## Voting on the Budget Deficit

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VOTING ON THE BUDGET DEFICIT

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## VOTING ON THE BUDGET DEFICIT

## ABSTRACT

This paper analyzes a model in which different rational individuals vote over the composition and time profile of public spending. Potential disagreement between current and future majorities generates instability in the social choice function that aggregates individual preferences. In equilibrium a majority of the voters may favor a budget deficit. The size of the deficit under majority rule tends to be larger the greater is the polarization between current and potential future majorities. The paper also shows that the ex-ante efficient equilibrium of this model involves a balanced budget. A balanced budget amendment, however, is not durable under mafority rule.

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## 1. INTRODUCTION

Opinion polls show that American voters are well aware of the Federal budget deficit, and disapprove of it. However, there are clear indications that it is politically very difficult to reach an agreement about how to balance the budget. In particular, several polls show that even though voters dislike deficits, they are not in favor of any specific measure which would reduce them. 1

Two explanations for this apparent inconsistency of opinions are commonly proposed. One is that voters do not understand the concept of budget constraint, and suffer from "fiscal illusion". However, it is quite difficult to reconcile this notion with the standard assumption of individual rationality. ${ }^{2}$ The other is that disagreement amongst voters generates cycling and prevents the existence of a stable majority. As a result, individual preferences about intertemporal fiscal policy cannot be aggregated in a consistent way, and no action can be taken to balance the budget. However, this argument fails to explain why the absence of a stable political equilibrium should result in a budget deficit, rather than in a surplus or a balanced budget: in principle, it seems than any outcome could be observed if the political equilibrium is indeterminate.

This paper provides an alternative explanation of budget deficits which is not based on either individual irrationality or non existence of equilibrium. The central ingredient of our explanation is the inability of current voters to bind the choices of future voters. This inability, coupled with potential disagreement between the current and future majorities, introduces a time inconsistency in the dynamic social choice problem that determines the size of budget deficits: the policies desired by the current majority would not be carried out if future majorities exhibit different preferences. The awareness of this possibility may induce the current majority to run a budget deficit in excess of what would be ex-ante optimal for society as a whole. This explains why it is hard to agree on how to eliminate deficits, even if there is a consensus that they may be socially sub-optimal.

Our results have a simple economic intuition. Consider a rational voter who is presented with a number of options on how to allocate the budget amongst alternative uses. Suppose further that he is uncertain about how
future majorities would choose amongst these same options in subsequent periods. In such a situation, the rational current voter may be in favor of budget deficits. The costs of running current deficits are not fully internalized by today's voter, not because of his irrationality, but because of his awareness that future policy choices might not reflect his preferences. In other words, the expected marginal disutility of having to reduce spending in the future, to repay the debt incurred today, is not sufficiently high. As a result, he votes in favor of fiscal deficits.

The paper also shows that, in this model, a balanced-budget is the exante optimal policy. That is, if the size of the deficit is chosen behind a "veil of ignorance" on how the debt proceeds are spent, then the voters are unanimous in choosing a balanced-budget. This implies that current voters would like to precommit governments in the distant future to a balanced budget rule. However, no current majority wants the rule to be binding on itself. Thus, a balanced-budget rule is enforceable only if a qualified majority is required to abrogate it. This constraint may imply a suboptimal lack of flexibility in reacting to unexpected events. Therefore, in this situation as in many others, society has to choose on the tradeoff between "rules and discretion."

Our results are related to those of other papers on intertemporal politico-economic models of fiscal policy. In particular, Alesina-Tabelini (1987a) and Tabellini (1987) analyze a general equilibrium model in which two ideologically motivated parties randomly alternate in office and disagree on the optimal composition of public spending. Alesina-Tabellini (1987b) study a similar problem for an open economy in which the disagreement is about the level of transfers and taxation of different constituencies. Persson-Svensson (1987) consider a "conservative" government who is certain to be replaced in the future by a more "liberal" successor. In these earlier papers as in the present one, public debt is a strategic variable used by the current policymaker to influence the actions of future policymakers. In this earlier work, however, either the political equilibrium was exogenously given, or voters had to choose between two alternative policies, presented to them by two ideological candidates. In equilibrium both candidates chose the same deficit, even though they chose a different level and/or composition of government spending and taxation. Thus, voters did not have a choice on the deficit. The present paper improves the characterization of the political
equilibrium by assuming that there are no constraints on the policy options that can be voted on. In particular, here voters explicitly vote on the deficit. To put it differently, in this paper there is "free entry" of candidates with different policy proposals.

