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Volume, Volatility, and Periodic Closure with Information Uncertainty

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ABSTRACT

This paper investigates the effects of opening and closing on transactions demand, volume, and volatility of options prices and their underlying assets. We use an extension of the models in Merton (1971) as in Brock and Kleidon (1992), who consider a similar issue with respect to equity markets. The transactions demand at open and close in the underlying assets markets are studied in the presence of information costs using the main concepts in Merton's (1987) model of capital market equilibrium with incomplete information. As in other studies by Brock and Kleidon (1992), Smith and Webb (1994), Hong and Wang (2000), Bellalah and Zhen (2002), we show that periodic market closure leads to periodic changes in the demand for transaction services. We present some empirical work regarding the patterns in volume, volatility and spreads using a dataset for the Paris Bourse. The predictions of periodic demand with high volume at open and close in options markets and their underlying assets are consistent with other markets. They confirm also the more recent results in Hong and Wang (2000) and Bellalah and Zhen (2002).

JEL: *G1, G12, G13, G14, G15, F3*
Keywords: *Volume; Volatility; Periodic closure*

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I. INTRODUCTION

As in other markets, an options dealer or specialist in the Paris Bourse sets bid and ask spreads, provides immediacy and trades as a public in the underlying asset. The quote information disseminated to the public during the open period concerns the bid and offer prices. This information is no longer disseminated in the close period and there is an abrupt change from a regime of continuous trading to a regime of no trading. The question of how the trading behaviour at open and close is affected in stock markets is analyzed in Admatti and Pfeiderer (1988), Brock and Kleidon (1992), Smith and Webb (1994), Hsieh and Kleidon (1996), Hong and Wang (2000), Bellalah and Zhen (2002), etc. While these papers are interested only in stock markets, our work gives an answer to similar question simultaneously in options markets and their underlying assets markets. This work provides a model for market closure and tests some of its implications. There are several reasons why periodic trading demand shifts at the open and the close of the trading, most of which are based on the effect of the periodic inability to trade.

We analyze some of these reasons and show that there is a greater demand to trade at open and close than at the other times of the trading day. The inability to trade modifies the optimal portfolio of investors. Investors engage expenses to collect, to analyze and get informed about the domestic and foreign markets. Therefore, information may play a central role in explaining the high demand to trade at market closure. This situation fits well with Merton's (1987) model of capital market equilibrium with incomplete information. Merton (1987) assumes that investors hold only securities of which they are aware. This assumption is motivated by the observation that portfolios held by investors include only a small fraction of all available traded securities.

Following the work of Brock and Kleidon (1992), Merton (1971), Bellalah and Zhen (2002) and the foundations of Merton's (1987) model, we investigate the implications of the discontinuity in trading regimes in the presence of incomplete information. We extend the model in Brock and Kleidon (1992) to allow for the periodic market closures by showing an increased and less elastic demand to trade at these points in time. Our analysis accounts for the shadow costs of incomplete information in the spirit of Merton (1987)[1].

The extended model implies higher transactions demand at open and close, confirming the higher volume around closure. Using a new data set, our empirical evidence gives some support to the implications of the model. In particular, we find that volume traded in the MONEP (Marche des Options Negociables de la Bourse de Paris) is concentrated at open and close. Also, that volatility patterns are associated with times of high volume on the options market[2]. Hence, the first motivation in this paper is to develop a model for periodic closure in options markets and the underlying assets markets. The second motivation is to use a new data set in the Paris Bourse to provide new results regarding market closure[3].

The theoretical contribution of this paper is a simple model for the study of periodic closure in the presence of information costs. The empirical contributions are that: (a) there may not be greater demand just at the open of trading in the option market relative to the middle of the day; (b) high transactions demand at the close of trading in the option market need not coincide with narrower bid/ask spreads; (c) the simultaneous examination of the option market and the underlying asset markets; and (d) several descriptive statistics regarding volume, volatility and spreads in the Paris bourse are provided. Even if these results are known for other markets, they are presented for the first time for the case of the Paris Bourse. This allows to study the implications of our model.

This paper is organized as follows. Section II develops a model in the lines of Merton (1971, 1987), Brock and Kleidon (1992) and Bellalah and Zhen (2002) to show how periodic market closure can lead to periodic changes in the demand for transaction services. The model reveals an increased demand to trade and less elastic transactions around closure in the presence of information costs in financial markets. Section III studies the implications of our model in the presence of information costs. In particular, it presents our findings of periodic market demand changes at open and close in the Paris bourse. The empirical work is done in the lines of Brock and Kleidon (1992), using a new data set for the period 1992-1998 in which transactions, volumes and transaction prices are given. The empirical results seem similar to those reported in other markets for half an hour time intervals. It is clear that the number of transactions in calls and puts is highest at the beginning and the end of the day. The pattern is clearly U-shaped. The results give support to those reported in Hong and Wang (2000) and Bellalah and Zhen (2002).

II. A MODEL FOR PERIODIC DEMAND SHIFTS AND MARKET CLOSURE IN THE PRESENCE OF SHADOW COSTS OF INCOMPLETE INFORMATION

To explain the periodic market closure, Brock and Kleidon (1992) extend Merton's (1971) model by accounting for an alternate regime. However, they do not account directly for the effect of incomplete information at the open and the close of the financial markets as in Bellalah and Zhen (2002). This is our main contribution in this section. In fact, Brock and Kleidon (1992) propose different explanations for the increased transactions demand at the open and the close of the stock exchange. However, their analysis ignores the role of incomplete information in the explanations of the greater demand to trade around closure in stock markets. Their first argument explaining the greater demand to trade at open and close concerns the effect of the periodic inability to trade. Their second argument refers to the ability to trade on an alternate market if the primary market is closed.

To illustrate the first arguments in their analysis, Brock and Kleidon (1992) and Bellalah and Zhen (2002) divide the calendar day into two periods. The first period of length T allows continuous trading. The second period of length N is characterized by the absence of trading. Hence, the first day of trading is in the interval [0, T] and the first closed period is in [T, T + N]. Trading starts again the next day over [T + N, 2T + N], and so on. At the open of trading at time [T + N], if traders want to be at the same position as in the close, they have to execute their overnight trades. This explains the high opening volume. However, as it appears in the work of Brock and Kleidon (1992), the effect of a closed stock market on trading before the close is more difficult to determine. The main argument that appears in several studies is that traders want to close positions by day's end to avoid potential exposure over the night.

