

VLBA Scientific Memorandum No. 32

Multi-frequency ASTROMETRY with VSOP-2:

An Application of  
SOURCE/FREQUENCY PHASE REFERENCING  
(SFPR) techniques

**María J. Rioja and Richard Dodson**

International Centre for Radio Astronomy Research (ICRAR), UWA, Australia  
Observatorio Astronómico Nacional (OAN), Spain

November 24, 2009

## ABSTRACT

This document describes the advantages of applying SOURCE/FREQUENCY PHASE REFERENCING (SFPR) techniques to the analysis of VLBI observations with VSOP-2, for high precision astrometric measurements and/or increased sensitivity. The SFPR calibration technique basics and a demonstration of the method applied to highest frequency VLBA observations are described in detail in VLBA Scientific Memo 31.

Here we outline its importance in the context of space VLBI astrometry with VSOP-2, where errors in the satellite orbit determination and rapid tropospheric phase fluctuations set extreme challenges for the successful application of CONVENTIONAL PHASE REFERENCING techniques, specially at the higher frequencies. SFPR is ideally suited for full calibration of those - regardless of the orbit determination accuracy - and, in general, of any non-dispersive terms. The requirements for application of SFPR techniques are fully compatible with current technical specifications of VSOP-2. Hence we foresee that SFPR will play an important role in helping expanding the scientific outcome of the space VLBI mission.

### The SFPR technique

In this section we include a brief review of the basics of the SOURCE/FREQUENCY PHASE REFERENCING (*hereafter* SFPR) technique, emphasizing the aspects which are relevant for space VLBI. A full description is given in Dodson & Rioja (2008, 2009).

This calibration approach relies on *fast frequency switching* combined with *slow source switching* observations, between two frequencies ( $\nu^{high}$  and  $\nu^{low}$ ) and two sources ( $A$  and  $B$ ), respectively. The former provides extended coherence time, and increased sensitivity, at the higher frequency; when combined with the latter it gives astrometric capability to observations with VSOP-2, irrespective of the uncertainties in the orbit reconstruction which set an insurmountable limiting factor for using CONVENTIONAL PHASE REFERENCING (*hereafter* PR) techniques.

The SFPR observations consist of alternating scans between the two frequencies, with a duty cycle within the coherence time at the lower frequency. The analysis is much simplified if the frequencies have an integer ratio,  $R = \frac{\nu^{high}}{\nu^{low}}$ .

Following standard nomenclature, the residual visibility phases for observations at the  $\nu^{high}$  frequency of source  $A$  are expressed as a compound of geometric, tropospheric, ionospheric, instrumental and structural (non-zero for extended sources) terms:

$$\phi_A^{high} = \phi_{A,geo}^{high} + \phi_{A,tro}^{high} + \phi_{A1,ion}^{high} + \phi_{A,inst}^{high} + \phi_{A,str}^{high} + 2\pi n, \quad n \text{ integer} \quad (1)$$

where  $2\pi n$  stands for the phase ambiguity term. The terms in equation (1) result from the differences between the true (*labelled* “true”) and *a priori* estimated

(labelled “mod”) values for the parameters used in the models involved in the data analysis. For example, the geometric contribution accounts for:

$$\phi_{geo}^{high} = 2\pi(\vec{D}_\lambda^{true} \cdot \vec{s}^{true} - \vec{D}_\lambda^{mod} \cdot \vec{s}^{mod})$$

where  $\vec{D}_\lambda$  is the baseline vector in units of observed wavelength, and  $\vec{s}$  is the unit vector in the direction of the source. A simplified general expression for the tropospheric contribution induced in an interferometer is:

$$\phi_{tro}^{high} = 2\pi \nu^{high} (z_1 \cdot f_1 - z_2 \cdot f_2)/c$$

where  $z_1$  and  $z_2$  are the excess path length in the zenith directions (e.g.  $z = z^{true} - z^{mod}$ ), and  $f_1$  and  $f_2$  are the mapping functions corresponding to the source directions at the two antennas. For baselines between a ground and a satellite antenna, where the effect of the propagation medium needs to be considered only for the ground component, the expression above is simplified to include only one adding term.