The idea that state variables can be used to influence future voting outcomes can provide important insights in understanding other public choice problems, besides those concerning budget deficits. For example, Glazer (1987) exploits this insight to investigate the choice of durability in public capital projects. He shows that uncertainty about future voting outcomes generates a bias towards overinvesting in long run projects. Several other potentially fruitful applications of this idea come to mind, such as to provatization decisions or to defense policy issues.

Finally, it should also be noted that our argument is markedly different from the idea that deficits occur because the current generation does not internalize the costs of future generations: in our model everybody has the same time horizon. Cukierman-Meltzer (1986) provide a different political explanation of budget deficits, based upon intergenerational redistributions. They show that today's voters choose deficits to tilt the time profile of disposable income in their favor. These two approaches are by no means contradictory, and they can both contribute to a politico-economic theory of budget deficits based upon general equilibrium optimizing models.

The rest of the paper is organized as follows. Section 2 describes the basic model. The political equilibrium is computed in Section 3. Section 4 discusses the implications of these results for the issue of budget balance amendments. The last section briefly summarizes the results and suggests several extensions.

## 2. THE MODEL

A group of heterogeneous individuals has to decide by majority rule on the production of two kinds of public goods, $g$ and $f$. The group is endowed with one unit of output in each period, and it can borrow or lend to the rest of the world at a given real interest rate, for notational simplicity taken to be 0 . The world lasts two periods, and all debt outstanding has to be repaid in full at the end of the second period. Thus, the group faces the intertemporal constraint:

$$
\begin{equation*}
\alpha_{2}^{m} u^{\prime}\left(g_{2}\right)-\left(1-a_{2}^{m}\right) u^{\prime}\left(1-b-g_{2}\right)=0 \tag{3}
\end{equation*}
$$

Equations (3) and (1b) implicitly define the equilibrium values $g_{2}^{*}$ and $f_{2}^{*}$ as a function of $a_{2}^{m}$ and $b$. Let us indicate these functions as $g_{2}^{*}=G\left(a_{2}^{m}, b\right)$ and $f_{2}^{*}=1-b-g_{2}^{*} \equiv F\left(\alpha_{2}^{m}, b\right)$. Applying the implicit function theorem to (3) and (1b), it can be shown that, for $1>\alpha_{2}^{m}>0, G_{\alpha}=-F_{\alpha}>0$, and $-1<G_{b}<0,-1<F_{b}<0$, where $G_{\alpha}, G_{b}, F_{\alpha}$ and $F_{b}$ denote the partial derivatives of $G(\cdot)$ and $F(\cdot)$ with respect to $\mathrm{a}_{2}^{\mathrm{m}}$ and b respectively. It can also be shown that:

$$
\begin{equation*}
G_{b}=\frac{-R\left(f_{2}\right)}{R\left(g_{2}\right)+R\left(f_{2}\right)} \quad ; \quad F_{b}=\frac{-R\left(g_{2}\right)}{R\left(g_{2}\right)+R\left(f_{2}\right)} \tag{4}
\end{equation*}
$$

where $R(\cdot)$ is the coefficient of absolute risk aversion of $u(\cdot)$; thus $R(\cdot)=$ $-u^{\prime \prime}(\cdot) / u^{\prime}(\cdot)$.

For future reference, this interior optimum is illustrated in the diagram of Figure 1. The downward sloping line denotes the opportunity set faced by the median voter in period 2, as a function of the debt inherited from the past, b. The equilibrium in period 2 occurs at point $E$, where the median voter's indifference curve is tangent to his budget line. The upwards sloping line $E P_{2}$ is the median voter's income expansion path. It traces out the equilibrium combinations of $g_{2}$ and $f_{2}$ as $b$ varies. The income expansion path is not necessarily linear: Using (4), its slope is given by $R\left(\mathrm{~g}_{2}^{*}\right) / R\left(\mathrm{f}_{2}^{*}\right)$. With decreasing absolute risk aversion ( $R^{\prime}<0$ ), this slope is greater than 1 if $E$ lies above the $45^{\circ}$ line (i.e., if $a_{2}^{m}>\frac{1}{2}$, as in Figure 1 ); it is smaller than 1 if $E$ lies below the $45^{\circ}$ line (i.e., if $\alpha_{2}^{\mathrm{m}}>\frac{1}{2}$ ). The opposite holds if $u(\cdot)$ exhibits increasing absolute risk aversion. Thus, except in the limit case of constant absolute risk aversion, the equilibrium composition of public spending is affected by the size of debt inherited from the past, b. In the plausible case of decreasing absolute risk aversion, a larger value of $b$ implies a more balanced composition of $g_{2}^{*}$ and $f_{2}^{*}$, for any given value of the median voter preferences (except in the case $\alpha_{2}^{m}=\frac{1}{2}$ ). Conversely, a smaller value of $b$ drives the equilibrium away from the $45^{\circ}$ line, and hence brings about a more unbalanced composition of $g_{2}^{*}$ and $f_{2}^{*}$, for any given $a_{2}^{m} \neq \frac{1}{2}$. Throughout the paper, we define a more (less) balanced, composition as one defined by a point in Figure 1, less (more) distant from the $45^{\circ}$ line. This
effect of pubiic debt on the second period equilibrium composition of expenditures piays a major role in the next section, where the incentives to issue public debt in the first period are analyzed.

