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Vortices in a Strongly Magnetized Electron–Positron–Ion Plasma

P. K. Shukla[1,*] A. A. Mamun[1,**] and L. Stenflo[2]

[1] Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany
[2] Department of Physics, Umeå University, SE-90187 Umeå, Sweden

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Abstract

A theoretical investigation has been presented for the linear and nonlinear properties of obliquely propagating coupled low-frequency electrostatic drift and ion-acoustic (ED-IA) waves in a strongly magnetized nonuniform electron–positron–ion plasma in the presence of sheared ion flow. A result from our linear analysis is that the ED-IA waves can be unstable due to the ion sheared flow. In addition, it is shown that the nonlinear equations governing the dynamics of weakly interacting ED-IA waves admit vortex solutions of two different classes viz. a vortex chain and a double vortex.

1. Introduction

An electron–positron plasma, a fully ionized gas composed of electrons and positrons having equal masses and charges with opposite polarity, is considered not only as a building block of our early universe [1], but also as an omnipresent ingredient of a number of astrophysical objects, such as active galactic nuclei [2], pulsar magnetospheres [3], solar flares [4], fireballs producing γ-ray bursts [5], etc. Electron–positron plasmas are also observed in laboratory experiments in which the positrons can be used to probe the particle transport in tokamak plasmas [6–8]. Processes of electron–positron pair production can occur during intense short laser pulse propagation in plasmas [10]. However, because of the rather long lifetime of positrons, most of the astrophysical [1,4,5] and laboratory [6–8] plasmas becomes an admixture of electrons, positrons, and ions.

Recently, the wave propagation in such a three component electron–positron–ion (e–p–i) plasma has attracted much interest [9–14]. Rizzato [9] considered weakly nonlinear circularly polarized electromagnetic waves in a cold e–p–i plasma with stationary ions. Berezhiani *et al.* [10,11] investigated the nonlinear propagation of intense electromagnetic radiation in a magnetized e–p–i plasma. Rizzato [9] and Berezhiani *et al.* [10,11] found that such a three-component plasma supports radiation driven humped electrostatic potentials, which can be used to accelerate charged particles. Berezhiani and Mahajan [12] described the formation of large amplitude electromagnetic solitary structures associated with the radiation driven compressional potentials in a cold e–p–i plasma. However, in a warm e–p–i plasma one can obtain acoustic-like waves [13] in which the inertia comes from the ion mass and the restoring force comes from the thermal pressures of the inertialess electrons and positrons. Very recently, Shukla *et al.* [14] studied the nonlinear interaction between intense electromagnetic waves and acoustic-like waves [13] that are reinforced by the ponderomotive force of electromagnetic waves in an unmagnetized e–p–i plasma.

In this paper, we consider a magnetized nonuniform e–p–i plasma with sheared ion flow, and investigate the linear and weakly nonlinear properties of obliquely propagating coupled low-frequency electrostatic drift and ion-acoustic (ED-IA) waves. We shall show here that due to the effect of sheared ion flow, the linear ED-IA waves become unstable, and that weakly nonlinear ED-IA waves can give rise to two different types of vortex structures, viz. a vortex chain and a double vortex.

2. Governing equations and instability

We consider a strongly magnetized electron–positron–ion plasma consisting of electrons, positrons, and ions. The equilibrium magnetic field is assumed to be along the z-direction, i.e., $\boldsymbol{B}_0 = \hat{\boldsymbol{z}} B_0$, where $\hat{\boldsymbol{z}}$ is the unit vector along the z-direction. We limit ourselves to the propagation of electrostatic waves satisfying the conditions $\omega \ll \omega_{ce}$, $k_z^2 \omega_{ce}/k_{ne,np} k_y$ and $\omega/k_z \ll v_{te}, v_{tp}$, where ω is the wave frequency, $\omega_{ce} = eB_0/m_e c$ is the electron gyrofrequency, m_e is the electron mass, e is the magnitude of the electron charge, c is the speed of light in vacuum, $k_{ne,np} = n_{e0,p0}^{-1} \partial n_{e0,p0}/\partial x$, $n_{e0}(n_{p0})$ is the equilibrium electron (positron) number density, $k_y(k_z)$ is the $y(z)$ component of the wave vector $\boldsymbol{k}$, and $v_{te}(v_{tp})$ is the thermal speed of the electron (positron). The almost inertialess electrons and positrons can therefore establish an equilibrium in the potential ϕ of the electrostatic waves under consideration. The pressure gradient of the electrons or positrons is therefore balanced by an electrostatic force. This leads to Boltzmann electron and positron number densities which are, respectively,

$$N_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right), \tag{1}$$

$$N_p = n_{p0} \exp\left(-\frac{e\phi}{T_p}\right), \tag{2}$$

where $T_e(T_p)$ is the electron (positron) temperature in units of the Boltzmann constant. We assume that the perturbation wave phase speed is much larger than the ion thermal speed. The ion fluid dynamics in the presence of such

*Also at the Department of Plasma Physics, Umeå University, SE-90187 Umeå, Sweden.
**Permanent address: Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh.

perturbation waves is then governed by

$$\frac{\partial N_i}{\partial t}+\nabla\cdot(N_i\boldsymbol{U_i})=0, \tag{3}$$

$$\frac{\partial \boldsymbol{U_i}}{\partial t}+\boldsymbol{U_i}\cdot\nabla\boldsymbol{U_i}=-\frac{e}{m_i}\nabla\phi+\boldsymbol{U_i}\times\hat{\boldsymbol{z}}\omega_{ci}, \tag{4}$$

$$\nabla^2\phi=4\pi e(N_e-N_p-N_i), \tag{5}$$

where N_i is the ion number density, $\boldsymbol{U_i}$ is the ion fluid velocity, $\omega_{ci}=eB_0/m_ic$ is the ion gyrofrequency, and m_i is the ion mass.

