

# VORTEX-LATTICE FORTRAN PROGRAM 

 FOR ESTIMATING SUBSONIC AERODYNAMIC CHARACTERISTICS OF COMPLEX PLANFORMSby Richard J. Margason and John E. Lamar
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## CONTENTS

Page
SUMMARY ..... 1
INTRODUCTION ..... 1
SYMBOLS ..... 2
BASIC CONCEPTS AND LIMITATIONS ..... 8
PROGRAM DESCRIPTION ..... 10
PART I - GEOMETRY COMPUTATION ..... 10
Section 1. Reference Planform ..... 11
Section 2. Configuration Computations ..... 11
Section 3. Horseshoe Vortex Lattice ..... 13
PART II - VORTEX STRENGTH COMPUTATION ..... 14
PART III - AERODYNAMIC COMPUTATION ..... 20
Section 1. Lift and Moment Using Entire Horseshoe Vortex ..... 20
Section 2. Lift and Pitching and Rolling Moments Using Only Spanwise Filament of Horseshoe Vortex ..... 26
Section 3. Output Data Preparation ..... 28
EFFECT OF VORTEX-LATTICE ARRANGEMENT ON COMPUTED AERODYNAMIC CHARACTERISTICS ..... 32
SAMPLE CASES ..... 34
CONCLUDING REMARKS ..... 34
APPENDIX A - INPUT DATA ..... 36
APPENDIX B - OUTPUT DATA ..... 40
APPENDIX C - SAMPLE CASES ..... 45
APPENDIX D - FORTRAN PROGRAM LISTING ..... 94
REFERENCES ..... 122
FIGURES ..... 124

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## SUMMARY

A FORTRAN computer program has been developed for estimating the subsonic aerodynamic characteristics of complex planforms. The program represents the lifting planforms with a vortex lattice. These complex planforms include wings with variablesweep outer panels, wings with several changes in dihedral angle across the span, wings with twist and/or camber, and a wing in conjunction with either a tail or a canard. The aerodynamic characteristics of interest are lift and pitching moment for both the flat and/or twisted wing, drag-due-to-lift parameter, leading-edge thrust, leading-edge suction, distributions of leading-edge thrust and suction coefficients, distributions of several span loading coefficients, distribution of lifting pressure coefficient, damping-in-pitch parameter, damping-in-roll parameter, and lift coefficient due to pitch rate.

This paper is intended as a user's guide for program application and sample cases are included to illustrate most of the options available for use in the program. Also included is a study of the effect of the vortex-lattice arrangement on some of the computed aerodynamic characteristics along with some recommendations for specifying vortex-lattice arrangements for particular types of planforms.

## INTRODUCTION

In recent years, some wings have become very complex because of the varied speed regimes in which they are required to operate. Such wings may have variable sweep, several changes in dihedral angle across the spair, or even a variable dihedral angle near the wing tip. Computing procedures for predicting the aerodynamic characteristics of these wings become very involved if an adequate representation of the planform is to be made. The problem becomes more involved when the body or body and tail are included in the representation. In order to solve this problem for preliminary designs or for parametric evaluations, a computer program has been developed for estimating the aerodynamic characteristics of these complex planforms.

In this FORTRAN computer program, the planform in steady subsonic flow is represented by a vortex lattice. Although this type of representation is not new (for example, refs. 1 to 12 ), the present program has several useful features that are not found together in other generally available programs of either the vortex-lattice or pressure-doublet type (refs. 13 to 15).

The program uses a minimum of input data to describe relatively complex planforms. These planforms may be described by up to 24 line segments on a semispan. They may have an outboard variable-sweep panel or they may have several dihedral angles across the span. In addition, two planforms may be used together to represent a combination of wings and tails or wing, bodies, and tails. The analysis in the present paper has been extended to handle planforms in a sidewash field. These velocities occur when a planform has dihedral or when a second planform is placed at a different height from the first planform.

The program described in the present paper was developed from a basic program written several years ago, which has had considerable use at the Langley Research Center. In recent years this basic program has also been used in industry. The results have shown good correlation with experimental data.

The present paper is intended to serve both as a description of the program and as a user's guide for its application. This paper describes in detail the program input data (appendix A) and output data (appendix B) and provides examples and typical running times of various types of configurations which can be handled (appendix $C$ ) along with a FORTRAN program listing (appendix D). In addition, the results of parametric applications of this program are presented to provide guidance in specifying vortex-lattice arrangements which can be expected to give acceptable results.

## SYMBOLS

The geometric description of planforms is based on the body-axis system with the origin on the planform center line. (See fig. 1 for positive directions.) The planform is replaced by a vortex lattice which is in a wind-axis system with the origin in the planform plane of symmetry. (See sketch (d) in text for details.) The axis system by which the geometric influence of a given horseshoe vortex is computed is wind oriented and referred to the origin of that horseshoe vortex (fig. 1). The units used for the physical quantities defined in this paper are given both in the International System of Units (SI) and in the U.S. Customary Units. For the purpose of the computer program, the length dimension is arbitrary for a given case; angles associated with planform are always in degrees. The symbols used for input data in the computer program are described in appendix $A$. The symbols used in the description of the program are defined as follows:

A aspect ratio; listed as AR in computer program output
$\mathrm{B}_{\mathrm{k}} \quad$ element of boundary-condition matrix, $4 \pi \alpha_{\mathrm{k}}$
b wing span, m (ft)
$C_{D, i} \quad$ induced drag coefficient, $\frac{\text { Induced drag }}{q_{\infty} S_{r e f}}$
$C_{D, i} / C_{L}{ }^{2}$ induced drag parameter based on Munk's far-field solution
$C_{D, i i} / C_{L}{ }^{2}$ induced drag parameter based on near-field solution
$C_{L} \quad$ lift coefficient, $L / q_{\infty} S_{\text {ref }}$
$\mathrm{C}_{\mathrm{L}, \tau} \quad$ lift coefficient based on additional loading and actual planform area
$\mathrm{C}_{\mathrm{Lq}} \quad$ lift coefficient due to pitch rate, $\frac{\partial \mathrm{C}_{\mathrm{L}}}{\partial\left(\frac{\mathrm{q}_{\mathrm{ref}}}{2 \mathrm{U}}\right)}$, per rad
$C_{L_{\alpha}} \quad$ lift-curve slope, $\left(\frac{\partial C_{L}}{\partial \alpha}\right)_{0}$, per deg or per rad
$C_{l} \quad$ rolling-moment coefficient, $\frac{\text { Rolling moment }}{q_{\infty} S_{r e f} b}$
$C_{l_{p}} \quad$ damping-in-roll parameter, $\frac{\partial C_{l}}{\partial\left(\frac{\mathrm{pb}}{2 \mathrm{U}}\right)}$, per rad
$C_{m} \quad$ pitching-moment coefficient about $\hat{Y}$-axis, $\frac{\text { Pitching moment }}{q_{\infty} S_{r} f_{f} \mathbf{c}_{\mathbf{r}} f}$
$\partial C_{m} / \partial C_{L}$ longitudinal stability parameter
$\mathrm{C}_{\mathrm{m}} \quad$ damping-in-pitch parameter, $\frac{\partial \mathrm{C}_{\mathrm{m}}}{\partial\left(\frac{\left(\mathrm{C}_{\mathrm{ref}}\right.}{2 \mathrm{U}}\right)}$, per rad
$C_{n}$
element of circulation term matrix, $\quad \Gamma_{n} / \mathrm{U}$
$\Delta C_{p}$
incremental pressure coefficient, $\frac{\mathrm{p}_{\text {lower }}-\mathrm{p}_{\text {upper }}}{\mathrm{q}_{\infty}}=\frac{\Delta \mathrm{p}}{\mathrm{q}_{\infty}}$
$C_{S}$
$\mathbf{C}_{T}$
c
$c_{a v}$
${ }^{c}$
$c_{d, i i} \quad$ section induced drag coefficient based on near-field solution
$c_{l} \quad$ section lift coefficient
$c_{\text {ref }} \quad$ reference chord, m (ft)
$c_{S} \quad$ section leading-edge suction coefficient
$c_{t}$
$\mathrm{d}_{\mathrm{ii}}$
F influence function which geometrically relates influence of single horseshoe vortex to a quantity which is proportional to velocity induced at a point, $\mathrm{m}^{-1}\left(\mathrm{ft}^{-1}\right)$
$\bar{F} \quad$ sum of influence function $F$ at a control point on wing caused by two symmetrically located horseshoe vortices, one on left half of wing and one on right half of wing, $\mathrm{m}^{-1}$ ( $\mathrm{ft}-1$ )
$\mathrm{G}_{\mathrm{n}, \mathrm{k}} \quad$ element of influence function matrix, $\bar{F}_{\mathrm{w}, \mathrm{n}, \mathrm{k}}-\overline{\mathrm{F}}_{\mathrm{V}, \mathrm{n}, \mathrm{k}} \tan \phi_{\mathrm{n}}$

L
$l$
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| $\tilde{\imath}$ | lift per unit length of vortex filament, $\mathrm{N} / \mathrm{m}$ ( $\mathrm{lb} / \mathrm{ft}$ ) |
| :---: | :---: |
| $\hat{\imath}$ | lift generated along a finite length of vortex filament, N (lb) |
| $\mathbf{M}_{\mathbf{Y}}$ | pitching moment for entire wing about $\hat{\mathrm{Y}}$-axis, $\mathrm{m}-\mathrm{N} \quad$ (ft-lb) |
| $\mathrm{M}_{\infty}$ | free-stream Mach number |
| $\mathrm{m}_{\mathbf{Y}}$ | pitching moment about $\hat{Y}$-axis due to lift developed on elemental panel, m-N ( $\mathrm{ft}-\mathrm{lb}$ ) |
| N | maximum number of elemental panels on entire wing |
| $\bar{N}_{c}$ | number of elemental panels in a chordwise row |
| $\overline{\mathbf{N}}_{\text {S }}$ | number of chordwise rows of elemental panels on wing semispan |
| p | roll rate, $\mathrm{rad} / \mathrm{sec}$; also, pressure, $\mathrm{N} / \mathrm{m}^{2} \quad\left(\mathrm{lb} / \mathrm{ft}^{2}\right)$ |
| q | pitch rate about $\hat{\mathrm{Y}}$-axis, rad/sec |
| $\mathrm{q}_{\infty}$ | free-stream dynamic pressure, $\mathrm{N} / \mathrm{m}^{2} \quad\left(\mathrm{lb} / \mathrm{ft}^{2}\right)$ |
| $\mathrm{S}_{\text {ref }}$ | reference area, $\mathrm{m}^{2}$ (ft2) |
| $\mathrm{S}_{\tau}$ | actual planform area, m2 (ft2) |
| S | horseshoe semiwidth in plane of horseshoe vortex, m (ft) |
| $T=S_{r e f} /\left(2 s_{n} \cos \phi c_{a v}\right)$ |  |
| t | section leading-edge thrust per unit span, $\mathrm{N} / \mathrm{m}$ ( $\mathrm{lb} / \mathrm{ft}$ ) |
| U | free-stream velocity, m/sec (ft/sec) |
| u | backwash velocity, m/sec (ft/sec) |
| V | resultant velocity, $\mathrm{m} / \mathrm{sec}(\mathrm{ft} / \mathrm{sec}$ ) |
| v | sidewash velocity, m/sec (ft/sec) |

downwash velocity, $\mathrm{m} / \mathrm{sec}(\mathrm{ft} / \mathrm{sec})$
$X, Y, Z \quad$ axis system of a given horseshoe vortex (see fig. 1)
$\bar{X}, \bar{Y}, \bar{Z} \quad$ body-axis system for planform (see fig. 1)
$\hat{\mathbf{X}}, \hat{\mathrm{Y}}, \hat{\mathrm{Z}} \quad$ wind-axis system
$\mathbf{x}, \mathrm{y}, \mathrm{z}$ distance along $\mathrm{X}-, \mathrm{Y}-$, and Z -axis, respectively, m (ft)
$\bar{x}, \bar{y} \quad$ distance along $\overline{\mathrm{X}}$ - and $\overline{\mathrm{Y}}$-axis, respectively, $\mathrm{m} \quad(\mathrm{ft})$
$\hat{\mathbf{x}}, \hat{\mathrm{y}}, \hat{\mathbf{z}} \quad$ distance along $\hat{\mathbf{X}}-, \hat{\mathbf{Y}}-$, and $\hat{\mathbf{Z}}$-axis, respectively, m (ft)
$\bar{x}_{c / 4} \quad$ midspan $\bar{x}$-location of quarter-chord of elemental panel, $m$ (ft)
$\bar{x}_{3 c / 4} \quad$ midspan $\bar{x}$-location of three-quarter-chord of elemental panel, $m$ ( $f t$ )
$\mathrm{X}^{\prime}=\mathrm{x} / \beta$
$\mathrm{y}_{\mathrm{cp}}$
$\alpha$
$o_{i} \quad$ induced angle of attack, rad
$\beta \quad$ Prandtl-Glauert correction factor to account for effect of compressibility in subsonic flow, $\sqrt{1-M_{\infty}{ }^{2}}$
$\Gamma$
vortex strength, $\mathrm{m} 2 / \mathrm{sec}\left(\mathrm{ft}^{2} / \mathrm{sec}\right)$
$\gamma$
$\Delta \Gamma$
nondimensional lift, $\frac{\Gamma}{b U}$ or $\frac{c_{l} c}{2 b}$
net vortex strength along left trailing leg of elemental panel, $\mathrm{m}^{2} / \mathrm{sec}$ (ft2/sec)
nondimensional spanwise coordinate, $\hat{y} /(b / 2)$
$\rho \quad$ density, $\mathrm{kg} / \mathrm{m}^{3} \quad$ (slugs $/ \mathrm{ft}^{3}$ )
$\phi \quad$ dihedral angle, in $\bar{Y}-\bar{Z}$ plane, deg
$\Lambda$
planform leading-edge sweep angle, in $\bar{X}-\bar{Y}$ plane, deg
$\psi \quad$ quarter-chord sweep angle of elemental panel; because of the small angle assumption, also used as sweep angle of spanwise horseshoe vortex filament, in $\mathrm{X}-\mathrm{Y}$ plane, deg
$\psi^{\prime}=\tan ^{-1}((\tan \psi) / \beta)$

Subscripts:
a additional; or angle of attack

B twist and/or camber at $C_{L}=0$ for chordwise row of elemental panels
b twist and/or camber at $C_{L}=0$ for elemental panel
d desired
i
index for elemental panel in chordwise row
j maximum number of elemental panels in chordwise row
k index for control point
$l \quad$ left half of wing
lower lower surface
n index for elemental panel on wing semispan
o value taken at $C_{L}=0$
r
rad
right half of wing
per radian angle of attack
spanwise bound vortex element
t
chordwise bound vortex element
tc twist and/or camber
u backwash
upper upper surface
v sidewash
w
downwash

## BASIC CONCEPTS AND LIMITATIONS

The vortex-lattice method is used in this computer program to determine the aerodynamic characteristics of planforms at subsonic speeds. This method is an extension of the finite step lifting-line method originally described in reference 16 and applied in reference 11. This method assumes steady, irrotational, inviscid, incompressible, attached flow. The effects of compressibility are represented by application of the Prandtl-Glauert similarity rule to modify the planform geometry. Potential flow theory in the form of the Biot-Savart law is used to represent disturbances created in the flow field by the lift distribution of the planform. It is assumed that in any plane parallel to the $\hat{X}-\hat{Z}$ plane the vertical displacements which occur in the wing or wake are neglected, except when the boundary conditions at the control points are determined.

The planform is divided into many elemental panels. Each panel is replaced by a horseshoe vortex. This horseshoe vortex has a vortex filament across the quarter-chord of the panel and two filaments streamwise, one on each side of the panel starting at the quarter-chord and trailing downstream in the free-stream direction to infinity. Figure 1 shows a typical horseshoe-vortex representation of a planform. The boundary condition for each horseshoe vortex is satisfied by requiring the inclination of the fluid streamlines to match the angle of attack at the three-quarter-chord point of its elemental panel. The circulations required to satisfy this tangent flow boundary condition is then determined by solving a matrix equation. Then, the Kutta-Joukowski theorem for lift from a vortex filament is used to determine the lift from each elemental panel. These lift results are then summed appropriately to obtain lift, pitching moment, and other aerodynamic characteristics. A similar procedure called the near-field solution is used to compute leadingedge thrust, suction, and induced drag. This program ignores the effect of thickness.

The lifting-surface planform is represented for the computer program by a series of up to 24 straight segments which are positioned counterclockwise around the perimeter of the left half of the planform. Lateral symmetry is presumed. The lines start at the leading edge of the plane of symmetry, go along the leading edge to the left tip of the planform, return along the trailing edge, and end at the trailing edge of the plane of symmetry. The preciseness of the $\bar{x}$ and $\bar{y}$ Cartesian coordinates and dihedral angles, given as input data, determines the accuracy of the planform representation. It is recommended that the planform coordinates listed in the second group of the geometry output data given in appendix $B$ be plotted and examined after each computation to verify the accuracy of the planform representation. This check should be made before using the aerodynamic output data.

There are a number of restrictions and limitations in the application of this computer program. These limitations are discussed in detail in the program description and are noted with the appropriate input variables in appendix $A$. For the convenience of the program user, a complete list of restrictions and limitations is presented.

The restrictions in the first group apply to all planforms and are as follows:
(1) A maximum of two planforms may be specified. For examples, see sample case 1 for one planform and sample case 2 for two planforms.
(2) A maximum of 24 straight-line segments may be used to define the left half of a planform. The lateral separation of the ends of these lines can be critical when the horseshoe vortices are laid out by the computer program. For details of the lateral separation requirements, see pages 12 and 13 .
(3) The maximum number of horseshoe vortices on the left side of the configuration plane of symmetry is 120 . When two planforms are specified, the sum total of the vortices in both is limited to 120 . Within this limit, the number of horseshoe vortices in any chordwise row may vary from 1 to 20 and the number of chordwise rows may vary from 1 to 50 . For examples, see the sample cases in appendix $C$.

The limitations that apply only to variable-sweep planforms are (1) there should always be a fixed-sweep panel between the root chord and the outboard variable-sweep panel, (2) the pivot cannot be canted from the vertical, and (3) no provisions have been made for handling dihedral in the geometry calculations for the variable-sweep panel or at the intersection of this panel with the fixed portion of the wing.

The limitations that apply only to planforms which have nonzero dihedral angles or to two planforms which do not lie in the same plane are (1) the variation in local chord must be continuous from the tip chord to the root chord of each planform specified, (2) the number of horseshoe vortices in each chordwise row must be at least two, and (3) the number of horseshoe vortices must be constant over the semispan of each planform.

Restrictions on allowed values or codes for individual items of input data are described in appendix A .

The calculations presented herein were made with a computer which used approximately 15 decimal digits. For other computers with fewer significant digits, it may be necessary to use double precision for some of the calculations. In addition, it may be necessary to change some of the tolerances used in the program. These tolerances are mentioned in either the text or the program listing.