Finally, if instead $\alpha_{2}^{m}=1$ or $\alpha_{2}^{m}=0$, then the median voter of period 2 is at a corner. He sets $g_{2}^{*}=1-b$ and $f_{2}^{*}=0$ if $\alpha_{2}^{m}=1$; and conversely he sets $G_{2}^{*}=0$ and $f_{2}^{*}=1-b$ if $a_{2}^{m}=0$. Thus, if $\alpha_{2}^{m}=1$, we al so have $G_{b}=-1$ and $F_{b}=0$ and if $a_{2}^{m}=0$, we have $G_{b}=0$ and $F_{b}=-1$. In this case, the income expansion path is given by the horizontal or vertical axis of Figure 1 , if $a_{2}^{m}=1$ or $a_{2}^{m}=0$, respectively.

### 3.2 The First Period: Preliminary Results

In period $l$ there is uncertainty about the identity of the median voter of period 2. As a result, from the point of view of the voters in period 1, the parameter $\alpha_{2}^{m}$ in (3) is a random variable. The policy most preferred by the median voter of period 1 (whose preferences are denoted by $\alpha_{\uparrow}^{m}$ ) can be found by solving the following optimization problem, where $E(\cdot)$ is the expectation operator over the random variable $a_{2}$.

$$
\operatorname{Max}_{g_{1}, b}\left\{\alpha_{1}^{m} u\left(g_{1}\right)+\left(i-\alpha_{1}^{m}\right) u\left(i-g_{1}+b\right)+E\left[a_{;}^{m} u\left(G\left(\alpha_{2}^{m}, b\right)\right)+\left(1-\alpha_{1}^{m}\right) u\left(F\left(a_{2}^{m}, b\right)\right)\right]\right\}
$$

The current median voter maximizes an expected utility function, since he is aware that, in the subsequent period, $g_{2}$ and $f_{2}$ will be chosen by a median voter with possibly different preferences. The expectation operator is taken with respect to all possible types of future median voters, knowing how each type would behave. Thus, today's median voter chooses the value of the state variable $b$ so as to influence the policy choices of future median voters, based on the distribution of the future preference parameter, $a_{2}$. 5

The median voter of period 1 makes two choices: he chooses the composition of public goods in period 1 and the amount of government borrowing (lending). If,$>a_{1}^{m}>0$, the first order condition relative to $g_{1}$ is:

$$
\begin{equation*}
a_{1}^{m} u^{\prime}\left(g_{1}\right)-\left(1-a_{1}^{m}\right) u^{\prime}\left(1+b-g_{1}\right)=0 \tag{6}
\end{equation*}
$$

Equation (6) implicitly defines the optimal values $g_{1}^{*}$ and $f_{1}^{*}$, as a function of $\alpha_{1}^{m}$ and $b: g_{1}^{*}=g\left(a_{q}^{m}, b\right), f_{q}^{*}=f\left(a_{q}^{m}, b\right)$. Using the same notation as before, it


EIGURE 1
can be shown that, for $1>\alpha_{1}^{m}>0,1>g_{b}>0$ and $f_{b}=1-g_{b}$. If instead $\alpha_{1}^{m}=1$ (or $a_{1}^{m}=0$ ), then the median voter in period 1 is at a corner and chooses respectively $f_{1}^{*}=0$ and $g_{1}^{*}=1+b\left(o r g_{1}^{*}=0\right.$ and $f_{1}^{*}=1+b$ ).

