Vote Buying: General Elections

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We examine the consequences of vote buying, assuming this practice were allowed and free of stigma. Two parties compete in a binary election and may purchase votes in a sequential bidding game via up-front binding payments and/or campaign promises (platforms) that are contingent on the outcome of the election. We analyze the role of the parties' and voters' preferences in determining the winner and the payments to voters.

I. Introduction

The practice of vote buying appears in many societies and organizations, and in different forms. Obvious examples include direct payments to

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voters, donations to a legislator's campaign by special-interest groups, the buying of the voting shares of a stock, and the promise of specific programs or payments to voters conditional on the election of a candidate. Our purpose here is to explore the consequences of vote buying. The aim is both to enhance the understanding of those forms of vote buying that are widely practiced, such as making campaign promises, and to shed light on the hypothetical question of what might happen if vote buying were allowed where it is currently prohibited. The latter question can of course help us think about the rationale behind current social conventions. To do so we study how vote buying would function in an environment in which it is allowed and free of stigma.

We inquire how voters' preferences over outcomes and parties' valuations of winning affect the outcome of the election, how the institutional environment—whether parties can purchase votes with up-front payments or can only make campaign promises—affects the outcome, and how vote buying affects the efficiency of the outcome.

We address these questions using the following model. We initially focus on a complete-information environment but later allow for some incomplete information. There is a finite population of voters choosing between two competing parties. Each of the parties has a value for winning and is interested in obtaining a majority of the votes while spending as little as possible. We examine two scenarios: one in which the parties compete only in campaign promises (that are contingent on the outcome of the election but not on the actual vote) and the other in which parties compete in up-front vote buying (where the payment is contingent on the vote but not on the outcome). In both scenarios the parties make offers in a sequential and alternating bidding process.

The answers to the first two questions raised above are intertwined. The identity of the winning party and the distribution of payments to voters depend not just on voter preferences and party valuations, but also critically on whether up-front vote buying is permitted or only campaign promises are allowed. When parties compete only through campaign promises, the total payments received by voters tend to be substantially higher than under up-front vote buying. Moreover, when parties compete only through campaign promises, the voters whose preferences matter are a specific subset of the voters near the median voter.

Both these features are broadly consistent with the analysis of Anderson and Tollison (1990), who claim that vote buying was widespread (though never fully legal) in Britain and the United States prior to the introduction of secret ballots toward the end of the nineteenth and beginning of the twentieth centuries.[1] They claim that when vote buying

[1] We should emphasize that vote buying is not solely something of the past, but continues today. See, e.g., Callahan and McCargo (1996) for a study of such activity in elections in Thailand in 1995.

occurred, the sums involved were quite small. Moreover, they argue that the elimination of vote buying contributed to the historical rise in government expenditures on social policies. The low payments with up-front vote buying also seem consistent with the observation that the price of stocks with voting rights is generally similar to that of nonvoting stocks (Lamont and Thaler 2003).[2]

The answer to the efficiency question is that with no vote buying, with campaign promises, or with up-front vote buying, the outcome could be Pareto efficient or inefficient. This independence of efficiency from the trading environment follows since in all three situations voters' preferences are not fully accounted for in determining the winner of the election.

There are several lines of related literature: the study of Colonel Blotto games (e.g., Laslier and Picard 2002); the political science literature on lobbying (e.g., Groseclose and Snyder 1996), common agency (Bernheim and Whinston 1986), campaign promises (e.g., Lindbeck and Weibull 1987; Myerson 1993), and vote buying (e.g., Buchanan and Tullock 1962; Anderson and Tollison 1990; Piketty 1994); and the finance literature on corporate control and takeover battles (e.g., Grossman and Hart 1988; Harris and Raviv 1988). We also have a companion paper (Dekel, Jackson, and Wolinsky 2006*b*) with a related but distinct model. Discussing how our conclusions relate to those in the literature will be easier after the presentation of our model and results, so we defer this discussion to Section V.*F*.

II. A Model of Vote Buying

Two "parties," X and Y, compete in an election with an odd number, N, of voters. We may think of the parties as candidates in the election or supporters of two alternatives. A party needs $m = (N+1)/2$ votes to win the election. Prior to the election the parties try to influence the voting by offering money payments to voters. Each voter i is characterized by parameters U_i^X and U_i^Y that are interpreted as the utility she obtains from a victory of X and Y, respectively. Let $U_i = U_i^X - U_i^Y$, and label voters so that U_i is nonincreasing in i. Under this labeling, we refer to voter m as the median voter and, without loss of generality, suppose that voter m is a supporter of party X ($U_m > 0$). There is a smallest money unit $\varepsilon > 0$, so offers can be made only in multiples of ε. To avoid dealing with ties, it is assumed that U_i is not an integer multiple of ε.

[2] The reason is that one can "buy" votes without buying shares in the firm by buying shares with voting rights and selling (short) shares without voting rights. Any difference in returns between the shares must arise from the ability to vote. Of course, in the case of the firm, one is purchasing the right to all future votes (not just one vote), so the small difference indicates an even smaller price per vote.

The vote-buying games.—We consider two types of offers that parties can make to voters:

1. up-front payments: a binding agreement that gives the party full control of the vote in exchange for an up-front payment to the voter;
2. campaign promises: a promise that has to be honored by the party if it is elected; the voter maintains control of the vote.[3]

The parties alternate in making offers. In the up-front buying game, party k announces in its turn an offer to buy up to m votes at price $p^k \geq 0$; in the campaign promises game, party k announces campaign promises $c_i^k \geq 0$ to be given to voter i if k is elected.

These games share the following common features:

- A fresh price offer (or a promise) made to a voter cannot be lower than those previously made by the same party to the same voter.
- When a party moves, it observes all past offers and promises by each party to each voter.
- The bidding process ends when two consecutive offers (one by X and one by Y) go by without any change in who would win if the game ended in those rounds.
- Once the bidding process ends, voters tender their votes to the parties, and the party that collects more than half the votes wins.

Voters are not modeled as players. In the description of each of the games below we make direct assumptions on how voters tender their votes given their preferences and the final bids they face.

Party k has a utility W^k for winning, so party k's (net) payoff is the probability of k winning times W^k less the total payments by k to voters. To avoid dealing with ties, it is assumed that W^k is not an integer multiple of ε.

Thus, the parties' payoffs are modeled like payoffs to bidders in an auction. This corresponds to a view that control of the government is an economic asset and that political competition is a contest of profit maximizers to obtain this asset at minimal cost. This is a stark view of political competition. We elaborate on it in Section V.*E*., where we also contrast it with alternative views. As more fully discussed there, political competition of this sort does not necessarily imply that the government does not provide benefits. It is consistent with a government being constrained to provide no less than a benchmark level of benefits and to levy no more than a benchmark level of taxes. In the campaign

[3] Thus in case 1 payments are contingent on the individual's vote but not on the outcome of the election, and under case 2 the opposite holds. There are other possibilities, such as having the payments be contingent on both, which we do not analyze.

promises scenario, promises are then made to individual voters to lower their tax rates and/or increase their benefits relative to those benchmark levels, with the winning party capturing any excess in the budget for itself. In the up-front payments scenario, the benchmark levels of taxes and benefits are unchanged by the vote-buying competition. Instead, voters may receive direct transfers before the election. For simplicity, we have normalized the minimum levels of taxes and benefits to be zero, without any effect on the analysis.

We focus on the complete-information version of the games in which the parties' and the voters' preferences are known to the parties when they bid. In order to identify robust conclusions, we also consider the case in which the parties' utilities are imperfectly known. Strategies for the parties are defined in the obvious way in each case. In the complete-information game, we study a subgame-perfect equilibrium; we discuss the solution concept for the incomplete-information case when we apply it below.

III. Campaign Promises

We begin by studying the game in which only campaign promises are permitted. Here party k's net payoff is $W^k - \sum_i c_i^k$ if k wins having made promises $(c_1^k, \ldots, c_N^k)$ to the voters, zero if k loses, and $-\infty$ if the game never ends.[4]

We assume that voter i will vote for X if and only if $c_i^X + U_i^X > c_i^Y + U_i^Y$, where (c_i^X, c_i^Y) are the final promises of the parties to voter i. Recall that $U_i = U_i^X - U_i^Y$, and let $n = |\{i : U_i > 0\}|$ be the number of a priori supporters of X. The analogous number for Y is simply $N - n$. Since $U_m > 0$ and U_i is nonincreasing in i, it follows that $m \leq n$. Given a positive number z, let $\lceil z \rceil^\varepsilon$ be the smallest multiple of ε greater than z, and let $\lfloor z \rfloor_\varepsilon$ be the largest multiple of ε smaller than z. For a negative z, $\lceil z \rceil^\varepsilon = -\lceil |z| \rceil^\varepsilon$ and $\lfloor z \rfloor_\varepsilon = -\lfloor |z| \rfloor_\varepsilon$.

Let $\bar{U} = \sum_{i=m}^{n} \lceil U_i \rceil^\varepsilon > 0$ be the minimal sum that Y has to promise to voters in order to secure the support of a minimal majority, in case X does not promise anything. Thus $\bar{U}$ is one possible measure of the preference advantage that X enjoys over Y. The step function in figure 1 is $\lceil U_i \rceil^\varepsilon$. It crosses the axis at n, the long vertical segment is at m, and the triangular area enclosed between $[m, n]$ and the step function is $\bar{U}$.

Proposition 1. There exists an equilibrium in the campaign promises game. In any equilibrium Y wins if and only if $\lfloor W^Y \rfloor_\varepsilon \geq \lfloor W^X \rfloor_\varepsilon + \bar{U}$.

