visualization focus-



**Figure 7:** Enhanced topology visualization combining (a) streamline-based glyphs, (b) direct flow visualization, (c) solution trajectories and (d) streambands [HG00].

ing on dense and texture-based techniques by Laramée et al. [LHD\*04]. Post et al. [PVH\*03] give an overview of feature extraction methods for flow fields.

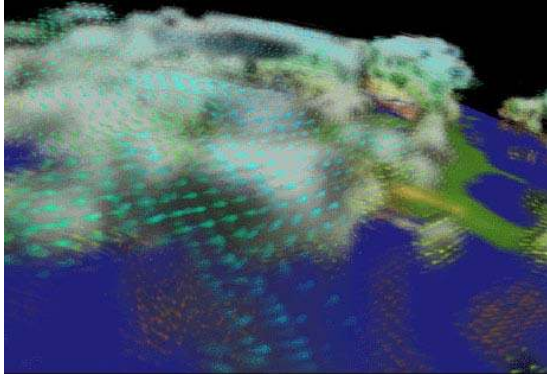
#### 3.1. Techniques in the processing, filtering and visualization mapping stage

A basic technique in flow visualization is to match the attributes of a data set to physically appropriate representations ('how' matched to 'what'). For example shock waves are mapped to surfaces, dispersed particles are mapped to particle traces or points. Therefore we will not repeat every application that uses combinations of standard flow visualization techniques such as lines [ZSH96, KM96], surfaces [Wij93], sub volumes [SVL91] or dense techniques [IG97].

Laramée et al. [LGSH06] discuss the application of texture advection on surfaces for visualization of vector fields defined at the stream surface. In this application tumbling motion of the flow in the combustion chamber of a diesel engine is visualized by seeding a surface that depicts the swirling motion of the flow. This is based on work by van Wijk and Laramée on image space advection [Wij03, LvWJH04]. In their approach parameterization of the surface is not necessary and advection is not computed for pixels occluded by other parts of the surface. The main steps are:

1. compute flow vectors at vertices of the surface mesh
2. project the vector field onto the image plane
3. advect texture properties according to the projected vector field
4. add shading to the image to convey shape information

This approach allows interactive frame rates for animated



**Figure 8:** Direct volume rendering combining both realistic cloud rendering and splatted directional glyphs and color coding [CM93]. (Image courtesy of R. A. Crawfis)

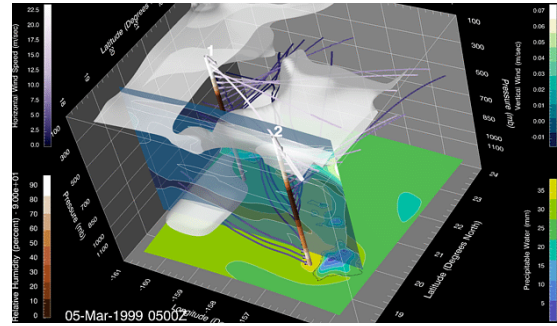
flow textures. Both the shape of the surface and the texture can transport meaning to the user (see Figure 6).

Since topology based visualization techniques feature sparse and economic screen usage, there is ample space left for additional information. Hauser and Gröller suggest a two step approach [HG00]. In the first step topology information is computed. Examples are fixed points and their Jacobins and higher order attractors. This is the classical step in topology visualization and in most cases the second step is not very intricate: different types of topological elements are visualized by different glyphs representing attracting, repelling and saddle points and separation lines [HH91]. This second step is now augmented by showing a visualization of the flow structure in a neighborhood of the critical point or visualizing the Poincaré map (see Fig. 7).

There is a lot of work in how the components of multi-variate data can be visualized. Sauber et al. [STS06] present multifield-graphs that deal with the question how the correlations between the components in the data can be shown. They introduce derived correlation fields that describe the strength of correlation between two variables at each point in space. The user can visualize correlation between scalar fields and vector fields. This also shows that the inherent information in multi-variate field that groups several variables to vectors and tensors can be useful when deriving additional information.

### 3.2. Rendering stage techniques

There is a number of flow visualization methods that render multi-valued flow data. Splatting is a very versatile technique that allows to integrate vector fields into scalar rendering by adding tiny vector particles into the splat texture [CM93, CSM00]. The examples by Max and Crawfis [MCW93] combine surface geometries representing cloudiness with colored glyphs representing wind ve-



**Figure 9:** A weather visualization combining streamribbons, arrows, slices, contour bands and isosurfaces [Tre99]. (Image courtesy of L. Treinish)

locity. This is an example, where a single rendering technique shows different types of data and still uses appropriate visualizations for the components. In a data type oriented manner the ground is rendered as a surface, while the clouds have a volumetric look giving a good feeling of orientation in space. Directions and altitude are visualized as colored glyphs, showing that they do not represent physical objects in space (see Figure 8).