His intertemporal choice is described by the first order condition of problem (5) relative to $b$, which, for $b<1$, is:

$$
\begin{equation*}
\alpha_{1}^{m} u^{\prime}\left(g\left(\alpha_{1}^{m}, b\right)\right)+E\left[\alpha_{1}^{m} u^{\prime}\left(G\left(\alpha_{2}^{m}, b\right)\right) G_{b}+\left(1-\alpha_{1}^{m}\right) u^{\prime}\left(F\left(\alpha_{2}^{m}, b\right)\right) F_{b}\right]=0 \tag{7}
\end{equation*}
$$

where it is understood that $G_{b}$ and $F_{b}$ are functions of $\alpha_{2}^{m}$ and $b$. Despite the concavity of $u(\cdot)$, the second order conditions are not satisfied unless an additional, very mild, condition is imposed. We assume throughout the paper that such a condition is always satisfied for any value of $\alpha_{2}^{m}$ and $\alpha_{1}^{m} .6$

The interpretation of (7) is straightforward. The first term on the left hand side is the marginal gain of issuing one more unit of debt; at the optimum, this must coincide with the marginal utility of spending one extra unit on either of the two public goods (good $g$ in (7)). The second term of (7) is the expected marginal disutility of having to repay the debt, by curtailing public spending tomorrow. This in turn is computed by taking into account that the future composition of public spending depends on the random parameter $\alpha_{2}^{m}$. The solution to equation (7) determines the equilibrium value of debt, $b^{*}$, chosen by the median voter in period 1.

In order to assess the sign of $b^{*}$, in the next subsection we consider equation (7) at the point $b=0$. If at this point equation (7) is satisfied, then $b^{*}=0$. If instead at $b=0$ the left hand side of (7) is positive, then by the second order condition we know that in equilibrium $b^{*}>0$. And conversely, if at $b=0$ the left hand side of (7) is negative, then the second order conditions imply $b^{*}<0$.

### 3.3 The Equilibrium Level of Debt

First of all, consider what happens if the median voter at time 1 is certain that he will also be the median voter in period 2 (i.e., if $\alpha_{1}^{m}=\alpha_{2}^{m}$ with certainty). In this case, the second term in (7) reduces to $\alpha_{1}^{m} u^{\prime}\left(G\left(\alpha_{1}^{m}, b\right)\right.$, so that $b^{*}=0$ is the only solution to (7) for any value of $\alpha_{1}^{m}$. This should come as no surprise: since its rate of time discount coincides with the real interest rate (they are both zero), in the absence of political uncertainty the median voter chooses to spend an equal aggregate
amount in both periods. It is easy to show that $b^{*}=0$ is also the policy that would be chosen by a benevolent social planner maximizing a weighted sum of individual utilities (for any choice of weights). Thus, with no uncertainty and no disagreement between current and future majorities, the political equilibrium lies on the Pareto frontier. ${ }^{7}$

The remainder of this section investigates the case in which $\alpha_{2}^{m} \neq \alpha_{1}^{m}$ with positive probability. It is convenient to break down the second term on the left hand side of (7) into the weighted average of two conditional expectations: the expectation conditional on the event that $1>a_{2}^{m}>0$; and the expectation conditional on the event that $a_{2}^{m}=1$ or $\alpha_{2}^{m}=0$.

Consider this second case first. Thus, suppose that future median voters are expected to always be at a corner, so that they produce only one kind of public good (only $g_{2}$ if $\alpha_{2}^{m}=1$, and only $f_{2}$ if $\alpha_{2}^{m}=0$ ). Suppose further that $\alpha_{2}^{m} \neq \alpha_{1}^{m}$ with positive probability. We have:

## Proposition 1

If either $a_{2}^{m}=0$ or $a_{2}^{m}=1$, then $b^{*}>0$. Moreover, $b^{*}$ tends to be greater the larger is the difference between $\alpha_{1}$ and the expected value of $\alpha_{2}^{m}$.

## Proof:

Let $\alpha_{2}^{m}=1$ with probability $\pi$ and $\alpha_{2}^{m}=0$ with probability $1-\pi, 1>\pi>0$. Then, using (6), equation (7) can be rewritten as:

$$
\begin{equation*}
\alpha_{1}^{m} u^{\prime}\left(g_{1}^{*}\right)-\bar{a} u^{\prime}(1-b)=\left(1-\alpha_{1}^{m}\right) u^{\prime}\left(f_{1}^{*}\right)-\bar{a} u^{\prime}(1-b)=0 \tag{8}
\end{equation*}
$$

where $\bar{a}=\alpha_{1}^{m} \pi+(1-\pi)\left(1-\alpha_{1}^{m}\right)$. Clearly, $\bar{\alpha} \leq \operatorname{Max}\left(\alpha_{1}^{m},\left(1-\alpha_{1}^{m}\right)\right)$, with strict inequality if $\alpha_{1}^{m} \neq \frac{1}{2}$. Moreover, at the point $b=0, u^{\prime}(1-b) \leq u^{\prime}\left(g\left(\alpha_{1}^{m}, b\right)\right)$ and $u^{\prime}(1-b) \leq u^{\prime}\left(f\left(\alpha_{1}^{m}, b\right)\right)$, with strict inequality if $1>\alpha_{1}^{m}>0$. Hence, at the point $b=0$ the two left hand sides of (8) are always strictly positive. As argued above, by the second order conditions this implies that $b^{*}>0$.