A similar expression to equation (1) holds for the residual phases  $\phi_A^{low}$  from observations at  $\nu^{low}$ , the reference frequency. These are analysed using self-calibration techniques. The antenna based estimated corrections are linearly interpolated to the times when the  $\nu^{high}$  frequency is observed ( $\tilde{\phi}_{A,self-cal}^{low}$ ), scaled by the frequency ratio  $R$  (with  $R = \frac{\nu^{high}}{\nu^{low}}$ ), and applied as calibration of the observed phases at  $\nu^{high}$ , the target frequency, in equation (1). We name this step as FREQUENCY PHASE TRANSFER (FPT). This calibration strategy results in perfect cancellation of the non-dispersive rapid tropospheric residual phase fluctuation terms in equation (1), since:

$$\phi_{A,tro}^{high} - R \cdot \tilde{\phi}_{A,tro}^{low} = 0$$

and the same applies for any orbit errors, and in general antenna and source coordinate errors contributing to the geometric term ( $\phi_{geo}$ ) in observations of an achromatic source, since:

$$\phi_{A,geo}^{high} - R \cdot \tilde{\phi}_{A,geo}^{low} = 0$$

In general, for sources whose VLBI position is frequency dependent the previous expression includes a ca. 24-hour sinusoidal extra term whose amplitude depends on the magnitude of the shift between the  $\nu^{low}$  and  $\nu^{high}$  observed frequencies, also known as “core shift” ( $\vec{\theta}_A$ ), and the baseline length:

$$\phi_{A,geo}^{high} - R \cdot \tilde{\phi}_{A,geo}^{low} = 2\pi \vec{D}_\lambda \cdot \vec{\theta}_A + O(\Delta \vec{D}_\lambda \cdot \vec{\theta}_A)$$

and also, it includes an extra contribution proportional to the scalar product of the antenna position error (or orbit error) and the “core shift” vectors, which we include in the formulation for completeness. The effect of the extra term is negligible and can be completely ignored given the VSOP2 orbit errors, or any other VLBI antenna position errors, and the expected typical values for “core shifts” for the range of frequencies observed with VSOP2.

The resulting FREQUENCY PHASE TRANSFERRED (FPT) visibility phases are:

$$\phi_A^{FPT} = \phi_{A, str}^{high} + 2\pi \vec{D}_\lambda \cdot \vec{\theta}_A + \text{“ion”} + \text{“inst”} \quad (2)$$

which contain, among others, contributions from the radio structure of the source at  $\nu^{high}$ , and, if present, the astrometric “core shift”, which modulates each baseline with a ca. 24-hour sinusoid. We have omitted the  $2\pi$  phase ambiguity term for simplicity, assuming that  $R$  is an integer number. The rapid tropospheric fluctuations have been perfectly calibrated, but longer time scale, residual ionospheric and instrumental (dispersive) terms, named “ion” and “instr” remain. Middelberg et al. (2005) successfully applied the fast frequency switching technique to mm-wavelength VLBI observations as a means to extend the coherence time and allow detections of weaker sources. They used one extra step of self calibration to get rid of the residual dispersive terms in equation (2). But this step of self-calibration also eliminates the position information, and hence prevented the measurement of the “core shift”.

An alternative approach to correct the residual long timescale phase drift terms while preserving the astrometric information consists of using the interleaved observations of a second source, a nearby calibrator ( $B$ ). The angular separation and duty cycle are determined by the ionospheric isoplanatic patch size, and the slow temporal properties of the “ion” and “instr” terms in equation (2), respectively. For the case of interest here, the angular separation can be up to several degrees, and the duty cycle of several minutes (ca. 5 minutes). The observations of the calibrator source are first analysed using the same strategy as described above for source “ $A$ ”. Next, they are used to correct the target source observations, in a similar fashion as is done in conventional phase referencing observations of two sources (using a temporal interpolation of antenna based solutions for  $B$  source,  $\tilde{\phi}_{B, self-cal}^{FPT}$ ). The residual SOURCE/FREQUENCY REFERENCED visibility phases ( $\phi_A^{SFPR}$ ) are:

$$\phi_A^{SFPR} = \phi_A^{FPT} - \tilde{\phi}_{B, self-cal}^{FPT} = \phi_{A, str}^{high} + 2\pi \vec{D}_\lambda \cdot (\vec{\theta}_A - \vec{\theta}_B)$$

The combination of the frequency and source switching in SFPR results in calibrated phases free of long scale drift terms. Finally, a Fourier transformation of the phases, without further calibration, results in a map of the brightness distribution of the target source ( $A$ ) at  $\nu^{high}$  frequency, and where the offset of the peak with respect to the center of the map is astrometrically significant: a measurement of the relative “core shift” between both frequencies ( $\nu^{high}$  and  $\nu^{low}$ ), for both sources ( $A$  and  $B$ ).