Treinish investigated how specialized visualizations can be used to effectively visualize weather data using views of varying complexity [Tre99] and presented a multi-resolution technique for complex weather data [Tre00] (see Figure 9).

Since many of the existing flow algorithms are derived from physical models based on particles, the combination of particle and texture based flow visualization is a natural approach. Erlebacher et al. [EJW05] developed a spatiotemporal framework that encompasses many aspects of time-dependent flow visualization. Weiskopf et al. [WSEE05] apply the spatio-temporal framework to unsteady flow visualization. In the context of dense flow visualization they identify two important types of coherence within the animation: spatial coherence, which conveys the structure of a vector field within a single picture and frame-to-frame coherence which conveys the development of these structures over time. They employ two steps: the first step is basically a propagation of particles forward in time to construct a spacetime volume of trajectories. The second step applies convolution along paths through the spacetime volume that is done independently for each time step and texel. This hybrid particle and texture based approach combines advantages of particle-based representations with texture-based visualization. Particle systems are computational and memory efficient and allow accurate Lagrangian integration. Texture-based systems on the other hand have hardware acceleration for texture lookups and manipulations supported on modern graphic cards.

An approach that achieved not too much attention in the

literature is to use more than one rendering system at the same time. Yagel et al. [YESK95] suggested the use of four different renderers on a single CFD data set. Each is specialized to a specific task. Interactions can be visualized using a fast hardware-accelerated algorithm, high magnification images employ a specialized anti-aliasing technique. They use a ray casting algorithm specialized for the design of transfer functions while high-resolution and high-quality images are produced using a volumetric technique. Since today's computing machinery makes interactive manipulation of transfer functions, lighting parameters and other rendering attributes possible, the advantages of multiple combined renderers may be less obvious. Nevertheless an automatic selection of different rendering approaches and smooth transition between these would improve the visual experience for the user. This is an open research problem. Also, the integration of multiple renderers (e.g. illustrative and volumetric) into a single image at the same time is not investigated in much detail today. We believe new ways to integrate different rendering algorithms is a promising route for future research. Mueller et al. [SMK05] present an application that uses several renderers for prototyping, comparison and educational applications.

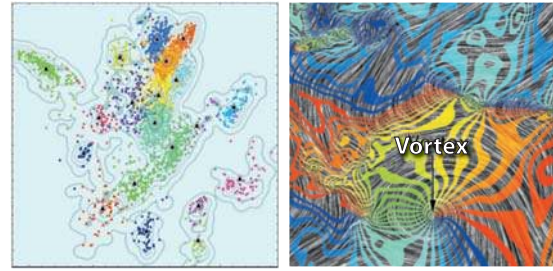
Stempel et al. [SLM02] explore the use of illustrative techniques to visualize multi-dimensional, multi-variate data sets on structured grids. They use silhouette enhanced shading and depth-based color variation to enhance the 3D results and reduce cluttering-related problems. Magnitude of temporal change is integrated using color modulation and flow properties such as direction, vorticity and velocity can be visualized using brush strokes.

### 3.3. Image stage techniques

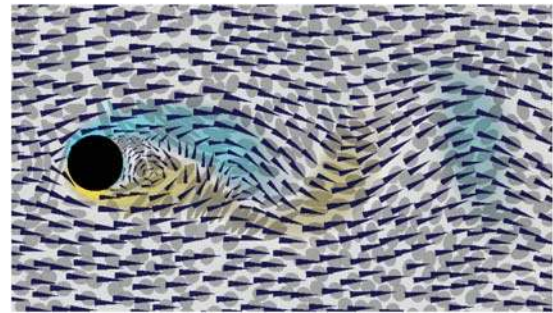
Crawfis and Allison [CA91] very early recognized the power of compositing several images of rendered objects together to do scientific visualization. Their graphic synthesizer could combine several images to generate multi-variate representations of two dimensional data sets. Wong et al. [WFK\*02] apply image compositing for visualization of multi-variate climate data. They present three image fusion techniques: opacity adjustments for see-through, filigreed graphics where portions of each layer are removed and elevation mapping where one scalar is mapped to the z-axis. In Figure 10 we see an example of layered glyph rendering (left) and a filigreed layering of color coded rendering and advection layered flow visualization (right).

Kirby [KKL04] gives an introduction to art-based layered visualization in the context of 2D flow visualization (see Fig. 11). A promising approach to visualize multiple aspects of high dimensional data sets is the combination of illustrative, art- and glyph-based rendering.