The analysis in Brock and Kleidon (1992) and Bellalah and Zhen (2002) can be extended to account for the shadow costs of incomplete information in the spirit of Merton's (1987) simple model of capital market equilibrium with incomplete information , CAPMI. The main difference between Merton's model and the CAPM lies in the shadow cost of incomplete information referred to in Merton's model as λ. This factor appears as an additional discount rate in the computation of the future value of risky cash flows. This model is used by several authors in different contexts[4]. Bellalah (1999) uses a continuous time version of Merton's (1987) model in the valuation of derivative securities[5].

Following Merton (1971, 1987), we extend the models in Brock and Kleidon (1992), Bellalah and Zhen (2002) and Bellalah (2001) to account for the effects of incomplete information. The main result regarding periodic market closure is that the demand to trade at open and close will be stronger and relatively inelastic at these special points in time. This situation may be explained by the absence of trading during the night. The accumulation of information during the evening and at open can induce trading. Besides, traders who want to close their positions at the end of the day can trade much more at the close of the exchange. Hence, the adjustment of portfolios at the close and the open gives an intuition of our result. Several other reasons will be later proposed to explain this situation.

To introduce the model, let us denote respectively by :
g_1 : the value of going from open with wealth W_0 to close with wealth W_T,
g_2 : the value of going from close with wealth W_T to open with wealth W_{T+N},
ρ : a discount factor, and
λ : an additional discount rate in the spirit of Merton's (1987) λ to account for information costs supported by investors.

Consider $J_1(W_0)$ the value of wealth W starting at the open and $J_2(W_0)$ the value of wealth starting at the close, or :

$$J_1(W_0) = \max\left\{Eg_1(W_0, W_T) + e^{-(\rho T)} EJ_2(W_T)\right\} \tag{1}$$

$$J_2(W_T) = \max\left\{Eg_2(W_T, W_{T+N}) + e^{-(\rho N)} EJ_t(W_{T+N})\right\} \tag{2}$$

Equation (1) can be written as follows:

$$J_1(W_0) = \max\left\{E\int_0^T e^{-\rho t}(c_1^a / a)dt + e^{-(\rho t)} EJ_2(W_T)\right\} \tag{3}$$

under the following conditions :

$$dW = W\left[\sum w_{1j}((\alpha_j + L_j)dt + \beta_j dZ_j)\right] - c_1 dt$$
$$W(0) = W_0, \sum w_{1j} = 1, dZ_i dZ_j = \rho_{ij} dt$$

$$\frac{dP_j}{P_j} = (\alpha_j + \lambda_j)dt + \beta_j dZ_j \tag{4}$$

where $c_1(t)$ and $w_{1j}(t)$ correspond to consumption and portfolio weights over the open period, α_j and β_j are time stationary over the open period and λ_j are information costs regarding the different assets traded in the market place. In the same context, equation (2) can be written as :

$$J_2(W_T) = \max\left\{\theta(N)c_2^a / a + e^{-(\rho N)} EJ_1 W_T \sum w_{2j} \frac{P_j(T+N)}{P_j(T)} - c_2 N\right\} \tag{5}$$

where $\theta(N) = \int_0^N e^{-\rho t} dt$

$$W(T+N) = \sum N_j(T)P_j(T+N) - c_2 N = W_T \sum w_{2j} \frac{P_j(T+N)}{P_j(T)} - c_2 N$$

$$W_{2j} = \frac{N_j(T)P_j(T)}{\sum N_i(T)P_i(T)} = \frac{N_j(T)P_j(T)}{W_T} \tag{6}$$

Remark 1: As in Brock and Kleidon (1992) and Bellalah and Zhen (2002), assuming a constant consumption rate c_2 during the "night" and neglecting compounded terms in equation (6), the conclusion is that volume changes abruptly at the close and at the open.

Remark 2: The univariate process for a stock j in the presence of an information cost λ_j is given by: $\frac{P_j(T+N)}{P_j(T)} = e^{(\frac{\alpha_j + L_j - \beta_j^2}{2})} N + \beta_j \widetilde{Z}(N)$ where $\widetilde{Z}(N) \approx N(0,1)$. In order to solve the open period problem in equation (3), let:

$$M(W_t, t, T) = \max E_t \left\{\int_t^T e^{-\rho s} \frac{c_1^a(s)}{a} ds + e^{-(\rho T - t)} J_2(W_T)\right\}$$

and

$$M(W_t, t, T) = b_1(t,T) W_t^a e^{-(\rho T - t)} = b_1 W_t^a e^{-(\rho T - t)} \tag{7}$$

then, we can show that $M(W_0, 0, T) = J_1(W_0)$ and $M(W_T, T, T) = J_2(W(T), T, T) = J_2(W(T))$.

Remark 3: Following Brock and Kleidon (1992) and Bellalah and Zhen (2002), using the price process in equation (4), M satisfies:

$$-M_t = \max\left(e^{-\rho t}u(c_1) + M_W \frac{d\overline{W}}{dt} + \frac{1}{2} M_{WW} \frac{d\overline{W}^2}{dt}\right)$$

$$\frac{d\overline{W}}{dt} = W(\sum w_{1j}(\alpha_j + \lambda_j)) - c_1$$

$$\frac{d\overline{W}^2}{dt} = W^2 \sum\sum w_{1i} w_{1j} \beta_i \beta_j \rho_{ij}$$

Using the utility function $U(c) = \frac{c^a}{a}$ and:

$$-M_t = M_W^{(\frac{a}{a-1})} \frac{1-a}{a} e^{\frac{\rho t}{a-1}} + M_W WA + M_{WW} WA + M_{WW} W^2 B \tag{8}$$

where $A = \sum \overline{w}_{1j}(\alpha_j + \lambda_j)$, $B = \frac{1}{2}\sum\sum \overline{W}_{1i}\overline{W}_{1j}\beta_i\beta_j\rho_{ij}$ and $\overline{W}_{1i}$ is solution to Merton's (1971) equation in the following form:

$$\frac{\partial}{\partial w_{1i}}\left[M_W W(\sum w_{1j}(\alpha_j + \lambda_j) + \frac{1}{2} M_{WW} W^2 (\sum\sum w_{1i} w_{1j} \beta_i \beta_j \rho_{ij}) + \gamma(1 - \sum w_{1j})\right] = 0 \tag{9}$$

Remark 4: For $dZ_i\, dZ_j = 0.\, dt$ for i different of j and 1.dt for i = j, we have :

$$\overline{w_{1j}} = \frac{(\alpha_j + \lambda_j) - \alpha_I - \lambda_I}{\beta_j^2} \tau$$

$$\tau = -\frac{M_W}{M_{WW} W} = \frac{1}{1-a}$$

where I risk-free.