The idea behind proposition 1 is easily explained. Party Y must spend at least $\bar{U}$ in order to secure a majority. If the two parties were to compete, they would compete over the minimum-cost voters. The competition

[4] Alternative interpretations are discussed in Sec. V.*E*.

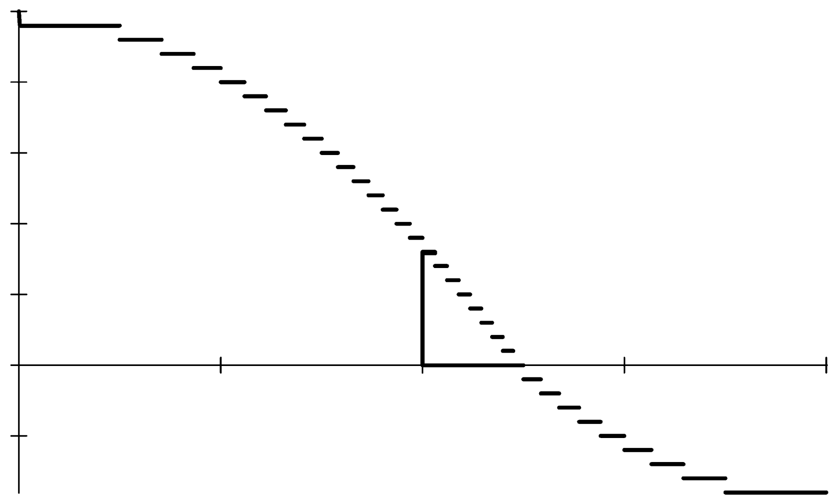

Fig. 1.—X's advantage in the campaign promises game

back and forth will lead to the winner being the party with the largest value once an expense of $\bar{U}$ has been incurred by Y.[5]

Example 1. There are three voters with $U_1 = 3.1$, $U_2 = 2.1$, and $U_3 = -5.1$. First consider the case in which $W^X = 2.2$ and $W^Y = 4.2$ and $\varepsilon = 1$. Thus, $\bar{U} = 3$, and according to proposition 1, X should win. In one equilibrium, Y offers nothing, that is, $(c_1^Y, c_2^Y, c_3^Y) = (0, 0, 0)$, and X responds with $(c_1^X, c_2^X, c_3^X) = (0, 0, 0)$ and wins. In another equilibrium, in each round the parties offer the minimum required to obtain a majority until Y reaches the maximum level it is willing to pay. Thus, Y starts with $(c_1^Y, c_2^Y, c_3^Y) = (0, 3, 0)$. Party X responds with $(c_1^X, c_2^X, c_3^X) = (0, 1, 0)$ (which will sway voter 2 to X's camp since $U_2 = 2.1$), Y increases to $(c_1^Y, c_2^Y, c_3^Y) = (0, 4, 0)$, and X increases to $(c_1^X, c_2^X, c_3^X) = (0, 2, 0)$. Now, Y quits since to regain the majority it would have to increase its commitment to voter 2 to at least 5, which exceeds W^Y. It is easy to see that there are also other equilibria, including some in which Y uses dominated strategies that generate promises in excess of W^Y. For instance, Y starts by offering $(c_1^Y, c_2^Y, c_3^Y) = (4, 3, 0)$. The sum of these offers exceeds W^Y but is still an equilibrium offer since X responds with $(c_1^X, c_2^X, c_3^X) = (0, 2, 0)$ and wins (as X does of course in all equilibria).

As the example shows, there are many equilibria in this game because

[5] Note that this characterization is easily extended to any voting rule, including ones that might be nonanonymous and/or nonneutral, and might include weights, veto players, or other special considerations. The critical calculation is the minimum expenditure that Y has to incur in order to secure a winning vote, and so one can calculate a corresponding $\bar{U}$ for any voting rule.

the loser's behavior is not pinned down, since it is certain to lose and will not have to honor the promises it makes. Note, however, that strategies that prescribe quitting below, or bidding above, one's value make sense only if one is certain of the other party's value and behavior. Hence we introduce uncertainty over the parties' values and consider a refinement that selects what seems to be the natural outcome. The outcome on which we focus arises when parties use least-expensive majority (LEM) strategies, in which each party purchases the least-expensive majority in turn, provided that their total commitment does not exceed their value (the second equilibrium in the example has this form). The identity of the winner would still be the same as above, but the total payment of the winner would be the loser's value adjusted by the magnitude $\bar{U}$, as spelled out in the proposition. Moreover, as discussed after proposition 2, in this case we can narrow down the subset of voters who might receive payments. Specifically, if realized party preferences are as in example 1, then this pins down the equilibrium to the one described in which only voter 2 receives offers, and offers to voter 2 increase one step at time until X wins with an offer of 2.

The refinement we consider is "ex post perfect equilibrium." A strategy for player k in this game of incomplete information specifies for each possible realization of type (W^k) for player k what that type will do after any sequence of offers. A pair of such strategies is an ex post perfect equilibrium if the strategies would constitute a subgame-perfect equilibrium when each player is told the realization of the opponent's type, and this has to hold for all possible realizations of the opponent's type.[6] Given our use of subgame perfection in the complete-information game, this seems to be a natural refinement for the incomplete-information game. While it is clear that such equilibria might not always exist in general environments, they are very robust and compelling equilibria when they do exist, which they do in our setting.

The values of each party are distributed on a finite set $\mathcal{W}$. The difference between any two adjacent values in $\mathcal{W}$ is no more than ε, and $\mathcal{W}$ does not include integer multiples of ε.

Proposition 2. Consider the campaign promises game with any full-support distribution over $\mathcal{W}$:

1. LEM strategies constitute an ex post perfect equilibrium.

[6] We believe that the result holds under much weaker assumptions (i.e., for more general solution concepts) but have not been able to prove such a conjecture. We have been able to show that the result also holds if we instead use an ex post Nash equilibrium in which players do not use weakly dominated strategies. That is neither a stronger nor a weaker solution than ex post perfect equilibrium. We know that it is false under the weaker refinements of excluding only weakly dominated strategies or considering only sequential equilibrium.

2. In any ex post perfect equilibrium Y wins if $\lfloor W^Y \rfloor_\varepsilon \geq \lfloor W^X \rfloor_\varepsilon + \bar{U}$, and X wins otherwise.
3. In any ex post perfect equilibrium if Y wins, then Y promises $\lfloor W^X \rfloor_\varepsilon + \bar{U}$; and if X wins, then X promises $\max\{\lfloor W^Y \rfloor_\varepsilon - \bar{U} + \varepsilon, 0\}$.
4. In any ex post perfect equilibrium, only voters between $\hat{m} = \{\min i : \lceil U_i \rceil^\varepsilon = \lceil U_m \rceil^\varepsilon\}$ and $\hat{n} = \{\min i : U_i > -\varepsilon\}$ can receive positive payments.

Thus, in ex post perfect equilibria, the loser promises an amount equal to its value to a subset of the "near-median" voters (those between $\hat{m}$, the first voter with the median preferences, and $\hat{n}$, the last voter whose preference for Y is marginal, i.e., less than ε). The winner commits—also to voters in this group—the minimal sum required to beat the loser. This sum amounts to the value of the loser plus or minus the magnitude $\bar{U}$ according to whether the winner is X or Y.[7]

While payments are concentrated among the voters between $\hat{m}$ and $\hat{n}$, the particulars of which voters get how much can differ across equilibria. For example, in one equilibrium using LEM strategies in a case in which $W^Y > W^X + \bar{U}$, the final outcome is that party X ends up offering $\lfloor W^X \rfloor_\varepsilon$ to a single voter, say voter m, and party Y ends up winning by offering $\lceil U_m \rceil^\varepsilon + \lfloor W^X \rfloor_\varepsilon$ to that voter and $\lceil U_i \rceil^\varepsilon$ to all voters $i \in [m+1, n]$. This happens by having the parties repeatedly outbid each other by a minimal amount for voter m. In another equilibrium with LEM strategies, X's budget is spread equally over voters $i \in [m, n]$, and Y matches all those bids and tops them off by $\lceil U_i \rceil^\varepsilon$ to compensate for these voters' initial preference for X.

One of the main objectives of this paper is to compare the equilibrium under campaign promises as described by proposition 2 with the equilibrium under up-front vote buying to be derived below. But the analysis of the present section also serves to complement the literature on campaign promises. Myerson (1993) considered a simultaneous move model of redistributive promises assuming symmetry among voters and between parties. The model above allows heterogeneity in the preferences of the parties and the voters and uses this heterogeneity to identify the winner, the magnitude of the promises, and the identity of the voters who benefit from them. As discussed further in Section V.*F*, the richer insights are made possible by the assumptions that the parties' promises are made sequentially and cannot be withdrawn. (This enables us to circumvent the technical difficulties encountered by Myerson and the earlier literature on "Colonel Blotto" games.)

Finally, notice that if there were only one voter, the campaign promises game would be an English auction in which the seller has a known

[7] If $W^Y < \bar{U}$, then any strategy by Y that involves promises amounting to less than W^Y is a LEM strategy, and no payments are made, although Y might still make promises.

preference for one buyer over the other. With many voters, this analogy is not exact, but the model and analysis still resemble those of the English auction, where there is competition over the "marginal" voters (the least-expensive voters whom the party that would lose in the absence of promises would have to obtain in order to win the election). The equilibrium in LEM strategies is the counterpart of the standard equilibrium in undominated strategies of the English auction.