## PROGRAM DESCRIPTION

This FORTRAN program is used to compute the following aerodynamic characteristics: $\mathrm{C}_{\mathrm{L} \alpha}, \mathrm{C}_{\mathrm{L}}$ at $\alpha=0, \alpha$ at $\mathrm{C}_{\mathrm{L}}=0, \mathrm{y}_{\mathrm{cp}}, \quad \mathrm{C}_{\mathrm{m}_{\mathrm{O}}}, \quad \partial \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}, \quad \mathrm{C}_{\mathrm{D}, \mathrm{i}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$, $\mathrm{C}_{\mathrm{D}, \mathrm{ii}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$, spanwise distribution of additional wing loading, spanwise distribution of wing loading due to twist and camber, and spanwise distribution of basic wing loading. In addition, the following aerodynamic characteristics are computed for a specified lift coefficient: the incremental pressure coefficient for each elemental panel, the spanwise distribution of the combined basic and additional wing loadings, the configuration angle of attack, and the contribution of the major planform to lift coefficient and induced drag coefficient. At an angle of attack of 1 rad , the induced drag, leading-edge thrust, and suction coefficients are computed for the entire configuration by using a near-field solution. This program can also be used to compute $C_{l_{p}}$ or both $C_{L_{q}}$ and $C_{m_{q}}$ (rotary derivatives). These quantities are described in detail in Part III of the Program Description.

The computation in this program for the aerodynamic characteristics is divided into three parts: Part I contains the required geometric calculations, Part II contains the circulation term calculations, and Part III contains the final output terms, calculations, and answer listings. These three parts coincide with the three overlays in the FORTRAN computer program. The input data are described in detail in appendix $A$, and the output data are described in detail in appendix B. Several sample cases are given to illustrate the use of the program. Listings of the input data and computed results for these sample cases (appendix C), along with the FORTRAN computer program (appendix D) are given.

## PART I - GEOMETRY COMPUTATION

The first part of the program is used to compute the geometric arrangement required to represent the planform by a system of horseshoe vortices and is divided into three sections. In Section 1, a description of the planform (group one of the input data in appendix A) is read into the computer. In Section 2, configuration details (group two of the input data) are read into the computer. In Section 3, the horseshoe vortex lattice is
laid out. When two planforms are used to describe a wing-body-tail configuration, each of these sections is repeated for the second planform. At the beginning of the geometry computation, a data card is read which describes the number of planforms (either 1 or 2 ), the number of configurations for which values are to be computed, and the reference values for chord and area.

## Section 1. Reference Planform

The planform is described by a series of straight lines which are projected onto the $\bar{X}-\bar{Y}$ plane from the deflected planform as shown in figure 1 for a double-delta planform. The primary geometric data are the locations of the intersections of the perimeter lines, the dihedral angles, and an indication as to whether the lines are on a fixed or movable panel. The pivot location is also required for a variable-sweep planform. These data are described in group one of the input data (appendix A). For variable-sweep wings, the planform used for input should be the configuration with the movable panel in a position where the maximum number of lines required to form its perimeter are exposed.

## Section 2. Configuration Computations

The particular configuration for which aerodynamic characteristics are sought is described by group two input data which are read here. These data include the following quantities: An appropriate configuration number, the number of horseshoe vortices chordwise, the nominal number of vortices spanwise, the Mach number, the particular lift coefficient at which the total span load distribution is desired, the sweep angle of the outboard panel for variable-sweep wings, a code to indicate whether $C_{l_{p}}$ should be computed, a code to indicate whether $\mathrm{C}_{\mathrm{L}_{\mathrm{q}}}$ and $\mathrm{C}_{\mathrm{m}_{\mathrm{q}}}$ should be computed, and a code for each planform to indicate whether it is flat or whether it has twist and/or camber. The foregoing data are punched on one card for each configuration as described in appendix $A$.

The number of horseshoe vortices used in each chordwise row (SCW) can be constant across the span or it can vary. If it is constant, simply indicate the number on the configuration card and this value will be used on each planform of the group one input. If it varies, use 0 and add the required input cards to define the table of values (TBLSCW (I)) described in appendix A. However, it is usually desirable to use a constant value the first time a planform is used in the program. For all but the most simple planforms, the program adds some extra rows of horseshoe vortices. (This is described in Part I, Section 3.) As a result, the number of chordwise rows actually laid out (SSW) is usually greater than the nominal number of rows (VIC) and it takes one run through the program to determine the exact number and location of the rows.

The lift coefficient at which the total span load distribution (basic loading plus additional loading) is desired will usually be between 0 and 1 . However, if a value of 11 is
specified, an induced drag polar is computed. In this case, the program will provide values of $C_{D, i}$ for 11 values of $C_{L}$ from -0.1 to 1 , as well as values of $\Delta C_{p}$ and the total span load distribution at a $\mathrm{C}_{\mathrm{L}}$ of 1 .

If a planform has twist and/or camber, additional data cards are required with the group two input data. These data are the local angles of attack in radians at the control points when the root-chord angle of attack is $0^{\circ}$. The control point of each elemental panel is at the midspan three-quarter-chord line. Generally, it is necessary to compute the vortex-lattice arrangement for the planform without twist and camber to determine the locations at which the local angles of attack are required. The order in which these data are provided is described in detail in appendix A. If a planform has no twist and/or camber, no additional cards are required for group two input twist data because the program will assign 0 for the values of the local angles of attack. If variations in the basic wing planform are desired for additional computer cases, they may be obtained by repeating only the group two input data with appropriate changes in any of the aforementioned variables.

For a variable-sweep planform, the angle which describes the sweep should be on the leading edge of the movable panel adjacent to the fixed portion. The intersection points and slopes for the planform in the desired position are then computed. For a fixed planform, the sweep-angle specification is not required because the program will use the unaltered basic planform. The planform breakpoints are checked to see whether any consecutive pair in the spanwise direction is less than (b/2)/2000 apart. If this occurs, the points are adjusted to coincide with each other. The adjustment is necessary to avoid a poorly conditioned matrix which could result in biased results for the circulation terms. Although this adjustment is usually adequate for planforms with no dihedral, it may not be sufficient for wings having dihedral or for use of this program in computers which have fewer than 15 significant decimal digits. This problem is discussed in detail in Part I, Section 3.

When two planforms are specified, the progr'am compares the spanwise location of the breakpoints on both planforms inboard of the tip of the planform with the shorter semispan. If all the breakpoints coincide spanwise, no action is taken. However, if one planform has a breakpoint which does not occur on the other planform, an additional breakpoint is added to the other planform on its leading edge. This is done to force all trailing legs from the horseshoe vortices to occur at the same spanwise location, which keeps a trailing leg from one planform from passing close by a control point on the other planfurm and prevents unrealistic induced velocities at that control point.

The program determines the planform area and span projected to the $\overline{\mathrm{X}}-\overline{\mathrm{Y}}$ plane and uses these values to compute the average chord. Planforms which have a constant angle of dihedral from the root chord to the tip chord have an average chord which is independent
of dihedral angle. However, wings with more than one dihedral angle have an average chord which is dependent on the individual dihedral angles.

## Section 3. Horseshoe Vortex Lattice

In this section, the procedure by which the horseshoe vortex lattice is laid out is described. The planform is divided chordwise and spanwise along the surface into trapezoidally shaped elemental panels; one horseshoe vortex is assigned to represent each panel. The horseshoe vortices are similar to those described in references 11 and 16 and are sketched in figure 2 for a typical panel. The horseshoe vortex is composed of three vortex lines: a bound vortex which is swept to coincide with the elemental-panel quarter-chord sweep angle in the plane of the wing and two trailing vortices which extend chordwise parallel to the free stream to infinity behind the wing. Figure 1 shows a typical chordwise row of horseshoe vortices on an arbitrary planform. The nominal width of these horseshoe vortices is the total semispan in the plane of the wing divided by the variable VIC. (See appendix A.)

The procedure for laying out the horseshoe vortices and the elemental panels is to begin at the left tip with the first chordwise row of vortices and then proceed toward the wing root. The actual spanwise locations of the chordwise rows of horseshoe vortices are adjusted so that there is always a trailing vortex filament at points where there are intersections of lines with breakpoints of the planform. This adjustment may cause the horseshoe vortex width to be narrower or wider than the nominal width. When a horseshoe vortex has one trailing vortex filament which coincides with a breakpoint, the width of the horseshoe vortex may vary from 0.5 to 1.5 times the nominal width. When both trailing legs coincide with breakpoints, the width may vary from a maximum of 1.5 times the nominal width to a minimum width of ( $b / 2$ )/2000, as described previously in Section 2. For wings with zero dihedral angles, good results can be expected for horseshoe vortices of these widths. However, for planforms having dihedral, the span loading results may be poor when narrow (less than 0.5 times the nominal width) horseshoe vortices exist. Hence, special care must be used in describing a planform with dihedral so that these narrow horseshoe vortices will not be used. The number of chordwise rows actually laid out is given by the variable SSW.

In the chordwise direction, the horseshoe vortices are distributed uniformly and the number of vortices is given by either the variable SCW or TBLSCW (I). The maximum number of horseshoe vortices in the chordwise direction is 20 and in the spanwise direction the maximum number is 50 on a semispan. However, the total number of horseshoe vortices (either the product of SCW and SSW or the sum of TBLSCW (I)) permitted by the program is 120 on a semispan. The exact number generated by the program depends on the values of VIC and SCW and on the details of the planform. As many as one additional
chordwise row of horseshoe vortices may be generated by the program at each breakpoint outboard of the root. Wings with dihedral must always have at least two horseshoe vortices chordwise; wings without dihedral may have only one. The most desirable spanwise-to-chordwise horseshoe-vortex ratio is examined in that portion of the paper entitled "Effect of Vortex-Lattice Arrangement on Computed Aerodynamic Characteristics."

The Prandtl-Glauert correction factor is applied to the $\bar{x}$-coordinates and the tangents of the sweep angle of the horseshoe vortices at this point to account for compressibility effects.

Parametric studies can be performed on optional features selected by repeating the group two input data. These parameters include Mach number, vortex-lattice arrangement, desired lift coefficient, distribution of twist and camber, and sweep angle for a variable-sweep planform. The optional features include the computation of the rotary derivatives $\mathrm{C}_{l_{\mathrm{p}}}$ or $\mathrm{C}_{\mathrm{L}_{\mathrm{q}}}$ and $\mathrm{C}_{\mathrm{m}_{\mathrm{q}}}$. This computation is accomplished by repeating the information required by group two of the input data for each additional case. Any number of additional cases may be used for a given initial wing planform set. A few limitations for variable-sweep planforms which should be noted are (1) the pivot cannot be canted from the vertical, (2) no provisions have been made for handling dihedral in the geometry calculations for the variable-sweep panel or at the intersection of this panel with the fixed portion of the wing, and (3) there should always be a fixed-sweep panel between the root chord and the outboard variable-sweep panel.

## PART II - VORTEX-STRENGTH COMPUTATION

The vortex lattice laid out in Part I is now used in place of the real wing to generate the same flow field as the wing and to determine the forces and moments acting on the real wing. To perform these functions, the flow must be constrained so that it does not pass through the vortex lattice at specified points. These points are called control points and are at the midspan three-quarter-chord line of each elemental panel. This flow constraint is called the "no flow" condition and is equivalent to requiring that the flow be tangent to the real wing mean-camber surface. Simultaneous matching of the no flow condition at all the control points is used to compute the required vortex strengths. This can be conveniently expressed in matrix form as

$$
\begin{equation*}
\{C\}=[G]^{-1}\{B\} \tag{1}
\end{equation*}
$$

where $C_{n}, G_{n, k}$, and $B_{k}$ are the elements of these matrices.
The matrix $\{B\}$ represents the numerical values satisfying the boundary conditions whịch are presented in sketches (a) to (d) and equations (2) to (4). The traditional
representation for flat wings is shown in sketch (a) of a wing chord.


Sketch (a)

$$
\begin{equation*}
\mathrm{w} \cos \alpha-\mathrm{U} \sin \alpha=0 \tag{2}
\end{equation*}
$$

This boundary condition may be extended to represent wings with dihedral. This extension is shown in sketch (b), which is a view looking upstream toward the trailing edge of the left half of the wing span.


Sketch (b)
$\mathrm{w} \cos \alpha \cos \phi_{l}-\mathrm{v} \sin \phi_{l}-\mathrm{U} \sin \alpha \cos \phi_{l}=0$
A view looking upstream toward the trailing edge of the right half of the wing span (sketch (c)) presents a somewhat different combination of velocity vectors for the no flow condition from that just considered.


Sketch (c)

$$
\begin{equation*}
\mathrm{w} \cos \alpha \cos \phi_{r}+\mathrm{v} \sin \phi_{r}-U \sin \alpha \cos \phi_{r}=0 \tag{4}
\end{equation*}
$$

In the geometry convention for this paper

$$
\phi=\phi_{l}=-\phi_{\mathbf{r}}
$$

This relationship can be used to show that equations (2) and (3) are identical and have the form

$$
\begin{equation*}
\mathrm{w} \cos \alpha \cos \phi-\mathrm{v} \sin \phi-\mathrm{U} \sin \alpha \cos \phi=0 \tag{5}
\end{equation*}
$$

or, for small angles of attack,

$$
\begin{equation*}
\mathrm{w}-\mathrm{v} \tan \phi \approx \mathrm{U} \alpha \tag{6}
\end{equation*}
$$

In the present formulation of a vortex lattice, the angle of attack in equation (5) refers to the flow at the control point for each elemental panel. The vortex lattice is located in a plane parallel to the free stream as shown in sketch (d).


Sketch (d)
The downwash velocity for a particular horseshoe vortex can be expressed as

$$
\begin{equation*}
\mathrm{w}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\frac{\Gamma}{4 \pi} \mathrm{~F}_{\mathrm{w}}\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right) \tag{7}
\end{equation*}
$$

where the downwash influence coefficient is

$$
\begin{align*}
F_{w}\left(\mathbf{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right)= & \left.\frac{\left(\mathrm{y} \tan \psi^{\prime}-\mathrm{x}^{\prime}\right) \cos \phi}{\left(\mathrm{x}^{\prime}\right)^{2}+(\mathrm{y} \sin \phi)^{2}+\cos ^{2} \phi\left(\mathrm{y}^{2} \tan 2 \psi+\mathrm{z}^{2} \sec ^{2} \psi^{\prime}-2 \mathrm{yx}\right.} \mathrm{x}^{\prime} \tan \psi^{\prime}\right)-2 \mathrm{z} \cos \phi \sin \phi\left(\mathrm{y}+\mathrm{x}^{\prime} \tan \psi^{\prime}\right) \\
& \times\left\{\frac{\left\{\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right) \cos \phi \tan \psi^{\prime}+(\mathrm{y}+\mathrm{s} \cos \phi) \cos \phi+(\mathrm{z}+\mathrm{s} \sin \phi) \sin \phi\right.}{\left[\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right. \\
& \left.-\frac{\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right) \cos \phi \tan \psi^{\prime}+(\mathrm{y}-\mathrm{s} \cos \phi) \cos \phi+(\mathrm{z}-\mathrm{s} \sin \phi) \sin \phi}{\left[\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\} \\
& -\frac{\mathrm{y}-\mathrm{s} \cos \phi}{(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}}\left\{1-\frac{\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}}{\left[\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\} \\
& +\frac{\mathrm{y}+\mathrm{s} \cos \phi}{(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}}\left\{1-\frac{\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}}{\left[\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\}
\end{align*}
$$

and the sidewash velocity can be expressed as

$$
\begin{equation*}
\mathrm{v}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\frac{\Gamma}{4 \pi} \mathrm{~F}_{\mathrm{v}}\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right) \tag{9}
\end{equation*}
$$

where the sidewash influence coefficient is

$$
\begin{align*}
& F_{v^{\prime}}\left(x^{\prime}, y, z, \mathbf{s}, \psi^{\prime}, \phi\right)=\frac{x^{\prime} \sin \phi-z \cos \phi \tan \psi^{\prime}}{\left(x^{\prime}\right)^{2}+(y \sin \phi)^{2}+\cos ^{2} \phi\left(y^{2} \tan ^{2} \psi^{\prime}+z^{2} \sec ^{2} \psi-2 y x^{\prime} \tan \psi^{\prime}\right)-2 z \cos \phi \sin \phi\left(y+x^{\prime} \tan \psi^{\prime}\right)} \\
& \times\left\{\frac{\left(x^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right) \cos \phi \tan \psi^{\prime}+(\mathrm{y}+\mathrm{s} \cos \phi) \cos \phi+(\mathrm{z}+\mathrm{s} \sin \phi) \sin \phi}{\left[\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right. \\
& \left.-\frac{\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right) \cos \phi \tan \psi^{\prime}+(\mathrm{y}-\mathrm{s} \cos \phi) \cos \phi+(\mathrm{z}-\mathrm{s} \sin \phi) \sin \phi}{\left[\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\} \\
& +\frac{\mathrm{z}-\mathrm{s} \sin \phi}{(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}}\left\{1-\frac{\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}}{\left[\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\} \\
& -\frac{\mathrm{z}+\mathrm{s} \sin \phi}{(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}}\left\{1-\frac{\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}}{\left[\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\} \tag{10}
\end{align*}
$$

Then, by using equations (7) and (9) equation (6) can be rewritten as

$$
\begin{equation*}
\frac{\Gamma}{4 \pi}\left(F_{W}-F_{V} \tan \phi\right)=U \alpha \tag{11}
\end{equation*}
$$

For a vortex lattice of N elements, equation (11) can be expressed for a particular control point by

$$
\begin{equation*}
\sum_{\mathrm{n}=1}^{\mathrm{N}}\left(F_{\mathrm{w}, \mathrm{n}}-F_{\mathrm{v}, \mathrm{n}} \tan \phi_{\mathrm{n}}\right) \frac{\Gamma_{\mathrm{n}}}{\mathrm{U}}=4 \pi \alpha \tag{12}
\end{equation*}
$$

For symmetrical aerodynamic loading on each half of the wing, equation (12) may be expressed as

$$
\begin{equation*}
\sum_{n=1}^{N / 2}\left(\bar{F}_{w, n}-\bar{F}_{v, n} \tan \phi_{n}\right) \frac{\Gamma_{n}}{U}=4 \pi \alpha \tag{13}
\end{equation*}
$$

where
and

$$
\begin{equation*}
\overline{\mathrm{F}}_{\mathrm{v}, \mathrm{n}}=\mathrm{F}_{\mathrm{v}, \mathrm{n}}\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right)_{\text {left }}^{\text {panel }} \underset{\mathrm{v}, \mathrm{~N}+1-\mathrm{n}}{ }\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right)_{\text {right }}^{\text {panel }} \tag{15}
\end{equation*}
$$

Figure 1 shows the locations of elemental panels $n$ and ( $N+1-n$ ). The matrix which is solved by the program is then

$$
\begin{equation*}
\left[\bar{F}_{\mathrm{w}, \mathrm{n}, \mathrm{k}}-\overline{\mathrm{F}}_{\mathrm{v}, \mathrm{n}, \mathrm{k}} \tan \phi_{\mathrm{n}}\right]\left\{\frac{\Gamma_{\mathrm{n}}}{\mathrm{U}}\right\}=4 \pi\left\{\alpha_{\mathrm{k}}\right\} \tag{16}
\end{equation*}
$$

where $\alpha_{k}$ describes the local angle of attack in radians at the control point. For the first solution, $\alpha_{k}$ is that angle of attack due to twist and camber when the root-chord angle of attack is zero; for the second solution, the angle of attack $\alpha_{k}$ is 1 rad for all the control points.