We follow the methodology in Brock and Kleidon (1992) and Bellalah and Zhen (2002) to solve explicitly for the state valuation function. Substitute equation (7) into (8) and conjecture

$$J_2(W(T)) = b_2(T) W(T)^a$$

gives: $\rho b_1 - b_1 = (ab_1)^{(\frac{a}{a-1})} \frac{1-a}{a} + ab_1(A + (a-1)B)$ and $b_1(T) = b_2(T)$ (10).

where b_1 represents the derivative of b_1 with respect to time.

Remark 5: Let us look for a solution to the closed period problem in equation (5):

$$J_2(W_T) = b_2(T)W_T^a = \max_{c_2, w_2}\left\{\theta(N)\frac{c_2}{a} + b_1(T+N)e^{-(\rho N)} * E\left[W_T \sum w_{2j}\frac{P_j(T+N)}{P_j(T)} - c_2 N\right]^a\right\} \quad (11)$$

Setting the Lagrangian, and relating $b_1(T+N)$ to $b_1(0)$ through the relation :

$$b_2(T)W_T^a = \theta(N)\frac{c_2^{*a}}{a} W_T^a + b_1(0)e^{-(\rho N)} * E\left[\sum w_{2j}\frac{P_j(T+N)}{P_j(T)} - c_2^* N\right]^a W_T^a \quad (12)$$

Differentiating equation (12) gives:

$$\frac{\partial}{\partial c_2^*} = 0 : \theta(N)c_2^{*a-1}W_T^a + b_1(0)e^{-(\rho N)} a W_T^a * E\left[\left(\sum w_{2j}\frac{P_j(T+N)}{P_j(T)} - c_2^* N\right)^{a-1}(-N)\right] = 0 \quad (13)$$

$$\frac{\partial}{\partial c_2^*} = 0 : b_1(0)e^{-(\rho N)} a W_T^a * E\left[\left(\sum w_{2j}\frac{P_j(T+N)}{P_j(T)} - c_2^* N\right)^{a-1}\frac{P_j(T+N)}{P_j(T)}\right] = \gamma \quad (14)$$

If you put $w_{2j}^* = w_{2j}$ for i = 1, 2,...I, and $\gamma = \gamma_2^* W^a$ into equation (14), this gives:

$$ab_1(0)e^{-(\rho N)}E\left[\left(\frac{w_{2i}^* P_j(T+N)}{P_j T} - c_2^* N\right)^{a-1}\frac{P_j(T+N)}{P_j(T)}\right] = \gamma^*$$

$$w_{2j}^* = w_{2j}(\gamma_2^*) \quad (15)$$

Now, for each γ_2, solve equation (15) and choose γ_2^* so that: $\sum w_{2i}(\gamma_2^*) = 1$. If you put $w_{2j}^* = w_{2j}(\gamma_2^*)$ and insert into equation (11), this gives:

$$j_2(w_T) = b_2(T)w_T^a = \theta(N)\frac{(c_2^*)^a}{a} w_T^a + b_1(0) * E\left[e^{-(\rho N)} a w_T^a\left(\sum w_{2j}^*\frac{P_j(T+N)}{P_j(T)} - c_2^*\right)^a\right] = 0$$

$$b_2(T)_T^a = \theta(N)\frac{c_2^* b_1(0)}{a} + b_1(0) * E\left[e^{-(\rho N)}\left(\sum w_{2j}^* b_1(0)\frac{P_j(T+N)}{P_j(T)} - c_2^* b_1(0)N\right)^a\right] \quad (16)$$

where $c_2^* b_1(0)$ and $w_{2j}^* b_1(0)$ solve equations (13), (14) and (15).

Remark 6: Since the parameters α_j and β_j regarding the price process are not constrained to equal those over the open period, then the optimal weights in our case will differ from those over the open period[6]. Now, the valuation function can be found from equations (16) and (10) using: $b_2(T) = \varphi_T(b_1(0)) =_1 (T), \quad b_1(0) = \varphi_T^{-1}(b_1(T))$.

For the open periods and every j, we have:

$$N_j(t) = \frac{\overline{w}_{1j} W(t)}{P_j(t)}$$

$$\frac{dW}{W} = \sum \overline{w}_{1j}((\alpha_j + \lambda_j)dt + \beta_j dZ_j) - \overline{c}_1 dt = \sum (\overline{w}_{1j}\alpha_j + \lambda_j - \overline{c}_1)dt + \sum \overline{w}_{1j}\beta_j dZ_j \quad (17)$$

where $c(W) = \overline{c}W$. From equation (17) over open periods, on and

$$\hat{t} \in \psi \equiv [T, T+N] \cup [2T+N, 2T+2N] \cup \ldots,, N_j(\hat{t}) = \frac{w_2^* W(\hat{t})}{P_j(\hat{t})} \quad \text{and}$$

$$W(\hat{t}) = W(T) \sum w_{2j}^* \frac{P_j(\hat{t})}{P_j(T)} \quad (18)$$

Remark 7: The volume of trade in shares of j abruptly changes at open and closes since w_j changes at the open and close while the ratio of wealth to P_j is modified continuously.

Remark 8: A similar derivation shows that during open periods:

$$w_{1j} = \frac{(\alpha_j + \lambda_j) - (\alpha_I + \lambda_I)}{\beta_j^2}$$

When the "Sharpe indices" change a lot, w_{1j} changes and this gives high volume. These indices account for the incremental return resulting from expenses in information.

Remark 9: At open/close, transactions demand must be relatively inelastic and strong. Hence, the urge to modify w_{1j} is especially strong at market closure open/close/open. In this analysis, the main result regarding periodic market closure is that the demand to trade at open and close will be stronger and relatively inelastic at these special points in time.