IV. Up-Front Vote Buying

We now consider the situation in which up-front vote buying is permitted. In this game each firm in its turn offers a price p^k that constitutes a commitment to buy up to m votes at this price. Again, voters are not formally modeled as players in this game. Instead, it is assumed that, once the bidding ends, all voters try first to tender their votes to the highest bidder and those who are rationed by the winner tender their votes to the loser. Thus, if $p^X > p^Y$ at the end of the bidding, X ends up getting the minimal majority of m voters that it needs at p^X per vote, and the remaining $N - m$ voters who are rationed out by X sell to Y at p^Y per vote. If when the bidding is over $p^X = p^Y$, the ties are broken using the voters' fundamental preferences captured by the parameters U_i^X and U_i^Y: if $U_i^X > U_i^Y$, voter i will try first to tender to X. Party k's net payoff is then $W^k - mp^k$ if k wins, $-(N - m)p^k$ if k loses, and $-\infty$ if the game never ends.

This is somewhat artificial. Besides assuming that the parties' offers are commitments that can only be increased—assumptions that are shared with the campaign promises model—the up-front buying model embodies a number of additional assumptions. First, the voters try to sell at the higher price, ignoring their potential of being pivotal. Second, the parties make the same restricted offers to all voters. Third, voters wait to the end of the bidding process before tendering their votes.

The main purpose of adopting this model is to simplify the analysis. Consider first the decision to assume away pivot considerations. Since U_i^X and U_i^Y are the utility that i obtains from a victory of X and Y, respectively, then a strategic voter i would compare

$$p^X + \Pr(X \text{ wins}|\text{tender to } X)U_i^X + \Pr(Y \text{ wins}|\text{tender to } X)U_i^Y \quad (1a)$$

with

$$p^Y + \Pr(X \text{ wins}|\text{tender to } Y)U_i^X + \Pr(Y \text{ wins}|\text{tender to } Y)U_i^Y \quad (1b)$$

and try to sell to X if (1a) is larger than (1b). Note that the probability of being pivotal is

$$\Pr(X \text{ wins}|\text{tender to } X) - \Pr(X \text{ wins}|\text{tender to } Y) =$$
$$\Pr(Y \text{ wins}|\text{tender to } Y) - \Pr(Y \text{ wins}|\text{tender to } X).$$

If this probability is negligible, then the comparison between (1a) and (1b) reduces to a comparison between p^X and p^Y. Thus, the assumption that voters try to sell to the highest bidder is a simple way of encapsulating the assumption that endogenous pivot probabilities do not play an important role in the situations we would like to consider. We explain this further in Section V.*C* by arguing that in a more complete model, pivot considerations are inconsequential in this setting even when voters are fully strategic.

Proposition 3. In the uniform-offer up-front vote-buying game, if $W^j > W^k + (m+1)\varepsilon$, $j \neq k$, party j wins in (every) equilibrium and j's total payments are bounded above by $m\varepsilon$.

Proposition 3 says that, modulo some ε's, the party with the higher value wins and makes negligible payments to voters. In contrast, when the competition between the parties is restricted to campaign promises, the voters' preferences have a direct effect on the outcome and some near-median voters might get substantial transfers. In a sense this confirms a popular view that vote buying would give more power to the vote buyers and not benefit the voters in comparison with competition via campaign promises.

The proposition is illustrated in the following example.

Example 2 (Revisiting example 1). Reconsider the example with three voters with $U_1 = 3.1$, $U_2 = 2.1$, and $U_3 = -5.1$. Again, let $W^X = 2.2$ and $W^Y = 4.2$ and $\varepsilon = 1$. Now, Y will win by bidding 1, and all voters will tender to Y and two of them will be selected. If X attempts to bid, then Y will be willing to increase the bid up to 2, whereas X will not be willing to follow. Anticipating this, X will not bid (since it will be forced to buy from one voter despite losing).

The assumption that the parties make uniform restricted-price offers is not made in our companion paper (Dekel et al. 2006*b*), where—among other differences in the modeling—the parties make direct offers to individuals rather than announce a uniform price. In that model the assumption that voters wait for the end is more compelling since it is weakly dominant for them to do so. The alternative model yields the same insights as the present one but is more complex to analyze and requires adding a (negligible) bidding cost per period. If we incorporated such a cost throughout this paper, it would complicate the results of the campaign promises model. Nevertheless, it is worth noting that such a model (with the bidding costs) would yield the same conclusion as below.

Notice that the up-front buying model is closely related to an all-pay auction: any outstanding promises must be paid regardless of whether a party ends up winning. It is not exactly an all-pay auction since the winner pays m times the last price it offered and the loser pays $m-1$ times the last price it offered. In contrast, campaign promises are not binding unless a candidate wins, and hence the interaction there resembles an English auction instead of an all-pay auction. Thus, when the parties compete through up-front buying, it is not worthwhile for a party to make substantial offers if it is unlikely to win; but when the competition occurs through campaign promises, it is worthwhile to bid even when the probability of winning is small.

Up-front buying with incomplete information about parties' values.—In the campaign promises game we identified a subset of equilibrium strategies that were robust. Here we show that the equilibrium outcome of proposition 3 is robust to the introduction of some (small) uncertainty about the values. To see this consider the up-front buying game under the assumption that the parties are uncertain about the valuations of the other party. That is, W^k is now private information of party k. We show that, when there is sufficiently "little" incomplete information, there is a perfect Bayesian equilibrium (PBE) outcome that is close to the complete-information outcome.[8]

PROPOSITION 4. Assume that the W^k's are independent, that they have a common finite support, that $\Pr(W^k = \tilde{W}^k) \geq 1 - \eta$, and that $\tilde{W}^Y > \tilde{W}^X + (m+1)\varepsilon$. For any $\delta > 0$, there exists $\eta(\delta) > 0$ such that if $\eta < \eta(\delta)$, then there is a PBE in which players use only undominated strategies with an outcome that coincides with the complete-information outcome (i.e., Y wins paying no more than $m\varepsilon$) with a probability of at least δ.

We believe that this equilibrium is not unique. Since we are interested in robustness rather than finding additional equilibria that disappear when there is complete information, we have not verified this conjecture.[9]

[8] The equilibrium we construct is not an ex post perfect equilibrium. In the games with up-front vote buying, parties prefer not to make any payments if they lose, which leads to different strategic properties than in the campaign promises case in which losers never have to make any payments.

[9] In a single-unit all-pay auction with jump bids, Dekel et al. (2006*a*) construct an equilibrium in which the voters receive, on average, significant positive payments (not only on the order of ε) and the losing bidder may pay significant amounts. Although that model is essentially the single-voter special case of the present model, the extension to $N > 3$ and some small differences in the specification do not allow us to just assert that such an equilibrium exists in the present model as well. However, it seems very likely that such an equilibrium exists here as well.

V. Discussion

A. *Insights*

The main insights of our analysis can be summarized as follows. First, with campaign promises, the party with the highest value, adjusted by the voters' preferences measure $\bar{U}$, wins and pays out the second-highest value, subject to the same adjustment, to a group of near-median voters. Second, with up-front vote buying and no uncertainty, there will be only minimal spending in equilibrium. Third, our analysis highlights some important differences between competition through up-front vote buying and through campaign promises, both in terms of the expected cost of winning and in the determination of the winner. The outcome of competition in campaign promises is affected by the preferences of the voters and might involve substantial transfers to the voters, whereas the outcome of up-front vote buying is not affected by the voters' preferences and the voters receive only minimal transfers.[10]

As mentioned in the introduction, all these features are broadly consistent with descriptive work on vote buying. Anderson and Tollison (1990) claim that during a period in which vote buying was common, the payments were small, and the elimination of vote buying led to an increase in social policies. Low payments in up-front vote buying also seem consistent with the observation that the price of stocks with voting rights is generally similar to that of nonvoting stocks (Lamont and Thaler 2003), and hence the additional payment that is solely for voting rights is minimal, as in our model.

B. *Efficiency*

We ask now how vote buying affects welfare, where welfare is measured by the sum of voters' values plus the vote buyers' values. We identify efficiency with maximization of this welfare measure. In the absence of any mechanism for buying and selling votes, the outcome of voting will in general be inefficient. There is simply nothing to make voters take into account the effect of their vote on others. A natural hypothesis then might be that the opening of trade will lead to efficient outcomes. Our analysis shows that the outcome of a vote-buying equilibrium is in general inefficient. In the up-front buying scenario, only the parties' valuations matter: If voters strongly support X but W^Y is larger than W^X, Y still wins. In the campaign promise scenario, only the preferences

[10] These relatively sharp insights are facilitated by modeling assumptions that have been discussed in the paper, including the sequential bidding with offers that cannot be withdrawn and the (almost) complete information with regard to the parties' valuations.

of voters near the median group affect the outcome, and hence, the outcome does not reflect the preferences of all voters.

Under what circumstances will vote buying result in efficiency? In the up-front vote-buying game, the equilibrium will be (approximately) efficient if the parties' valuations are proportional to the true surpluses, that is, if W^X stands in the same proportion to $\sum_i [U_i]^e$ as W^Y is in proportion to $\sum_i [-U_i]^e$. This would be the case if the party's valuation perfectly aggregated the values of its supporters.[11]

More fundamentally, the main source of inefficiency is that the voters are not pivotal with respect to the decision.[12] A nonpivotal voter will sell her vote regardless of how she values the parties. Hence, it is clear that vote buying cannot take such a person's preferences into account and thus would not be efficient.[13]

Do vote buying and selling entail greater welfare loss than would occur in their absence? It is easy to construct examples that generate higher or lower overall utility with vote buying than with campaign promises or with neither. What we learn from our model is that vote buying may lead to parties' valuations rather than voter preferences being the driving force that determines the winner. Thus, if we think of a party's valuation as reflecting the profit that a certain narrow group will derive from taking over the government, then the opening of vote trading will elevate the relative importance of such groups, but of course nothing can be said in general on whether these biases are likely to produce lower total utility than simple voting.