As previously mentioned, this program can be used to compute the rotary stability derivatives $C_{l_{p}}, C_{m_{q}}$, and $C_{L_{q}}$. This computation is accomplished by following the method outlined in reference 17 where the values of the boundary conditions of the second solution are changed to an equivalent quasi-steady-state rolling or pitching motion. For steady-state rolling at zero angle of attack, the boundary conditions lead to a linear twist whose angle variation across the span is

$$
\begin{equation*}
\alpha_{\mathrm{k}}(2)=\frac{-\mathrm{p} \hat{y}}{\mathrm{U}} \tag{17}
\end{equation*}
$$

For this computation, if the tip angle $\mathrm{pb} / 2 \mathrm{U}$ is specified to be $5^{\circ}$, then equation (17) can be written as

$$
\begin{equation*}
\alpha_{\mathrm{k}}(2)=\frac{-\mathrm{pb}}{2 \mathrm{U}}\left(\frac{\hat{\mathrm{y}}}{\mathrm{~b} / 2}\right)=\frac{-5 \pi}{180}\left(\frac{\hat{\mathrm{y}}}{\mathrm{~b} / 2}\right) \tag{18}
\end{equation*}
$$

For pitching motion, the $\hat{Y}$-axis is the center of rotation. It is recommended that the perimeter points be specified so that the $\hat{\mathrm{Y}}$-axis coincides with either the center of gravity or the wing quarter-chord. For steady pitching motion, the boundary conditions lead to a parabolic camber as can be seen from

$$
\begin{equation*}
\alpha_{k}(2)=\frac{-q \hat{x}}{U}=\frac{-\partial \hat{Z}}{\partial \hat{\mathbf{x}}} \tag{19}
\end{equation*}
$$

Specifying that

$$
\begin{equation*}
\frac{q}{U}=\frac{5 \pi}{180} \tag{20}
\end{equation*}
$$

leads to

$$
\begin{equation*}
\alpha_{\mathrm{k}}(2)=\frac{-5 \pi \hat{\mathrm{x}}}{180} \tag{21}
\end{equation*}
$$

If any of the rotary derivatives are to be computed, the program assigns zero values for the $\alpha_{k}(1)$ terms and the appropriate boundary condition values for the $\alpha_{k}(2)$ terms.

In addition to solving for the circulation, solutions for section induced drag and leading-edge thrust are made at this point in the program by using a near-field approach. A detailed description of this implementation is given in Part III, Section 3.

## PART III - AERODYNAMIC COMPUTATION

The circulation terms $\Gamma_{n} / U$ computed in Part II are used in this part of the program to compute the lift and pitching-moment data for planforms with dihedral. A simplified procedure is used for zero-dihedral planforms. Then, the final form of the output data is obtained and printed for both planforms.

The procedure described in Section 1 is used for planforms with dihedral and for wing-tail planforms where the planforms are not at the same elevation. A special treatment is needed for both types of planforms because there are local sidewash and backwash velocities in addition to the free-stream velocity. The interaction of these velocity components with the spanwise bound vortex provides an additional lift force and the interaction of the sidewash with the chordwise bound vortex (that portion of the horseshoe vortex trailing leg ahead of the wing trailing edge) results in another and new lift force. Because of the computation procedure used in Section 1, these types of planforms must have a continuous variation in local chord from the wing tip to the wing root. As a result, streamwise perimeter edges can only be used at the wing tip or tip of the tail for these planforms.

## Section 1. Lift and Moment Using Entire Horseshoe Vortex

The lift, pitching-moment, and rolling-moment output data for planforms which have a nonzero dihedral angle over any portion of the planform or for two planforms at different elevations are computed here by using the local sidewash and backwash velocities in addition to the free-stream velocity.

The procedure described herein for computing lift and pitching-moment data is performed twice: first, for the circulation terms due to twist and camber and, second, for the circulation terms due to an angle of attack of 1 rad. The lift, pitching-moment, and spanwise center-of-pressure data are computed for all elemental panels in a particular chordwise row; the procedure is then repeated for each chordwise row until the entire left half of the wing has been taken into account. For each elemental panel, the lift developed along the left chordwise bound vortex is computed first and then the lift along the spanwise bound vortex is computed. The Kutta-Joukowski theorem for lift per unit length of a vortex filament is used to compute lift for wings with dihedral and is given by the
following equation:

$$
\begin{equation*}
\tilde{l}=\rho \mathrm{V} \Gamma \tag{22}
\end{equation*}
$$

The circulation and velocity values used in equation (22) by this computer program are described in the discussion that follows.

The lift developed along the chordwise bound vortices in a chordwise row of horseshoe vortices varies from leading edge to trailing edge of the wing because of the longitudinal variation of both the sidewash velocity and the local value of vortex strength. In figure 3 , it can be seen that there is no circulation along the chordwise bound vortex from the leading edge of the wing to the quarter-chord of the first elemental panel. As a result, no lift can be generated here. On the chordwise bound vortex from the quarterchord of the first elemental panel to the quarter-chord of the second elemental panel, there is a constant value of circulation and a varying value of sidewash velocity. A special case occurs for the first elemental panel at the left wing tip; there the value of circulation just equals that of the first elemental panel of the first chordwise row of horseshoe vortices. Inboard from the tip, this chordwise bound vortex lies between two chordwise rows of horseshoe vortices, and its circulation is equal to the difference between the circulations of the first elemental panel of each row. The sidewash velocity used is the one computed at the three-quarter-chord on the left chordwise bound vortex of the first elemental panel.

The next lift to be computed is that developed along the chordwise bound vortex between the quarter-chord of the second elemental panel and the quarter-chord of the third elemental panel. This lift is computed in a manner similar to that of the first horseshoe vortex but there are differences and these are now explained. At the left wing tip, the sum of the circulation values of the first two elemental panels is used. Inboard from the tip between two chordwise rows of horseshoe vortices, the circulation is equal to the sum of the difference between the circulations of the first elemental panel of each row and the difference between the circulations of the second elemental panel of each row. The sidewash velocity used is the one computed at the three-quarter-chord on the left chordwise bound vortex of the second elemental panel.

This procedure continues through the last elemental panel in a chordwise row. However, the final chordwise bound vortex extends from the quarter-chord of the last elemental panel to the trailing edge of the wing so that its length is equal to only threequarters of the length of the other chordwise bound vortices in the same chordwise row of horseshoe vortices. The sidewash velocity described in the foregoing procedure is given by the following equation:

$$
\begin{equation*}
\frac{\mathrm{v}}{\mathrm{U}}=\frac{1}{4 \pi} \sum_{\mathrm{n}=1}^{\mathrm{N} / 2} \frac{\Gamma_{\mathrm{n}}}{\mathrm{U}} \bar{F}_{\mathrm{v}, \mathrm{n}} \tag{23}
\end{equation*}
$$

Horseshoe vortex filaments or their extensions which go through the point at which the velocity is being computed are eliminated in the computer program from equation (23) since a line vortex filament cannot induce a velocity on itself. The lift generated along an elemental length of chordwise bound vortex divided by free-stream dynamic pressure and reference wing area is given by

$$
\begin{equation*}
\frac{\hat{l}_{\mathrm{t}}}{\mathrm{qS} \mathrm{~S}_{\mathrm{ref}}}=\frac{2}{S_{\mathrm{ref}}} \frac{\Delta \Gamma}{U} \mathrm{c}_{\mathrm{c}} \frac{\mathrm{v}}{\mathrm{U}} \tag{24}
\end{equation*}
$$

where $\Delta \Gamma$ is the local value of circulation as described in the preceding paragraph and $c_{c}$ is the chord or elemental length of the chordwise bound vortex. No lift is computed along the chordwise bound vortex at the root because the sidewash velocity is zero for symmetric loading and geometry.

The lift along the spanwise bound vortex depends on the values of free-stream, backwash, and sidewash velocities and on the circulation at the elemental panel. The sidewash velocity is given by equation (23) and the backwash velocity is computed from

$$
\begin{equation*}
\frac{\mathrm{u}}{\mathrm{U}}=\frac{1}{4 \pi} \sum_{\mathrm{n}=1}^{\mathrm{N} / 2} \frac{\Gamma_{\mathrm{n}}}{\mathrm{U}} \overline{\mathrm{~F}}_{\mathrm{u}, \mathrm{n}} \tag{25}
\end{equation*}
$$

where

$$
\begin{equation*}
\overline{\mathrm{F}}_{\mathbf{u}, \mathrm{n}}=\mathbf{F}_{\mathbf{u}, \mathrm{n}}\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right)_{\underset{\text { pant }}{\text { eft }}}+\mathrm{F}_{\mathrm{u}, \mathrm{~N}+1-\mathrm{n}}\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathrm{~s}, \psi^{\prime}, \phi\right)_{\text {right }} \tag{26}
\end{equation*}
$$

and the backwash influence coefficient is

$$
\begin{align*}
\mathbf{F}_{\mathrm{u}}\left(\mathrm{x}^{\prime}, \mathrm{y}, \mathrm{z}, \mathbf{s}, \psi^{\prime}, \phi\right)= & \left.\frac{\mathrm{z} \cos \phi-\mathrm{y} \sin \phi}{\left(\mathrm{x}^{\prime}\right)^{2}+(\mathrm{y} \sin \phi)^{2}+\cos ^{2} \phi\left(\mathrm{y}^{2} \tan ^{2} \psi+\mathrm{z}^{2} \sec ^{2} \psi-2 \mathrm{yx}\right.}{ }^{\prime} \tan \psi^{\prime}\right)-2 \mathrm{z} \cos \phi \sin \phi\left(\mathrm{y}+\mathrm{x}^{\prime} \tan \psi^{\prime}\right) \\
& \times\left\{\frac{\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right) \cos \phi \tan \psi^{\prime}+(\mathrm{y}+\mathrm{s} \cos \phi) \cos \phi+(\mathrm{z}+\mathrm{s} \sin \phi) \sin \phi}{\left[\left(\mathrm{x}^{\prime}+\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}+\mathrm{s} \cos \phi)^{2}+(\mathrm{z}+\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right. \\
& \left.-\frac{\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right) \cos \phi \tan \psi^{\prime}+(\mathrm{y}-\mathrm{s} \cos \phi) \cos \phi+(\mathrm{z}-\mathrm{s} \sin \phi) \sin \phi}{\left[\left(\mathrm{x}^{\prime}-\mathrm{s} \cos \phi \tan \psi^{\prime}\right)^{2}+(\mathrm{y}-\mathrm{s} \cos \phi)^{2}+(\mathrm{z}-\mathrm{s} \sin \phi)^{2}\right]^{1 / 2}}\right\} \tag{27}
\end{align*}
$$

Equations (8), (10), and (27) represent an extension of the original formulation by Glauert (ref. 16) for rectangular horseshoe vortices, the later formulation by Campbell (ref. 11) for a spanwise vorticity filament with sweep, and the recent formulation by Blackwell (ref. 12) for a rectangular horseshoe vortex with dihedral. In contrast, the present equations represent a subset of the formulation by Rubbert (ref. 3) in that the trailing legs are constrained to the free-stream direction.

A spanwise bound vortex filament is shown in figure 4 and the lift generated along this vortex filament comes from both the total axial velocity interacting with the component of the vortex filament parallel to the $\hat{\mathrm{Y}}$-axis ( $2 \mathrm{~s} \cos \phi$ ) and the sidewash interacting with the component of the vortex filament parallel to the $\hat{\mathbf{X}}$-axis ( $2 \mathrm{~s} \tan \psi \cos \phi$ ). The expression for this lift divided by free-stream dynamic pressure and reference area is

$$
\begin{equation*}
\frac{\hat{l}_{\mathrm{S}}}{\mathrm{q}_{\infty} \mathrm{S}_{\text {ref }}}=\frac{2}{S_{\text {ref }}} \frac{\Gamma}{U}(2 \mathrm{~S})\left[\left(1-\frac{\mathrm{u}}{\mathrm{U}}\right)+\frac{\mathrm{v}}{\mathrm{U}} \tan \psi\right] \cos \phi \tag{28}
\end{equation*}
$$

The contribution of the lift of the elemental panel to pitching moment is given by

$$
\begin{equation*}
\frac{m_{Y}}{q_{\infty} S_{r e f} c_{r e f}}=\frac{\hat{l}_{S}}{q_{\infty} S_{\text {ref }}} \frac{\hat{x}_{S}}{c_{\text {ref }}}+\frac{\hat{l}_{t}}{q_{\infty} S_{\text {ref }}} \frac{\hat{x}_{t}}{c_{\text {ref }}} \tag{29}
\end{equation*}
$$

To get the total wing lift and pitching-moment coefficients, these terms are summed over all the elemental panels which represent the wing in the following manner:

$$
\begin{align*}
& C_{L}=\frac{L}{q_{\infty} S_{r e f}}=2 \sum_{n=1}^{N / 2}\left(\frac{\hat{l}_{S}}{q_{\infty} S_{r e f}}\right)_{n}+\left(\frac{\hat{l}_{t}}{q_{\infty} S_{\text {ref }}}\right)_{n}  \tag{30}\\
& C_{m}=\frac{M_{Y}}{q_{\infty} S_{\text {ref }} c_{\text {ref }}}=2 \sum_{n=1}^{N / 2}\left(\frac{m_{Y}}{q_{\infty} S_{r e f}{ }^{c_{r e f}}}\right)_{n} \tag{31}
\end{align*}
$$

There are two values for each of these quantities; one for the surface loading due to twist and camber and the other for the surface loading at 1 rad angle of attack. From these quantities, four output terms are obtained. The lift-curve slope per radian is the value given by equation (30) (i.e., the lift coefficient at 1 rad angle of attack). The lift-curve slope per degree is

$$
\begin{equation*}
\mathrm{C}_{\mathrm{L}_{\alpha}}=\left(\frac{\mathrm{L}}{\mathrm{q}_{\infty} \mathrm{S}_{\mathrm{ref}}}\right) / \mathrm{a} / 57.29578 \tag{32}
\end{equation*}
$$

The longitudinal stability parameter about the origin of the $\mathbf{X}$-axis for the wing is given by

$$
\begin{equation*}
\frac{\partial C_{m}}{\partial C_{L}}=\frac{\left(\frac{M_{Y}}{q_{\infty} S_{r e f} C_{r e f}}\right)_{a}}{\left(\frac{L}{q_{\infty} S_{r e f}}\right)_{a}} \tag{33}
\end{equation*}
$$

The pitching moment at zero lift is

$$
\begin{equation*}
C_{m_{o}}=\left(\frac{M_{Y}}{q_{\infty} S_{r e f} c_{r e f}}\right)_{t c}-\frac{\partial C_{m}}{\partial C_{L}}\left(\frac{L}{q_{\infty} S_{r e f}}\right)_{t c} \tag{34}
\end{equation*}
$$

The center of pressure in a spanwise direction is computed from the following expression:

$$
\begin{equation*}
\mathrm{y}_{\mathrm{cp}}=\frac{\sum_{\mathrm{n}=1}^{\mathrm{N} / 2}\left[\left(\frac{\hat{l}_{\mathrm{s}}}{q_{\infty} S_{\mathrm{ref}}}\right)_{\mathrm{a}, \mathrm{n}} \hat{\mathrm{y}}_{\mathrm{s}, \mathrm{n}}+\left(\frac{\left.\left.{\hat{l_{\mathrm{t}}}}_{q_{\infty} \mathrm{S}_{\mathrm{ref}}}\right)_{\mathrm{a}, \mathrm{n}} \hat{\mathrm{y}}_{\mathrm{t}, \mathrm{n}}\right]}{\frac{1}{2}\left(\frac{\mathrm{~L}}{\mathrm{q}_{\infty} \mathrm{S}_{\mathrm{ref}}}\right)\left(\frac{b}{2}\right)}\right.\right.}{\left.\frac{\mathrm{a}}{2}\right)} \tag{35}
\end{equation*}
$$

The span-load coefficients are obtained from the lift along the spanwise and chordwise bound vortices of each horseshoe vortex. Before converting the lift expressions to span-load coefficients, a few basic definitions should be emphasized. The lift in equations (24) and (28) is lift in units of force developed over a span equal to the width of a horseshoe vortex. Therefore, lift per unit length of span is

$$
\begin{equation*}
l=\frac{\hat{l}}{2 \mathrm{~s} \cos \phi} \tag{36}
\end{equation*}
$$

The span-load coefficient for an elemental panel is developed as follows:

$$
\begin{equation*}
\frac{c_{l} c}{C_{L} c_{a v}}=\frac{\left(\frac{l}{q_{\infty} c}\right) c}{C_{L} c_{a v}}=\left(\frac{\hat{l}}{q_{\infty} s_{r e f}}\right) \frac{s_{r e f}}{C_{L} 2 s_{n} \cos \phi c_{a v}} \tag{37}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{c}_{\mathrm{av}}=\frac{\mathrm{S}_{\tau}}{\mathrm{b}} \tag{38}
\end{equation*}
$$

and

$$
\begin{equation*}
T=\frac{S_{\text {ref }}}{2 s_{\mathrm{n}} \cos \phi c_{\mathrm{av}}} \tag{39}
\end{equation*}
$$

so that

$$
\begin{equation*}
\frac{c_{l} c}{C_{L} c_{a v}}=\frac{\hat{l}}{q_{\infty} S_{r e f}} \frac{T}{C_{L}} \tag{40}
\end{equation*}
$$

At a particular spanwise location, each of these lifts are summed chordwise and converted to span-load coefficients by the following equations: For lift along the spanwise bound vortex filament,

$$
\begin{equation*}
\left(\frac{c_{l} \mathrm{c}}{\mathrm{C}_{\mathrm{L}} \mathrm{c}_{\mathrm{av}}}\right)_{\mathrm{s}}=\mathrm{T} \sum_{\mathrm{i}=1}^{\mathrm{j}}\left(\frac{\hat{l}_{\mathrm{s}}}{\overline{\mathrm{q}}_{\infty} \mathrm{S}_{\mathrm{ref}}}\right)_{\mathrm{i}} \frac{1}{\mathrm{C}_{\mathrm{L}}} \tag{41}
\end{equation*}
$$

For lift along the chordwise bound vortex filament,

$$
\begin{equation*}
\left(\frac{c_{l} c}{C_{L} c_{a v}}\right)_{t}=T \sum_{i=1}^{j}\left(\frac{\hat{l}_{t}}{a_{\infty} S_{r e f}}\right)_{i} \frac{1}{C_{L}} \tag{42}
\end{equation*}
$$

Figure 5 shows the spanwise distribution of the span-load coefficients obtained from equations (41) and (42) for a wing with dihedral. The results of these equations must now be combined to get the final distribution. It is assumed that the span-load coefficient should be zero at the wing tip, a result which cannot be obtained by direct combination of the results of equations (41) and (42). Since the vortex-lattice procedure is a finite approximation for the continuous variation of circulation across the wing span, each value of circulation represents the average value over the width of one horseshoe vortex. For this calculation, it is assumed that the circulation terms or span-load terms are correct only at the center of each row of horseshoe vortices. The lift along the spanwise bound vortices is computed here and is used directly; whereas, the lift along the chordwise bound vortices is interpolated linearly to determine its value at the midpoint of each row. These two values of lift are then combined as illustrated in figure 5 to give the final spanwise distribution of span-load coefficients.