They introduce the concept of layering in a similar way as done in oil-paintings: underpainting contains a



**Figure 10:** A layered combination of glyphs, color coding and isolines (left) and a filigreed layered visualization of flow data combining texture advection and color coding [WFK\*02]. (Image courtesy of P. C. Wong)



**Figure 11:** An visualization using multiple layers to visualize scalar, vectorial and tensorial information [KML99]. (Image courtesy of M. Kirby)

low-frequency and low color-range coloring of a one-dimensional scalar vorticity value. Then two data layers follow: ellipses and arrows, depicting the most important aspects of the data. A final mask layer gives black context information (see Figure 11). By carefully selecting the order of layers it is possible to weight different aspects of the data differently and can suggest a viewing order for different parts of an image. Sobel [Sob03] presents a descriptive language for modeling layered visualizations, that allows to design and share visualization parameters for layering algorithms.

## 4. Tensor field visualization

Visualization of multi-variate data containing only tensor information is a difficult problem already. The interpretation of tensor information suffers if it is reduced to scalar information or if parts are visualized separately (e.g. in different images). Tensorial information has to be visualized fully or meaning and comprehensibility can be lost. When speaking of tensor field visualization we typically refer to second order tensors (three by three matrices). Depending on the application these tensors can be symmetric or non-symmetric. A symmetric tensor allows to calculate three orthonormal eigenvectors and corresponding eigenvalues.



Non-symmetric tensor fields can be decomposed to a symmetric tensor and a vector field. Because of these properties most visualization applications focus on the visualization of symmetric tensor data - this already involves six variables at each point simultaneously. Because tensorial information is difficult to comprehend and structure, multi-style visualization techniques are common in this field. An example would be a layered visualization combining diffusion tensor glyphs and a CT reference image slice to show the organ geometry. It is also common to show basic geometry cues (e.g. the shape of the brain or the kidney) as context information in form of a wire frame or silhouette rendering.

Important sources for tensor data are

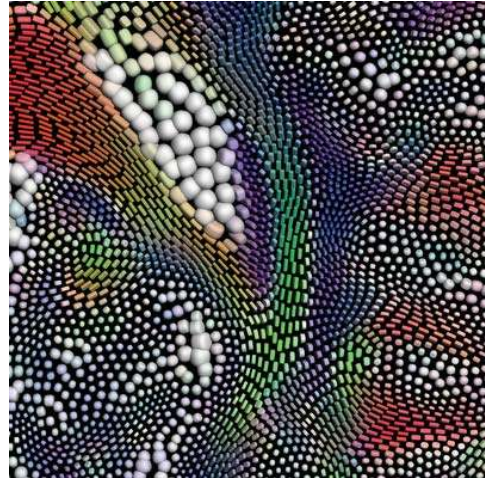
- medical applications working with measured MRI diffusion tensors. Their visualization is the field of Diffusion Tensor Imaging (DTI) and deals with symmetric tensors with positive eigenvalues.
- materials science and geomechanics working with stress and strain tensor fields. Related tensors are symmetric with signed eigenvectors.
- fluid dynamics where several properties are tensor valued. Examples are the vorticity tensor and the fluid momentum gradient tensor.
- general relativity theory simulations, where gravity is expressed as a rank two tensor and the electro-magnetic field tensor in special relativity.

Zhang et al. [VZKL05] give an extensive introduction and a state of the art overview of diffusion tensor visualization. Wünsche [Wün99] gives a basic introduction into stress and strain tensor fields suitable for the Computer Scientist.

#### 4.1. Techniques in the processing, filtering and visualization mapping stage

For tensor fields glyph-based visualization is the most common technique. Glyphs of stresses and strains is surveyed by Hashash et al. [HYW03]. One basic question that many publications state is "visualize all the information of one tensor in some places or only some part of it everywhere". The first would lead to some kind of glyph-based visualization where the information is visualized using a glyph that can represent all the degrees of freedom. Glyph-based visualization of tensor fields mainly uses the three eigenvectors (major, medium and minor) to generate a shape showing the direction of the eigenvectors. The most common is the ellipsoid, since it is possible to include all eigenvectors in a straightforward manner. Other glyphs are the Haber Glyph and the Reynolds Glyph [HYW03].

A classification of tensor shapes was given by Westin [WMM\*02]. A diffusion tensor is isotropic when the eigenvalues are about equal ( $\lambda_1 \approx \lambda_2 \approx \lambda_3$ ), planar anisotropic where two eigenvalues are about the same and larger than the third ( $\lambda_1 \approx \lambda_2 \geq \lambda_3$ ) or linear anisotropic where one eigenvalue is larger than the others



**Figure 12:** Sensible layout of glyphs allows to convey the location of fiber structures in the data. This shows how much automated parameter setting approaches (layouting) can improve the visualization [KW06]. (Image courtesy of G. Kindlmann)

( $\lambda_1 \geq \lambda_2 \approx \lambda_3$ ). The corresponding ellipsoids are spherical, disk- or needle-shaped respectively. Westin introduced the shape factors to measure which of these cases is dominant:

$$c_{linear} = \frac{\lambda_1 - \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}} \quad c_{planar} = \frac{2(\lambda_2 - \lambda_3)}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

$$c_{spherical} = \frac{3\lambda_3}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

The three shape factors sum to one and define barycentric coordinates, that can be used for glyph geometry assignment [WMM\*02], opacity mapping [KWH00], color coding, or glyph culling [ZKL04].