The first reason concerns the optimal portfolio weights allocated to different securities at the end of the trading day. If these weights were constant at all time, the investor will trade at the open because overnight price changes will imply changes in these proportions in order to preserve the assumed constant portfolio weights. These proportions are constant during the open period in the model in the presence of HARA utility functions. These proportions are affected by the presence of information costs engaged by market participants for the different securities in their portfolios. In fact, investors acquire and diffuse information even during the closed period since information is fundamental in the allocation of their wealth. (Remark 4).

There will be in general unusually high trade at open since the process describing the dynamics of asset prices evolves overnight. Hence, prices at open may be different from those at close implying a change in the optimal proportions. However, the direction of the price will depend on whether there is net buying or net selling. The univariate process of stock j shows in its deterministic return a shadow return reflecting information costs on the asset. This shows that investors handle information continuously and that this information affects the expected return of investors' portfolios. (Remarks 2 and 4).

The second reason concerns the optimal weights for the overnight period. The portfolio weights during the closed period depend on the optimal choice just before the market closes. The changes in the dynamics of assets prices induce a change in the optimal weights overnight. These weights are affected also by the levels of the shadow costs of information during these periods. (Remarks 2, 4 and 6).

The abrupt changes in the optimal portfolio weights in the presence of a continuous process induce a spike in demand for trading at the open and the close. The abrupt changes in the portfolio weights depend heavily on the severity of the shadow costs of incomplete information. Information at these two instants of time is different from that in the other times of the day. (Remarks 7 and 9).

Equations (11) to (16) show the computation of the optimal portfolio weights in the presence of shadow costs of incomplete information regarding the traded assets. Brock and Kleidon (1992) investigate the possibility to apply their analysis for increased volume at closure when the assets specificities do not change over the open and closed periods. Their interesting question raises issues dealing with the role of the market in the investor's portfolio choice. They refer to the work of Duffie and Huang (1985) to show the presence of different opportunities to adjust portfolios to provide optimal hedging in the sense of Black and Scholes's (1973) perfect hedge. Therefore, they explain the argument that "the optimal portfolio during the open period with potential continuous adjustment across assets differs from the optimal portfolio during the closed period". Their results are especially correct in the context of our proposed extension in the presence of shadow costs of incomplete information[7]. The extension of the main results in the models of Merton (1971, 1987) and Brock and Kleidon (1992) to a regime of periodic closure shows that the demand to trade will be stronger and relatively inelastic at closure mainly because of the presence of shadow costs of incomplete information. The conclusion can also be applied to a portfolio of derivative assets. The extended model shows an increased volume of trade around closure in the presence of incomplete information. This result is central to the analysis of a portfolio of securities and derivative assets. The evidence proposed in the empirical section confirms our results. The model can be extended as in Brock and Kleidon (1992) and Bellalah and Zhen (2002) to account for transaction costs by assuming that investors have different risk tolerances[8]. Mayshar (1979) defines fixed transaction costs as a fixed charge that has two components: an objective cost and a subjective cost. This cost involves the cost of gathering information and decision making for each asset (as in Merton (1987)). It may vary from one investor to another depending upon his ability to gather information and keep track a given asset[9].

The extended model would be appropriate for the description of the institutional features of financial markets. The lack of information regarding the night induces traders to avoid potential risks by liquidating their positions at the close. Short-sellers and day traders avoid the overnight exposure of their positions by closing them at the close and reestablishing them at the open[10].

If investors have the possibility to trade elsewhere during the closed period [T, T + N], they try to manage their positions in different markets in order to hold optimal portfolios. In the absence of transaction costs, it may be optimal to liquidate the portfolio in one market and reinvest in another market. This strategy leads also to an increasing volume around closure across markets. However, these operations cannot be done without information about these markets. The information costs in our model reflect this possibility.

Brock and Kleidon (1992) recognize that for small investors, it may be optimal to remain within one market. However, for institutional investors, the fixed costs of conserving worldwide accounts can be effectively amortized and therefore, they can trade continuously. In our extension of their models, even institutional trading can imply high information costs, which are not expected to disappear. In fact, Merton (1987) shows that the shadow costs of information are important even for institutional investors.

The ability to trade and manage portfolios in other markets when national markets are closed can lead to some anomalies. The fact that investors appear to invest only in their home country, ignoring in general, foreign opportunities is referred to as the "home bias puzzle". The explanations of this bias are based on barriers to international investment such as governmental restrictions on foreign and domestic capital flows, foreign taxes and high transactions costs[11]. Our model accounts for the shadow costs of incomplete information and might explain the real costs associated with changing from one market to another. Brock and Kleidon (1992) propose two other institutional features that might explain the strong trading demand at certain times of the day. The first reason is that brokers need to fill the remaining orders as close approaches. The second reason is related to differential demands across trading times when investors receive payoffs that depend on the time of the day. This is the case for example for fund managers who are evaluated at closing prices. This might provide an incentive to trade[12].

III. DATA AND EMPIRICAL EVIDENCE ON VOLUME AND VOLATILITY

This section provides some of our model implications. It presents some evidence regarding the volume, the volatility and the spreads in options markets and their underlying assets markets in the Paris Bourse.

A. The Data for Options Contracts and the Underlying Assets

The Paris Bourse's satellite-based, automated data dissemination system enables MONEP trading activity members to have immediate access to information. The database comprises daily data and intraday data. The daily data regarding short term index options (PX1) and long term index options (PXL) are taken from the MONEP database for the period from January 1992 to June 1998. For each day, the data regarding calls and puts show the number of transactions, the number of traded contracts and the amounts of capital exchanged. The data covered 1632 trading days. The intraday data concerns calls, puts, the time of quotation, the degree of parity and the time to maturity. The data covers 653,225 trades. The data regarding the underlying index and the futures index contracts are also available for the same period[13]. We first present some descriptive statistics regarding the Paris Bourse. Then, we examine the implications of the model.