In order to prove the second part of the Proposition, note that the expected value of $a_{2}^{m}$ here is just $\pi$. Fix $a_{1}^{m}$, and consider $b^{*}$ as a function of $\pi$. We have:

$$
\begin{equation*}
\frac{d b^{*}}{d \pi}=\frac{d b^{*}}{d \tilde{\alpha}} \frac{d \tilde{\alpha}}{d \pi}=\frac{d b^{*}}{d \dot{a}}\left(2 \alpha_{1}^{m}-1\right) \tag{9}
\end{equation*}
$$

Applying the implicit function theorem to (8), we obtain that $\frac{d b^{*}}{d \alpha}<0$. Hence,

$$
\begin{equation*}
\frac{d b^{*}}{d \pi} \frac{1}{3} 0 \quad \text { as } \quad a_{1}^{m} \frac{\geq}{<} \frac{1}{2} \tag{10}
\end{equation*}
$$

Thus, if $a_{1}^{m}>\frac{1}{2}$, a lower value of $\pi$ (a higher likelinood that $\alpha_{2}^{m}=0$ ) increases $b^{*}$. And conversely, if $a_{1}^{m}<\frac{1}{2}$, a higher value of $\pi$ (a higher likelihood that $a_{2}^{m}=1$ ) also increases $b^{*}$. Hence, $b^{*}$ tends to be larger when the difference between $a_{1}^{m}$ and the expected value of $\alpha_{2}^{m}$ is greater. Q.E.D.

This result has a simple intuition. An increase in debt today implies a reduction of aggregate spending tomorrow. But since $a_{2}^{m}$ lies outside the $(0,1)$ interval, tomorrow only one kind of public good will be provided. Hence, with positive probability (and with probability 1 if $1>a_{1}^{m}>0$ ), this reduction of spending will affect only the good that will have a low marginal utility from the point of view of today's median voter. Thus, the median voter of period 1 does not fully internalize the cost of issuing debt: he finds it optimal to spend in excess of the current aggregate endowment. Moreover, this incentive to borrow is stronger the lower is the marginal utility of the future public. good. This is more likely to happen if the future median oter is more likely to exhibit very different tastes from the current median voter. That is, if $\alpha_{1}^{m}$ is large and the probability of having $\alpha_{2}^{m}=1$ is small, or viceversa.

Next, consider the case in which $\alpha_{2}^{m}$ lies in the open ( 0,1 ) interval. With no loss of generality, suppose further that over this interval $a_{2}^{m}$ is distributed according to a continuous probability function $H(\cdot)$, where $H(a) \equiv$ prob ( $\alpha_{2}^{m} \leq \alpha$ ). Then (7) can be rewritten as:

$$
\begin{equation*}
\int_{0}^{1}\left[\alpha_{1}^{m} u^{\prime}\left(g_{1}^{*}\right)-v\left(\alpha_{2}^{m}\right)\right] d H\left(\alpha_{2}^{m}\right)=0 \tag{11}
\end{equation*}
$$

where by (4) and (7), $v\left(\alpha_{2}^{m}\right)$ is:

$$
\begin{equation*}
v\left(a_{2}^{m}\right) \equiv \frac{a_{1}^{m} u^{\prime}\left(g_{2}^{*}\right) R\left(f_{2}^{*}\right)+\left(1-a_{1}^{m}\right) u^{\prime}\left(f_{2}^{*}\right) R\left(g_{2}^{*}\right)}{R\left(g_{2}^{*}\right)+R\left(f_{2}^{*}\right)} \tag{12}
\end{equation*}
$$

and $g_{2}^{*}=G\left(\alpha_{2}^{m}, b\right), f_{2}^{*}=F\left(a_{2}^{m}, b\right)$. As above, $R(\cdot)$ denotes the coefficient of absolute risk aversion of $u(\cdot)$.