While it is natural to ask how vote buying and campaign promises fare in terms of efficiency, our goal was not to find a mechanism that yields efficiency. That mechanism design question is trivial in the context studied here, where the parties have complete information. Rather we wanted to take the voting as given and explore the implications of permitting trade.

[11] For example, we could consider a stage taking place before the vote-buying game, where voters could contribute to the two parties. The vote-buying game would then be one in which the parties can spend up to the budgets at their disposal (where budgets substitute for valuations, as discussed below). For certain specifications of such a contribution game, there exist some equilibria in which the winning party would be the one whose supporters had a higher total valuation (following a logic similar to that behind the results of Bernheim and Whinston [1986]).

[12] Piketty (1994) presents an example illustrating a different sort of inefficiency that can emerge in vote-trading environments. His point is that when there are private signals about common values, voters may fail to account for the informational externalities concerning lost information when they sell their vote.

[13] Buchanan and Tullock (1962) and Neeman (1999) make the point that, if decisions require unanimity, then vote trading could lead to efficiency, since then every voter is pivotal.

C. Voter Behavior

Assuming, in the up-front buying model, that voters sell to the party that offers the higher p^k is a shortcut that embodies the assumption that pivot probabilities play a negligible role. If the voters were modeled as players, who at the end of the bidding decide simultaneously to which party to tender, then the behavior that we have assumed—that everybody tries first to tender to the party that offers the higher price—will still be an equilibrium behavior in the tendering subgame. But there might also be other equilibria that rely on pivot considerations. For example, there might be an equilibrium in the tendering subgame in which exactly $(N+1)/2$ voters tender to party X although $p^X < p^Y$, since for each of these voters $U_i > p^X - p^Y$. We think that pivot considerations of this sort are not truly important in the situations we would like to consider. In large elections there is inevitably sufficient noise to make the pivot probability of an individual voter insignificant. This can be modeled formally by introducing some "noise voters" into the model. The magnitude of such noise can be made small relative to the size of the electorate, hence leaving intact the essence of the analysis conducted above. At the same time, the noise can be significant enough to make the pivot probabilities negligible.[14]

To see this formally, suppose that there is an odd number N of strategic voters and an even number L of noise voters each of whom tenders her vote randomly and independently with equal probability to each of the parties. Consider now the up-front buying of Section IV. The minimal majority required now for winning is $(N+L+1)/2$. Each of the N strategic voters tenders to X if

$$p^X + U_i \Pr(X \text{ wins} \mid i \text{ tenders to } X) >$$
$$p^Y + U_i \Pr(X \text{ wins} \mid i \text{ tenders to } Y).$$

Consider now a tendering subgame that takes place after the bidding stops with prices $p^X < p^Y$. If there is an equilibrium in which some voters tender to X with positive probability, in this equilibrium it must be that

$$U_i[\Pr(X \text{ wins} \mid i \text{ tenders to } X) - \Pr(X \text{ wins} \mid i \text{ tenders to } Y)] \geq p^Y - p^X. \tag{2}$$

[14] See Dal Bo (2007) for alternative arguments behind why pivot considerations are not an issue.

Now

$$
\begin{aligned}
&\Pr(X \text{ wins}|i \text{ tenders to } X) - \Pr(X \text{ wins}|i \text{ tenders to } Y) \\
&\quad = \Pr\left(\text{exactly } \frac{N+L-1}{2} \text{ voters other than } i \text{ tender to } X\right) \\
&\quad = \sum_{k=0}^{L} \Pr(k \text{ noise voters tender to } X) \\
&\qquad \times \Pr\left(\frac{N+L-1}{2} - k \text{ strategic voters tender to } X\right) \\
&\quad = \sum_{k=0}^{L} \binom{L}{k}\left(\frac{1}{2}\right)^{L} \Pr\left(\text{exactly} \frac{N+L-1}{2} - k \text{ strategic voters tender to } X\right) \\
&\quad \leq \binom{L}{L/2}\left(\frac{1}{2}\right)^{L}.
\end{aligned}
$$

The last inequality follows from the fact that $k = L/2$ maximizes $\binom{L}{k}$ and the fact that the probabilities $\Pr(\text{exactly } [(N+L-1)/2] - k$ strategic voters tender to $X)$ sum up to less than one.

Since $\binom{L}{L/2}(\frac{1}{2})^{L} \to 0$ as $L \to \infty$, there is L' such that, for all $L \geq L'$,

$$
\binom{L}{L/2}\left(\frac{1}{2}\right)^{L} < \frac{\varepsilon}{\max_i U_i}.
$$

Thus, for $L \geq L'$,

$$
\begin{aligned}
U_i[\Pr(X \text{ wins}|i \text{ tenders to } X) - \Pr(X \text{ wins}|i \text{ tenders to } Y)] < \\
\varepsilon \leq p^{Y} - p^{X}, \qquad (3)
\end{aligned}
$$

where the last inequality follows from the assumption that $p^{Y} > p^{X}$. But (3) implies that (2) is violated. Therefore, for $L \geq L'$, there is no equilibrium in which strategic voters tender to the lower price.

Now, if N is large, the fraction of noise voters to strategic ones can be negligible, yet pivot considerations never affect the considerations of the strategic voters. The analysis of the parties' bidding competition will remain exactly the same as in Section IV.

The bottom line is that we think that, for the purposes of our analysis, it is appropriate to abstract away from pivot considerations. We chose to do so in a straightforward way. As the preceding paragraph explains, this can be done in a more sophisticated way. However, if we were to adopt such an approach and carry it throughout, the complexity of the analysis would increase substantially without any gain in substance.

D. Budgets

Throughout the above analysis the parties were not subjected to budget constraints. We argue below that our main results have immediate analogues in the case in which the parties are constrained by budgets. Suppose that the parties have budgets B^X and B^Y, respectively. The constraint is that a party's offers at any point in the game are such that its liability if the game ended at that time would not exceed its respective budget. Let us retain the assumptions about parties' payoffs made above and assume that $B^X \leq W^X$ and $B^Y \leq W^Y$. That is, the parties are willing to spend up to their budgets in order to win but prefer spending less to more.

In the campaign promises case, the analogues of propositions 1 and 2 are obtained by replacing W^k by B^k everywhere in the statements. That is, Y wins if and only if $B^Y \geq B^X + \bar{U}$, and with some uncertainty, the payments end up as in proposition 2 and only the voters in the interval $[\hat{m}, \hat{n}]$ ever receive promises in equilibrium. In the case of up-front buying, the analogues of propositions 3 and 4 again hold, with budgets replacing the valuations for winning, and thus the party with the larger budget (modulo some ε's) wins with a negligible total payment. The proofs of these results are simpler than their counterparts without budget constraints, since the budget constraints together with the ε grid bound the depth of the game tree.

Notice, however, that if we introduce budget constraints, the meaning of the comparison between campaign promises and up-front buying is less clear than it was when the focus was on valuations alone as it was throughout the analysis. The reason is that it is not obvious that the same budget constraints should apply to these two scenarios, whereas it is natural to assume that parties' valuations are independent of the mode of competition.

E. The Nature of Political Competition and Redistribution

1. Our Model of Political Competition

When we refer to the "utility of a party," W^k, we mean the benefits accruing to a narrow group of individuals who control it.[15] The utility W^k aggregates the intangible benefit of being in power and the value of resources that party k obtains from being in power.

[15] In our analysis we do not model these individuals as voters. Including them explicitly as voters would have no effect on the up-front vote-buying analysis. In the case of campaign promises, if the insiders are those whose intangible utility is greater than the utility any voter gets from their winning (i.e., it is greater than U^1 for X party members and greater than $-U^N$ for Y party members), then the analysis of campaign promises would also be essentially the same.

The basic premise is that the winner is a residual claimant to the government resources that have not been committed. This does not mean that the government can do anything it wishes. It is reasonable to assume that the government is constrained (by law, custom, or fear of rebellion) to levy no more than a benchmark level of taxes and to provide no less than a benchmark level of benefits. The maximum sum that the party in power can capture is the difference between the maximal tax revenue it can collect and the cost of the provision of the mandated benefits. This, combined with the intangible benefit, makes up W^k.[16] The payoff of the winner is W^k minus the value of the commitments and payments made during the contest. The commitments made in the campaign promises scenario can then be viewed as reductions of the maximal tax and/or enhancements of the minimum benefits that are targeted at given voters. In the up-front payments scenario, voters may get transfers before the election, but the benchmark levels of taxes and benefits are not changed by the competition. The fact that the benchmark levels of taxes and benefits were not included explicitly in the formal model was just a normalization of both to zero. Nothing would change if we introduced nonzero benchmark levels into the model.[17]

This modeling approach, that casts the winner as a residual claimant, enables the direct comparison between the up-front purchase of votes and campaign promises: whether the payments are made up-front or later from the government resources, they are paid out of the same source, namely out of the government resources that the winning party extracts.