B. Evidence on Volume and Volatility Patterns

We use three measures for the volume of transactions: the number of trades, the number of traded contracts and the amounts of capital exchanged. For intraday data, these measures are standardized by the total value each day. For the range of parity of options, we calculate each day for each transaction, the difference between the index level at that time and the strike price. This difference approximates for the degree of parity. We define five levels or ranges of parity for short term and long-term options. The difference between two successive strike prices for short-term options is 25 points. The difference for long-term options is 150 points of the index. The range of parity for CAC 40 PX1 options is defined as the difference $X = K - S$ where K stands for the option strike price and S for the implicit index level. We attribute the values -2 to +2 for the following ranges for PX1 options:
-2 when $-100 \leq X$, -1 for $-100 \leq X \leq -50$, 0 for $-50 \leq X \leq +50$, +1 for $+50 \leq X \leq +100$, and +2 when $+100 \leq X$.

This is a standard way in the empirical tests of option pricing model. The distinction between options by their degree of parity does not mean that we discard data. This simply shows the specific features of options and their repartition in the sample (in-the-money, at-the-money and out-of-the-money options). In the same context, we define three levels for the maturity dates as a function of the number of remaining days to the maturity date T. The levels of maturity in months for CAC 40 PX1 options are respectively 1 when $T \leq 1$ month, 2 for $1 < T \leq 2$ months and 3 for $2 \leq T$ (This is the case when the maturity is longer than two months). This distinction between the option degree of parity and time to maturity allows a deeper study of options properties according to their range of parity and time to maturity. We do not use longer maturities because they are less traded.

To get an idea of the trading activity in the Paris Bourse, we present some descriptive statistics. Since index options are traded each day from 10 H to 17 H Paris time, we divide the trading day into 28 time intervals of length 15 minutes.

Figure 1 represents for CAC 40 PX1 options the number of traded contracts and the capital exchanged each year for the period 1992-1998. The number of traded contracts seems to be the highest in 1992. However, the amount of capital is highest in 1998. In order to appreciate the mean measures for the number of transactions, the number of traded contracts and the capital exchanged, Figure 2 reports the number of transactions as a function of the degree of parity and the maturity date for PX1 options. It shows also the number of traded contracts as a function of the degree of parity and the maturity date. Note that the highest values for the number of transactions and the number of traded contracts are observed for at-the-money options. The two measures of volume are highest for shorter-term PX1 options (less than one month). The number of transactions and the number of traded contracts seem to be U-shaped for PX1 options. This result is not different from those reported in other markets.

We tried to detect systematic patters in the number of traded contracts and the amounts of capital exchanged for the last 10 days preceding the option's maturity dates, according to the days of the week and the months of the year. Figure 3 reports the effect of the maturity date on the statistics of volume for the period 1992-1998. The volume is highest (number of traded contracts and the amount of exchanged capital) for the period of ten days before the option's maturity date. The pattern in these two variables seems to follow an inverted U-shaped curve. We tried to detect systematic patterns in the number of traded contracts and the amounts of capital exchanged according to the days of the week.

The list of major anomalies in stock returns corresponds to the size/January effect, the monthly effect, the weekend effect, etc. The weekend effect describes the tendency for Monday stock returns to be negative. It was documented by French (1980) and Gibbons and Hess (1981) and studied by several authors[14].

We analyze the effect of the days of the week on the mean volume for the period 1992-1998. We find that the volume is highest (number of traded contracts and the amount of exchanged capital) in Fridays and lowest in Mondays for the whole period 1992-1998 in the PX1 option market. The pattern in these two variables seems to be systematic for the whole period. This result may be explained by the fact that market participants prefer to close their positions in Fridays before the week-end.

Figure 1

The Number of traded contracts, Nt, and capital exchanged, CA, each year for the CAC 40 PX1 options for the period (1992–1998)

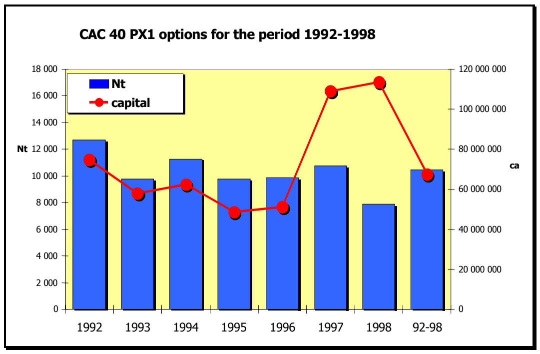
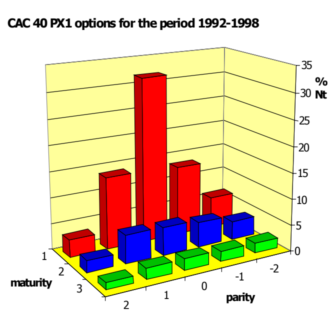
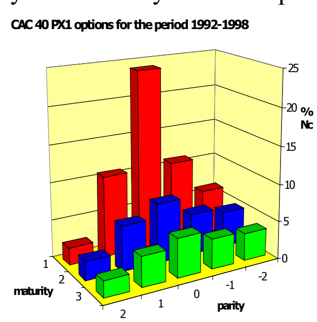


Figure 2

The number of transactions, Nt, the number of traded contracts, Nc, and capital exchanged, ca, as a function of the degree of parity and maturity for PX1 options



(a)



(b)

Figure 3

The effect of the maturity date on volume, number of traded contracts Nt and exchanged capital, ca for PX1 options

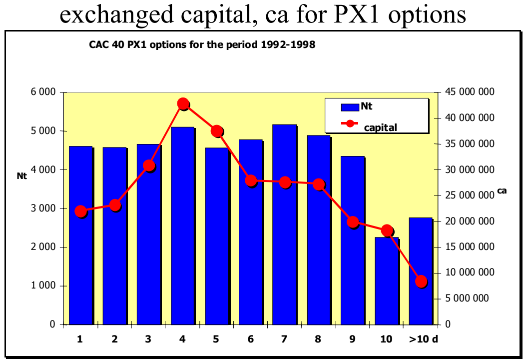


Figure 4
The year-end-effect on the mean volume, number of traded contracts Nt and exchanged capital, ca for PX1 options