The appendix proves that, at the point $b=0, \alpha_{1}^{m} u^{\prime}\left(f_{1}^{*}\right)-v\left(a_{2}^{m}\right)>0$ for any $\alpha_{2}^{m} \neq a_{1}^{m}$, if the utility function $u(\cdot)$ satisfies the following condition: 8

$$
\begin{equation*}
u^{\prime}(x) / R(x) \text { is increasing in } x \text {, for } 1>x>0 \tag{c}
\end{equation*}
$$

Hence, under this condition, at the point $b=0$ the left hand side of (11) (and hence of (7)) is strictly positive. Thus:

## Proposition 2

Given that $\alpha_{2}^{m} \varepsilon(0,1), b^{*}>0$ if (c) holds

Next, let us define the probability distribution $H\left(\alpha_{2}^{m}\right)$ as "more polarized" than the distribution $K\left(\alpha_{2}^{m}\right)$ if, for any continuous increasing function $f(\cdot)$, the following condition is satisfied:

$$
\begin{equation*}
\int_{0}^{1} f\left(\left|\alpha_{2}^{m}-\alpha_{1}^{m}\right|\right) d H\left(\alpha_{2}^{m}\right)>\int_{0}^{1} f\left(\left|\alpha_{2}^{m}-\alpha_{1}^{m}\right|\right) d K\left(\alpha_{2}^{m}\right) \tag{13}
\end{equation*}
$$

That is, a more polarized probability distribution assigns more weight to values of $\alpha_{2}^{m}$ that are further apart from $\alpha_{1}^{m}$. The appendix also proves that, if condition (c) holds, then for any $b>0$, the expression $\left[\alpha_{1}^{m} u^{\prime}\left(g_{1}^{*}\right)-v\left(\alpha_{2}^{m}\right)\right]$ is an increasing function of $\left|\alpha_{2}^{m}-\alpha_{1}^{m}\right|$ (strictly increasing if $\left|\alpha_{2}^{m}-\alpha_{1}^{m}\right|>0$ ). Then, using (11) and appealing to the second order conditions, we also have: 9

## Proposition 3

Under the same conditions of Proposition $2, b^{*}$ is larger the more polarized is the probability distribution of $\alpha_{2}^{m}$ over the interval ( 0,1 ).

If $u^{\prime}(x) / R(x)$ is everywhere decreasing (constant) for $1>x>0$, then Propositions 2 and 3 hold in reverse: $b^{*}<0\left(b^{*}=0\right)$ and $b^{*}$ is more negative if $H\left(\alpha_{2}^{m}\right)$ is more polarized. If $u^{\prime}(x) / R(x)$ is not monotonic over $1>x>0$, then the sign of $b^{*}$ is ambiguous.

Condition (c) is stronger then decreasing absolute risk aversion: it says that the coefficient of absolute risk aversion, $R$, must fall more rapidly than marginal utility as $x$ increases. Nonetheless, this condition is satisfied for a large class of utility functions, such as any member of the HARA class which also has decreasing absolute risk aversion. This family includes commonly used utility functions, like the power function.

In order to gain an intuitive understanding of the role played by condition (c), consider the diagram of Figure 2. The downward sloping line
denotes the opportunity set faced by the median voters in both periods if $b=0$. A positive value of $b$ shifts this line to the right in period 1 , and to the left in period 2. A and $B$ denote the points chosen in periods 1 and 2 by the median voters of type $a_{1}^{m}$ and $a_{2}^{m}$ respectively, again for $b=0$. For concreteness, it has been assumed that $\alpha_{1}^{m}>\frac{1}{2}>\alpha_{2}^{m}$. The indifference curves for the median voter of type $\alpha_{i}^{m}$ in periods 1 and 2 are labelled I and II respectively. Finally, the upwards sloping lines $E P_{1}$ and $E P_{2}$ denote the income expansion paths of types $a_{1}^{m}$ and $\alpha_{2}^{m}$. As noted in Section 3.1 , their divergent slopes reflects the assumption of decreasing absolute risk aversion. Thus, decreasing absolute risk aversion implies that the divergence between the choices of the two types of median voter (points $A$ and $B$ ) increases with income. To put it differently, with decreasing absolute risk aversion, disagrement concerning the optimal composition of $g$ and $f$ is a luxury good: it grows with the overall size of public spending. 10

The ambiguity in the sign of $b^{*}$ for $1>a_{2}^{m}>0$ is due to the opposite influence of the two following countervailing forces. By running a surplus (by setting $b<0$ ), the median voter in period 1 moves $A$ to the left along $E P$; and $B$ to the right along $E P_{2}$; this has the effect of reducing the distance between the indifference curves labeled I and II. Hence, setting b<0 "buys insurance" for the median voter of period 1 , in the sense that it tends to equalize the level of utility in the two periods. This is the force that works in the direction of making $b<0$ more desirable.