We assume that (*i*) at equilibrium the ion flow velocity $u_0\hat{\boldsymbol{z}}$ and the ion number density n_{i0} are not constant, but have gradients along the x-direction, (*ii*) the wave frequency ω is much smaller than the ion gyrofrequency ω_{ci}, i.e., $\omega \ll \omega_{ci}$, (*iii*) $e\phi/T_{e,p}\ll 1$, and (*iv*) $|\partial/\partial t|, (c/B_0)|\hat{\boldsymbol{z}}\times\nabla\phi\cdot\nabla| \gg U_z|\partial/\partial z|$, where U_z is the z-component of the ion fluid velocity $\boldsymbol{U_i}$. Using these assumptions in (1)–(5) we obtain

$$\left[D_t(1-\rho_a^2\nabla_\perp^2)+V_n^*\frac{\partial}{\partial y}\right]\phi+C_a^2\frac{m_i}{e}\frac{\partial u_z}{\partial z}=0, \tag{6}$$

$$D_tu_z=-\frac{e}{m_i}\left(\frac{\partial}{\partial z}-\alpha\frac{\partial}{\partial y}\right)\phi, \tag{7}$$

where $D_t=\partial/\partial t+(c/B_0)\hat{\boldsymbol{z}}\times\nabla\phi\cdot\nabla$, u_z is the perturbation of U_z, $\alpha=\omega_{ci}^{-1}(\partial u_0/\partial x)$, $\rho_a=C_a/\omega_{ci}$, $C_a=[n_{i0}T_e/n_{e0}m_i\times(1+\delta)]^{1/2}$, $\delta=n_{p0}T_e/n_{e0}T_p$, $V_n^*=-k_nC_a^2/\omega_{ci}$, $k_n=n_{i0}^{-1}\partial n_{i0}/\partial x$, and n_{i0} is the equilibrium ion number density. At equilibrium, we have $n_{i0}=n_{e0}-n_{p0}$. In deriving (6) we have assumed that $\omega_{pi}^2\gg\omega_{ci}^2$, where ω_{pi} is the ion plasma frequency.

We first analyze the dispersion properties of low-frequency electrostatic waves by a normal mode analysis, i.e., we neglect the nonlinear terms in (6) and (7), and assume that ϕ and u_z are proportional to $\exp(-i\omega t+ik_yy+ik_zz)$. Thus, from (6) and (7) we readily obtain

$$\mu_k\omega^2-k_yV_n^*\omega-(k_z-\alpha k_y)k_zC_a^2=0, \tag{8}$$

where $\mu_k=1+k_y^2\rho_a^2$. It is obvious that for a uniform plasma ($k_n=0$), (8) gives modified ion-acoustic waves defined by $\omega=k_zC_a(1-\alpha k_y/k_z)^{1/2}/\sqrt{\mu_k}$, and that for $\omega\gg k_zC_a$, (8) represents drift waves defined by $\omega=k_yV_n^*/\mu_k$. However, for a finite value of k_z from (8) we have

$$\omega=\frac{1}{2}\left[\omega_d\pm\sqrt{\omega_d^2+4\omega_a^2\left(1-\alpha\frac{k_y}{k_z}\right)}\,\right], \tag{9}$$

where $\omega_d=k_yV_n^*/\mu_k$ and $\omega_a=k_zC_a/\sqrt{\mu_k}$. Equation (9) with + sign represents an accelerated mode, whereas with − sign it represents a retarded mode. Our interest is in the accelerated mode which is stable for $\alpha<\alpha_c$ and unstable for $\alpha>\alpha_c$, where

$$\alpha_c=\frac{k_z}{k_y}\left(1+\frac{k_y^2V_n^{*2}}{4k_z^2C_a^2\mu_k}\right). \tag{10}$$

It is obvious that for $\alpha\gg\alpha_c$ the growth rate reaches a maximum value (γ_m) which can be approximated as $\gamma_m/\omega_a\simeq\sqrt{\alpha\cot\theta}$, where θ is the angle between the directions of the external magnetic field and the wave propagation vector. This means that the maximum growth rate (normalized to ω_a) is directly proportional to the square root of α and $\cot\theta$.