## SFPR observations with VSOP-2

VSOP-2 is the second space VLBI mission planned by the Institute of Space and Astronautical Science (ISAS). Its goal is placing a satellite radio telescope in orbit for joint VLBI observations with ground radio telescopes. It will be equipped with a 9.3 meter off-axis parabolic antenna, and dual polarization receivers at 8.4, 22, and 43-GHz.

The analysis of data from its predecessor VSOP mission, with the HALCA satellite, showed that the calibration of space VLBI observations involves additional difficulties arising from the relatively poor sensitivity achievable with small orbiting antennas, smaller correlated source flux densities at the higher resolution of space baselines, and large geometric delay errors introduced by uncertainties in the spacecraft orbit that prevent long integration times (Rioja & Porcas, 2007). Additionally, VSOP-2 is designed to carry out astrometric observations, which impose more severe demands on the phase calibration.

The use of conventional PR calibration techniques for space VLBI data analysis, especially at the higher frequencies, will be rather challenging, due to the shortage of suitable reference sources (i.e. compact and strong enough on space baselines), rapid tropospheric phase fluctuations and satellite orbit determination errors. Asaki and collaborators (2007) have carried out a comprehensive study on the feasibility of conventional PR observations with VSOP-2, under a range of different weather conditions and orbit determination accuracy of the satellite, among other parameters. Their simulations demonstrate that astrometric observations are expected to achieve a good performance at 8.4 GHz, while at the higher frequencies the best possible weather would be required and the probability of finding suitable calibrators (particularly at 43 GHz) is greatly reduced.

Alternatively, VSOP-2 could benefit from the use of the SFPR technique for astrometric studies and/or increasing coherence time at the higher frequencies. It encompasses observations at two frequencies, ideally with an integer ratio, and of two sources. The VSOP-2 mission specifications are compatible with the observing schedule requirements for successful application of our SFPR calibration method, that is, to have the capability to alternate observations between frequency bands, e.g. between 21.5 and 43 GHz, with a ca. 1 minute cycle (*fast frequency switching*), and between sources with a much longer cycle, of several minutes (*slow source switching*).

The SFPR route holds great promise for space VLBI since any orbit errors (as well as ground antennae coordinate errors) are fully removed in the analysis, therefore alleviating the constraint on orbit determination accuracy; also, the conditions for a suitable SFPR calibrator source are much more relaxed than in conventional PR, and the angular separation between sources can be up to several degrees, and the observing source duty cycle up to several minutes.

Figures 1 to 3 show the estimated error budget for astrometric analysis of VSOP-2 observations using conventional PR, and SFPR techniques, for comparison. The individual mis-modelled contributions are shown in separate columns, and labelled following standard nomenclature: uncertainties arising from satellite orbit error determination (ODDA), errors in coordinates of reference source (Ref-Source), unmodelled propagation effects through the troposphere (TROP) and ionosphere (ION), and finally the limitation imposed by the finite interferometric