One way to get from glyphs to hyperstreamlines [DH93] is to place ellipsoids close to another along the direction of the major eigenvector. From any seed point, three hyperstreamlines can be generated using one of the three eigenvector fields for the streamlines and the other two for the cross section. This leads to a connected line along the major direction that encodes the other two eigenvalues in the cross section of the streamline. A hyperstreamline visualizing a tensor field can be enhanced to show other properties by coloring. For non-symmetric tensors, the rotational components can be encoded as 'wings' along the main hyperstreamlines [ZP03].

Zhang et al. [ZKL04] use stream tubes and stream surfaces for the tensor aspect of the data and contours and volumes as anatomical landmarks. The authors do not use the different visualization techniques to visualize different com-

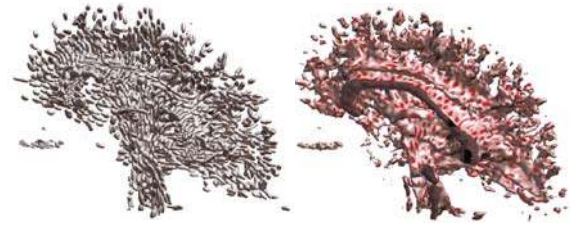
ponents of the data but show data ranges differently. Since in the brain, regions of high linear anisotropy very often correlate with regions containing densely fiber tracks, tensors having high linear anisotropy are adequately visualized using stream tubes, while tensors of high planar anisotropy are visualized using stream surfaces. This way both techniques can be used for the type of data they work best for.

In an adaption of image-based flow visualization Zhang et al. [ZHT07] visualize topological properties of tensor fields on curved surfaces. They discuss properties of critical points and an approach to extract flow directions to apply advection vectors. Furthermore they show applications to painterly rendering of 2D images.

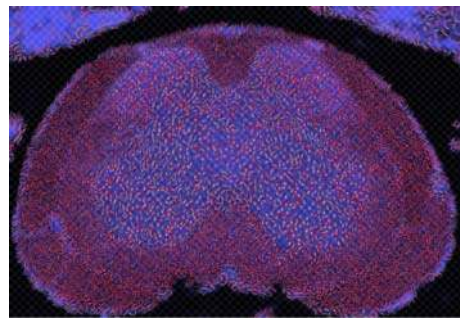
Merhof et al. [MSE\*06] present a hybrid rendering technique combining point sprites and triangle strips to display fiber tracts in the brain. They show that combining two rendering techniques can improve the comprehensibility of the visualization. This is an example how thinking about the 'what' part of the visualization (fiber tracts) can give clues to improving the 'how' approach.

#### 4.2. Rendering stage techniques

Visualizing parts of the tensor information in a continuous fashion is done in volume rendering of tensor fields. Sigfridsson et al. [SEHW02] present a dense technique that filters noise along the eigenvector directions to produce a continuous representation of tensor fields that produce results similar to LIC. The basic tasks in volume rendering tensor fields - determining opacity, calculating shading and assigning material color - can be done by specific mappings of tensor properties based on the shape factors. Opacity is determined by using a barycentric opacity map (e.g. high opacity for linear anisotropy). Lighting is determined by using a heuristic that refers to the shape an ellipsoid glyph would have in the same position: in case of planar anisotropy the lighting model is the same as with traditional surface modeling, in the linear case the lighting model is similar to lighting of illuminated streamlines. Cases in between are interpolated. In the simplest setting color coding is done by using a colored ball and choosing color depending on the direction of the major eigenvector. This basic setting allows improvement using additional visualization techniques. Kindlmann et al. [KWH00] present a reaction-diffusion texture that can visualize a tensor field alone but also integrate it with the volume-rendered tensor-field visualization (see Figure 13). The idea of a reaction-diffusion texture is to simulate a system of two differential equations. One describes the diffusion governed by Fick's second law of two morphogens where the resulting concentration of these morphogenes determines the color at each position. The other differential equation measures how much the two substances react and neutralize each other. The initial condition is that both have the same concentration everywhere. Applying diffusion relative to the given tensor field at each position generates a



**Figure 13:** (a) The reaction diffusion texture allows natural glyph placement and geometry. (b) Alternative combined use of the texture for coloring the volume [KWH00]. (Image courtesy of G. Kindlmann)