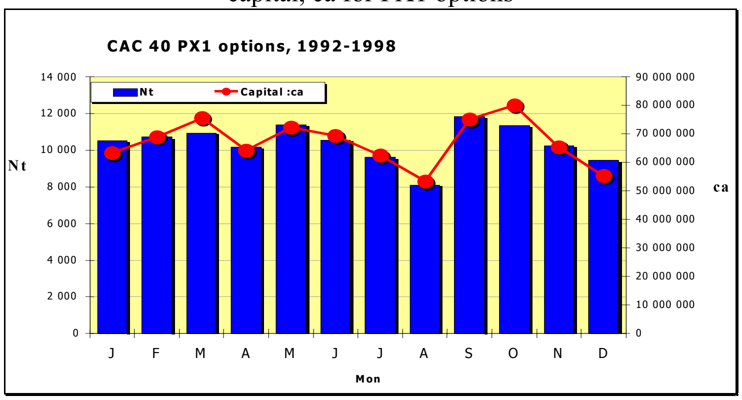


Figure 5
Intraday pattern in volume of transactions Nt and the put/call ratio (P/C) for PX1 options

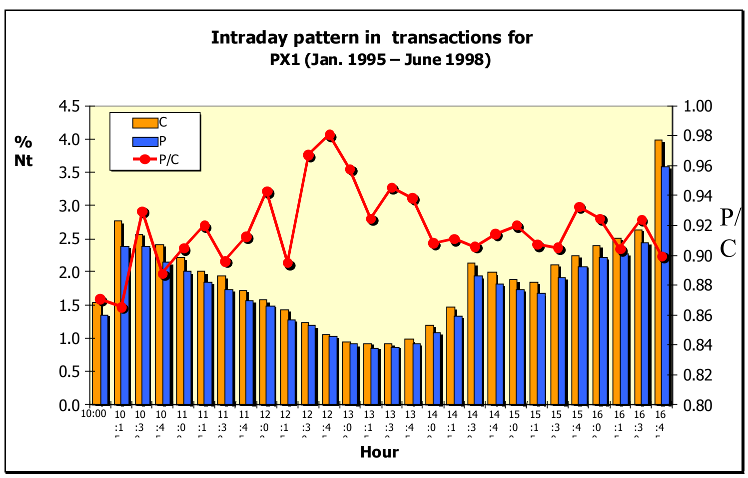


Figure 4 reports the effect of the months of the year on volume statistics in the period 1992-1998 for PX1 options. The figure shows that the volume is lowest (number of traded contracts and the amount of exchanged capital) in December for the whole period 1992-1998 in the PX1 option market. The pattern in these two variables seems to be systematic for the whole period.

We run some regressions using the standard OLS method to appreciate the significativity of the results for the effects of the days of the week and the months of the year on volumes for PX1 options. As in other markets, the results are statistically significant. In fact, they confirm those reported in other places regarding the weekend effect and the year–end effect. These results are rather expected and do not affect our conclusions regarding market closure[15].

C. Analysis of the Intraday Pattern in Volume and Volatility

Figure 5 shows the intraday patterns in volume for PX1 options during the period 1995-1998. It gives for each time interval of 15 minutes from the opening to the closing of the Paris Bourse the intraday pattern in the number of transactions as well as the put/call ratios. The put/call ratio defined with respect to the number of transactions has an inverted U-shaped form. It is clear that the number of transactions in calls and puts is highest at the beginning and the end of the day. As in other markets, the pattern is U-shaped.

Admati and Pfeiderer (1988) show that spreads will narrow with high volume. Brock and Kleidon (1992) find that the underlying assets spreads narrow with low volume and vice-versa. By studying the intraday volume on similar time intervals on the period 1995-1998, we find that average trading volume is highest during the first half hour each day. It declines then between the third and fourth hour and increases again between the fifth and the seventh' hours. The frequency of trading is highest at the open and the close and is lowest around the lunch hour. This increased volume around closure is consistent with our model of periodic transactions demand at open and close.

Figure 6 reveals the intraday pattern in the number of traded contracts and the put call ratios for PX1 options for the same period. A same pattern is observed as in Figure 5. In fact, the number of traded contracts in calls and puts seems to be U-shaped during the trading day with a high demand at open and close.

Figure 6

Intraday pattern in the number of traded contracts Nt and the put/call ratio (P/C) for PX1 options