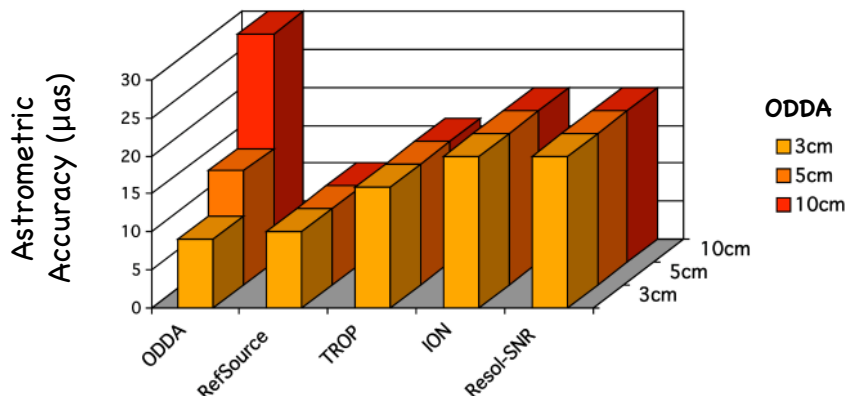


Figure 1: *Error budget for CONVENTIONAL PR-astrometric analysis of space VLBI observations at 8.4 GHz of a pair of sources  $2^\circ$  apart, for increasing orbit errors (shown with different colours). The columns account for individual contributions arising from satellite orbit errors (ODDA), reference source coordinates, tropospheric and ionospheric residuals and the instrumental resolution and SNR criteria. Orbit errors equal to 3, 5, and 10 cm, and “typical values” for the rest of the parameter model errors (see text). Tropospheric/Ionospheric terms are among the dominant ones with orbit errors less than 5 cm, at 8.4 GHz.*

resolution and noise in the observables (RESOL-SNR).

Figure 1 shows the astrometric error budget for conventional PR observations at 8.4 GHz, of a pair of sources  $2^\circ$  apart with a one-minute duty cycle, with a space baseline and different orbit errors. The plot shows the ODDA contributions arising from increasing orbit errors of 3, 5 and 10 cm; we used “typical values” for the parameter model errors, as listed in Asaki et al. (2007), to estimate the rest of the contributions. The current plan for precise orbit determination for ASTRO-G, at the time of writing this memo, is described in Asaki et al. (2008). Residual tropospheric/ionospheric errors are the dominant contributions with orbit errors less than 5 cm, at 8.4 GHz. All residuals scale with the pair angular separation, except for the thermal noise contribution. Also, since all residuals scale with baseline length (except for the “RefSource” contribution) the long space baselines result in an improvement with respect to a ground array.

Figure 2 shows the astrometric error budget for conventional PR observations at 8.4, 22 and 43 GHz, with an orbit error of 3cm - all other parameters are as for Figure 1. Contributions from non-dispersive terms, such as geometric contributions (e.g. orbit and reference source coordinate errors) remain identical at the 3 frequency bands. Instead, the ionospheric term is dispersive, e.g. decreases with increasing frequency, and is much reduced at 22 and 43 GHz, with respect to 8.4 GHz observations.

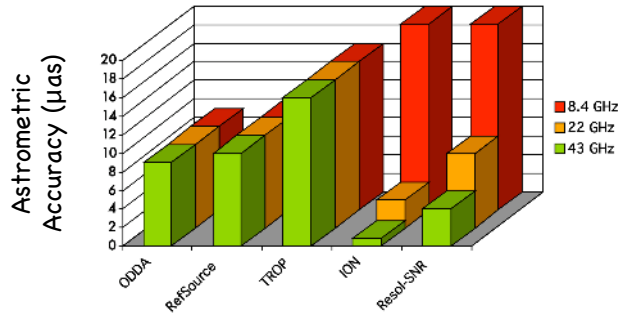


Figure 2: *Error budget for CONVENTIONAL PR-astrometric analysis of space VLBI observations at 8.4, 22 and 43 GHz, assuming an orbit error of 3cm. Contribution from non-dispersive terms remains identical irrespective of the observing band  $\sim 20\mu\text{as}$ . The ionospheric term is dispersive, and is much reduced at 22 and 43 GHz.*

Figure 3 shows a compilation of SFPR-astrometric error budget estimates for observations of eligible frequency pairs observed with VSOP-2, with a one-minute frequency switching cycle. The first striking aspect is the perfect compensation of the non-dispersive terms, that is, the orbit errors are cancelled out in the analysis.