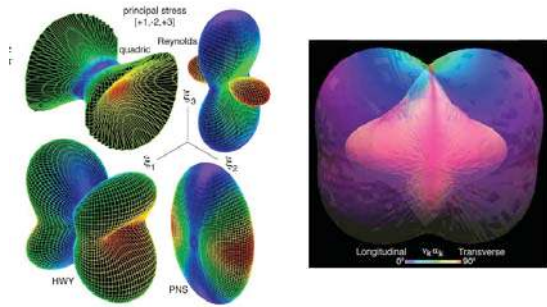


**Figure 14:** A visualization of the mouse spinal cord based on artistic techniques using multiple layers and glyphs [LAK\*98]. (Image courtesy of D. H. Laidlaw)

texture that can show information about the tensor field in its own right. The authors suggest color modulation or bump mapping to combine volume rendering and the volumetric texture. The result is similar to a surface rendering of geometry combined with diffusion glyphs, but has several advantages. The most important is that the resulting ellipsoids are distributed more naturally and are packed in a way that represents features of the data. Also the empty space between tensor ellipsoids is reduced. Furthermore it avoids the common problem of gridded ellipsoid layouting to give the false impression of structure in the data.

#### 4.3. Image stage techniques

For tensor visualization an image stage technique has been published by Laidlaw et al. [LAK\*98]. They show that a lot of information can be conveyed in a single image using brush based glyphs and layering. Combining varying brush strokes and layering allows to display many components of the data locally, while the underpainting can show form. Contrast is used to create depth. Stroke sizes, texture and contrast help to define a focus within each image (see Figure 14). In a recent publication Wenger et al. [WKZ04] combine volume rendering techniques and layering into a multilayer volume rendering approach. Their method is related to two level vol-



**Figure 15:** Advanced glyphs for stress tensor visualization using color and geometry for information coding (left) and transparency (right) [KGM95]. (Image courtesy of R.D. Kriz)

ume rendering [HMBG00] which will be discussed in the second part of this paper. They combine densely packed threads of colored and haloed streamlines with direct volume rendered context information. To deal with occlusion problems they designed interactive controls to change visualization parameters like thread length or opacity. Also, they heavily use transfer functions. This interesting publication is a good example of how to layer volumetric rendering successfully to visualize different portions of information.

## 5. General approaches to multi-dimensional visualization

In this section we will give an overview of the techniques we have identified to deal with complex data sets. It has become a widely accepted perspective to view visualization as a path from data to understanding [DKS\*05]. We have identified a wide and diverse range of general approaches to multi-variate or complex data visualization. The following subsection cannot give a comprehensive enumeration of all the related work, but is thought to be an introductory overview. We also do not distinguish between 'how' and 'what' approaches, since several of the techniques can be used both ways.

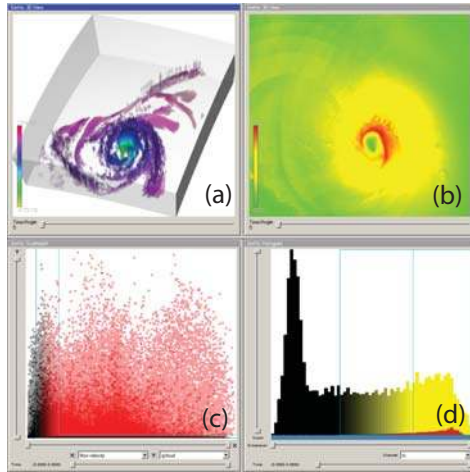
**Derivations** or derived quantities are used because visualizing the measured data directly might not be useful for understanding it. Kirby et al. [KML99] show that in flow visualization for example showing additional components, that do not give information that was not already available from the velocity field, helps understanding the situation. In flow visualization useful derive quantities are vorticity, the rate-of-strain tensor, the rate-of rotation tensor, turbulent charge and turbulent current. Smoothing the data to remove noise or calculating gradients to improve lighting will very often result in more pleasing visualizations that are easier to work with. Hauser [Hau06] discusses the use of differential information to improve scientific visualization.

**Glyphs** (also referred to as icons) are a powerful communication item. A large number of data dimensions can be incorporated into the attributes of a single shape or symbol (see Figure 15). The particular mappings may also be customized to reflect semantics relevant to specific domains to facilitate interpretation. Since glyphs are generally not placed in dense packings, the free space between them allows the visualization of additional information. They interact therefore nicely with other visualization algorithms and are frequently added to visualization applications. Wittenbrink et al. [WPL96] suggest glyphs for uncertainty in vector fields. Kindlmann and Westin [KW06] have presented a technique for packing glyphs in a way that their alignment conveys additional information. Hashash gives an overview of stress and strain tensor glyphs [HYW03]. (See also [Kin04, War02, KGM95, WL93].)