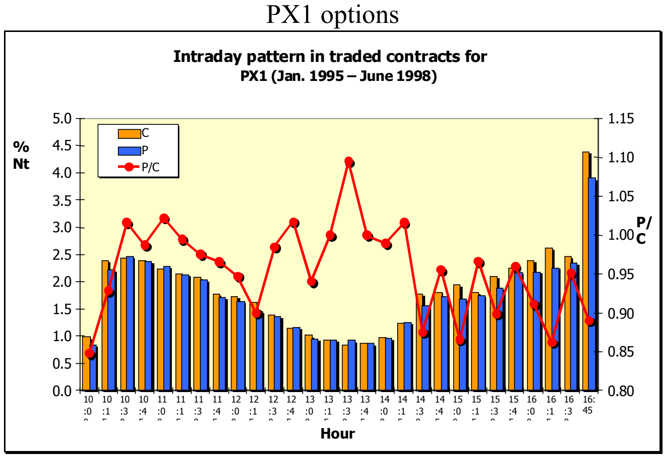


Figure 7
Intraday pattern in the amounts of capital exchanged CA for PX1 options

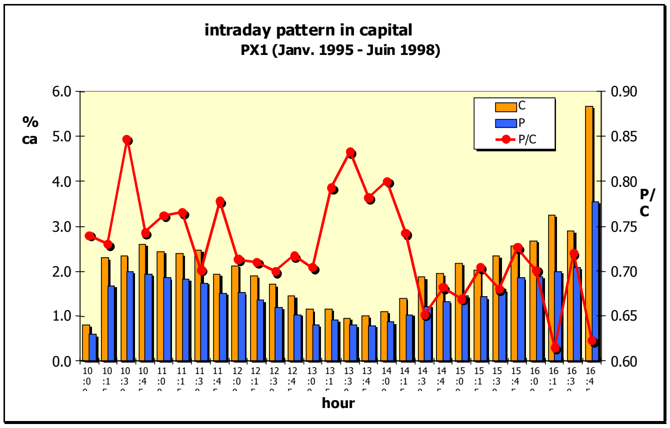


Figure 8
The volume of trades V for PX1 calls for the period 95-98

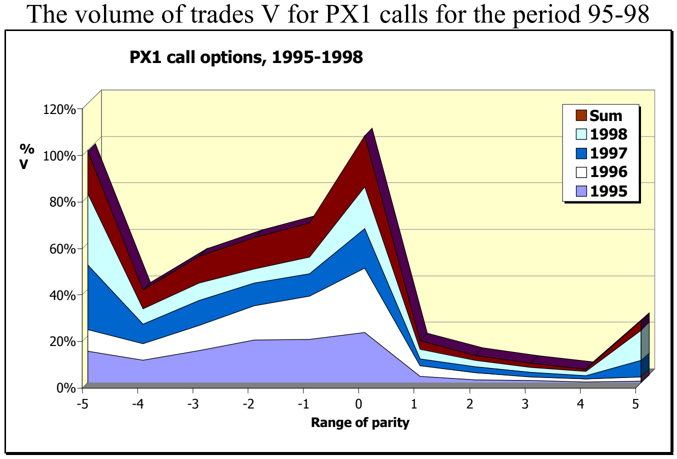
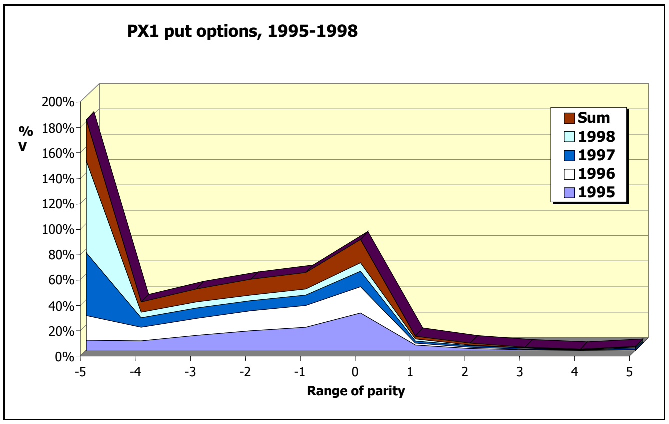


Figure 7 shows the intraday pattern in the amount of capital exchanged in each quarter of an hour time interval. An U-shaped curve is also observed for calls and puts. It is important to note that in each case, the demand is very high around the close of the stock exchange.

We have also used several intervals to define the degree of parity of options. The parity is defined with respect to zero, and then in-the-money options are defined for the intervals from -1 to -5. Out-of-the-money options are defined in the intervals + 1 to + 5. The choice of a higher number of intervals allows the observation of a similar pattern for the dynamics of the volume of trades. Figure 8 shows the volume of trades of PX1 calls according to the range of parity for the period 1995-1998. Figure 9 reports the same information for PX1 puts. Figures 8 and 9 show that the volume is very low for out-of-the-money calls and puts. The volume is high for at and in-the-money calls and puts. This result confirms those obtained in other markets.

Figure 9
The volume of trades V for PX1 puts for the period 95-98



Many authors report evidence of significantly greater volatility in NYSE stock returns at the open and close of trading. The reader can refer to Stoll and Whaley (1990), Amihud and Mendelson (1987), Webb and Smith (1994) and the references therein. Several studies report greater volatility of stock returns at the open and close of trading than at other times of the trading day. Webb and Smith (1994) examine whether the observed patterns in volatility are also characteristic of other financial markets. Using Eurodollar futures prices, they find greater volatility during the opening of trading than during all other intervals. They find also a significant market closing for the CME.

To proxy for the CAC 40 volatility rate, volatility estimates are implied from each transaction, each day, using a modified lattice approach as in Bellalah (2001). Each day, implied volatilities are aggregated with respect to the degree of parity. Hence, we obtain each day 11 average implied volatilities corresponding to different degrees of parity. To get an idea about the index volatility estimates, we calculate an implied ratio of volatility. This ratio is defined as follows:

$$Rv_{K,t} = \frac{\sigma_{K,t}}{\sigma_{0,t}}$$

for K= -5,...5. By construction, this ratio is equal to one for at-the-money options.

Figure 10 reports put/call ratios according to the degree of parity for PX1 and PXL calls and puts for the period 1995-1998. The figure shows that in and at-the-money options are more traded than out-of-the-money options. Figure 11 shows the mean ratios of implied volatility according to the degree of parity for PX1 calls and puts for the period 1994-1998. Volume and volatility show nearly inverted curves with respect to each other. Figure 11 reveals clearly the presence of a smile effect for call and put options. The shape of the volatility smile reveals higher volatilities for out-of-the money options. It is nearly U-shaped for calls and puts traded in the Paris Bourse. The results confirm those observed in other markets since the frequency of trading is highest at the open and the close and is lowest around the lunch hour. The increased volume around closure is consistent with our model of periodic transactions demand at open and close. Our empirical results for the underlying assets markets give also support to those in Hong and Wang (2000).

Figure 10

Put/Call ratios P/C for CAC 40 options, 1995-1998

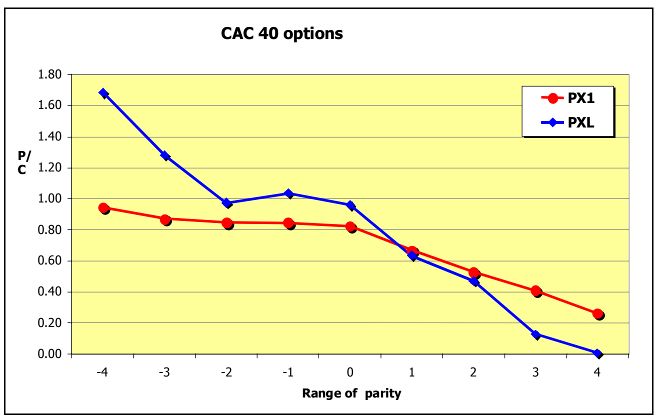
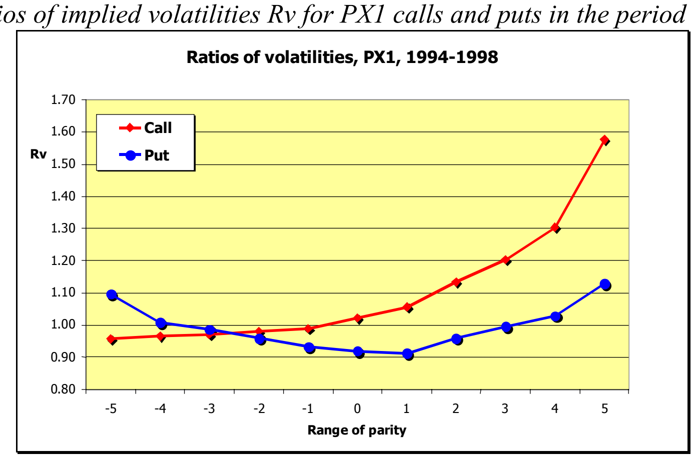


Figure 11

Mean ratios of implied volatilities Rv for PX1 calls and puts in the period 1994-1998



IV. SUMMARY

This paper examines whether the open and close of trading represent special moments in financial markets because of a discontinuity between a continuous trading regime and the absence of trading in a context of incomplete information. This question is studied simultaneously for options markets and their underlying assets markets. While most studies in the literature are interested in the underlying assets markets, we present a simple model for both the options markets and the underlying markets by accounting for the effects of information uncertainty.