2. An Alternative View of Political Competition

Under an alternative view, government resources cannot be appropriated by the winner. The winner may derive some utility from being in power, but it can only redistribute the government resources among the voters. This view is embodied, for example, in Myerson's (1993) model. Our model can be modified as follows to consider this view. Let B^k denote the government budget that will be available for distribution if party $k = X, Y$ wins. The budgets B^k may differ across the parties as a result of different abilities to manage the government or different fun-

[16] Like the intangible benefits, the value of being a residual claimant may vary across the parties, e.g., owing to different managerial abilities, which translate to different costs.

[17] Notice, however, that the campaign promises model cannot be simply extended to allow the parties to commit to new taxes in each round along with their other promises, since the possibility of offsetting promises by taxes would violate the assumption that the parties' promises cannot be withdrawn. Given our monotonicity constraints on offers, taxes could be incorporated as described above, with a benchmark tax level that will be levied on any voter (to the extent that it is not reduced by campaign promises).

damental commitments on resources that are external to this model.[18] The utilities of winning, W^k, are the direct (what we called before intangible) benefits for the winner of being in power, which do not come out of the budgets B^k. The winner here is not a residual claimant to the government resources. The process of bidding (alternate offers of prices p^k in the up-front purchase case and of promises c_i^k in the campaign promises case) and the rules for its termination remain the same as before. Note that under this view a promise made to one voter comes indirectly at the expense of other voters, whereas in the previous view, a promise to a voter comes at the expense of the winner by virtue of its being the residual claimant.

The analysis of the up-front vote buying case remains exactly the same. Regardless of how voters expect the parties to distribute the government budget upon winning, the tendering decisions are still based only on the up-front prices p^k. Therefore, the winner is still the party with the higher W^k, and it does so at minimal cost. The only difference from before is in the interpretation of the W^k's.

The analysis of the campaign promises case is somewhat different since the outcome will depend on the B^k's and on how voters expect the parties to distribute the portions of their budgets that have not been committed in the bidding process. First, the parties are constrained only to make promises c_i^k that satisfy the budget it would have once in control, that is, $\sum c_i^k \leq B^k$. Second, suppose that the bidding stops with outstanding promises c_i^k and that voters expect party k to distribute fraction α_i^k of the committed portion of the budget to voter i. Then voter i votes for X if

$$U_i^X + c_i^X + \alpha_i^X\left(B^X - \sum c_i^X\right) > U_i^Y + c_i^Y + \alpha_i^Y\left(B^Y - \sum c_i^Y\right).$$

The outcome of the competition depends on the expectations regarding the distribution of the uncommitted resources, as embodied in the α_i^k's, and this is also the main difference between this model and the model we analyzed throughout.

If these uncommitted resources are expected to be distributed to a narrow group of close affiliates, say $\alpha_1^X = \alpha_N^Y = 1$ and $\alpha_i^k = 0$ otherwise, then the model is essentially the model used throughout the paper, as was discussed in the preceding subsection. In this case the party and its close affiliates are again the residual claimants to the government resources, and the analysis with B^k is essentially as in Section V.*D* on budget-constrained parties.[19]

[18] In contrast to the preceding discussion of budgets, now the budget is derived solely from government revenue. The source of revenue was irrelevant to the preceding discussion, and it maintained our perspective of parties that profit directly from any surpluses.

[19] Assuming that party insiders are as described in the last paragraph of Sec. V.*E*.1.

If, alternatively, these uncommitted resources are expected to be distributed more evenly, say $\alpha_i^k = 1/N$ for all i and k, then the situation is somewhat different. However, it is fairly straightforward to see that the identity of the winner is determined by the same sort of consideration as in our original analysis (or more precisely as in the budget-constrained analysis of Sec. V.*D*). That is, X wins if $B^X + \overline{U} > B^Y$ and Y wins if the reverse inequality holds. This can be verified by letting the designated winner play a LEM strategy with respect to the explicit promises c_i^k. Suppose, for example, that $B^Y > B^X + \overline{U}$. If Y plays LEM, then at any stage after it has outbid X by obtaining a majority in the least-expensive way possible, then the residual budgets satisfy $B^Y - \sum c_i^Y > B^X - \sum c_i^X$. If the bidding stops at that point, Y will continue to have the advantage in the subgame in which they can just spend these uncommitted resources. It then follows from the analysis of Section III that by using a LEM strategy Y can guarantee a profitable win.

3. Which Is the Correct View?

Both views are somewhat extreme. It is difficult to think of examples of regimes in which the people in charge act completely as residual claimants to the government resources. It is similarly difficult to think of regimes that do not appropriate some government resources to benefit narrow groups of affiliates. Realistic situations combine both elements. As can be inferred from the discussion just above, our model can be modified to deal with such intermediate situations without changing much of the analysis or results.

F. Related Literature

We discuss below three literatures that dealt with vote buying and relate them to our analysis.

1. Colonel Blotto Games

In a "Colonel Blotto game," two opposing armies simultaneously allocate forces among n fronts. Any given front is won by the army that committed a larger force to that front, and the overall winner is the army that wins a majority of the fronts. This model has been also interpreted as a model of electoral competition, where each party wins the voters to whom it made the larger promise and the overall winner of the election is the party that managed to win a majority of the votes (Gross and Wagner's [1950] continuous version of a Colonel Blotto game is perhaps the earliest contribution adopting this interpretation). A simultaneous version of our campaign promises game with budget constraints (as explained earlier in this section) is also a Colonel Blotto game.

The problem is that Colonel Blotto games are notoriously difficult to solve, even in the simplest settings.[20] The existing analyses are of symmetric mixed-strategy equilibria in which voters are treated identically (from an ex ante point of view) and the parties are equally likely to win.

Myerson (1993) circumvents some of the technical difficulties of Colonel Blotto games by allowing candidates to meet the budget constraint on average rather than exactly. In particular, Myerson considers a simultaneous move game that is similar to the campaign promises game we analyze, but in which parties can offer random payments to each voter and the payments need only meet the budget in expectation. As in the previous Colonel Blotto literature, Myerson assumes that voters and parties are symmetric and derives a symmetric mixed-strategy equilibrium in which parties exhaust their budgets.

Our work circumvents the technical difficulties of this literature by making the bidding sequential and irreversible (past promises cannot be withdrawn or lowered). While the irreversibility may not always apply, these features permit a rich analysis. This enables us to consider heterogeneous voters and parties and examine how such heterogeneity affects the outcomes.

2. Other Vote-Buying Models

Groseclose and Snyder (1996) present a model of vote buying in a legislature. Their model can be thought of as a two-round version of our campaign promises or our alternative up-front vote-buying model.[21] The restriction to two rounds gives the second mover a substantial advantage. The first mover has to purchase a supermajority of voters in order to successfully block the response of the second mover. Thus, for example, if all voters were indifferent between the parties, the first mover would need to make promises totaling twice the value (or budget) of the second mover in order to win, since the second mover should not be able to purchase the least-expensive 50 percent. As is evident from the above analysis, our more symmetric bidding process neutralizes the effect of the order of moves and consequently gets significantly different results with respect to the identity of the winner, how much it pays,

[20] See Laslier and Picard (2002), Szentes and Rosenthal (2003), and Weinstein (2005) for some characterizations of equilibria.

[21] Given that each party moves only once in Groseclose and Snyder's model, it is irrelevant to the outcome whether the game incorporates up-front vote buying or campaign promises.

which voters it buys, and the mode of competition (up-front vote buying or campaign promises).[22]

Another feature of vote buying in legislative settings that differs from that of general elections is that legislators may care (substantially) about how they cast their vote independent of the outcome. In a companion paper (Dekel et al. 2006*b*), we analyze alternating-move vote-buying games similar to the ones analyzed here, but in contexts in which voters care about how they cast their vote and not just about the eventual outcome. For instance, a legislator might strongly prefer to vote against a certain bill even if the bill is sure to pass, given that his or her constituents might pay attention to the legislator's voting record in future campaigns. This changes the behavior of legislators (voters) significantly vis-à-vis the analysis in this paper and hence also has a substantial impact on the strategic interaction of the vote buyers. For instance, the up-front vote-buying game with complete information can involve substantial payments by the winner, and the identity of the winner depends in a subtle way on both the buyer's willingness to pay and the voters' preferences. That contrasts sharply with the analysis of general elections in this paper. The companion paper also has a different focus: it studies the impact of budget constraints on vote buyers. We refer the interested reader to the companion paper for more details.

3. Corporate Control

The related literature on corporate control (Grossman and Hart 1988; Harris and Raviv 1988) examines settings in which two alternative management teams—an incumbent and a rival—are competing to gain control of a corporation through acquisition of a majority of the shareholders' votes. The alternative teams are the counterparts of our parties, and their private benefits from controlling the corporation are the counterparts of the parties' valuations for being elected. The shareholders are the counterparts of our voters with a special form of identical preferences based on the difference in share value that will be generated under the two teams. The model of Harris and Raviv[23] resembles a two-round version of our up-front restricted price offers model. Harris and Raviv characterize an equilibrium in which the efficient team wins, that is, the team that maximizes the total shareholder value plus its private

[22] Other articles that address similar issues are sufficiently distant in terms of their focus and framework to be considered largely complementary to our discussion, and it does not seem useful to try to relate them to our analysis. These include Buchanan and Tullock (1962), Tobin (1970), Philipson and Snyder (1996), Kochin and Kochin (1998), and Baron (2006).