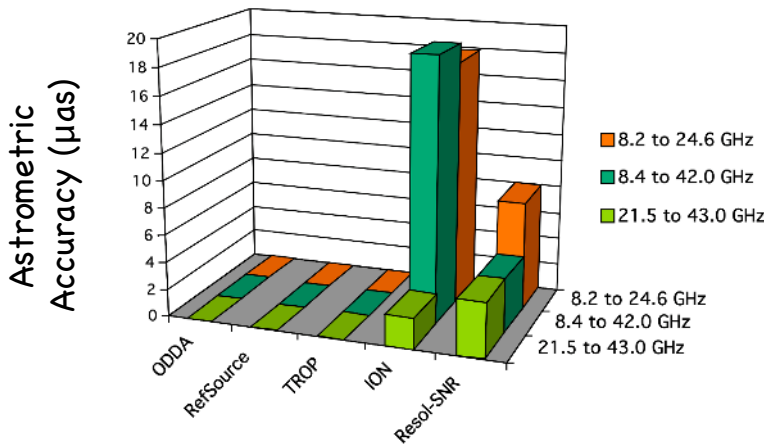


Figure 3: *Error budget estimates for SFPR-astrometric analysis of space VLBI observations, switching between pairs of frequencies in VSOP-2 with a duty cycle of 60 seconds. Parameter model errors as in Figures 1 and 2. Note the perfect compensation of non dispersive terms. The residual ionospheric contribution is approximately that from the the reference (lower) frequency scaled by the frequency ratio. The ionospheric contribution is the dominant one when 8 GHz is the reference frequency. The improvement becomes noticeable in the combination 22 to 43 GHz, when the 22 GHz ionospheric contribution is very much reduced.*

The residual ionospheric contribution depends on the reference frequency ( $\nu^{low}$ ), and the frequency ratio ( $R$ ); the higher the reference frequency and the smaller the frequency ratio, the smaller the contribution to the error budget. The mis-modelled ionospheric contribution is the dominant source of error when using 8.4 GHz as the reference frequency. The improvement becomes noticeable between 22 to 43 GHz, when the residual ionospheric contribution is very much reduced at 22 GHz.

## Summary

The current technical specifications of VSOP-2 are compatible with the requirements to carry out successful SFPR observations in order to achieve astrometric measurements and/or increase the coherence time in VLBI observations with the space mission. Moreover, the SFPR technique naturally addresses the main factors which limit the application of conventional phase referencing to VSOP-2 observations, i.e. orbit uncertainties and tropospheric fluctuations, specially at the higher frequencies. A list of benefits from applying SFPR to VSOP-2 space mission follows:

- SFPR technique relies on fast switching between frequencies, rather than between sources as in conventional phase referencing, a less demanding operation for VSOP-2.
- Satellite Orbit uncertainties, which set a hard constraint for using conventional phase referencing, are not an issue for the SFPR technique.
- A larger number of calibrators available for SFPR since wider angular separation is acceptable.
- SFPR techniques result in increased sensitivity at 43 GHz, and recoverable astrometric information regardless of the orbit determination accuracy.
- The implementation of SFPR techniques are compatible with VSOP-2 technical specifications.

Summarizing, we think this new technique will prove its potential in the calibration of observations at the highest frequency, 43 GHz, either using the second harmonic of 21.5 GHz, or the fifth harmonic of 8.4 GHz as “reference” observations. The current mission specifications do not allow the calibration of the 22 GHz band with the 8.4 GHz band, unless the latter is expanded to cover 7.5 GHz.

## Acknowledgements

The authors express their gratitude to Craig Walker, Ed Fomalont, Asaki Yoshiharu and Richard Porcas for providing useful comments on this manuscript.



## References

- [1] Asaki, Y., Sudou, H., Kono, Y., Doi, A., Dodson, R., Pradel, N., Murata, Y., Mochizuki, N., Edwards, P., Sasao, T., and Fomalont, E. 2007. Verification of the Effectiveness of VSOP-2 Phase Referencing with a Newly Developed Simulation Tool, ARIS. PASJ 59, 397.
  
- [2] Asaki, Y., Takeuchi, H., Yoshikawa, M., 2008, Next Space-VLBI Mission, VSOP-2, and the Precise Orbit Determination with GNSS Navigation and SLR, Proc. of ION GNSS 2008
  
- [3] Dodson, R. & Rioja, M. 2008, Informe Técnico, IT-OAN-2008-03
  
- [4] Dodson, R. & Rioja, M. 2009, VLBA Scientific Memorandum No. 31
  
- [5] Middelberg, E., Roy, A. L., Walker, R. C., Falcke, H. 2005. VLBI observations of weak sources using fast frequency switching. *Astronomy and Astrophysics* 433, 897-909.
  
- [6] Rioja, M. & Porcas, R. 2007. Astrometry with VSOP. “Approaching Micro-Arcsecond Resolution with VSOP-2: Astrophysics and Technology”, Eds. Y.Hagiwara, E.Fomalont, M. Tsuboi, and Y.Murata (ASP Conference Series vol 402, 2009).