On the other hand, by running a deficit (by setting $b>0$ ), the median voter of period 1 moves $B$ to the left along $E P_{2}$. Since the slopes of $E P_{1}$ and $E P$, are divergent, this has the effect of moving point $B$ closer to point $A$; that is, it moves the future composition of public spending towards the point that is preferred by today's median voter. This is the force that provides the incentive to issue public debt today.

Condition (c) guarantees that this second effect prevails over the first one. This condition is more likely to be satisfied if the slopes of $E P{ }_{1}$ and $E P_{2}$ are very divergent from each other (that is, if the coefficient of absolute risk aversion is decreasing very rapidly); or if the indifference curves are very flat (that is if the utility function is not very concave), because in this case the indifference curves labeled I and II are already close to one another.

Combining the results of Propositions $1-3$, we can conclude that a


EIGURE 2
positive equilibrium level of debt is more likely if: i) the future median voter is likely to have extreme preferences and be at a corner (i.e., $a_{2}^{m} \notin(0,1)$ is likely); or ii) condition (c) on $u(\cdot)$ is likely to be satisfied. Moreover, in both cases, the size of debt is larger the greater is the probability mass assigned to values of $a_{2}^{m}$ that are very different from $a_{1}^{m}$; that is, using the previous terminology, the more polarized is the distribution of the future median voter's preferences.

It is worth noting that, in a sense, (i) is the limit case of case (ii); if the future median voter is at a corner, then its income expansion path coincides with either the vertical or the horizontal axis. With reference to Figure 2, then, leaving some debt to the future has always the effect of moving the future composition of public spending in the desired direction, as in the less extreme case (ii).

Furthermore, in a more general model, the future median voter could find itself at a corner even for values of $\alpha_{2}^{m}$ belonging to the interior of the $[0,1]$ interval. For instance, this would happen if the utility function $u(\cdot)$ did not satisfy the Inada conditions, so that the indifference curve of Figure 1 could intersect either the horizontal or the vertical axis. Alternatively, it could happen if the public goods $g$ and $f$ had to be provided in some minimum positive amounts (for instance, because of survival reasons): in this case too, the future decisionmaker could find itself at a corner with respect to either $g_{2}$ or $f_{2}$, despite the fact that $1>a_{2}^{m}>0$. The expectation of this event, in turn, would induce the current median voter to run a budget deficit, just like in the case of Proposition 1. Obviously, this event would be more likely to happen the closer is $\mathrm{a}_{2}^{m}$ to either 0 or $1 .{ }^{11}$

### 3.4 Positive Implications

Propositions 1-3 relate the size of budget deficits to the instability of the median voter's preferences over time. This type of instability depends upon the distribution of individual preferences within society. In the remainder of this section we show that, the more "homogenous" are the preferences of different individuals, ceteris paribus the more stable are the median voter preferences over time.

Consider a family of density functions indexed by $\varepsilon, \gamma(a, \varepsilon)$. For a given $\varepsilon$, let $\gamma(\alpha, \varepsilon)$ be the frequency distribution of $\alpha$ over the $[0,1]$ interval, where $a$ is the parameter that summarizes the preferences of a specific
individual in equation (2). Different density functions are associated to different values of $\varepsilon$. Thus, $\varepsilon$ can be thought of as a perturbation of the distribution of the voters' preferences, associated with random shocks to the voting participation or to the eligibility of the voting population.

To any realization of $\varepsilon$ is associated a value of the median voter's preferences, $\alpha^{m}(\varepsilon)$, defined implicitly by:

$$
\begin{equation*}
\int_{0}^{\alpha^{m}} r(\alpha, \varepsilon) d \alpha-1 / 2=0 \tag{14}
\end{equation*}
$$