Our model is an extension of Merton (1971, 1987) and Brock and Kleidon (1992) to account for the effects of the shadow costs of incomplete information. Incomplete information is introduced with respect to Merton's (1987) simple model of capital market equilibrium with incomplete information. We study periodic closure for options markets and the underlying assets markets in the presence of information costs. We show the presence of a strong and inelastic demand for trade at closures. This corresponds to the actions of market participants who seek to achieve optimal portfolio proportions or manage overnight risk in the presence of incomplete information.

Periodic market closure leads to periodic changes in the demand for transaction services showing an increased demand and less elastic transactions around closure. The analysis is conducted in the presence of information costs and applies to securities markets as well as to derivative markets.

In line with previous research, our study confirms the main findings using the recent options and assets prices data on the MONEP in the Paris Bourse. The effect of periodic market closure on transactions demand and volatility patterns of options prices is studied. Empirical tests on the Paris Bourse show that the frequency of trading is highest at the open and the close and is lowest around the lunch hour. This increased volume around closure is consistent with our model of periodic transactions demand at open and close. Our main contribution to the literature concerns the study of closure simultaneously for options markets and the underlying asset markets in the presence of shadow costs of incomplete information. The empirical results confirm those reported in other markets as in Hong and Wang (2000). Our analysis provides a starting point for the study of periodic closure in options markets. From an empirical viewpoint, our analysis documents new results regarding the Paris Bourse for a new data set.

ENDNOTES

1. The first explanation of periodic closure is the accumulation of information overnight. This leads portfolios at open to deviate from optimal holdings. The acquisition of costly information leads market participants to reallocate their portfolios at the first opportunity to trade.
2. For more details about the Paris bourse the reader can refer toSolnik and Bousquet (1990), Biais, Hillion and Spatt (1995) and Bellalah (2001), etc.
3. As it appears in Hong and Wang (2000), the literature on the empirical patterns of stock returns and trading activities in relation to market closures reveals that:
   (a) the intraday mean return and volatility are U-shaped;

(b) the intraday trading volume is also U-shaped;
(c) open-to-open returns are more volatile than close-to-close returns; and
(d) returns are more volatile over trading periods with comparison to non-trading periods.

4. Edwards and Wagner (1999) study the role of information in capturing the research advantage and how to incorporate trading information into the decision process of active investment management. Their results are consistent with evidence in Arbel and Strebel (1982), Barry and Brown (1984) and Amihud and Mendelson (1986) where it is shown that investors demand a higher premium for higher trading costs and for holding stocks with less available information.
5. I have shown in that paper that the discounted payoffs of different claims in a risk-neutral word must be computed at the riskless rate plus the shadow cost of incomplete information.
6. There could be cases where the optimal proportions remain the same from open to close for particular utility functions.
7. Bellalah (1999) proposed option valuation formulas and hedging portfolios in the presence of information costs by combining the main results in Merton's (1987) model and Black and Scholes (1973) theory.
8. The reader can refer to the analysis in Dumas (1989).
9. As shown in Bellalah and Zhen (2002), models in the line of Brock and Kleidon (1982) are not really general equilibrium models as the one in Hong and Wang (2000). These authors realize that Brock and Kleidon (1992) analysis is done in a partial equilibrium setting. Hong and Wang (2000) study how market closures affect investors' trading policies and the corresponding return generating process. They show that closures generate U-shaped patterns in the mean and volatility of returns over trading periods and that there is a higher trading activity around the close and open. They find also that closures can make prices more informative about future payoffs. General equilibrium feedbacks are not likely to neutralize the result that there is a burst of volume at open and close. Since trading services are not in perfectly elastic supply, it is likely that the same "peak load" effect at open and close will affect the equilibrium price of trading at open and close. The direction of the price will depend on whether there is net buying or net selling.
10. In fact, option market makers follow delta-neutral strategies and prefer at the end of the trading day to reestablish delta-neutral hedges before "going to bed".
11. These explanations appear in Brennan and Cao (1997) and the references therein.
12. In some countries, like in France, the high demand to trade in index options leads the market authority to extend the hours of trading (in the exercise of options) with a specific period, 5:00 p.m. to 5:45 p.m. This leads also to a higher trading activity at the close as in the market for the underlying asset. For index options, this additional time of 45 minutes gives rise to a wildcard optionvalued in Bellalah (2001).
13. However, we have only the spreads for the year 1998 since the MONEP database is actually in construction.
14. Connolly (1991) reports a posterior odds evaluation of the day-of-the week and weekend effect that reverses earlier findings. Connolly's (1991) paper presents

several contributions to the literature on the weekend effect. He shows that the tests of the weekend effect are sensitive to the assumed error distribution. He does not find systematic evidence that returns vary by day of the week. The results in Connolly (1991) confirm the findings in Connolly (1989) where it is shown that robust tests for day-of-the-week and weekend effects do not support these apparent anomalies. The author finds only weak evidence of a weekend effect.

15. Two statistically significant patterns in stock market returns are the weekend effect and the turn-of-the year effect. French (1980) studied daily returns on the S&P 500 index and found negative returns on Monday, which were highly significant. One explanation of this pattern is that firms wait until after the close of the market on Fridays to announce bad news. Another explanation is that these negative returns are generated by a general "market-closed" effect. Stock returns decline in December of each year, then the prices increase during January. Roll (1983) studied this year-end-effect for the period 1963-1980. Similar results are reported in the Paris bourse. Detailed results can be provided from the author on request.

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