In order to determine the damping-in-roll parameter of wings with dihedral, the lift distribution which results from the antisymmetrical span loading must be combined with the appropriate spanwise moment arm. This combination can be expressed as

$$
\begin{equation*}
C_{l}=\frac{2}{q_{\infty} S_{r e f} b}\left[\sum_{n=1}^{N / 2}\left(\hat{l}_{t} \hat{y}_{t}\right)_{n}+\sum_{n=1}^{N / 2}\left(\hat{l}_{s} \hat{y}_{s}\right)_{n}\right] \tag{43}
\end{equation*}
$$

and, thus,

$$
\begin{equation*}
\mathrm{C}_{l_{\mathrm{p}}}=\frac{\partial \mathrm{C}_{l}}{\partial\left(\frac{\mathrm{pb}}{2 \mathrm{U}}\right)} \approx \frac{\mathrm{C}_{l}}{5 \pi / 180} \tag{44}
\end{equation*}
$$

Section 2. Lift and Pitching and Rolling Moments Using Only Spanwise Filament of Horseshoe Vortex

The computation of the lift, pitching-moment, and rolling-moment output data for wings which have no dihedral over any portion of the wing is described in this section. All the lift is generated by the free-stream velocity crossing the spanwise vortex filament since there will be no sidewash or backwash velocities. For a single elemental panel, the lift per unit length of vorticity is

$$
\begin{equation*}
\tilde{l}=\rho U \Gamma \cos \psi \tag{45}
\end{equation*}
$$

Since the length of vorticity is $2 s / \cos \psi$, the resultant lift is given by

$$
\begin{equation*}
\hat{\imath}=\tilde{\imath} \frac{2 \mathrm{~s}}{\cos \psi} \tag{46}
\end{equation*}
$$

Then, the lift per unit of span is defined by

$$
\begin{equation*}
l=\frac{\hat{l}}{2 \mathrm{~s}}=\rho U \Gamma \tag{47}
\end{equation*}
$$

and is nondimensionalized in the following form for later use as

$$
\begin{equation*}
\frac{l}{q_{\infty} c_{a v}}=\frac{2}{c_{a v}} \frac{\Gamma}{U} \tag{48}
\end{equation*}
$$

For a chordwise row

$$
\begin{equation*}
\frac{\mathrm{c}_{2} \mathrm{c}}{\mathrm{c}_{\mathrm{av}}}=\sum_{\mathrm{i}=1}^{\mathrm{j}}\left(\frac{l}{\mathrm{q}_{\infty} \mathrm{c}_{\mathrm{av}}}\right)_{\mathrm{i}} \tag{49}
\end{equation*}
$$

The total lift coefficient is obtained by integrating the lift over the span as given by

$$
\begin{equation*}
\mathrm{C}_{\mathrm{L}}=\frac{\mathrm{S}_{\tau}}{\mathrm{S}_{\mathrm{ref}}} \int_{0}^{1} \frac{\mathrm{c}_{l} \mathrm{c}}{\mathrm{c}_{\mathrm{av}}} \mathrm{~d}\left(\frac{\hat{\mathrm{y}}}{\mathrm{~b} / 2}\right) \tag{50}
\end{equation*}
$$

or approximately by

$$
\begin{equation*}
C_{L}=\frac{8}{S_{\text {ref }}} \sum_{n=1}^{N / 2} \frac{\Gamma_{n}}{U} s_{n} \tag{51}
\end{equation*}
$$

The lift-curve slope per radian is obtained from a lift coefficient based on the circulation terms obtained at 1 rad angle of attack.

The longitudinal stability about $\hat{\mathbf{Y}}$-axis is given by

$$
\begin{equation*}
\frac{\partial C_{m}}{\partial C_{L}}=\frac{1}{c_{\mathrm{ref}}} \frac{\sum_{\mathrm{n}=1}^{\mathrm{N} / 2} \frac{\Gamma_{\mathrm{a}, \mathrm{n}}}{\mathrm{U}} \hat{x}_{\mathrm{S}, \mathrm{n}} \mathrm{~s}_{\mathrm{n}}}{\sum_{\mathrm{n}=1}^{\mathrm{N} / 2} \frac{\Gamma_{\mathrm{a}, \mathrm{n}}}{\mathrm{U}} \mathrm{~s}_{\mathrm{n}}} \tag{52}
\end{equation*}
$$

The pitching moment at zero lift is

$$
\begin{equation*}
C_{m_{0}}=\frac{8}{c_{r e f} S_{r e f}} \sum_{n=1}^{N / 2} \frac{\Gamma_{t c, n}}{U} \hat{x}_{S, n^{S_{n}}}-\frac{\partial C_{m}}{\theta C_{L}} C_{L, t c} \tag{53}
\end{equation*}
$$

The center of pressure in a spanwise direction is

$$
\dot{y}_{\mathrm{cp}}=\frac{1}{\mathrm{~b} / 2} \frac{\sum_{\mathrm{n}=1}^{\mathrm{N} / 2} \frac{\Gamma_{\mathrm{a}, \mathrm{n}}}{\mathrm{U}} \hat{\mathrm{y}}_{\mathrm{s}, \mathrm{n}} \mathrm{~s}_{\mathrm{n}}}{\sum_{\mathrm{n}=1}^{\mathrm{N} / 2} \frac{\Gamma_{\mathrm{a}, \mathrm{n}}}{\mathrm{U}} \mathrm{~s}_{\mathrm{n}}}
$$

The span-load coefficient is

$$
\begin{equation*}
\frac{c_{l} c}{C_{L} c_{a v}}=\frac{\frac{b}{2} \sum_{i=1}^{j} \frac{\Gamma_{i}}{U}}{2 \sum_{n=1}^{N / 2} \frac{\Gamma_{n}}{U} s_{n}} \tag{55}
\end{equation*}
$$

The same procedure used to compute the damping-in-roll parameter for wings with dihedral can be used to compute $C_{l p}$ for zero-dihedral wing planforms except that the contribution of the chordwise bound vortex is eliminated. Thus, equation (43) becomes

$$
\begin{equation*}
c_{l}=\frac{2}{q_{\infty} S_{r e f} b}\left[\sum_{n=1}^{\mathrm{N} / 2} 2\left(\frac{\Gamma}{\mathrm{U}}\right)_{\mathrm{n}} \hat{\mathrm{y}}_{\mathrm{S}, \mathrm{n}^{2} \mathrm{~s}_{\mathrm{n}}}\right] \tag{56}
\end{equation*}
$$

and likewise

$$
\begin{equation*}
\mathrm{C}_{l_{\mathrm{p}}} \approx \frac{\mathrm{C}_{l}}{5 \pi / 180} \tag{57}
\end{equation*}
$$

Section 3. Output Data Preparation
This section of the program is used to compute the last portion of the data listed in the final output. These data include the damping-in-pitch parameter, the lift coefficient due to pitch rate, the induced drag parameter, the angle of attack for zero lift, the angle of attack for the desired lift coefficient, the basic span load distribution, and the additional span load distribution.

The pitch derivatives can be computed by using the vortex strengths obtained with the boundary condition values which represent a constant pitching motion. These vortex strengths are employed to compute $\mathrm{C}_{\mathrm{L}}$ and $\mathrm{C}_{\mathrm{m}}$ which, in turn, are used as follows:

$$
\begin{equation*}
\mathrm{C}_{\mathrm{m}}=\frac{\partial \mathrm{C}_{\mathrm{m}}}{\partial\left(\frac{\mathrm{qc}}{2 \mathrm{U}}\right)} \approx \frac{\mathrm{C}_{\mathrm{m}}}{\frac{5 \pi}{180} \frac{\mathrm{c}_{\mathrm{ref}}}{2}} \tag{58}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{C}_{\mathrm{L}_{\mathrm{q}}}=\frac{\partial \mathrm{C}_{\mathrm{L}}}{\partial\left(\frac{\mathrm{qc}}{2 \mathrm{U}}\right)} \approx \frac{\mathrm{C}_{\mathrm{L}}}{\frac{5 \pi}{180} \frac{\mathrm{c}_{\mathrm{ref}}}{2}} \tag{59}
\end{equation*}
$$

In this paper, induced drag parameters are computed by both far-field and nearfield methods. The far-field method is based on the lifting-line concepts employed in the Treffetz plane by Munk and the induced drag parameter thereby obtained can be expressed mathematically as

$$
\begin{equation*}
\frac{\mathrm{C}_{\mathrm{D}, \mathrm{i}}}{\mathrm{C}_{\mathrm{L}}{ }^{2}}=\frac{\mathrm{b}^{2}}{\mathrm{C}_{\mathrm{L}}{ }^{2} \mathrm{~S}_{\mathrm{ref}}} \int_{-1}^{1} \gamma \alpha_{\mathrm{i}} \mathrm{~d} \eta \tag{60}
\end{equation*}
$$

This equation has been reformulated by Multhopp using, in part, his quadrature formula and is programed here in the form presented by equation (146) in reference 18. Equation (60) can give good results for wings without dihedral but should be used only as a guide for wings with dihedral, since no vertical displacement of the span loadings is taken into account. For wings having dihedral, a method such as that developed in reference 19 or the near-field method should be used to compute the induced drag. Even for wings without dihedral, good results can only be expected for the far-field method when a large number of chordwise rows of horseshoe vortices are specified since the interpolating procedure chosen to represent the variation of $\gamma$ with $\sin ^{-1} \eta$ was a linear curve fit between consecutive pairs of data points. This curve fit requires that a sufficient number of data points be available near the wing tip where the gradient of the $\gamma-\sin ^{-1} \eta$ curve is the greatest.

The near-field computation for the induced drag is based on combining for each elemental panel the lift and leading-edge thrust as follows:

$$
\begin{equation*}
\frac{\mathrm{d}_{i i}}{\mathrm{q}_{\infty}}=\alpha \frac{l}{\mathrm{q}_{\infty}}-\frac{\mathrm{t}}{\mathrm{q}_{\infty}} \tag{61}
\end{equation*}
$$

where the lift per unit of span $l / q_{\infty}$ is computed by equation (48) for planforms without dihedral and by equations (24) and (28) for planforms with dihedral. The leading-edge thrust per unit of span is computed by using the Kutta-Joukowski theorem where the induced and free-stream velocity components parallel to the $\overline{\mathrm{Y}}-\overline{\mathrm{Z}}$ plane interact with the spanwise bound vortex filament as follows:

$$
\begin{equation*}
\frac{t}{q_{\infty}}=-2\left(\frac{w}{U}-\frac{v}{U} \tan \phi-\alpha\right)\left(\frac{\Gamma}{U}\right)_{a, \mathrm{rad}} \tag{62}
\end{equation*}
$$

There is no contribution of the chordwise bound vortex filaments to the leading-edge thrust. In contrast, however, there is a contribution of the lift due to the chordwise bound vortex filament included in the induced drag term. (See eqs. (6) and (24).) It should be noted that this equation is evaluated at an angle of attack of 1 rad and that the circulation used is the one due to the additional loading only.

These results are then summed along each chordwise row to get the following section leading-edge thrust:

$$
\begin{equation*}
\frac{c_{t} c}{2 b}=\frac{1}{2 b} \sum_{i=1}^{j}\left(\frac{t}{q_{\infty}}\right)_{i} \tag{63}
\end{equation*}
$$

From equation (63) the section suction coefficient is computed as

$$
\begin{equation*}
\frac{c_{s} c}{2 b}=\left(\frac{c_{t} c}{2 b}\right) / \cos \Lambda \tag{64}
\end{equation*}
$$

Then, the section induced drag for a chordwise row of horseshoe vortices is

$$
\begin{equation*}
\frac{c_{\mathrm{d}, \mathrm{ii}} \mathrm{c}}{2 \mathrm{~b}}=\alpha\left(\frac{\mathrm{c}_{\imath} \mathrm{c}}{\mathrm{C}_{\mathrm{L}} \mathrm{c}_{\mathrm{av}}}\right) \frac{\mathrm{c}_{\mathrm{av}} \mathrm{~S}_{\mathrm{ref}}\left(\mathrm{C}_{\mathrm{L}_{\alpha}}\right)_{\mathrm{rad}}}{2 \mathrm{bS} \tau}-\frac{\mathrm{c}_{\mathrm{t}} \mathrm{c}}{2 \mathrm{~b}} \tag{65}
\end{equation*}
$$

Finally, the near-field solution for the induced drag parameter is

$$
\begin{equation*}
\frac{C_{D, i i}}{C_{L}^{2}}=\frac{4 b}{S_{r e f}\left(C_{L_{\alpha}}\right)_{r a d}^{2}} \sum_{k=1}^{\bar{N}_{S}}\left(\frac{c_{d, i i}}{2 b}\right)_{k} 2 s_{k} \cos \phi_{k} \tag{66}
\end{equation*}
$$

In addition, the leading-edge thrust and suction coefficients are computed similarly as

$$
\begin{equation*}
C_{T}=\frac{2}{S_{r e f}} \sum_{k=1}^{\bar{N}_{S}}\left(\frac{c_{t} c}{2 b}\right)_{k} 2 s_{k} \cos \phi_{k} \tag{67}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{C}_{\mathrm{S}}=\frac{2}{\mathrm{~S}_{\mathrm{ref}}} \sum_{\mathrm{k}=1}^{\overline{\mathrm{N}}_{\mathrm{S}}}\left(\frac{\mathrm{c}_{\mathrm{S}} \mathrm{c}}{2 \mathrm{~b}}\right)_{\mathrm{k}} 2 \mathrm{~s}_{\mathrm{k}} \cos \phi_{\mathrm{k}} \tag{68}
\end{equation*}
$$

The angle of attack for zero lift is computed by

$$
\begin{equation*}
\alpha_{0}=-\frac{\mathrm{C}_{\mathrm{L}, \mathrm{tc}}}{\mathrm{C}_{\mathrm{L}_{\alpha}}} \tag{69}
\end{equation*}
$$

The angle of attack required for the additional loading and basic loading combined to produce the input value of the desired lift coefficient is

$$
\begin{equation*}
\alpha_{d}=\frac{\mathrm{C}_{\mathrm{L}, \mathrm{~d}}}{\mathrm{C}_{\mathrm{L}_{\alpha}}}+\alpha_{0} \tag{70}
\end{equation*}
$$

The basic load due to twist and/or camber is the load on the wing when the lift coefficient is zero. This load is obtained from the values of $c_{l} c / c_{a v}$ for each elemental panel as follows:

$$
\begin{equation*}
\left(\frac{l}{q_{\infty} c_{a v}}\right)_{b}=\left(\frac{l}{q_{\infty} c_{a v}}\right)_{t c}-\left(\frac{l}{q_{\infty} c_{a v}}\right)_{a} \frac{C_{L, t c}}{C_{L, a}} \tag{71}
\end{equation*}
$$

Equation (71) is then summed for each chordwise row for the span load distribution of basic load to give

$$
\begin{equation*}
\left(\frac{c_{l} c}{c_{a v}}\right)_{B}=\sum_{i=1}^{j}\left(\frac{l}{q_{\infty} c_{a v}}\right)_{i, b} \tag{72}
\end{equation*}
$$

The span load distribution at the input value of desired lift coefficient is

$$
\begin{equation*}
\left(\frac{c_{l} c}{c_{a v}}\right)_{d}=\left(\frac{c_{l} c}{c_{a v}}\right)_{B}+\sum_{i=1}^{j}\left(\frac{l}{q_{\infty} c_{a v}}\right)_{i, a} \frac{c_{L, d}}{C_{L, a}} \tag{73}
\end{equation*}
$$

In addition, the span load distribution $c_{l} \mathbf{c} / \mathrm{C}_{\mathrm{L}, \tau} \mathrm{c}_{\mathrm{av}}$ and local lift-coefficient ratio $c_{l} / C_{L, \tau}$ are listed where the lift coefficients are based on the lift due only to additional loading and the total lift coefficient $C_{L, \tau}$ is based on the true planform area $S_{\tau}$. Also listed is the distribution of local chord ratio $c / c_{a v}$.

The incremental pressure coefficient is defined as

$$
\begin{equation*}
\Delta C_{p, n}=\frac{\left(\mathrm{p}_{\text {lower }}-\mathrm{p}_{\text {upper }}\right)_{\mathrm{n}}}{\mathrm{q}_{\infty}} \tag{74}
\end{equation*}
$$

Since the pressure is assumed to be uniform over an elemental panel,

$$
\begin{equation*}
\Delta \mathrm{C}_{\mathrm{p}, \mathrm{n}}=\frac{(l / \mathrm{c})_{\mathrm{n}}}{\mathrm{q}_{\infty}} \tag{75}
\end{equation*}
$$

which is used in the program. For planforms without dihedral, equation (75) can be expressed as

$$
\begin{equation*}
\Delta C_{p, n}=\frac{\rho U \Gamma_{n} / c_{n}}{q_{\infty}}=\frac{2}{c_{n}} \frac{\Gamma_{n}}{U} \tag{76}
\end{equation*}
$$

## EFFECT OF VORTEX-LATTICE ARRANGEMENT ON COMPUTED <br> AERODYNAMIC CHARACTERISTICS

Several sets of lifting-surface planforms have been investigated to determine the effect of the vortex-lattice arrangement on the computed aerodynamic characteristics. The first four sets of planforms had two prescribed leading-edge sweep angles in combination with three different taper ratios for aspect ratios of 2, 4.5, and 7. Calculated results for these planforms show that for different vortex-lattice arrangements, smaller variations of $y_{c p}$ and $C_{D, i} / C_{L}{ }^{2}$ are produced than of $C_{L_{\alpha}} \quad \partial C_{m} / \partial C_{L}$, and $\mathrm{C}_{\mathrm{D}, \mathrm{ii}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$. The variation of $\mathrm{y}_{\mathrm{cp}}$ with vortex-lattice arrangement is presented for unswept wings of taper ratio 1.0 in figure 6. These data indicate that increasing $\overline{\mathrm{N}}_{\mathrm{S}}$ leads toward converging results for $y_{c p}$ for all $\bar{N}_{c}$.