**Hybrid rendering** is the application of several visualization techniques for the same image. This is useful especially for segmented data sets where background information is applicable to choose the appropriate rendering technique for different subregions of the data [HMBG00]. There are many examples for this approach: Jesse and Isenberg [JI03] describe a hybrid rendering scheme that combines photo-realistic and illustrative rendering to highlight parts of a volume for presentation. Kreeger and Kaufmann [KK99] describe a fast method to combine volume rendering and translucent polygons to render mixed scenes. Laramee et al. [LJH03, LvWJH04] and van Wijk [Wij03] present rendering algorithms to visualize flow on surfaces. Wegenkittl et al. [WGP97] combine surfaces, tubes and particles to visualize the behavior of a dynamical system.

**Interaction** is probably the most important tool for understanding complex data. Possible interactions are changing viewing parameters, transfer function manipulation, seeding point selection, streamline culling, streamline queries, graphical model exploration, region of interest selection and many others. An emerging trend is to use concepts from interactive visual analysis for data exploration. In Figure 16 we see an example of multiple linked views that work together to help understanding the data. In the attribute view (c) and (d) linking is helps to understand how different attributes are related: the data elements selected by the brush are shown red, while the elements selected in the other view are colored yellow.

**Layering and Fusion** has been used extensively in scientific visualization to show multiple items. Fusion-based methods combine different rendering styles in image space [WFK\*02]. Layering is a generalization of this approach where multiple layers of information are visualized on top of each other. This is most applicable for two dimensional visualization but there is work where transparent stroked textures show surfaces without completely obscuring what is behind them [IFP97, Int97]. Several other layering tech-

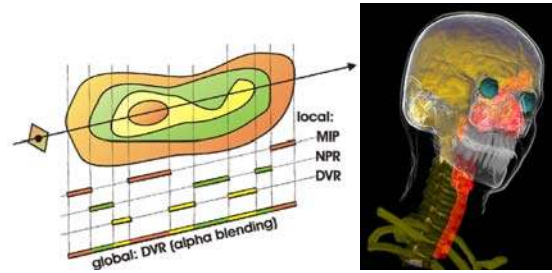


**Figure 16:** An example of combined attribute and volumetric views. The 3D view (a) shows the location of data points in space with pressure mapped to color. A 2D slice (b) shows the velocity close to the eye of the storm. Two attribute views (scatterplot of velocity vs. cloud density (c) and a histogram of temperature (d)) are used to select which cells are shown.

niques have been discussed in the first section of this paper, see [CA91, LAK\*98, WFK\*02, KKL04].

**Two-level volume rendering** Hauser et al. [HMBG00] and Hadwiger et al. [HBH03] present a two-level approach that combines different rendering methods for volume rendering of segmented data sets (see Figure 17). Each of the segmented regions can be rendered using a specific rendering method like NPR, DVR or MIP during ray accumulation. Since most users perceive three dimensional scientific data sets as built up from individual objects, the authors use the segmentation information to generate images that take this into account (one per object intersection). To compute the representative values for the objects different rendering techniques can be used, The authors also use the technique to visualize dynamical systems. This gives a hint at a more general applicability of their approach. Since the decision what rendering method to choose is given to the user, it becomes possible to use the most adequate in the given moment. We believe this approach is well suited to visualize multi-dimensional data sets by combining different rendering methods that are most appropriate for different features inside the data. (See also [WKZ04].)

**Multiple Views** present the information in several different views that encourage comparison, give contrast and help to generate a correct understanding. Roberts [Rob00] describes the generation and presentation of multi-form visualizations in an abstract way and gives an introduction to multi-view visualization. Yagel et al. [YESK95] discuss to group volume renderers that have different quality and rendering speed tradeoffs. Van Wijk and van Liere’s hyperslic-



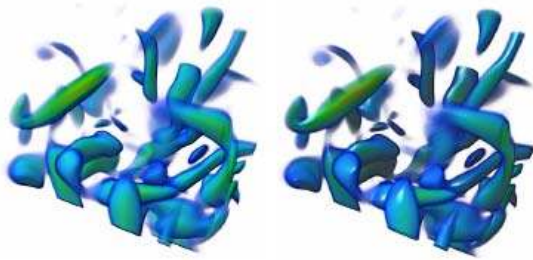
**Figure 17:** Two-level volume rendering allows to combine multiple rendering techniques using different compositioning methods locally (left). A multi-level volume rendering of a human head using tone shading (brain), contour enhancement (skin), shaded DVR (eyes and spine), unshaded DVR (skull, teeth, and vertebrae) and MIP (trachea) [HBH03]. (Image courtesy of M. Hadwiger)

ing approach uses multiple views to display a large set of possible projections of the data [WL93].