[23] The related model of Grossman and Hart does not seem to have an explicit equilibrium model for the case that would be close to our model (what they call competition in restricted offers between parties with significant private benefits).

benefit. This equilibrium relies critically on every voter believing that his or her tendering decision will be completely pivotal. In this sense the Harris-Raviv model takes a view opposite of ours. Whereas we assume away the pivot considerations on the grounds that they are marginal, these considerations are the central element of their model. Owing to this approach, the Harris-Raviv equilibrium is very fragile in the sense that uncertainty about the number of shares, actions of other voters, or offers could destabilize it.[24] We believe that their game has stable equilibria in which shareholders are not pivotal and the team with the larger private benefit wins.[25] These stable equilibria are the counterpart of the equilibrium we derive, except that the limitation to two rounds means that the price paid by the winner depends on whether it moves first or second (as in the analysis of Groseclose and Snyder [1996]).[26]

Appendix

This appendix contains proofs of those results not proved in the main body of the paper.

Proof of Proposition 1

The proof is based on the following lemma, in which we characterize the outcomes resulting when at least one player follows LEM strategies. These are strategies such that each party in its turn acquires the least-expensive majority as long as its total commitment does not exceed its value.

Let C^{kl} denote the total promises made by party $k = X, Y$ up to some node l of the game, and $\bar{U}^l > 0$ the minimal amount needed by Y to obtain a majority at that point. At the initial node, $C^{jl} = 0$ and $\bar{U}^l = \bar{U}$.

LEMMA 1.

1. If $\lfloor W^Y \rfloor_\varepsilon - C^{Yl} \geq \lfloor W^X \rfloor_\varepsilon - C^{Xl} + \bar{U}^l$ and, for $k = X, Y$, $\lfloor W^k \rfloor_\varepsilon \geq C^{kl}$, then the following conditions are satisfied.
   a. If X's strategy is LEM from l onward, then with a LEM strategy from l onward, Y wins and spends $\lfloor W^X \rfloor_\varepsilon - C^{Xl} + C^{Yl} + \bar{U}^l$.
   b. If X's strategy is LEM from l onward, then to win Y must spend at least $\lfloor W^X \rfloor_\varepsilon - C^{Xl} + C^{Yl} + \bar{U}^l$.

[24] Their model has a continuum of voters and so is not quite a closed game-theoretic model. It appears that a large finite approximation to this equilibrium could be built, but the equilibrium would be unstable in that any shift in bidders' beliefs would lead to a change in their tendering strategies and thus a movement to another equilibrium in the subgame (the one conjectured next in the text).

[25] These are equilibria that we conjecture but are not mentioned by Harris and Raviv. We do not provide a formal analysis since it would take a good deal of space to set up the model for a relatively tangential point.

[26] Those equilibria do not exhibit the second-mover advantage of the Groseclose and Snyder (1996) equilibria, since Harris and Raviv's model has restricted price offers whereas Groseclose and Snyder's model has targeted offers.

c. If Y's strategy is LEM from l onward, then X cannot win without spending more than W^X.

2. If $\lfloor W^Y \rfloor_\varepsilon - C^{Yl} < \lfloor W^X \rfloor_\varepsilon - C^{Xl} + \bar{U}^l$ and, for $k = X, Y$, $\lfloor W^k \rfloor_\varepsilon \geq C^{kl}$, then

a. If Y's strategy is LEM from l onward, then with a LEM strategy from l onward, X wins and spends $\lfloor W^Y \rfloor_\varepsilon - C^{Yl} + C^{Xl} + \bar{U}^l + \varepsilon$.

b. If Y's strategy is LEM from l onward, then to win X must spend at least $\lfloor W^Y \rfloor_\varepsilon - C^{Yl} + C^{Xl} - \bar{U}^l + \varepsilon$.

c. If X's strategy is LEM from l onward, then Y cannot win without spending more than W^Y.

Proof of lemma 1. Parts 1*a* and 2*a* follow immediately from the nature of the LEM strategies: Y initially must buy sufficiently many voters at cost $\bar{U}^l$ (the notion of "buying voters" stands here for making promises that would convince these voters to vote for the buying party if the bidding stops immediately after those promises were made); X then must buy one voter with an additional cost of ε; Y then must buy a voter back at additional cost ε; and so on. Iff $\lfloor W^Y \rfloor_\varepsilon - C^{Yl} \geq \lfloor W^X \rfloor_\varepsilon - C^{Xl} + \bar{U}^l$, this process will reach a point where Y has promised not more than W^Y; in order stay in the game, X has to increase its total outstanding promises to more than W^X, and hence, by the hypothesis that X plays LEM, X stops.

Part 1*b* is proved by induction on $\lfloor W^X \rfloor_\varepsilon$ as follows. By definition of $\bar{U}^l$, part 1*b* is true for $\lfloor W^X \rfloor_\varepsilon = 0$ and any C^{Xl}, C^{Yl}, and $\bar{U}^l$. Suppose that it is true for $\lfloor W^X \rfloor_\varepsilon \leq K\varepsilon$ and for all C^{Xl}, C^{Yl}, and $\bar{U}^l$, and consider $\lfloor W^X \rfloor_\varepsilon = (K+1)\varepsilon$. Let $\bar{U}$ be the sum promised by Y in its first move after l. Clearly, $\bar{U} \geq \bar{U}^l$. Following its LEM strategy, X promises some S such that $\varepsilon \leq S \leq \bar{U} - \bar{U}^l + \varepsilon$. After X's promise, at a node we denote by l', we have $C^{Yl'} = C^{Yl} + \bar{U}$, $C^{Xl'} = C^{Xl} + S$, and $\bar{U}^{l'} = \varepsilon$. But this situation is equivalent to a configuration with $\bar{U}' = \varepsilon$, $C^{Yl'} = C^{Yl}$, and $C^{Xl'} = C^{Xl}$ and with values $V'^Y = W^Y - \bar{U}$ and $V'^X = W^X - S \geq W^X - (\bar{U} - \bar{U}^l + \varepsilon)$. Since $\lfloor V'^X \rfloor_\varepsilon \leq K\varepsilon$, by the inductive assumption, Y's overall expenditure will be at least $\lfloor V'^X \rfloor_\varepsilon - C^{Xl} + C^{Yl} + \varepsilon + \bar{U}$. Now, this and $V'^X \geq W^X - (\bar{U} - \bar{U}^l + \varepsilon)$ imply that Y's overall expenditure is at least

$$\lfloor W^X \rfloor_\varepsilon - C^{Xl} + C^{Yl} - (\bar{U} - \bar{U}^l + \varepsilon) + \varepsilon + \bar{U} = \lfloor W^X \rfloor_\varepsilon - C^{Xl} + C^{Yl} + \bar{U}^l.$$

For all $j = a, b, c$, part 2j is the counterpart of 1j. In particular, part 2*b* is analogous to 1*b*. Finally, part 1*c* follows from 2*b*. This completes the proof of the lemma. QED

The existence of equilibrium follows from the following lemma.

LEMMA 2. LEM strategies for both parties constitute an equilibrium.

Proof of lemma 2. For $\lfloor W^Y \rfloor_\varepsilon \geq \lfloor W^X \rfloor_\varepsilon + \bar{U}$, parts 1*a* and 1*b* of lemma 1 imply that Y's LEM strategy is a best response against X's LEM strategy. Part 1*c* implies that X's LEM strategy is a best response against Y's LEM strategy. Analogously, parts 2*a*–2*c* of lemma 1 imply that X's and Y's LEM strategies are mutual best responses when $\lfloor W^Y \rfloor_\varepsilon < \lfloor W^X \rfloor_\varepsilon + \bar{U}$. This demonstrates that LEM strategies constitute an equilibrium. QED

To conclude the proof of proposition 1, first observe that in any equilibrium there is a unique winner. To see this, suppose the contrary. Note that the equilibrium path hits only a finite number of nodes, since play will end in any subgame-perfect equilibrium at any node where both players have made promises that exceed their values. Since there is not a unique winner, there must be a last node where some player mixes along the equilibrium path and is the winner along one path that follows and the loser along another path. Since a player's value is different from any level of payments that he could promise, the path that leads the player to be the winner must result in either a strictly positive or a strictly negative payoff; exiting results in a zero payoff. This cannot be since the player will strictly prefer one of these pure outcomes. Next, note that in any subgame-perfect equilibrium, no player will follow a strategy in which he ends up paying more than his value. Thus, by parts 1c and 2c and by focusing on the initial node in which $C^{kl} = 0$, Y can guarantee a win if $\lfloor W^Y \rfloor_\varepsilon \geq \lfloor W^X \rfloor_\varepsilon + \bar{U}$, and X can guarantee a win otherwise. Thus, given that the equilibrium is such that all equilibrium paths lead to the same winner, the proposition must hold, since then the player who has a strategy that guarantees a win against any subgame-perfect equilibrium strategy of the other must have a positive utility and be the winner. QED

Proof of Proposition 2

Part 1 follows from lemma 2.

Part 2 follows from the definition of ex post perfect equilibrium and proposition 1.

Part 3: Assume to the contrary that in some ex post perfect equilibrium Y wins and with some probability promises less than $\lfloor W^X \rfloor_\varepsilon + \bar{U}$, say $\overline{W}$. Consider then the case in which Y with value W^Y such that $\lfloor W^Y \rfloor_\varepsilon = \overline{W}$ plays against X with value W^X. In an ex post perfect equilibrium, the strategies of Y with such a value against X with value W^X are an equilibrium of that complete-information game, and Y loses with certainty. However, by mimicking the strategy of the higher type that wins (only up to any node where the promises do not exceed $\overline{W}$), Y would win with positive probability and never pay more than his value W^Y and end up with strictly positive utility against X with value W^X. This is a contradiction.