The variations of $\mathrm{C}_{\mathrm{L}_{\alpha}}{ }^{\partial} \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}, \quad \mathrm{C}_{\mathrm{D}, \mathrm{i}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$, and $\mathrm{C}_{\mathrm{D}, \mathrm{ii}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$ with vortexlattice arrangement are presented in figure 7 for unswept planforms with a taper ratio of 1.0 and in figures 8 to 10 for planforms with a leading-edge sweep angle of $45^{\circ}$ and taper ratios of $1.0,0.5$, and 0 , respectively. These data indicate the following conclusions. A spanwise increase in the number of chordwise rows of horseshoe vortices $\overline{\mathrm{N}}_{\mathrm{S}}$ leads to converging answers. For these simple planforms, the $\overline{\mathrm{N}}_{\mathrm{S}}$ required for convergence of $C_{L_{\alpha}}$ to a particular value is sufficient for convergence of $\partial C_{m} / \partial C_{L}$ and $C_{D, i} / C_{L}{ }^{2}$ and should be 20 or larger. Also, the computed values of $C_{L_{\alpha}}, \partial C_{m} / \partial C_{L}$, and $\mathrm{C}_{\mathrm{D}, \mathrm{ii}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$ in most instances have a definite dependence upon $\overline{\mathrm{N}}_{\mathrm{C}}$. In particular, $\overline{\mathrm{N}}_{\mathrm{C}}$ controls the asymptotic levels that these aerodynamic characteristics attain with varying $\overline{\mathrm{N}}_{\mathrm{S}}$. These asymptotic levels approach'a converged result when $\overline{\mathrm{N}}_{\mathrm{c}}$ is increased. Differences between asymptotic levels which occur for consecutive $\overline{\mathrm{N}}_{\mathrm{c}}$ values decrease with increasing $\bar{N}_{c}$ and the largest difference in asymptotic levels is obtained by increasing $\overline{\mathrm{N}}_{\mathrm{c}}$ from 1 to 2. Therefore, an $\overline{\mathrm{N}}_{\mathrm{c}}$ value of 2 should be the minimum used. Higher values of $\overline{\mathrm{N}}_{\mathrm{c}}$ have little effect on $\mathrm{C}_{\mathrm{L}_{\alpha}}$; however, increasing $\overline{\mathrm{N}}_{\mathrm{c}}$ to 4 or more can provide additional improvement in $\partial \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}$ and $\mathrm{C}_{\mathrm{D}, \mathrm{ii}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$. In contrast, the calculated results indicate that $\bar{N}_{c}$ has little effect on $C_{D, i} / C_{L}{ }^{2}$. The asymptotic levels of $C_{D, i} / C_{L}{ }^{2}$ and $C_{D, i i} / C_{L}{ }^{2}$ when $\bar{N}_{S}$ is greater than 20 can be compared with those of $1 / \pi A$. This comparison shows that $C_{D, i} / C_{L}{ }^{2}$ converges to a value greater than $1 / \pi A$, as expected, whereas $C_{D, i i} / C_{L}{ }^{2}$ converges in a less uniform manner to a value less than $1 / \pi A$.

Since $C_{D, i i} / C_{L}{ }^{2}$ is computed by using equations (65) and (66) which are based on $c_{t}$ and $c_{2}$, these results indicate that $c_{t}$ may be overpredicted. However, a comparison can be made in figure 11 between the distribution of section thrust computed for an $A=4$ delta wing by the vortex-lattice and Wagner's (ref. 14) methods. It can be seen that the resulting magnitudes predicted by the two different methods compare closely in general shape and lead to comparable overall thrust results. From additional computer studies it has been found that the $\overline{\mathrm{N}}_{\mathrm{C}}=10$ and $\overline{\mathrm{N}}_{\mathrm{S}}=12$ pattern used for the results shown in figure 11 also provides reasonable results for other delta wings. The large number of chordwise stations is necessary on such wings so that the effect of the induced camber loading can be properly taken into account. Although the correct thrust coefficient can be obtained from the far-field induced drag and lift-curve slope directly, only by finding the appropriate combination of $\overline{\mathrm{N}}_{\mathrm{C}}$ and $\overline{\mathrm{N}}_{\mathrm{S}}$ will the induced-drag results be the same for both methods. This check provides a method by which the correct distribution of section thrust can be obtained. The results presented in figures 7 to 10 show how difficult it is to make this check even for some simple planforms.

To determine the effect of vortex-lattice arrangement on $C_{l_{p}}, C_{m_{q}}$, and $C_{L_{q}}$, additional computer studies were made with a cropped double-delta planform having an inboard leading-edge sweep angle of $83^{\circ}$, an outboard leading-edge sweep angle of $62^{\circ}$, and an aspect ratio of 1.49. Results of these studies showed two trends. For estimating $\mathrm{C}_{l_{\mathrm{p}}}$, a large value of $\overline{\mathrm{N}}_{\mathrm{S}}$ is desired with at least two horseshoe vortices ( $\overline{\mathrm{N}}_{\mathrm{c}}$ ) in each row. For estimating $\mathrm{C}_{\mathrm{m}_{\mathrm{q}}}$ and $\mathrm{C}_{\mathrm{L}_{\mathrm{q}}}$, a large value of $\overline{\mathrm{N}}_{\mathrm{c}}$ (8 or more) is desirable with a nominal value of $\overline{\mathrm{N}}_{\mathrm{S}}$ of 8 or 10 .

A final set of computer studies were made with the wing-body-tail configuration illustrated in sample cases 2,3 , and 4. The aerodynamic characteristics were computed for this complex configuration by using 22 different vortex-lattice arrangements which had a total number of vortices on a semispan ranging from 17 to 120 . Results showed very little variation of $\mathrm{C}_{\mathrm{L}_{\alpha}} \mathrm{y}_{\mathrm{Cp}}$, and $\mathrm{C}_{\mathrm{D}, \mathrm{i}} / \mathrm{C}_{\mathrm{L}} 2$ with changes in the vortex lattice. However, there is a very significant variation in $\partial \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}$ (fig. 12). Two different types of vortex patterns were employed to produce these variations. The first type used uniform values of $\overline{\mathrm{N}}_{\mathrm{C}}$ at each row of horseshoe vortices on the wing-body and on the tail. These $\overline{\mathrm{N}}_{\mathrm{c}}$ values were used in combination with three values of $\overline{\mathrm{N}}_{\mathrm{S}}$. The results with uniform distribution of $\overline{\mathrm{N}}_{\mathrm{C}}$ reveal a large variation of $\partial \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}$ with increasing $\overline{\mathrm{N}}_{\mathrm{C}}$. These results can be shown, by cross-plotting, to be similar to those in figure 7 because increasing $\bar{N}_{S}$ for a given value of $\bar{N}_{c}$ has little effect on $\partial C_{m} / \partial C_{L}$ but increasing $\bar{N}_{c}$ caused noticeable changes between asymptotic levels of $\partial C_{m} / \partial C_{L}$ for all values of $\overline{\mathrm{N}}_{\mathbf{S}}$ considered, especially at the smaller values of $\overline{\mathrm{N}}_{\mathrm{c}}$. The second type of vortex pattern used uniform values of $\overline{\mathrm{N}}_{\mathrm{c}}$ on the outboard wing panel and outboard
tail panel and then used an increased density of elemental panels on the inboard portion of the planform. The increased density is illustrated in the input data for sample case 2. The purpose of these additional inboard elemental panels was to make their chords more uniform. This type of vortex pattern virtually eliminated the variation of $\partial \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}$ with $\overline{\mathrm{N}}_{\mathrm{c}}$. These computed results agree with unpublished experimental data for this configuration to within $0.01 \mathrm{x} / \mathrm{c}_{\text {ref }}$ and indicate that good results can be obtained for complex planforms with large changes in chord by arranging the pattern of elemental panels so that the largest panel chords are no more than two to three times the smallest panel chords.

## SAMPLE CASES

Sample cases have been prepared to illustrate most of the program options available. Sketches of the sample cases along with corresponding input data and output data listings are provided in appendix C. The sample cases are as follows:

| Sample case | Configuration | Description | Page |
| :---: | :---: | :---: | :---: |
| 1 | 70 | Fixed sweep wing with dihedral and twist and camber | 46 |
| 2 | 13 | Wing-body-tail combination with variable $\overline{\mathrm{N}}_{\mathrm{c}}$ | 48 |
| 3 | 113 | Wing-body-tail combination with variable $\overline{\mathrm{N}}_{\mathrm{C}}$ and tail incidence of $-10^{\circ}$ | 48 |
| 4 | 110 | Wing-body-tail combination with variable sweep of wing outer panel | 48 |
| 5 | 15 | Cropped double-delta wing with variable $\overline{\mathrm{N}}_{\mathrm{c}}$ and twist and camber to illustrate drag polar option | 50 |
| 6 | 215 | Cropped double-delta wing to illustrate $C_{l p}$ computation | 50 |
| 7 | 315 | Cropped double-delta wing to illustrate $\mathrm{C}_{\mathrm{Lq}}$ and $\quad \mathrm{C}_{\mathrm{m}_{\mathrm{q}}}$ computation | 50 |

## CONCLUDING REMARKS

A FORTRAN computer program for estimating the aerodynamic characteristics of lifting surfaces in subsonic compressible flow has been described along with the input and output variables. Also, a detailed description of the program organization and programed equations has been given. The program has been used to compute the aerodynamic
characteristics for several configurations that were selected to show the range of planforms to which the program may be applied. In addition, results from parametric studies of the effects of vortex-lattice arrangement on some of the computed aerodynamic characteristics are presented. From these results, the following recommendations are provided as guidance in determining the number of spanwise rows of horseshoe vortices and the number of horseshoe vortices chordwise in each row to use to represent a simple wing planform or to represent a more complex planform such as a wing-body-tail combination:

1. For simple planforms, (a) use at least 20 spanwise rows and four horseshoe vortices chordwise for good values of $C_{L_{\alpha}} \quad \partial C_{M} / \partial C_{L}, y_{C p}$, and $C_{D, i} / C_{L}{ }^{2}$, and (b) use a vortex-lattice arrangement which gives similar answers for $C_{D, i}$ and $C_{D, i i}$ inasmuch as a desirable vortex-lattice arrangement for good values of $C_{D, i i}, C_{T}$, and $\mathrm{C}_{\mathrm{S}}$ is difficult to determine because it is very dependent on the planform.
2. For a rolling planform, use a large number of spanwise rows and at least two horseshoe vortices chordwise.
3. For a pitching planform, use eight to 10 spanwise rows and eight or more horseshoe vortices chordwise.
4. For wing-body-tail combinations, use at least 10 to 15 spanwise rows and vary the number of horseshoe vortices chordwise so that the local panel chords differ by no more than a factor of 2 to 3 from the smallest to the largest.

Langley Research Center,
National Aeronautics and Space Administration, Hampton, Va., October 28, 1970.

## APPENDIX A

## INPUT DATA

GROUP ONE

The input data required for the reference planform is described in the order that it is called for by the computer program. All coordinates and slopes should be given for the left half of the wing planform. The axis system used is given in figure 1 . The $\bar{y}=0$ intercept coincides with the root chord and is positive pointing along the right wing. Although the $\bar{x}=0$ intercept usually coincides with the intersection of the leading edge at the root chord, it may lie anywhere along the root chord; $\overline{\mathrm{X}}$ is positive pointing into the wind. All the cards use a format of ( 8 F 10.6 ) for group one data.

Data for the first card are to be supplied in the following order:
PLAN Number of planforms for the configuration; use 1 or 2
TOTAL Number of sets of group two data specified for the configuration
CREF Reference chord of the configuration
This chord is used only to nondimensionalize the pitchingmoment terms and must be greater than zero.

SREF
Reference area of the configuration This area is used only to nondimensionalize the computed output data such as lift and pitching moment and must be greater than zero.

The data required to define each planform are then provided by a set of cards. The initial card in this set is composed of the following data:

AAN (IT)

XS (IT)

YS (IT)
RTCDHT (IT)

Number of line segments used to define left half of a wing planform (does not include plane of symmetry) A maximum of 24 line segments may be used.
$x$ location of the pivot; use 0 on a fixed wing The axis system used is given in figure 1.
y location of the pivot; use 0 on a fixed wing
Vertical distance of particular planform being read in with respect to the wing root chord height; use 0 for a wing

## APPENDIX A

The rest of this set of data requires one card for each line segment used to define the basic planform (variable AAN (IT)). All data described below are required on all except the last card of this set; the last card uses only the first two variables in the following list:

XREG (I, IT) $\quad x$ location of ith breakpoint
The first breakpoint is located at the intersection of the left wing leading edge with the root chord. They are numbered in increasing order for each intersection of lines in a counterclockwise direction.

YREG (I, IT)
DIH (I, IT)

AMCD
y location of ith breakpoint
Dihedral angle (degrees) in $\overline{\mathrm{Y}}-\overline{\mathrm{Z}}$ plane of line from breakpoint $i$ to $i+1$; positive upward
Along a streamwise line, the dihedral angle is not defined; use 0 for these lines.

The move code
This number indicates whether the line segment $i$ is on the movable panel of a variable-sweep wing. Use 1 for a line which is fixed or 2 for a line which is movable.

GROUP TWO

Three sections of data may be used for group two data. The first section must always be included; it is a single card which describes the details of the particular configuration for which the loading is desired. This card requires a format of (8F5.1, F10.4, F5.1, F10.4). The second section is required when the number of horseshoe vortices used in each chordwise row is not the same; it consists of two or more cards. The third section is used when the wing has a twist and/or camber distribution and may consist of up to 15 cards, depending on the number of horseshoe vortices. The cards in the second and third sections use a format of (8F10.4).

Section one data are to be supplied in the following order:
CONFIG An arbitrary configuration number which may include up to four digits

SCW The number of chordwise horseshoe vortices to be used to represent the wing; a maximum value of 20 may be used If set to 0 , then a table of the number of chordwise horseshoe vortices from tip to root must be provided as TBLSCW (I). This SCW $=0$ option can be used only on wings without dihedral and for coplanar wing-tail combinations.

VIC

MACH

CLDES

PTEST

QTEST

TWIST (1)

SA (1)

The nominal number of spanwise rows at which chordwise horseshoe vortices will be located
The variable VIC must not cause more than 50 spanwise rows to be used by the program to describe the wing. In addition, the product of SSW and SCW cannot exceed 120. If SCW is 0, then the sum of the values in TBLSCW (I) cannot exceed 120. The use of the variable VIC is discussed in detail in Part I, Section 3 of the Program Description.

Mach number
Use a value other than 0 only if the Prandtl-Glauert compressibility correction factor $\beta=\sqrt{1-\mathrm{M}_{\infty}^{2}}$ is to be applied. It should be less than the critical Mach number.

Desired lift coefficient
The number specified here is used to obtain the span load distribution at a particular lift coefficient. If this answer is not required, use 1 for this quantity. If a drag polar for $C_{L}$ values from -0.1 to 1 is desired, use 11 for this quantity.
$C_{l p}$ indicator
If the damping-in-roll parameter is desired, use 1 for this quantity. Except for the incremental pressure coefficients and $C_{l p}$, all other aerodynamic data will be omitted. Use 0 if $C_{l p}$ is not desired.
$\mathrm{C}_{\mathrm{Lq}}$ and $\mathrm{C}_{\mathrm{m}_{\mathrm{q}}}$ indicator If these stability derivatives are desired, use 1 for this quantity. Except for $\Delta C_{p}, C_{L_{q}}$, and $C_{m_{q}}$, all other aerodynamic data will be omitted. It should be noted that both PTEST and QTEST cannot be set equal to 1 for a particular configuration. Use 0 if $\mathrm{C}_{\mathrm{L}_{\mathrm{q}}}$ and $\mathrm{C}_{\mathrm{m}_{\mathrm{q}}}$ are not desired.

Twist code for first planform
If this planform has no twist and/or camber, use a value of 0 . When this planform has twist and/or camber, use a value of 1 for this code and provide data for section three.

Variable-sweep angle for the first planform Specify leading-edge sweep angle (degrees) for the first movable line adjacent to the fixed portion of the planform. For a fixed planform, this quantity may be omitted.

## APPENDIX A

TWIST (2) Twist code for the second planform
SA (2) Variable-sweep angle for the second planform
Section two data are required if SCW is 0 . Data for the first variable go on the first card and data for the second variable go on the second and following cards. The data to be supplied are

STA Total number of spanwise rows of horseshoe vortices per semispan
This variable sets the number of values of TBLSCW (I) to be read in.

TBLSCW (I) Number of horseshoe vortices in each row starting at the row near the tip of the first planform and proceeding to the row near the root

If a second planform has been specified, the table of chordwise rows concludes with number of horseshoe vortices in each row of the second planform. For an example, see sample case 2.

Section three data are described as follows: If the configuration has no twist and/or camber, the local angles of attack are not specified since the program will set them equal to 0 . If the configuration consists of two planforms, local angles of attack may be specified for both or only one of the two planforms. The twist code describes the input to the computer.

ALP (NV) Local angles of attack in radians
These are the values at the control point for each horseshoe vortex on the wing when the root-chord angle of attack is $0^{\circ}$. These data will usually require several cards. For the first value on the first card, use the local angle of attack for the horseshoe vortex nearest the first planform leading edge at the tip; for the second value, use the angle of attack for the horseshoe vortex immediately behind in a chordwise direction. Continue with the rest of the chordwise row of horseshoe vortices at the tip; then continue inboard at the next chordwise row in the same manner to the root until local angles of attack for all the control points have been specified.

## APPENDIX B

## OUTPUT DATA

The printed results of this computer program appear in two sections: geometry data and aerodynamic data.

## GEOMETRY DATA

The geometry data are described in the order that they are found on the printout. The first group of data describes the basic planform: It states the numbers of lines used to describe the planform, root chord height, and pivot position and then lists the breakpoints, sweep and dihedral angles, and move codes. These data are a listing of the input data except for the sweep angle which is computed from the input data.

The second group of data describes the particular planform for which the aerodynamic data are being computed. Included are the configuration number, the sweep position, a listing of the breakpoints of the wing planform ( $\bar{x}, \bar{y}$, and $\bar{z}$ ), the sweep and dihedral angles, and the move codes. These data are listed primarily for variable-sweep wings to provide a definition of the planform where the outer panel sweep is different from that of the reference planform.

The third group of data presents a detailed description of the horseshoe vortices used to represent the planform. These data are listed in nine columns with each line describing one elemental panel of the wing in the same order that the twist and/or camber angles of attack are provided. (See ALP (NV) in appendix A.) The following items of data are presented for each elemental panel:

| X C/4 | x location of quarter-chord at horseshoe vortex midspan |
| :---: | :---: |
| X 3C/4 | $x$ location of three-quarter-chord at horseshoe vortex midspan <br> This is the x location of the control point. |
| Y | y location of horseshoe vortex midspan |
| Z | z location of horseshoe vortex midspan |
| S | Semiwidth of horseshoe vortex |
| $\begin{aligned} & \text { C/4 SWEEP } \\ & \text { ANGLE } \end{aligned}$ | Sweep angle of quarter-chord |

## APPENDIX B

DIHEDRAL ANGLE

LOCAL ALPHA IN RADIANS

DELTA CP AT DESIRED CL =

Dihedral angle of elemental panel

Local angle of attack at control point (X 3C/4,Y,Z)

The fourth group of data presents the following geometric data:

| REF. CHORD | Reference chord of wing |
| :--- | :--- |
| C AVERAGE | Average chord (true planform area divided by true span) |
| TRUE AREA | True area computed from planform listed in second <br> group of geometry data |
| REF. AREA | Reference area |
| B/2 | True semispan of planform listed in second group of <br> geometry data |
| REF. AR | Reference aspect ratio computed from reference plan- <br> form area and true span |
| TRUE AR | True aspect ratio computed from true planform area <br> and true span |
| MACH NUMBER | Mach number |

## AERODYNAMIC DATA

The aerodynamic data are described in the order that they are found on the printout. Note that $\mathrm{C}_{\mathrm{L}_{\alpha}}, \mathrm{C}_{\mathrm{L}, \mathrm{Twist}}, \quad \partial \mathrm{C}_{\mathrm{m}} / \partial \mathrm{C}_{\mathrm{L}}, \mathrm{C}_{\mathrm{m}_{\mathrm{O}}}, \mathrm{C}_{\mathrm{D}, \mathrm{i}} / \mathrm{C}_{\mathrm{L}}{ }^{2}$, and $\mathrm{C}_{\mathrm{L}, \mathrm{d}}$ are based on the specified reference dimensions.