**n-D viewing** is based on defining hyperplanes on the high-dimensional volume or direct projection. This is done very often for time-varying data sets, where time-coherency can be exploited for compression and acceleration. The major issue for projections from n-D is to determine occlusion because a front-to-back ordering is not clearly defined after projection. Feiner and Beshers [FB90] suggest the World within World approach to drill down on the data by iteratively slicing away dimensions (see also [WWS03, NM02, BPRS98, WL93] and references therein). Blaas et al. [BBP07] have developed a framework that uses interactive projection parameter specification for mapping multi-variate data values to scatterplots.

**Probing** is a general visualization approach for multi-variate data visualization. The user can state interest in a specific location or data range. Then a reduced amount of data is shown everywhere and for subsets of the data a local and more complex visualization conveys details. This avoids clutter and occlusion, is computationally efficient and helps the user to focus on specific aspects of the data. Examples for local detail global overview techniques are: focus and context visualization [DGH03], magic lenses [Kea99], level-of-detail [CMS98], clipping-probes [WEE03] or zooming.

**Reduction** Dimension reduction and de-noising can remove unwanted details in the data and remove obscuring structures that hinder the process of understanding. Also, presenting views that contain a reduced amount of information and clipping are examples for data reduction tools. The importance of data reduction is very well expressed in the saying that in the future the main question will not be what to show, but what not to show. There is a trend to include attribute views (such as scatterplots, parallel sets, etc.) for interactive visual analysis of the attributes of the data-



**Figure 18:** Visualizing flow data can benefit from using illustrative techniques. The image shows a closeup of turbulent vortex flow using silhouette and shading (left) and additionally gradient and depth enhancement (right) [SLM02]. (Image courtesy of Q.-L. Ma)

set. These views can benefit strongly from having automated clustering and reduction algorithms available.

## 6. Illustrative rendering

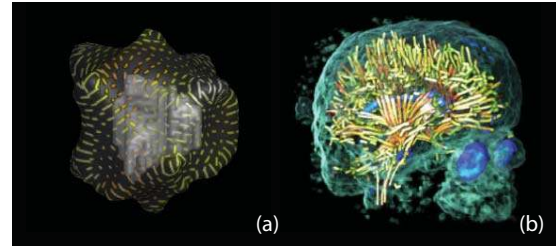
The visualization of multiple features and the accentuation of important structures and information has gained special attention in scalar volume rendering, especially in illustrative rendering. Illustrative rendering employs abstraction techniques to convey relevant information. In the context of scientific visualization non-photorealistic rendering refers to the adaption of techniques that have been developed by traditional artists for the generation of synthetic imagery. In the context of scientific visualization Stoppel et al. [SLM02] explore the use of illustrative techniques to visualize multi-dimensional, multi-variate data sets on structured grids. Bruckner et al. [BG05] have developed the VolumeShop framework for direct volume illustration.

**Depth color Cues** Svakhine and Ebert [SE03] describe depth based color variation. It gives intuitively understandable cues of the relative positions of different features in the data set. Distance color blending dims sample colors as they recede from the viewer. At the front of the volume, the voxel color remains unchanged. As the screen-depth value increases the color is gradually blended with the background color:

$$Color = (1 - depth) \cdot Color_{original} + depth \cdot Color_{background}$$

### Silhouette enhancement and boundary enhancement

Silhouette lines are particularly important in the perception of surface shape and the perception of volumetric features. In order to strengthen the cues provided by silhouettes, one increases the opacity of volume samples where the gradient is close to perpendicular to the view direction. Using a dark silhouette color can be effective for outlining of features. Levoy [Lev88] proposed to scale opacity using the magnitude of the local gradient. Many applications also use the lo-



**Figure 19:** (a) Transparent surfaces can allow layered visualization for three dimensional images [IG97]. (Image courtesy of V. Interrante) (b) Non-photorealistic rendering of tensor information using line-glyphs and DVR of context information [WKZ04]. (Image courtesy of A. Wenger)

cal gradient as a transfer function parameter [KKH01]. Ebert and Rheingans [ER00] suggest to add scaling parameters to boundary enhancement such that the gradient-based opacity of the volume sample becomes:

$$o_g = o_v(k_{gc} + k_{gs}(\|\nabla_f\|)_{ge}^k)$$

depending on the data  $o_v$  (original opacity),  $\nabla_f$  (the value gradient of the volume at the sample) and on user specified parameters  $k_{gc}$  (scales influence of original opacity),  $k_{gs}$  (scales influence of gradient enhancement) and the exponent parameter  $k_{ge}$  that allows the user to adjust the slope of the opacity curve.

**Enhanced transparent surfaces and stippling** Since many features can be visualized using surfaces, transparent surface rendering offers a good possibility to show the spatial relationship between two superimposed features. To improve shape recognition, Interrante [Int97] uses principal direction-driven LIC [IG97]. The surface shape and curvature are enhanced using a stroke texture. This enables the visualization of additional variables using colored strokes and varying stroke width and density. The surface information is stored in a texture and needs recomputation if stroke parameters are changed. Nagy et al. [NSW02] have developed a technique to do stippling of surfaces in real time using pre-processed principal directions. This allows to change stroke positions and appearance interactively but it needs an additional pass to render the strokes.