Now assume to the contrary that in equilibrium Y with value W^Y wins and promises more than $\lfloor W^X \rfloor_\varepsilon + \bar{U}$, say $\hat{U}$. Consider the case in which Y with value W^Y such that $\lfloor W^Y \rfloor_\varepsilon = \lfloor W^X \rfloor_\varepsilon + \bar{U}$ plays against X with value W^X. Note that in any equilibrium Y does not pay more than $\lfloor W^Y \rfloor_\varepsilon$. By part 2 this W^Y wins, and as was just noted it does not pay more than $\lfloor W^X \rfloor_\varepsilon + \bar{U}$. Thus, by mimicking W^Y against the strategy of W^X, type W^Y would win and pay $\lfloor W^X \rfloor_\varepsilon + \bar{U}$, which is less than what W^Y is paying in the supposed equilibrium (since Y is always paying at least this much by the above argument, and sometimes more by supposition), leading to a contradiction.

We now show part 4.

Definition 1. Assume that the minimal amount needed for Y to obtain a majority is $\bar{U} > 0$ and it is Y's turn to make an offer. Party Y's offer in the amount $c > \bar{U}$ is wasteful if $c - \bar{U} > \bar{U}' - \varepsilon$, where $\bar{U}'$ is the minimal amount needed for X to obtain a majority after Y offered c.

To understand this, note that an offer by Y can attain two objectives: achieving a majority and increasing the amount $\bar{U}'$ that X will subsequently need to offer in order to obtain a majority. An offer is wasteful if it is greater than the minimal amount needed to achieve majority plus the amount by which it increases $\bar{U}'$. The definition of a wasteful offer by X is analogous.

We now show that a wasteful offer can be made only as the last offer in any ex post perfect equilibrium.

Lemma 3. In an ex post perfect equilibrium, no party ever makes a wasteful offer.

Proof of lemma 3. Assume to the contrary that the ex post perfect equilibrium strategies lead to Y or X making a wasteful offer at some stage other than the last in the game when Y has value W^Y and X has value W^X, with $\lfloor W^X \rfloor_\varepsilon \leq \lfloor W^Y \rfloor_\varepsilon - \bar{U}$. Assume that along this path Y is the first to make a wasteful offer. Such an offer must occur before X has offered $\lfloor W^X \rfloor_\varepsilon \leq \lfloor W^Y \rfloor_\varepsilon - \bar{U}$. (If not, then the first wasteful offer of Y is made after X has offered $\lfloor W^X \rfloor_\varepsilon$ and hence after Y has offered $\lfloor W^X \rfloor_\varepsilon + \bar{U}$, and then Y wins with a promise of more than $\lfloor W^X \rfloor_\varepsilon + \bar{U}$, contradicting part 3.) Now consider the case in which Y faces an X with value W^X such that $\lfloor W^X \rfloor_\varepsilon = \lfloor W^Y \rfloor_\varepsilon - \bar{U}$. Such an X should lose against W^Y by part 2. But if W^X mimics W^X until Y makes the wasteful offer and then continues with LEM, then according to lemma 1, X will win, leading to a contradiction. QED

We now continue with the proof of part 4. After any offer is made, a new function describing the advantage that X holds over Y for each voter emerges. Specifically, given $U = (U_i)_{i=1}^N$, if X makes offers of c_i^X, then the new advantage of X over Y is given by U', where $U_i' = U_i + c_i^X$. We now clarify and develop further some aspects of the notation. The term U_m is the amount by which the median voter prefers X over Y, when each voter i prefers X over Y by U_i (and this U_i incorporates the basic preferences of i and the difference in promises that i has received up to but not including c_i^X). If the median voter under U is, say, voter 7, then U_m' is not necessarily $U_7 + c_7^X$, since under U' the preferences are different, and the median voter may change. So U_m' denotes the utility of the new median voter when each i prefers X over Y by the amount U_i'. Similarly, since the advantage that X holds over Y is changing, we replace the symbols of $\bar{n}$, $\hat{n}$, and $\hat{m}$ with the following functions for any U. Let $\tilde{U}$ be a reordering of U that is decreasing. Then $\hat{N}(U) = \{\max i : \tilde{U}_i > -\varepsilon\}$, $\hat{M}(U) = \{\min i : \lceil \tilde{U}_i \rceil^\varepsilon = \lceil \tilde{U}_m \rceil^\varepsilon\}$, and $\bar{N}(U) = \{\max i : \tilde{U}_i > 0\}$.

From the lemma we know that no party makes a wasteful offer during the game. We now use the fact that no wasteful offers are made to deduce that offers are made only to voters between $\hat{m}$ and $\hat{n}$. If $U_m > 0$, then there are three basic possibilities. If it is X's turn, then X quits. If it is Y's turn, Y can make an ineffective offer, c_i^Y, so that it remains the case that the median voter prefers X, that is, $U_i' = U_i - c_i^Y$ where $U_m' > 0$, so that X wins. The third possibility is that Y makes an effective offer, so that $U_m' < 0$. In this case, if Y's offer is not wasteful, then the following claims hold.

Claim 1. Y makes positive offers, $c_i^Y > 0$, only to $\hat{N}(U) - m + 1$ voters, and each voter i receiving an offer satisfies $-\varepsilon \leq U_i \leq \lceil U_m \rceil^\varepsilon$.

This implies that if $\lceil U_i \rceil^\varepsilon > \lceil U_m \rceil^\varepsilon$, then $c_i^Y = 0$; hence if $\lceil U_i' \rceil^\varepsilon = \lceil U_i \rceil^\varepsilon - c_i^Y$ is reordered to be decreasing, these individuals remain before $\hat{M}(U)$. It also implies that if $U_i < -\varepsilon$, then $c_i^Y = 0$; hence if $\lceil U_i' \rceil^\varepsilon = \lceil U_i \rceil^\varepsilon - c_i^Y$ is reordered to be decreasing, these individuals remain after $\hat{N}(U)$.

Claim 2. If $c_i^Y > 0$, then $\lceil U_m \rceil^\varepsilon \geq \lceil U_i' \rceil^\varepsilon = \lceil U_i \rceil^\varepsilon - c_i^Y \geq \max\{\lceil U_i \rceil^\varepsilon : U_i < -\varepsilon\}$.

The above two properties imply that for individuals who get positive offers, the advantage of X before and after Y's offers is in the range $[-\varepsilon, \lceil U_m \rceil^\varepsilon]$. They also imply $\hat{M}(U') \geq \hat{M}(U)$ and $\hat{N}(U') \leq \hat{N}(U)$.

Proof of claim 1. If Y makes an offer c_i^Y to any i where $c_i^Y < \lceil U_i \rceil^\varepsilon$, then it is wasteful. The reason is that this voter continues to prefer X and so does not increase the amount that X needs to spend to get a majority and does not help Y obtain a majority.

Making an offer to more than $\hat{N}(U) - m + 1$ voters is wasteful because X need not buy them all back and not all were needed to obtain a majority. Specifically, if instead Y did not make an offer to any one of them, then the amount offered would decrease, but $\bar{U} + \bar{U}'$ (the amount required to obtain a majority by Y, plus the amount that subsequently X is forced to spend to obtain a majority) is unchanged.

If Y makes an offer to $\bar{\imath}$ with $U_{\bar{\imath}} > \lceil U_m \rceil^\varepsilon$, consider the alternative in which instead Y makes the offer of $c_{i'}^Y = c_{\bar{\imath}}^Y - (\lceil U_{\bar{\imath}} \rceil^\varepsilon - \lceil U_m \rceil^\varepsilon) + \varepsilon \leq c_{\bar{\imath}}^Y$ to some other voter i' with $\lceil U_m \rceil^\varepsilon \geq U_{i'} > 0$ to whom Y was not making an offer (which exists by the preceding arguments). Then Y obtains a majority, and the amount that X is required to spend to obtain a majority increases. The reason for this increase is that X would have had to offer $\bar{\imath}$ an amount $c_{\bar{\imath}}^Y - U_{\bar{\imath}}$ and has to offer i' an amount $c_{i'}^Y - \lceil U_{i'} \rceil = c_{\bar{\imath}}^Y - (\lceil U_{\bar{\imath}} \rceil^\varepsilon - \lceil U_m \rceil^\varepsilon) + \varepsilon - \lceil U_{i'} \rceil^\varepsilon > c_{\bar{\imath}}^Y - \lceil U_{\bar{\imath}} \rceil^\varepsilon$. So the original offer to $\bar{\imath}$ was wasteful.