DESIRED CL

COMPUTED ALPHA

CL(WB)
Desired lift coefficient specified in input data for complete configuration
$\mathrm{C}_{\mathrm{L}, \mathrm{d}} / \mathrm{C}_{\mathrm{L}_{\alpha^{\prime}}}$ angle of attack where desired lift coefficient is developed

That portion of desired lift coefficient developed by the planform with the maximum span when two planforms are specified When one planform is specified, this is the desired lift coefficient.

## APPENDIX B

CDI AT CL(WB)
$\mathrm{CDI} /(\mathrm{CL}(\mathrm{WB}) * * 2) \quad$ Induced drag parameter computed from the two previous items

Induced drag parameter for an elliptic load distribution based on reference aspect ratio
Lift-curve slope per radian
Lift-curve slope per degree
Lift coefficient due to twist and/or camber at zero angle of attack

Angle of attack at zero lift in degrees Nonzero only when twist and/or camber is specified

Spanwise distance in fraction of semispan from root chord to center of pressure on left wing panel

Longitudinal stability parameter based on a moment center about $\hat{\mathrm{Y}}$-axis

Pitching-moment coefficient at $C_{L}=0$

At each chordwise row of horseshoe vortices the following data are presented:
$2 \mathrm{Y} / \mathrm{B} \quad$ Location of midpoint of each chordwise row of horseshoe vortices in fraction of semispan locations are listed sequentially from near left wing tip toward root
The next two columns of data describe the additional (or angle of attack) wing loading at a lift coefficient of 1 (based on the total lift achieved and the true wing area).

SL COEF
CL RATIO
C RATIO
LOAD DUE
TO TWIST
Span-load coefficient, $c_{l} c / C_{L} c_{a v}$
Ratio of local lift to total lift, $c_{l} / C_{L}$
Ratio of local chord to average chord, $c / c_{a v}$
Distribution of span-load coefficient due to twist and camber at $0^{\circ}$ angle of attack

## APPENDIX B

| ADD. LOAD AT $C L=$ | Distribution of additional span-load coefficient required to produce zero lift when combined with lift due to twist and camber This distribution is computed at $\mathrm{C}_{\mathrm{L}, \mathrm{tc}}$. |
| :---: | :---: |
| BASIC LOAD AT $C L=0$ | Basic span-load-coefficient distribution at zero lift coefficient <br> These data are the sum of the previous two columns of data. |
| SPAN LOAD AT DESIRED CL | Distribution of combination of basic span load and additional span-load coefficients at desired $\mathrm{C}_{\mathrm{L}}$ |
| SL COEF FROM CHORD BD VOR | Portion of span-load coefficient due to lift along chordwise bound vortices averaged at horseshoe vortex midspan |

In addition, at each chordwise row of horseshoe vortices, the following data are presented for induced drag, leading-edge thrust, and suction coefficient characteristics computed at an angle of attack of 1 rad from a near-field solution for the additional loading (see Part III, Section 3).

## L. E. SWEEP ANGLE

CDII C/2B

CT C/2B

CS C/2B

CDII

CT

CS

Leading-edge sweep angle in degrees
Nondimensional section induced-drag-coefficient term

Nondimensional section leading-edge thrustcoefficient term

Nondimensional section leading-edge suctioncoefficient term

Contribution to total drag coefficient from each spanwise row of horseshoe vortices, $\mathrm{c}_{\mathrm{d}, \mathrm{ii}}(2 \mathrm{~s} \cos \phi) /\left(\mathrm{q}_{\infty} \mathrm{S}_{\mathrm{ref}}\right)$

Contribution to total leading-edge thrust coefficient from each spanwise row of horseshoe vortices, $c_{t}(2 s \cos \phi) /\left(q_{\infty} s_{r e f}\right)$
Contribution to total suction coefficient from each spanwise row of horseshoe vortices, $c_{s}(2 s \cos \phi) /\left(q_{\infty} S_{r e f}\right)$

## APPENDIX B

Finally, the total coefficient values are listed.

CDII/CL**2

CT

CS

THIS CASE IS FINISHED

Induced-drag parameter computed from nearfield solution

Leading-edge thrust coefficient computed at 1 rad angle of attack
Leading-edge suction coefficient computed at 1 rad angle of attack
End of output for a particular configuration

For the case where PTEST is 1 , all the foregoing aerodynamic output data are omitted and only CLP is printed.

For the case where QTEST is 1, all the foregoing aerodynamic output data are omitted and only CMQ and CLQ are printed.

## APPENDIX C

## SAMPLE CASES

Input data, sketches, and output data for the sample cases described on page 34 are presented in the following order:

| Sample case | Configuration | Item | Page |
| :---: | :---: | :--- | :---: |
| 1 | 70 | Input data | 46 |
| 1 | 70 | Sketch | 47 |
| $2,3,4$ | $13,113,110$ | Input data | 48 |
| $2,3,4$ | $13,113,110$ | Sketch | 49 |
| $5,6,7$ | $15,215,315$ | Input data | 50 |
| $5,6,7$ | $15,215,315$ | Sketch | 51 |
| 1 | 70 | Output data | 52 |
| 2 | 13 | Output data | 59 |
| 3 | 113 | Output data | 67 |
| 4 | 110 | Output data | 74 |
| 5 | 15 | Output data | 80 |
| 6 | 215 | Output data | 86 |
| 7 | 315 | Output data | 89 |

These sample cases reflect the fact that the central processing time for a case is generally proportional to the square of the number of horseshoe vortices used to represent the left half of a planform. Some typical times for the sample cases with a Control Data 6600 computer system are as follows:

| Sample case | Number of horseshoe vortices | Time, sec |
| :---: | :---: | :---: |
| 1 | 100 | 62.6 |
| 2 | 89 | 28.7 |
| 3 | 89 | 28.7 |
| 4 | 52 | 7.4 |
| 5 | 61 | 12.1 |
| 6 | 57 | 9.0 |
| 7 | 96 | 34.8 |

## APPENDIX C

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APPENDIX C


## APPENDIX C




## APPENDIX C




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## APPENDIX C



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## APPENDIX C

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## APPENDIX C



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## APPENDIX C



APPENDIX C


## APPENDIX C

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$\stackrel{\sim}{0}$
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this case is finished
$\begin{array}{rr}.19416 & 68.14716 \\ -.07232 & 68.14716\end{array}$
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## APPENDIX C

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## APPENDIX C



## APPENDIX C




## APPENDIX C

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INOUCEO DRAG, LEADing edge thrust and suction coeffic ient characteristics
compured at one radian angie of artac frin
angle of attack from a near fielo solution
section coefficients contribut

|  | E. Sueep | section coefficients |  |  | CONTRIGUTIONS TO TATAL COEF. from each spanhise row |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2 Y / 8$ -.06667 | ANGLE | COII C/2B | Cr C/2B | CS C/28 | CD |  |  |
| -.96667 -.90000 | 24.73430 24.73430 | -.00038 | . 16725 | . 18414 | -. 00015 | CT | CS |
| -.83333 | 24.73430 | -.00024 | -. 236274 | -26014 | -. 00010 | . 094888 | -10446 |
| -. 76667 | 24.73430 | . 00192 | . 31625 |  | -00022 | . 11304 | . 12446 |
| -. 70000 | 24.73430 | . 00335 |  | -34820 | -00077 | -12700 | -13983 |
| . 6333 | 24.73430 | .00474 | . 37232 | -. 40992 | -00143 | -13879 | . 15281 |
| -. 56667 | 24.73430 | . 00407 | - 39877 | . 43905 | -00190 | -14951 | . 16462 |
| -. 50359 | 24.73430 | -. 00350 | . 42974 | -47314 | -00163 | -16014 | . 17631 |
| .44608 | 24.73430 | -.01073 | . 458849 | - 57344 | -. 00125 | -15396 | . 16951 |
| 38497 | 63.69569 | -.03036 | . 50372 | -13671 | -. 00359 | . 15343 | . 16893 |
| 31830 | 63.69569 | . 12328 | - 37675 | 1.1367 | -. 01219 | - 20228 | 45647 |
| 25163 | 63.69569 | - 20760 |  | - 85008 | . 04950 | . 15127 | 341 |
| 19575 | 63.69569 | -20760 | -31529 | . 71150 | . 08337 | . 12661 | 28572 |
|  | \%3.6956 | -46763 | . 07159 | . 16156 | . 12703 | . 01945 | 24389 |
|  | 7.48296 | . 33782 | . 21617 | 1.18431 | . 15960 | -10213 |  |
| 寿843 | 82.09284 | . 35632 | . 20428 | 1.48494 | -07014 | - 04021 | 仡 |
| 105 | 79. | . 47681 | . 08704 | -48885 | -17834 | - 04321 | .00000 |




## APPENDIX C



## APPENDIX C

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## APPENDIX C







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APPENDIX C

INDUCED DRAG，LEADING EDGE THRUST AND SUCTION COEFFICIENT CHARACTERISTICS
COMPUTED AT ONE RADIAN ANGLE OF ATTACK FROM A NEAR FIELO SOLUTION


## APPENDIX C



APPENDIX C

## APPENDIX C


SECOND PLANFORM HORSESHOE VORTEX DESCRIPTIONS

## APPENDIX C



## APPENDIX C



COMPLETE CONFIGURATION CHARACTERISTICS




## 

COMPLETE CONFIGURATION
DESIREO CL COMPUTED ALPHA


## APPENDIX C

INOUC ED DRAG. LEADING edge thrust ano suction coefficient characteristics
computed at one raolan ancle of attack from a near fielo solution

geometry data

| ROOT CHORD | HEIGHT = | 0.00,000 | REFERENCE PLANFORM HAS 14 Curves |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | variable | Eep pivot position | $x(s)=$ |
|  |  |  | break poin | for the reference | Planferm |
|  | PaINT | $x$ | $r$ | SWEEP | otredral |
|  |  | REF | REF | ANGLE | Angle |
|  | 1 | 33.32500 | 0.00000 | 82.51413 | C. 00000 |
|  | 2 | 25.90500 | -. 97500 | 90.00000 | c. 00000 |
|  | 3 | 18.10500 | -. 97500 | 73.96679 | c. 00000 |
|  | 4 | -6.44500 | -8.03000 | 68.42604 | c. 0 coso |
|  | 5 | -10.79500 | -9.75000 | 64.91246 | c. 00000 |
|  | 6 | -14.34500 | -11.41200 | 90.00000 | c. 00000 |
|  | 7 | -15.72500 | -11.41200 | 30.52577 | c. 00000 |
|  | 8 | -14.74500 | -9.75000 | 32.36329 | c. cc 000 |
|  | 9 | -13.65500 | -8.03000 | 35.58737 | C.00000 |
|  | 10 | -12.09500 | -5.85000 | 32.41231 | 0.00000 |
|  | 11 | -11.09500 | -4.27500 | 18.97041 | c. 06000 |
|  | 12 | -10.54500 | -2.67500 | 3.36646 | c. 00000 |
|  | 13 | -10.44500 | -. 97500 | 90.00000 | c. 00000 |
|  | 14 | -12.42500 | -. 97500 | 0.00000 | c. 00000 |
|  | 15 | -12.42500 | 0.00000 |  |  |

CONFIGURATION NO. 15



## APPENDIX C




## APPENDIX C



## APPENDIX C

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CONFIGURATION NO. 215
57 horseshoe vortices used on the left half of the configuration




$\begin{array}{ccc}\text { Planform } & \text { TOTAL } & \text { SPANWISE } \\ 1 & 57 & 19 \\ 3 & \text { HORSESHOE VORTICES IN EACH CHORCWISE ROH }\end{array}$

## APPENDIX C



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## APPENDIX C

CONFIGURATION NO. 315

| POINT | x | $\gamma$ | z | SWEEP ANGLE | CIrEDRAL ANGLE | MOVE CODE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 33.32500 | 0.00000 | 0.00000 | 82.51413 | 0.00000 | 1 |
| 2 | 25.90500 | -. 97500 | 0.00000 | 90.00000 | 0.00000 | 1 |
| 3 | 18.10500 | -. 97500 | 0.00000 | 73.96679 | 0.00000 | 1 |
| 4 | -6.44500 | -8.03000 | 0.00000 | 68.42604 | 0.00000 | 1 |
| 5 | -10.79500 | -9.75000 | 0.00000 | 64.91246 | 0.00000 | 1 |
| 6 | -14.34500 | -11.4120C | 0.00000 | 90.00000 | 0.00000 | 1 |
| 7 | -15.72500 | -11.4120C | 0.00000 | 30.52577 | 0.00000 | 1 |
| 8 | -14.74500 | -9.75000 | 0.00000 | 32.36329 | C. 60000 | 1 |
| 9 | -13.65500 | -8.03000 | 0.00000 | 35.58737 | 0.00000 | 1 |
| 10 | -12.09500 | -5.85000 | 0.00000 | 32.41231 | 0.00000 | 1 |
| 11 | -11.09500 | -4.27500 | 0.00000 | 18.97041 | 0.00000 | 1 |
| 12 | -10.54500 | -2.67500 | 0.00000 | 3.36646 | 0.00000 | 1 |
| 13 | - 10.44500 | -. 97500 | 0.00000 | 90.00000 | 0.00000 | 1 |
| 14 | -12.42500 | -. 97500 | 0.00000 | 0.00000 | 0.00000 | 1 |
| 15 | -12.42500 | 0.00000 | 0.00000 |  |  |  |



## APPENDIX C

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## APPENDIX C








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$\square$
this case is finished

APPENDIX C

## APPENDIX D

## FORTRAN PROGRAM LISTING

This program was written in FORTRAN IV language, version 2.3, for the Control Data series 6000 computer systems with the SCOPE 3.0 operating system and library tape. Minor modifications may be required prior to use with other computers. The program requires $65,000_{8}$ words of storage on the Control Data 6600 computer system and consists of the main program, three overlays, and four subroutines. Each program or subroutine is identified in columns 73 to 76 by a 4 -character identification. In addition, each of these parts is sequenced with a 4-digit number in columns 77 to 80 . The following table is an index to the program listing:

| Program or subroutine | Identification | Page |
| :---: | :---: | :---: |
| WINGTL | MAIN | 95 |
| INFSUB | INFS | 96 |
| GEOMTRY | GEOM | 97 |
| MATXSOL | MATX | 106 |
| AERODYN | AERO | 108 |
| CDICLS | CDIC | 116 |
| MATINV | MINV | 118 |
| FTLUP | TLUP | 120 |

## APPENDIX D



## APPENDIX D

```
    SLBRULTINE INFSUB {BOT,FUI,FVI,FWI\ INFS 10
    CCMMCN/INSUBZ3/PSII,APFII,XXX,YYY,ZZ2,SNN,TOLFNC INFS 20
    INFS 30
    FS =SIN(PSII) INFS 40
    FT =FS/FC
    FPC=CCS(APHII)
    FPS=SIN(APHII) INFS 70
    FPT=FPS/FPC INFS 80
    F = XXX+SNA*FT*FPC
    F2 =YYY+SNN#FPC INFS 100
    F3 = ZZZ+SNN*FPS INFS 110
    F4 =XXX-SNN*FT*FPC INFS 120
    F5 =YYY-SNN*FPC INFS 130
    F6 =ZZL-SNA*FPS INFS 140
    FFA= (XXX**2+(YYY*FPS)**2+FPC**2*((YYY*FT)**2*(ZZZ/FC)**2-2.* INFS 150
    1XXX*YYY*FT)-2.*ZZ2#FPC*(YYY*FPS+XXX*FT*FPS))
    FFB=(FI*F1+F2*F2*FZ*F3)**.5 INFS 170
    FFC=(F4*F4+F5*F5*F6*F6)**.5 INFS 180
    FFD=F5*F5+F6*F6 INFS 190
    FFE=F2*F2+F3*F3 INFS 200
    FFF=(F1*FPC*FT+FZ*FPC*F3*FFS)/FFB - (F4*FPC*FT*F5*FPC*F6*FPS)/ [NFS 210
    lFFC
    INFS 220
C
C
C
C
C
```



```
    the tClerance set at this pCint in the program may neec to be
    ChANGED FOR CCMPUTERS OIFER THAN THE CDC 6000 SERIES
    IF(ABS(FFA).LT.(BCT*15.E-5)**2) GO TO 262
    FLUNE=(ZZZ*FPC-YYY*FPS)*FFF/FFA
    FVONE=(XXX*FPS-ZZZ*FT*FPC)*FFF/FFA
    FWCNE={YYY*FT-XXX)*FFF/FFA*FPC
    GG TO 265
    262 FUCNE=FVONE=FWCNE=O.
    265 IF(ABS(FFD).I.T.TOLRNC) GC TC 263
    FWTWO= F6*(1.-F4/FFC)/FFC
    FWTWO=-F5*(1.-F4/FFC)/FFC
    GC TO 266
    263 FVTWO=FWTWO=0.
    266 IF(AbS(FFE).LT.TOLRNC) GC TG 2E4
    FVTHRE=-F3*(1.-F1/FFB)/FFE
    FWTHRE=F2*(1.-F1/FFB)/FFE
    GG TC 267
    FVTHRE=FWTHRE=C.
    FLI = FLCNE
    FVI = FVCNE +FVThC+FVIFRE
    FhI =FWCNE + FWTHO+FWTHRE
    RETURN
    EMD
```