**Feature halos** In line drawings it is common to depict the depth ordering of two crossing lines by inserting a short gap into the line behind the other such that the lines do not touch. Therefore halos are an important technique for visualization of streamlines. For scalar volumes Ebert and Rheingans [ER00] propose a method for creating halo effects for features during the illumination process. Halos are created primarily in planes orthogonal to the view vector by making regions just outside features darker and more opaque. They calculate the halo-intensity by weighting the neighbors gradient magnitude and halo-direction.

**Shadows and indirect illumination** Even though a feature in the volume (e.g. a vortex) may not cast a shadow in reality, a shadow can very well show spatial relationships between features in the data. Especially for glyphs and disconnected features lighting is a powerful way to provide better perceptual cues [GP06]. Also, shadowgraphs and schlieren [SJEG05] can depict important structures inside a volume. Conceptually, due to inhomogeneities inside a translucent volume, refraction within the affected region displaces light rays before they reach the screen. Therefore this is similar to boundary enhancement. The technique stems from flow photography but is applicable to visualize structures in other volumetric data sets as well.

**Cutaways and Ghosting** Feiner and Seligmann [FS92] introduced cut-away views and ghosting to 3D visualization. Cut-away views are a common tool of illustrators to show important structures in a volume that would be occluded otherwise. Viola et. al [VKG05] introduced importance-driven volume rendering to determine which parts of the volume are to be cut away (this notion of importance is also referred to as degree-of-interest (DOI) in other publications). Ghosting [GBKG04, BGKG05] is a technique very often combined with cutaway views. Instead of removing the obscuring region completely they keep the most important parts (e.g. edges) of the occluding structures to conserve context information.

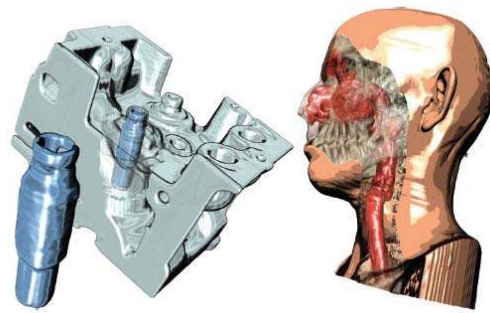
**Motion blur** is a widely used technique to show information from time-dependent data sets. For example Neophytou and Mueller [NM02] use a motion blurring effect to give hints on how the data will evolve.

## Conclusion

We have discussed new developments in visualization research and have outlined recent trends in the literature. Based on these trends we have presented an abstracted goal for multifield visualization algorithms that highlights open questions on different stages of the visualization pipeline:

- How can we capture all the relevant information?
- How can we combine relevant quantities of information belonging to the same location in space?
- What quantities can be derived from the data, that will help the user to understand the data?
- What are the features in the data and how can we combine automated and interactive techniques for finding them?
- Which visualization technique is most appropriate to the given data?
- How can we combine different visualization techniques for efficient and effective visualization?

In the first part of this paper, we have discussed applications and algorithms employing one or several useful techniques to visualize complex scientific data sets in two and three dimensions. In the second part we have given an overview



**Figure 20:** Techniques from traditional illustrative imaging can increase the information content in volume rendered images. Ghosting can be used to show important features in detail while cutaway views keep structural information independent of the viewing direction [BGKG05]. (Image courtesy of S. Bruckner)

of modern approaches to for multi-variate data visualization and analysis.

Conclusively, in the literature we found researchers taking three general approaches to deal with complex, high-dimensional data sets. We believe these approaches to be highly promising and research in how these can be combined and improved will increase the value of visualization systems:

- **Interaction:** The connection between elements of the data can be explored interactively. This approach works on the assumption that the user will understand the configuration of a dataset using different views and different levels of detail interacting with visualization parameters. Exploration of correspondence between different views on the data by visual comparison using the powers of the visual system to see what the data means. This is probably the most important key to understand complex multi-dimensional data and is used extensively in related fields like information visualization as well.
- **Complexity-adjusted rendering:** Due to the growth in computation power a valid approach to deal with the rising complexity of the data is to increase the computational efforts put into visualizing this data. Rendering complex visual clues (e.g. lighting models, shadows and distance clues) can make spatial interpretation easier for the user. We have seen that visualization research has produced tools to decrease clutter and to visualize complex and intertwined three-dimensional data features.
- **Multi-method visualization:** Is the approach to use a visualization technique only for the type of range of data it works best for. In the scope of this article we have discussed several publications following this approach and we believe a tight integration of multiple techniques in a

visualization gives a key advantage towards understanding the data.

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