3. Vortex solutions

We now focus on the long term steady state behavior of weakly interacting EI-ED waves. We then suppose that ϕ and u_z are functions of x and $\xi=y+\mu_zz-V_0t$, where μ_z and V_0 are constants. Thus, under this condition we can rewrite (6) and (7) as

$$D_\xi\left(1-\frac{V_n^*}{V_0}-\rho_a^2\nabla_\perp^2\right)\phi-C_a^2\frac{m_i\mu_z}{eV_0}\frac{\partial u_z}{\partial\xi}=0, \tag{11}$$

$$D_\xi u_z=\frac{e(\mu_z-\alpha)}{m_iV_0}\frac{\partial\phi}{\partial\xi}, \tag{12}$$

where

$D_\xi=\partial/\partial\xi-(c/V_0B_0)[(\partial\phi/\partial x)(\partial/\partial\xi)-(\partial\phi/\partial\xi)(\partial/\partial x)]$.

It can be shown that (12) is exactly satisfied by $u_z=e(\mu_z-\alpha)\phi/m_iV_0$. Substituting the latter into (11) we obtain

$$D_\xi(\nabla_\perp^2-\mu)\phi=0, \tag{13}$$

where $\mu=[1-V_n^*/V_0-C_a^2\mu_z(\mu_z-\alpha)/V_0^2]/\rho_a^2$. To find analytical solutions of (13) we consider two special cases, namely $\mu=0$ and $\mu>0$.

3.1. *Vortex chain*

When $\mu=0$, we find that (13) is satisfied by the Ansatz

$$\nabla_\perp^2\phi=\frac{4\phi_0C_0^2}{A_0^2}\exp\left[-\frac{2}{\phi_0}\left(\phi-\frac{V_0}{c}B_0x\right)\right], \tag{14}$$

where ϕ_0, C_0, and A_0 are arbitrary constants. The solution of (14) is

$$\phi=\phi_0\ln[2\cosh(C_0x)+2A_1\cos(C_0\xi)]+\frac{V_0}{c}B_0x, \tag{15}$$

where $A_1=(1-1/A_0^2)^{1/2}$. We note that for $A_0>1$ the vortex profile (15) resembles the Kelvin–Stuart "cat's eyes" that are chains of vortices in an electron–ion plasma [15–18]. The vortex chain speed is $V_0=[V_n^*\pm\sqrt{V_n^{*2}+4C_a^2\mu_z(\mu_z-\alpha)}]/2$.

3.2. *Double vortex*

We now present a double vortex [19] solution of (13) when $\mu\neq 0$. The outer solution ($r=\sqrt{x^2+\xi^2}>R$, where R is the radius of the vortex) of a double vortex is

$$\phi=\Phi_OK_1(\kappa_1r)\cos\theta, \tag{16}$$

where Φ_O is a constant, K_1 is the modified Bessel function of order one, $\kappa_1=\sqrt{\mu}$, and $\theta=\cos^{-1}(x/r)$. Since κ_1 ought to be positive, the formation of a double vortex is ensured provided that $\mu > 0$. On the other hand, the inner region solution ($r < R$) of a double vortex is

$$\phi = \Phi_I \left[J_1(\kappa_2 r) + \frac{C_I r}{\kappa_2^2} \right] \cos\theta, \tag{17}$$

where Φ_I is a constant, J_1 is the Bessel function of order one, $C_I = V_0 B_0(\kappa_1^2 + \kappa_2^2)/c$. The constant κ_2 is determined by the transcendental equation $K_2(\kappa_1 R)/\kappa_1 K_1(\kappa_1 R) = -J_2(\kappa_2 R)/\kappa_2 J_1(\kappa_2 R)$, which comes from the matching of the electric field at the vortex interface $r = R$. The other constants are given by $\Phi_O = R C_I(\kappa_1^2 + \kappa_2^2) K_1(\kappa_1 R)$ and $\Phi_I = -\kappa_1^2 R C_I(\kappa_1^2 + \kappa_2^2) J_1(\kappa_2 R)$. The rotational speed V_0 of the double vortex must satisfy $V_0^2 - V_n^* V_0 - C_a^2 \mu_z(\mu_z - \alpha) > 0$.

4. Summary

To summarize, we have studied the linear and nonlinear properties of obliquely propagating coupled low-frequency electrostatic drift and ion-acoustic (ED-IA) waves in a strongly magnetized nonuniform e–p–i plasma in the presence of sheared ion flow. We first carried out a normal mode analysis and showed from our general dispersion [cf. (8)] that (*i*) in a plasma with uniform ion density ($k_n = 0$) we have the modified ion-acoustic waves $\omega = k_z C_a(1 - \alpha k_y/k_z)^{1/2}/\sqrt{\mu_k}$, (*ii*) for $\omega \gg k_z C_a$ we have the drift waves $\omega = k_y V_n^*/\mu_k$, (*iii*) for a finite value of k_z we have accelerated and retarded modes [cf. (9)], and *iv*) the accelerated mode is unstable for $\alpha > \alpha_c$. We have analyzed the long-term behavior of the weakly interacting ED-IA waves and have shown that the nonlinear equations governing the dynamics of these waves admit vortex solutions of two different classes, viz. a vortex chain and a double vortex. The latter can be associated with coherent nonlinear structures in a magnetized electron–positron–ion plasma.

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