## APPENDIX D



## APPENDIX D

```
        ALY (l.E+13 
        PIT 
        IF ITCTAL.GT.O.I GCTG 80 GEOM 630
    C
    C
    C
        SET PLAN EQUAL TO l. FOR A hING ALCNE COMPUTAICN - EVEN FCR A
        SET PLAN EGUAL TO 2. fOR a hing - taIl combinatica
        SET TCTAL EQUAL IC THE NUMEER CF SETS
        OF GROUP TWO DATA PRCVIOEC
        OF GROUP TWO DATA PRCVIDEC
        IF IECF,5) 1CC6,4C
        40 IPLAN =PLAN
```



```
        DEFINE THE PLANFORM PERIMETER GF THE GER OF CURVES REQUIREC TO GEOM 790
        GEOM
        SURFACE (ITI, WhOSE PERINETER PCT CHCRD HEIGHT CF IFE LIFTING GEOM B2O
    RESPECT TO THE WING RCGT CHCRD FEIGHT BEING REAC IN, WITH GEOM 830
    WRITE (6,1)
    DC 58 IT = 1.IPLAN
    READ (5,3) AAN(IT);XSIITI,YS(IT),RTCCFI(IT)
        N Nl 
        MAK = O
        IF (IPLAN.EQ.I) PRTCON = 10H
        IF IIPLAN.EC.2 ANC. IT.EQ.1, PRTCON = 1OF
        IF IIPLAN.EO.2 .AND. IT.EO.2, PRTCON = ION
    WRITE (6,2I PRTCON,N,RTCLHT\IT),XS{IT),YS(IT)
    WRITE(6.17)
    DC 59 I=1,N1
    READ (5,3) XREG(I,IT), YREG(I,ITI, CIHII,ITI, AMCD
        MCD(I,IT) = AMCD
IF II.EQ.I) GO TO }5
        IF (MAK,NE.O -OR. MCC(I-I,IT).NE.2) GO TO 49
49 IF (ABSI YREG(I-I,IT)-YREG(I,II)).LT.YTCLIGO TO EC
    AREG(I-I.IT)= (XREG(I-I,IT)-XREG(I,ITI)/IYREG(I-I,IT)-YREG(I,IT))GGEOMIOMO
    AREG(I-I,IT)= (XREG(I-I,IT)-XREG(I,ITI)/IYREG(I-I,IT)-YREG(I,IT))GGEOMIO40
    GG TO 51 RAD
50 YREG{I,IT\= YREG(I-I,IT)
    AREG(I-I,ITI = AZY 
    AREGI I-I,ITI = AZY 
51 J = I - 1 GEOM1090
    WRITE PLANFGRM PERIMETER PCINTS AND ANGLES
    WRITE (G,14) J, XREG(J,ITI,YREG(J,II),ASWP,DIH(J,ITI,MCD(J,IT)
    O[H(J,IT) = TAN(CIF(J,IT)/RAD)
    5 9 ~ C O N T I N U E ~
    KFCISIIT)= MAK
    WRITE (6,14) N1, XREG(NI,ITI,YREG(N1,IT)
    5 8 ~ C G N T I N U E ~
C
        GEOM }64
        GEOM 650
        GEOM 650
        600
        GEOM 670
        GEOM 680
        GEOM 690
        GEOM 700
    GEOM 710
    GEOM }72
    GEOM 730
    GEOM 740
    GEOM 750
    GEOM 760
    GEOM }77
    GEOM }80
    GEOM 810
    RESPECT TO THE WING RCGT CHCRO FEIGHT GEING REAC IN, WITH GEOM 830
    GEOM 850
    GEOM 860
    GEOM B70
    N =AANIIT)
    GEOM 890
    GEOM 900
    GEOM 910
    GEOM 910
```



```
    WRITE (6,2I PRTCON,N,RTCLHT\IT),XS{IT),YS(IT)
    GEOM 950
    GEOM 960
    GEOM 960
    GEOM 980
    GEOM 980
IF (MAK.NE.O .OR. MCCII-I,ITJ.NE.2, SO GO TO 49 GEOMIOOO
GEOM1010
GEOM1020
IGEOM1040
GEOM 1050
50 YREG{I,IT\= YREG(I-I,IT)
    AREG(I-I,ITI = AZY 
51 J = I - 1* GEOM1090
GFOM1100
C
C
PAHT 1 - SECTICN 2
GEOM1110
GEOM1120
GEOM113C
GEOM1140
    59
    SET RTCDHTIIT) EQUAL IO IHE ROCT CHCRD HEIGHT CF IFE LIFTING
    Nl =N+1
5
*
```




## APPENDIX D

```
    114 A(I)=ALY GEOM1830
    113 SAR(I)=RSAR(I)
        X(NI)=XREF (N1)
        Y(N1)=YREF(N1)
        GC TO 103
C
C
    111 IF (MCL(K,II).NE.2)
        GC IC 1007
        KA=K-1
        DC B1 I=1,KA
        XII)=XREF\I)
        Y(I)=YREF(I)
    81 SAR(I)=RSAR(I)
    DETERNINE LEACING ECGE IATERSECTION EETwEEN FIXED AND VARIABLE
        SWEEP WING SECTICAS
    SAR(K)=SB
    A(K) = TAN(SB)
    SAI=SB-RSAR(K)
    X(K+1)=XS+(XKEF(K+1)-XS)*CCS(SAI)+(YREF(K+1)-YS)*SIN(SAI)
    Y(K+1)=YS +(YREF(K+1)-YS) &CCS(SAI)-(XREF(K+1)-XS)*SIN(SAI)
    IF ( ABS (SB - SAR(K-1) ) .LT. (.1/RAD) )
    Y(K)=X(K+1)-X(K-1)-A(K)*Y(K+1)+A(K-1)*Y(K-1)
    Y(K)=Y(K)/(A(K-1)-A(K))
    X(K)=A(K)*X(K-1)-A(K-1)#X(K+1) +A(K-1)*A(K)#(Y(K+1)-Y(K-1))
    x(K)=x(K)/(A(K)-A(K-1))
    GC TG &5
C
    84 X(K)=XREF(K-1)
        Y(K)=YREF (K-1)
        SAR(K) = SAR(K-1)
    85 K=K+1
    SWEEP THE RREAKPGINTS CA THE VARIABLE SWEEP PANEL
        (IT ALSC KEEPS SWEEP ANGLES IN FIRST OR FOURTH GUADRANTS)
    86 K=K+1
    SAR(K-1)=SAI+RSAR (K-1)
    99 IF (SAR(K-1).LE. PIT ) GO TO 102
    SAR(K-1)=SAR(K-1)-3.1415¢27
    GC TO 99
    102 IF ( SAR(K-1) .GE.{-PIT)) GO TC 106
    SAR(K-1)=S AR(K-1)+3.1415527
    GC TO 102
    106 IF(( SAR(K-1)).LT..C) GC TC IC8
    IF (SAR(K-1) - PIT ) 90,87,87
    108 IF ( SAR{K-1) + PIT ) 89.89.90
    87 A(K-1)=AZY
    GC TO 91
    89 A(K-1)=-AZY
    GC TO 91
    90 A(K-1)=TAN(SAR(K-1))
    91 KK = MCD(K,11)
        GC TO (93.52),KK
    92 Y(K)=YS+(YREF(K)-YS)*COS(SAI)-{XREF(K)-XS)*SIN(SAI)
        X(K)=XS+(XREF(K)-XS)*COS(SAI)+(YREF(K)-YS)*SIN(SAI)
        GC TO &6
C DETERMINE THE TRAILING ELGE IATERSECTICN
C BETHEEN FIXED AND VARIAELE SWEEP WING SECTIONS
    93 IF (AES (RSAR(K)-SAR(K-1)).LT. (.1/RAD) ) GC TO G6
    Y(K)=XREF(K+1)-X(K-1)-A(K)*YREF(K+1)+A(K-1)*Y(K-1)
    Y(K)=Y(K)/(A(K-1)-A(K))
    X(K)=A(K)*X(K-1)-A(K-1)*XREF(K+1)+A(K-1)*A(K)*(YREF(K+1)-Y(K-1))
```

GEOM 1830
GEOM1840 GEOM1 850 GEOM1 860 GEOM1870 GEUM1880 GEOM1890 GEOM 1900 GEOM1910 GEOM1920 GEOM1930 GEOM1940 GEOM 1950 GFOM1960 GEOM1970 GEOM1980 GEOM1990 GEOM2000 GEOM2015 GEDM2020 GEOM2030 GEOM2040 GEOM2050 GEUM2060 GEOM2070 GFOM 2080 GEOM 2090 GEOM2100 GEOM2110 GEOM2120 GEOM2130 GEOM2140 GEOM2150 GEDM 2160 GEOM2170 GEOM2180 GEOM2190 GEOM2200 GEOM2210 GEOM2220 GEOM2230 GEOM2240 GEOM2250 GEOM2260 GEOM2270 GEOM2280 GEOM2290 GEOM2300 GEOM2310 GEOM2320 GEOM2330 GEOM2340 GEOM2350 GEOM 2360 GEOM2370 GEOM2380 GEOM 2390 GEOM2400 GEOM2410 GEOM2420 GEOM 2430

## APPENDIX D

```
GEOM2440
x(K)=x(K)/(A(K)-A(K-1))
GG TO S7
    96 X(K)=XREF (K+1)
    Y(K)=YREF(K+1)
    97K=K+1
C
    STORE REFERENCE PLANFCRM CCCRCINATES CA INBOARD FIXED IRAILING
C
    ECGE
    DC 98 I=K,N1
    X(II=XREF{I)
    Y(I)=YREF(I)
    98 SAR(I-1)=RSAR(I-1)
    103 DC 101 I=1,N
        XX(I,IT)= X(I)
        YY(I,IT)=Y(I)
        MMCD(I;IT)= MCL(I,ITI
        TTWO(I,IT)=CIH(I,IT)
    101 AS (I,IT) = A(I)
        XX(NI;IT) = X(NI)
        YY(NI,ITT) = Y(NI)
        AN(IT) = AAN(IT)
    100 CCNTINUE
C
    299 BGISV(1)= BCTSV(2)=0.
    WRITE (6.16)
    OC 180 IT=I, I PLAN
    NIT=AN(IT)+I
    DC 178 ITT=1. IPLAN
    IF (ITT.EO.IT) GC TC 178
    NITT=AN(ITT)+I
    OC 176 I=1,NITI
    JF SV=0
    DC 166 JP=1,NIT 
    166 CCNTINUE
        OG 17C JP=1,NIT
        IF {YY(JP,IT\.LT.YY(I,IIII) GC TO 168
    170 CCNTINUE
    GG TG 176
    168 JFSV = JP
        IND = NIT - {JPSV-11
        DG 172 JP=1,IND
        K\hat{C}=NIT-JP +2
        KI = NIT - JP +1
        XX(K2,IT) = XX(KI,IT)
        YY(K2,IT) = YY(K1,IT)
        MMCD(K2,ITI= NMCD(K1,IT)
        AS(K2,IT) = AS(K1,IT)
    172 TTWD(K2,IT)=TTWD(K1,IT)
        YY(JPSV,IT)= YY(I,IIT)
        AS(JPSV,IT) = AS(JPSV-I,IT)
        TTWD(JPSV,IT)= TTWC(JPSV-1,IT)
        XX(JPSV,IT) = (YY(JPSVV,IT) - YY(JPSV-1,ITI) * AS(JPSV-1,IT)
        1 + XXIJPSV-I,ITI
            MMCD(JPSV,IT)= MMCC(JPSV-I,IT)
            AN(IT) = AN(III * 1.
            NIT = NIT + 1
    176 CCNTINUE
    178 CCNTINUE
C
```


## APPENDIX D



## APPENDIX D



APPENDIX D


## APPENDIX D

```
        BETA = 1. - MACF* NACF) **.5 GEOM4880
        NYTWO = O I M, MACR* NACH) ** 
        DC 354 IT=1.IPLAN
        DC 354 IT=1,IPLAN (T-II*NSV(1) O
        GEOM4910
        NVTWO = NVTWO + MSV(IT) GEOM4920
        IF (THIST(IT) LE.O.) GO TC 350 GEOM4930
        READ (5,3) (ALP(NV),ALP(AV+1),ALP(NV+2),ALP(NV+3),ALP(NV+4),ALP(NVGEOM4940
        1 +5),ALP(NV+E),ALP(NV+7),NV=NVONE,NVTWC,8) GEOM4950
            GC TO 354 GEOM4960
    350 DC 351 NV = NVCNE NVThC GEOM4970
```



```
    354 CENTINUE
    WRITE (6,24) M
    WRITE (6,25) (IT,MSV(IT),NSSWSVIIT), IT=1,IPLAN) GEOM5010
    GEOM5000
    IF I SCW.NE.O, WRITE (E,20) SCW GEOM5020
    IF ISCW.EQ.O.) WRITE (E.22) (TBLSCW(I),I=1,NSTA) GEOM5030
GEOM5040
C APPLY PRANOTL-GLAUERT CCRRECTICN
C
    DG 360 NV = 1,M
    OG 360 NV = I,M 
    PN (NV) = PN(NV) / BETA
    360 PV (NV) = PV(NV)/ BETA GEOM5100
    RETURN
    ICODECF = 1
1006 IRITE(6.11) CONFIG GEOM5130
    WRITE(6.1I) CONFIG GEOMS140
1007 ICODEGF = 2 K.IT GRITE(6.12) GEOM5160
    WRITE(6.12) K.IT
    RETURN
1008 ICODECF = 3 GEOM5180
    WRITE (6,15) PTEST,OTEST
    RETURN
    END
    GEOM5050
    GEOM5060
    GEOM5070
    GEOMS140
    GEOM5170
    \GEOM5210
```


## APPENDIX D

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | OVERLAY(WINGTL, 2,0 ) | MATX | 10 |
|  | PROGRAM MATXSOL | MATX | 20 |
|  | DIMENSION YY(2), FU(2), FV(2), FW(2), FVN(120, 120), IPIVOT(120), | MAIX | 30 |
|  | 1 INDEX (120,2) | MAİ | 40 |
|  | COMMON/ALL/ BOT,M,BETA,PTEST,OTEST, TBLSCW(50),01120),PN(120). | MA ${ }^{\text {P }}$ X | 50 |
|  | 1 PV(120), ALP(120), S(120), PSI (120), PHI (120), $2 \mathrm{H}(50)$ | Mait | 60 |
|  | COMMON/TOTHRE/ CIR(120,2), SECTRST (50) | MATX | 70 |
|  | COMMON/INSUBZ3/APSI,APHI -XX , YYY,ZZ ,SNN,TOLC | MATX | 80 |
| C |  | mati | 90 |
| C |  | mati | 100 |
| ${ }_{C}$ |  | matx |  |
| ${ }_{C}^{C}$ | PART 2 - COMPUTE CIRCULATION TERMS | matx | 120 |
| ${ }^{\text {c }}$ |  | Matx | 130 |
| C |  | matx | 140 $\times 120$ |
| c |  | matx | 150 |
|  | FPI $=12.5663704$ | MATX | 160 |
| C |  | MATX | 170 |
| C |  | MATX | 180 |
| C | the tolerance set at this point in the program may need to be | MATX | 190 |
| C | CHANGED FOR COMPUTERS OTHER THAN THE CDC 6000 SERIES | MATX | 200 |
| ${ }^{\text {c }}$ |  | Matx | 210 |
| C |  | MATX | 220 |
|  | TOLC= (B0T*15.E-05)**2 | mati | 230 |
|  | DO 6667 NUU=1,120 | MATX | 240 |
|  | Do 6667 NUT $=1,120$ | MATX | 250 |
|  | FVN(NUU, NUT) $=0$. | MatX | 260 |
| 6667 | CONTINUE | MATX | 270 |
|  | Do 308 NV=1,M | MATX | 280 |
|  | CIR(NV,1) $=12.5663704 * A L P(N V)$ | MATX | 290 |
|  | CIR(NV.2) $=12.5663704$ | MATX | 300 |
|  | IF (PTEST.NE.0.) CIR(NV,2) $=-1.0964155$ ( O(NV) / BOT | MATY | 310 |
|  | IF (OTEST.NE.0.) CIR(NV,2) $=-1.0964155$. PV(NV) BETA | MATX | 320 |
| 308 | CONTINUE | MATX | 330 |
|  | I $72=1$ | MATX | 340 |
|  | NNV=TBLSCW(IZ2) | MATX | 350 |
|  | Do $314 \mathrm{NV}=1 . \mathrm{M}$ | MATX | 360 |
|  | $\mathrm{I} 2=1$ | MATX | 370 |
|  | NnN=TBLSCW(IZ) | MATX | 380 |
|  | Do $316 \mathrm{NN}=1, \mathrm{M}$ | Matx | 390 |
|  | ADHI = ATAN(PHI (IZ)) | matx | 400 |
|  | ADSI $=$ PSI (NN) | mati | 410 |
|  | $X X=P V(N V)-P N(N N) \quad \$ Y Y(1)=Q(N V)-Q(N N) \quad \Phi Y Y(2)=Q(N V)+Q(N N)$ | MATX | 420 |
|  | ZZ=2H(IZZ)-ZH(IZ) | MatX | 430 |
|  | SNN $=$ S(NN) | MATX | 440 |
|  | Dn $261 \mathrm{I}=1,2$ | MATX | 450 |
|  | YYY = YY(I) | MATX | 460 |
|  | CALL INFSUB (ROT, FU(I),FV(I),FW(I)) | MATX | 470 |
|  | $A P H I=-A P H I \quad \& A P S I=-A P S I$ | MATX | 480 |
| 261 | CONTINUE | matx | 490 |
|  | IF(PTEST.NE.O.) GO TO 342 | matx | 500 |
|  | FVN(NV, NN $)=F W(1)-F V(1) * P H I(1 Z)+F W(2)-F V(2) * P H I(1 Z)$ | matx | 510 |
|  | Go T0 312 | matx | 520 |
| 342 | $F V N(N V, N N)=F W(1)-F V(1) * P H I(1 Z)-F W(2)+F V(2) * P H I(1 Z)$ | MATX | 530 |
| 312 | IF (NN.LT.NNN OR. NN.EQ.M) GO TO 316 | MATX | 540 |
|  | $\mathrm{IZ}=1 \mathrm{Z}+\mathrm{l}$ | Matx | 550 |
|  | NNN=NNN+TBLSCW(IZ) | matx | + 560 |
| 316 | Continue | MATX | x $\times 50$ $\times 50$ |
|  | IF INV.LT.NNV .OR. NV.EQ.M ) GO TO 314 | MATX | $\times 580$ $\times \quad 50$ |
|  | $\mathrm{I} 7 \mathrm{Z}=1 \mathrm{ZZ}+1$ | matx | $\times 90$ $\times 59$ |
|  | NNV $=$ NNV*TBLSCW(12Z) | matx | 600 |