Finally, making an offer to $\bar{\imath}$ with $U_{\bar{\imath}} < -\varepsilon$ is wasteful. That such an offer does not help Y obtain a majority is obvious. It also does not increase the subsequent cost to X in obtaining a majority. The minimal cost majority for X will not result in an offer to $\bar{\imath}$ unless $\lceil U_i \rceil^\varepsilon - c_{\bar{\imath}}^Y \leq \lceil U_{\bar{\imath}} \rceil^\varepsilon - c_{\bar{\imath}}^Y$ for all i with $\lceil U_m \rceil^\varepsilon \geq \lceil U_i \rceil^\varepsilon \geq -\varepsilon$, since otherwise X can obtain a majority by promising less to other voters. But if c_i^Y is such that all i with $\lceil U_m \rceil^\varepsilon \geq \lceil U_i \rceil^\varepsilon \geq -\varepsilon$ are brought to $U_i' \leq \lceil U_{\bar{\imath}} \rceil^\varepsilon - c_{\bar{\imath}}^Y$, then $c_i^Y > 0$ for all such i. Now, if for some such i, say $\hat{\imath}$, $\lceil U_{\hat{\imath}} \rceil^\varepsilon - c_{\hat{\imath}}^Y < \lceil U_{\bar{\imath}} \rceil^\varepsilon - c_{\bar{\imath}}^Y$, then X does not make an offer to $\hat{\imath}$. In that case, if Y were to lower the offer to $\hat{\imath}$ to $c_{\hat{\imath}}^{\prime Y}$ (so that $\lceil U_{\hat{\imath}} \rceil^\varepsilon - c_{\hat{\imath}}^{\prime Y} < \lceil U_{\bar{\imath}} \rceil^\varepsilon - c_{\bar{\imath}}^Y$), then the cost to X in obtaining a subsequent majority would not change, and the cost to Y would be lower. Hence in that case the offer is wasteful. If there is no such $\hat{\imath}$, then for all i with $\lceil U_m \rceil^\varepsilon \geq \lceil U_i \rceil^\varepsilon \geq -\varepsilon$, we have $\lceil U_i \rceil^\varepsilon - c_i^Y = \lceil U_{\bar{\imath}} \rceil^\varepsilon - c_{\bar{\imath}}^Y$. In that case not making the offer to $\bar{\imath}$ will not affect the amount X must offer to obtain a majority. QED

Proof of claim 2. The first inequality follows because offers by Y decrease U and are made only to the set of voters who receive positive offers as characterized in claim 1. The second inequality follows because making an offer that leads to $U_{\bar{\imath}}' < \max\{\lceil U_i \rceil^\varepsilon : U_i < -\varepsilon\}$ implies that the least-expensive way for X to obtain a majority will involve X not making an offer to $\bar{\imath}$ (since making an offer to $\arg\max\{\lceil U_i \rceil^\varepsilon : U_i < -\varepsilon\}$ is less expensive). But then if Y decreases the offer to $\bar{\imath}$, the cost of offers decreases, with Y retaining the majority and no change in the minimal cost for X to subsequently obtain a majority. QED

The above properties jointly imply that after a move by Y leading to U' from U, then a reordering of U' and U as decreasing functions has them coincide where either has values above $\lceil U_m \rceil^\varepsilon$ or below $-\varepsilon$; hence when one is in between those values, so is the other. For similar reasons the same is true after a move by X, which implies that this holds throughout the process: the only offers are made to individuals with values in the intermediate group. This concludes the proof of part 4, and hence of proposition 2. QED

Proof of Proposition 3

The proof is based on the following lemma.

LEMMA 4. Consider a subgame starting with a move by party i. If i increases its standing offer with positive probability, then it must be that in the equilibrium continuation $j \neq i$ drops out with positive probability at the next node.

Note that this implies that, in any equilibrium, the only node on the equilibrium path where the current bidder (if he has not won already) has a strictly positive expected payoff is the first node. Note also that if the bidding were to continue past the first node, it must involve mixing or dropping out completely at any subsequent node on the equilibrium path.

Proof of lemma 4. Suppose to the contrary that j stays in at the next move for sure. Let us go to the first subsequent node where some agent drops out with positive probability (such a node exists since the value of the infinite play is $-\infty$). That bidder must have zero expected utility at that node. That node is reached with probability one on the continuation. If that bidder is i, then i has a negative expected utility conditional on making a bid now. If that bidder is j, then j has a negative expected utility conditional on making a bid at the next turn. Thus, we reach a contradiction in both cases. QED

Assume that $W^Y > W^X$, and let ε be sufficiently small to satisfy $W^Y > W^X + (m+1)\varepsilon$.

Let p^l denote the last price offered by $l = X, Y$. We first prove that if $p^Y > p^X$, then X quits. This is obviously true if $p^Y > W^X$. Suppose, therefore, that $p^Y < W^X$. The proof proceeds by induction as follows.

Observe that when $mp^Y > (m-1)p^X + W^X$, X quits since beating Y would require X to increase its commitment by $mp^Y - (m-1)p^X > W^X$.

We next establish that if X quits when $p^Y > p^X$ and $mp^Y > (m-1)p^X + k\varepsilon$, then X also quits when $p^Y > p^X$ and $mp^Y > (m-1)p^X + (k-1)\varepsilon$.

So, suppose that $p^Y > p^X$ and $mp^Y > (m-1)p^X + (k-1)\varepsilon$. Let q^l denote the next price offer by party $l = X, Y$. Clearly,

$$mq^X \leq (m-1)p^X + W^X \tag{A1}$$

since otherwise it is better for X to quit. If we substitute from $mp^Y > (m-1)p^X + (k-1)\varepsilon$, it follows that

$$mq^X < mp^Y - (k-1)\varepsilon + W^X \tag{A2}$$

or

$$mp^Y > mq^X - W^X + (k-1)\varepsilon. \tag{A3}$$

Consider now a price offer q^Y that responds to q^X by increasing Y's total commitment by more than $W^Y - m\varepsilon$ but less than W^Y. That is, q^Y satisfies

$$(m-1)p^Y + W^Y - m\varepsilon \leq mq^Y < (m-1)p^Y + W^Y. \tag{A4}$$

We have

$$mq^X \leq (m-1)p^X + W^X < (m-1)p^Y + W^Y - (m+1)\varepsilon \leq mq^Y,$$

and hence $q^Y > q^X$, where the first inequality follows from (A1), the second from the hypotheses $W^Y > W^X + (m+1)\varepsilon$ and $p^Y > p^X$, and the third from (A4). We also have

$$\begin{aligned} mq^Y &\geq (m-1)p^Y + W^Y - m\varepsilon = mp^Y + W^Y - m\varepsilon - p^Y \\ &> mq^X - p^Y - W^X + (k-1)\varepsilon + W^Y - m\varepsilon \\ &\geq (m-1)q^X + (k-1)\varepsilon + (W^Y - W^X) - m\varepsilon > (m-1)q^X + k\varepsilon, \end{aligned}$$

where the second inequality follows from (A3), the third from $q^X \geq p^Y$, and the fourth from the hypothesis $W^Y > W^X + (m+1)\varepsilon$. Now, by the inductive hypothesis, $q^Y > q^X$ and $mq^Y > (m-1)q^X + k\varepsilon$ leads X to quit. This implies that X does not win with positive probability after q^X: if this were the case, then by lemma 4 Y must quit with positive probability after q^X. But as we have just seen, Y can do better. Thus, it has been established by induction that if $p^Y > p^X$, then X quits.

In any equilibrium party, X's first price offer must be no more than W^X/m. But if party Y responds by offering $p^Y = p^X + \varepsilon$, it follows from the above that it will win, and this will be profitable for Y since $mp^Y = m(p^X + \varepsilon) \leq W^X + m\varepsilon < W^Y$. Therefore, if party X moves first, it will offer 0 in equilibrium. If Y moves first, it will offer price ε and X will not match.

The case of $W^Y < W^X$ is almost identical. QED

Proof of Proposition 4

Without loss of generality, suppose that X has to move first. Consider an auxiliary game in which X's initial price offers are restricted to be either 0 or $\bar{p}^X$ such that $m\bar{p}^X \geq \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, and select a PBE of this game. Existence can be seen as follows. The only weakly dominated strategies for a type are to increase its bid by more than its value. So consider an extensive-form game with such strategies removed for all types. What remains is a finite extensive-form game, so a PBE exists. Consider extending a PBE of that game in any way to the original game (which only means describing continuation strategies at nodes precluded by one's own earlier actions). This will be an undominated PBE of the original game.

Construct now an undominated PBE for the full game as follows. If $W^X \leq \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, then X yields immediately. If $W^X > \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, then X follows the equilibrium strategy of the auxiliary game. If p^X is such that $mp^X < \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, then Y's belief is that W^X is the maximum between W^X and the smallest value of U greater than mp^X, both of which are below $\tilde{W}^Y - \varepsilon$, and Y plays the same strategy it would play in the equilibrium of the complete-information game against that type of X. If p^X is such that $mp^X \geq \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, then Y's belief and strategy are the same as in the selected equilibrium of the auxiliary game. By construction, in either case Y's behavior is a best response to its beliefs. To establish that this is an equilibrium, it has to be shown that, if $W^X > \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, it is not beneficial to X to deviate to $\underline{p}^X \geq \varepsilon$ such that $m\underline{p}^X < \lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$. Observe that, after $\underline{p}^X$, X's payoff is at most

$$(1-\eta)(W^X - m\underline{p}^X - \tilde{W}^Y) + \eta W^X \leq (1-\eta)(W^X - m\varepsilon - \tilde{W}^Y) + \eta W^X \quad \text{(A5)}$$

since in the event $W = \tilde{W}^Y$, which occurs with probability $1-\eta$, Y will continue after $\underline{p}^X$ under the belief that $W^X < \tilde{W}^Y - \varepsilon$, and in order to win, X will have to increase its bid later by at least $\lfloor \tilde{W}^Y \rfloor_\varepsilon$.

If instead X offers $\lceil \tilde{W}^Y \rceil^\varepsilon - m\varepsilon$, its payoff will be at least

$$(1-\eta)(W^X - \lceil \tilde{W}^Y \rceil^\varepsilon + m\varepsilon) + \eta(-\lceil \tilde{W}^Y \rceil^\varepsilon + m\varepsilon) \tag{A6}$$

since in the event $W = \tilde{W}^Y$, X will win immediately.

Clearly when η is sufficiently small, (A6) is larger than (A5). Hence, the above construction is indeed an equilibrium. It is immediate that when η is sufficiently small, the equilibrium outcome is near the complete-information outcome. QED

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