## APPENDIX D

```
314 CONTINUE MNV(FVN,M,CIR,Z,DETERM,IPIVOT,INDEX,120,ISCALE)
314 CONTINUE MNV(FVN,M,CIR,2,DETERM,IPIVOT,INDEX,120,ISCALE) MATX 62O
    IZZA = 1ZZ
    DO 320 NZ=1,IZZA
320 SECTRST(NZ) = 0.
IZZ=1
IZZ=1
    DO 614 NV*I&M
    I 7=1
    NNN=TBLSCW(IZ)
    VELIN = 0.
    D0 616 NN=1,M
    APHI = ATAN(PHI(IZ))
    APHI = PSI(NN) MPSI MATX 740
    XX=PN(NV)-PN(NN)
    YYY(1)=Q(NV)-Q(NN)
    YY(Z)=Q(INV)
    SNN = S(NN) MATX 790
    DO 661 I=1,2
    YYY = YY(1)
    CALL INFSUB (BOT,FU(I),FV(I),FW(I))
    APHI=-APHI
    APSI=-APSI
661 CONTINUE (FW(1)+FW(2))-(FV(1)&FV(2)) TAN(APHI) ) CCIR(NN,2)
    VELIN = I(FW(1)*FW
    IF (NN.LT.NNN OOR. NN.EQ.M) GO TO El6
    IT=1Z+1
    NNN=NNN+TBLSCW(IZ)
616 CONTINUE
    CTCP = - (VELIN - 1.) 2. # CIR(NV,2)
    SECTRST(IZZ) = SECTRST(ITZ) * CTCP
    IF (NV.LT.NNV.OR.NV.EO.M) GO TO 614
    IZZ=IZZZ+1
    NNV=NNV + TBLSCW(IZZ)
614 CONTINUE
614 CONTINN
    END
    MATX 610
MATX 630
MATX 630
MATX }64
MATX 650
MATX 660
    NNV=14LNWH1,M
    MATX 750
    MATX 670
    MATX 680
    MATX 680
    MATX }69
    MATX 700
    MATX 700
    MATX }72
    MATX 720
    M
    <m
MATX &OO
MATX 800
MATX 820
MATX 820
MATX E30
MATX 840
MATX 860
MATX 870
MATX 870
MATX A80
MATX 890
MATX 900
MATX 910
MATX 920
MATX 930
MATX 940
MATX }95
MATX 960
MATX 960
MATX 980
MATX 990
```

```
        OVFRLAY(WINGTL,3.0) AERO 10
        PROGRAN AERODYN AERO 20
```




```
        2), AC(2),CH(2,50),CCAV(?,50).CLCL(ح.501.CP(120),FW(2) AERO 50
        3.DIFCIPS(?5),YLEGSV(25),ZLEGSV(2G).CLPT(120,2),CLPR(120,2) AERO 60
        COMMON/ALL/ ROT.M.RETA,PTEST,OTEGT,TRLSCW(50).O(120),PN(120), AERO 70
        1 PV(120), ALP(120),S(1?n),DSI(120),PHI(120),ZH(50)
        AER\cap 80
        COMMON/TOTHRF/ CIR(1PO.2).SFCTPST(50) AERO 90
        COMMON/ONETHRF/TWIST(?),CREF,SREF,CAVE,CLDES,STRUE,AR,ARTRUE, AERO 100
    1 RTCOHT(2),CONFIG,NSSWSV(?).4SV(2),KBOT,PLAN,IPLAN,MACH AERO 110
    COMMON/THRECDI/SLOAD (3,50)
        COMMON/INSUB23/APSI,APHI ,XX,YYY,ZZ SNN,TOLCSO
    I FORMAT (/ 12X, SECONO PLANFORM YOROL
    *SECONO PLANFORM HORSESHOE VORTEX DESCRIPTIONS* / AFRO 150
    3 FORMAT (6F12.5) AERO 160
    4 FORMAT (1HI///58X,16HAERODYNAMIC DATA///54X, *CONFIGURATION AFRO ITO
        1NO.*F7.0 //, 
    AERO 180
    5 FORMAT (1HI.IRX*COMPLETE CONFIGURATION*3IX*WING-BODY CHARACTERISTICAERO IGO
        1S*/64X LIFT* 9X INDUCED ORAG (FAR FIELO SOLUTION)*// AKRO 200
        2 16x AR # CL COMPUTED ALPHA*I9X *CL(WB)* 7x *CDI AT CL(WR) AAERO ?10
    3 4X , 15HCDI/(CL(WB)**2)/88X 12H(1/(PI*AR) = F8.5*)*, AERO P20
    6 ~ F O R M A T ~ ( 1 I X , 2 F I 5 . 5 , 1 5 x . 3 F 1 5 . 5 ) ~
    7 FORMAT(/////4X,1IH REF. CHORD, 5X,?5HC AVERAGE TRUE AREA , PX AERO ?40
        1*REFERENCE AREA*9X*B/2* 8X,7HREF. AR,8X7HTRUE AR,4X,1IHMACH NUMFAERO }75
        2ER/I
    8 FORMAT (8F15.5)
    11 FORMAT (/// 47X *COMPLETE CONFIGURATION CHARACTERISTICS*/ AERO 270
    1 36X CL ALPHA* 8X *CL (TWIST) ALPHA AT CL=0 Y CP AERN P80
    2 CMO* / 2TX &PER RADIAN PER NEGREE* / 24X.7FI2.5 )
    12 FORMAT(//25x**ADOITIONAL LOADING*/24X*WITH CL BASED ON S(TRUEI* AERON ?lO
    1 /67\times34HLOAD DUE ADD. LOAD AT BASIC LOAD3X.27HSPAN LOAD AAERO 320
        2T SL COEF FROM/8H STATION6X5H 2Y/B9X9H SL COEF .4X8HCL RATIO,4XTAERO }33
    3HC RATIO.7X,14HTO TWIST CL=,F9.5.3X,7HAT CL=05X,26HDESIRED CL AERO 740
    4 CHORD BD VOR/) AERO 350
    13 FORMAT (/ 47X, CONTRIBUTION OF THE SECOND PLANFORM TO SPAN LOAD MAERD 360
    IISTRIBUTION*, )
    15 FORMAT ( }4X,14,F12,5,5X,3F12,5,3X,3F12,5,3X,2F12,5)
    16 FORMAT (IHI)
    18 FORMAT (/////55x, 21HTHIS CASE IS FINISHED)
    20 FORMAT (///5X*DELTA CP TERMS FROM LE TIP TO TE TIP THEN INBOARO
    AERO 410
    IENDING WITH THE TE OF ROOT CHORD *) AERO 420
    2FORMAT ( /54x*CMO AND CLO ARE COMPUTED*//) AERO 430
    22 FORMATI/38X#STATIC LONGITUDINAL AEROOYNAMIC COEFFICIENTS ARE COMPIJAFRO 440
    lTED*//)
23 FORMAT ( 159X*CLP 15 COMPUTED*//1) AERO 450
*1)
25 FORMAT (/20X*X* 11X*X* 11X *Y* 11X *Z* 12X *S* 5X *C/4 SWEEP* 4XAERO 480
    I DIHEDRAL* 2X LOCAL ALPHA# 2X #DELTA CP AT DESIRED* / AERO 490
    2 19X C/4* 9X 3C/4* 42X ANGLE* 7X,*ANGLE* 4X**IN RADIANS* 4X AERO 500
```



```
303 FORMAT(12X.9F12.5)
AERO 520
1013 FORMAT(/47X*CONTRIBUTION OF THE SECOND PLANFORM TO THE CHORD GR DFAERO 530
    IAG FORCE*/)
AERO 540
1070 FORMAT (///// 30X. "INDUCED DRAG* LEADING EDGE THRUST AND SURTIO'AFRO 550
    1 COEFFICIENT CHARACTERISTICS*/ 
    2 34X #COMPUTED AT ONE RADIAN ANGLE OF ATTACK FROM A NEAR FIELO SOI AERO 57%O
    3UTION*// / NOM
    4 58x #SCTION COEFFICIENTS* 12X CONTRIBUTIONS TO TOTAL COEF.*/ AERO 590
    5 92X &FROM EACH SPANWISE ROW*// AERO SOO
    6 38x L. E. SWEEP*/
    AERO A10
```


## APPENDIX D



## APPENDIX D



## APPENDIX D



## APPENDIX D



## APPENDIX D



## APPENDIX D



## APPENDIX D

```
5000 WRITE (6,6) CLDES,ALPD,CLWB,CDI,CDIWB
    WRITE(6.11) CLA(2),CLAPD,CLT,ALPO,YCP(2),CMCL,CMO
    WRITE (6,12) CLT
    NR = J=0
    DO 1004 NV=1,NSSW
    BCLCC=BADLAE=BASLD=0.
    NSCW = TBLSCW(NV)
    NP = NR * I
    NR = NR * NSCW
    DO 1002 I =NP,NR
    ADLAE=CLCC(I, 2)*CLT/CLNT
    BSLD=CLCC(I,1)=ADLAE
    BCLCC=BCLCC*CLCC(1,1)
    BADLAE = BADLAE +ADLAE
    BASLD=BASLD*BSLD
1002 CONTINUE
    J =J + NSCW
    YQ =Q(J) / BOT
    IF (NV.EO.(NSSWSV(1)+1)) WRITE(6.13)
1004 WRITE (6, 15) NV,YQ,SLOAD(2,NV),CLCL(2,NV), CCAV (2,NV),BCLCC,BADLAE,AERO4460
    1 BASLD,SLOAD(3,NV),SLDT(NV) AERO4480
    WRITE (6,1070) AERO4490
    CTHRUST = CSUCT = CDRAG =0. AERO4500
    NNx1 AERO4510
    00 1050 NV=1,NSSW AER04520
    SSCTRST = SECTRST (NV) / (4.*BOT) SREF * CLA(2) / (STRUE * 4. * BOT)AERO4530
    SSCDRAG = SLOAD (2,NV) CAVE SREF * CLA(2) / (STRUE * 4. BOT)AERO4540
    1 - SSCTRST AERO4550
    CSSWWA = COS ( ATAN (SSWWA(NV))) AER04560
    SSCSUCT = SSCTRST / CSSWWA AERO4570
    IF (NV.EQ.1) GO TO 1060 AERO4580
    NN = NN * TBLSCW(NV-1) AERO4590
1060 PHIPR = ATAN (PHI(NV))
    CDRAGS = SSCORAG*4.*BOT*2.*S(NN)*COS(PHIPR)/SREF AER04610
    CDRAG = CDRAG 2.0 CDRAGS A COS(PHIPR)/ SRFF AER04620
    CTHRUSS = SECTRST(NV)*2.#S(NN)*COS(PHIPR) / SRFF AERO4530
    CTHRUST = CTHRUST * 2.0 CTHRUSS AFR04640
    CSUCTS = CTHRUSS / CSSWWA INE IEADING EDGE SWEEP ANGLE IS GREATER AERO4641
C IF THE ABSOLUTE VALUE OF THE LEADING EDGE SWEEP ANGLE IS GREATER AERO4R42
C IF THE ABSOLUTE VALUE SUCTION CONTRIRUTION IS COMPUTED CSGES NO SOCTS = O. AERO4R42
    IF ( CSSWWA .LT. 0.17365) CSUCTS = 0. CSSWWA,NV AERO4G44
    IF (CSSWWA LT. O.) WRITE (6.1074) CSSWWA,NV AERO4650
    CSUCT = CSUCT - 2.0 CSUCTS AERO4660
    SWALE = ATAN(SSWWA(NV)) RAD AERO4670
    YO =Q(NN)/ BOT AERO4FRO
    IF(NV.EQ.(NSSWSV(1)+1)) WRITE (6,1013)
    1050 WRITE (6,1071) NV,YQ,SWALE,SSCDRAG,SSCTRST,SSCSUCT,CDRAGS,CTHRUSS,AEROMKYO
            1 CSUCTS
                        AER04710
            CDRAGP = CDRAG/(CLA(2)*CLA(2))
            WRITE (6.1072) CDRAGP,CTHRUST,CSUCT AERO4730
    4444 WRITE (6,18)
    AER04740
    WRITE(6.16) AER04750
    RETURN AER04760
    END
```

```
        SURROUTINE COICLS (AR,ARTRUE.TSEMSP,MTOT,NSV,CDI.CDIT) CDIC IO
        DIMENSION ETAN(51),GAMPR(51,1),ETA(41),GAMMA(41),VF(41),R(41), CDIC ZO
    IFVN(41.41)
        COMMON/ALL/ROT,M,DETA,PTFST,OTFST,TRLSCW(50),0(120),PN(1ว0),
```



```
    COMMON/THRECDI/SLOAD(3.50)
    DO 15 T=1,41
    DO 15 J=1,41
15 FVN(I.|=0
    SPAN=2. ROT
    CAVB=SPAN/AQTDUE
    PI=.3141592.65F+01
    NST=ISFMSP+1
    NN=MTOT
    DO 101 N=1. ISFMSP
    NM=NSV - N
    NSCW=TPLSCW(NM)
    NN=NN-NSCW
    ETAN(N)=ASIN(-O(NN)*2./SPAN)
    GAMPR(N,1)=SLOAD(3,NM)#CAVB/(?.#SPAN)
101 CONTINIIE
    ETAN(NST)= PT/P.
    GAMPR(NST,1)=0
    DO }7\textrm{NP}=1.4
    ANP=NP
    7ETA(NP)= (ANP-21.)#DI/42.
    DO 102 JK=21.41
    CALL FTLIIP(ETA(JK),GAMMA(JK), 1,NST,ETAN,(FAMPR)
102 CONTINUE
    DO 600 NY=22.41
    ETA(NY)=SIN(ETA(NY))
    NR=42-NY
    ETA(NR)=-ETA(NY)
600 GAMMA(NR)=FIAMMA(NY)
    D0 589 NU=21.41
    ANU=NU
    DO 14 N=1,41
    AN=N
    NNUD=IABS(N-NIJ)
    VE(N)=COS(((AN-21.)#PI)/4 ).)
    IF (NNUN.NE.0) GO TO 9
    B(N)=(42.)/(4.0*COS(((ANU-21.)*PT)/42.))
    GO TO 14
    9 IF(MOD (NNUD,?).EQ.0) GO TO I?
    B(N)=VF(N)/((42.)*(ETA(N)-ETA(NU))**?)
    GO TO 14
12 B(N)=0.0
14 CONTINIJE
    DO 589 ND=21,41
    NUST =TARS(NU-71)
    IF(NUST.FO.0) GO TO 589
    IF(MOD(NUST,?).EQ.O) GO TO 589
    NPST=IARS(NP->0)
    IF(MOD(NPST.?).EQ.O) GO TO 5RQ
    NONUD=IARS (NP-NU)
    IF (NPNUD.EQ.0) GO TO 599
    IF(MOD (NPNUD,2).EQ.0) GO TO 589
    FVN(NU,NP)=2.0*B(NP)/21.*COS((ANU-2l.)*PI/42.)
    IT=42-NU
```


## APPENDIX D

```
    ITT=42-NO
    FVN(NU.ITT)=2.0&R(ITT)/21.*COS((\DeltaNU-?1.)*PI/4?.)
    FVN(IT,ND)=FVN(NU,TTT)
    FVN(IT,ITT)=FVN(NU,ND)
5R9 CONTINIIE
    CCC=0.:
    DO 10 N=1.41
10CCC=CCC*(GA'MMA(N)*GAMMA(N))
    CCD=0.0
    DO 11 NUP=1.41
    DO 11 N!=1,41
    CCO=CCN->.0$FVN!(NUP,N)*(GAMMA (IUUP)#GAMMA(N))
11 CONTINIJE
C\capI=PI*AR/4.*(CCC+CCO)
COIT=1./(DI*AO)
RFTURN
ENO
```

CDIC -10
CDIC -20
COIC $43 n$
CDTC 440
CDIC 550
CDIC 660
COIC -70
CDIC 490
CDIC 90
CDTC 700
COIC 710
CDIC 720
COIC 730
CDIC 740
CDIC 750
CDIC $76 n$
CDIC 770
CDIC 780

APPENDIX D


```
    ISCALE=ISCALE+1 MINV 610
    IF(ABS(DETERM)-R1)1060,1020,1020 MINV 620
    1020 DETERM=DETERM/RI MINV 630
            ISCALE=ISCALE+1
            GO TO 1060
    1030 IF(ABS (DETERM)-R2)1040,1040,1060
    1040 DETERM=DETERM*R1
            I SCALE=I SCALE-1
            IF(ABS(DETERM)-R2)1050,1050,1060
    1050 DETERM=DETERM*R1
    ISCALE=1SCALE-1
    1060 IF{ABS(PIVOTI)-R1)1090,1070,1070
    1070 PIVOTI=PIVOTI/R1
        I SCALE =I SCALE+I
        IF(ABS(PIVOTI)-R1)320,1080,1080
    1080 PIVOTI=PIVOTI/RI
        ISCALE=ISCALE+1
    GO TO 320
    1090 IF(ABS(PIVOTII-R2) 2000,2000,320
    2000 PIVOTI=PIVOTI*R1
        I SCALE=ISCALE-I
        IF(ABS(PIVOTI)-R2)2010,2010,320
    2010 PIVOTI=PIVOTI*RI
            ISCALE=ISCALE-1
    320 DETERM= DETERM*PIVOTI
C
C
    330 A(ICOLUM,ICCLUM)=1.0
    340 D O 350 L=1,N
    350 A(ICOLUM,L)=A(ICOLUN,L)/PIVOT
    355 IF(M) 380, 380, 360
    360 DO 370 L=1,M
    370 B(ICOLUM,L)=E(ICOLUN,L)/PIVOT
C
    380 DO 550 LI=1,N
    390 IF(LI-ICOLUM) 400, 550,400
    400 T=A(LI,ICOLUM)
    420 A(LI,ICOLUM)=0.0
    430 D0 450 L=1,A
    450 A(LL,L)=A(LI,L)-A(ICOLUM,LI*T
    455 IF(M) 550, 550, 460
    460 00 500 L=I,M
    505 B(LL,L)=B(LI,L)-B(ICCLUM,L)*T
    550 CONTINUE
C
C
    INTERCHANGE CCLUMNS
    600 00 710 I= I,N
    6 1 0 ~ L = N + 1 - 1
    620 IF (INDEX(L,1)-INDEX(L,2I) 630, 710, 630
    6 3 0 ~ J R O N = I N D E X ( L , 1 ) ~
    640 JCOLUM=INDEX(L, 2)
    650 D0 705 K=1,N
    665 SWAP=A(K,JROW)
    670 A(K,JROW)=A(K,JCCLUM)
    70J A(K,JCOLUM)=SWAP
    705 CONTINUE
    71J CONTINUE
    740 RETURN
        END
```



## APPENDIX D

```
    200 IF (N.EQ.2) GO TO 2
    IF (I.EQ.(N-1)) GO TO 209
        IF (I.EQ.1) GC TO 201
C* PICK THIRD PCINT
    SK= VARI(I+1)-VARI(I)
    IF ((SK*(X-VARI(I-1))|.LT.ISK*(VARIII+2)-X)|) GO TO 209
    L=I
    GO TO 702
209 L=I-I
702 V(1)=VARI(L)-X
    V(2)=VARI(L+1)-X
    V(3)=VARI(L+2)-X
    YY(1)={VARD(L)*VIR)-VARD(L+1)*V(1))/(VARI(L+I)-VARI(L))
    YY(2)={VARD(L+1)*V(3)-VARD(L+2)*V(2))/(VARI (L+2)-VARI(L+1))
        Y=(YY(1)*V(3)-YY(2)*V(1))/(VARI(L+2)-VARI(LI)
    800 II(LI)=I.
    RETURN
    END
```


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Figure 1.- General layout of axis systems, elemental panels, and horseshoe vortices for a typical wing planform.


Figure 2.- Variables used to describe the geometry of an elemental panel.


Figure 3.- This detailed sketch of a chordwise row of horseshoe vortices illustrates the velocities and circulations used to compute lift and pitching moment on the elemental panels of a wing with dihedral. Note that the velocity terms and circulations which are shown with each horseshoe vortex are different. (See Part III, Section 1 for discussion.)


Figure 4.- Spanwise bound vortex filament at an arbitrary orientation in the flow.


Lift computed on trailing vortex filament Data from output listing
DDDD7 Lift from spanwise vortex filament QZ77ZD Lift from trailing vortex filament


Figure 5.- Span-load-coefficient data for a wing with dihedral illustrating linear interpolation of lift generated along trailing vortex filaments and the combination of these interpolated values with lift generated along spanwise filament of vorticity to obtain final span load distribution.


Figure 6.- Effect of vortex-lattice arrangement on $y_{c p}$ for rectangular wings at $M_{\infty}=0$.








Figure 9.- Concluded.






Figure 12.- Effect of vortex-lattice arrangement on $\partial C_{m} / \partial C_{L}$ for a wing-body-tail combination at $\mathrm{M}_{\infty}=0$.


[^0]:    *For sale by the National Technical Information Service, Springfield, Virginia 22151