The extent to which $a^{m}$ varies as $\varepsilon$ takes different values depends on the properties of the density function $Y(\alpha, \varepsilon)$. Specifically, applying the implicit function theorem to (14) one obtains:

$$
\begin{equation*}
\frac{d a^{m}}{d \varepsilon}=-\frac{\int_{0}^{a^{m}} Y_{\varepsilon}(\alpha, \varepsilon) d \alpha}{r\left(\alpha^{m}, \varepsilon\right)} \tag{15}
\end{equation*}
$$

where $\gamma_{E}(\alpha, \varepsilon)=\frac{\partial \gamma(\cdot)}{\partial \varepsilon}$. The numerator of (15) is the area underneath the density function that is shifted from one side to the other of $a^{m}$ as $\varepsilon$ varies. According to (15), for a given value of the numerator, the term $\frac{d a^{m}}{d e}$ is larger in absolute value the smaller is $\gamma\left(\alpha^{m}, \varepsilon\right)$. That is, if there are relatively few individuals in the population that share the median voter's preferences (i.e.: if $\gamma\left(\alpha^{m}, \varepsilon\right)$ is small for all $\varepsilon$ ), then $\alpha^{m}$ tends to vary a lot as the distribution is perturbed by random shocks. Conversely, if the median voter preferences are representative of a large part of the population, (i.e.: if $\gamma\left(\alpha^{m}, \varepsilon\right)$ is large , then $\alpha^{m}$ tends to be stable even in the face of large perturbations to the underlying distribution of voters preferences.

This result is illustrated in Figure 3. Consider the top distribution first. When $\varepsilon$ goes from $\varepsilon_{0}$ to $\varepsilon_{1}$, a fraction of individuals corresponding to the area $A$ is moved from the right to the left of $a_{0}^{m}=a^{m}\left(\varepsilon_{0}\right)$, to the area $A^{\prime}$ $=A$. This area is the numerator of (15). The new median voter, $\alpha_{1}^{m}=\alpha^{m}\left(\varepsilon_{j}\right)$, is found by equating the area between $\alpha_{0}^{m}$ and $a_{1}^{m}, B$, to the area $A$. Consider now repeating the same perturbation to the distribution in the bottom of Figure 3. Clearly, the same area $B$ now corresponds to a much larger horizontal distance between $\alpha_{0}^{m}$ and $\alpha_{1}^{m}$ : since the frequency of the population around $\alpha^{m}$ is relatively small, the median voter's preferences here have to shift by much



Figure 3
more than in the case of the top distribution of Figure 3. This is the sense in which a more polarized distribution of societies preferences (such as in the bottom of Figure 3) tends to be associated with more instability and more polarization in the induced probability distribution of the median voter's preferences.

These considerations are suggestive of an empirically testable interpretation for the results of Propositions 1-3. Namely, that more polarized and unstable societies have larger budget deficits than more homogeneous societies. As suggested by the foregoing discussion, in a more polarized and unstable political system there is a higher probability that future majorities will choose policies that are very different from those chosen by today's majority. And zocording to Propositions $1-3$, it is precisely in this case that the current majority is in favor of budget defieits. Moreover, according to Proposition 1, debt is more likely to be issued when the future majorities are likely to have extreme preferences (for then the likelihood of being at a corner is greater). Finally, note that even condivion (c) itself (which implies that debt will always be issued in the presence of political uncertainty) can be interpreted as an instance of polardsation: if the utility function $u(\cdot)$ satisfies condition (c), then different types of median voters have divergent income expansion paths. In this case, current and future majorities tend to choose different policies in the following sense: they allocate changes in their budget (cuts or increments) to very different items. According to Proposition 2, this kind of "incremental" polarization always creates incentives in favor of issuing public debt.

## 4. CONSTITUTIONAL CONSTRAINTS ON THE BUDGET DEFICIT

The previous results state that in equilibrium a majority of the voters may be in favor of a budget deficit. This section investigates tine efficiency properties of this equilibrium.

Section 3.4 showed that a social planner certain of being reappointed would always choose $b^{*}=0$, for any weighting of individual preferences. That is, a balanced budget is always a component of the first best policy. On this ground, it is tempting to conclude that a budget deficit is inefficient in this model. However, this argument would be wrong, or at least misleading.

By assumption, a social planner can precommit to choosing the composition of both periods 1 and 2 public goods according to a stable social welfare function. This assumption is violated in the political equilibrium of the model and in any real world political regime: the current majority cannot precommit the spending choices of future majorities. In other words, the solution to the social planner's optimum is not necessarily the optimal social contract to write for a group of individuals who cannot also precommit the expenditure choices of future governments.

In order to characterize such an optimal social contract consider the following conceptual experiment. Suppose that the collective decision process in period 1 is separated in two stages. First the group chooses a level of debt, b. Then it chooses the composition of the public good in terms of $g_{1}$ and $\mathrm{f}_{1}$. The decision taken at this second stage depends on the value of a for the median voter, $\alpha_{1}^{m}$, as in equation (6). Suppose further that at the first stage, when choosing $b$, the group ignores the value of $\alpha_{1}^{m}$ corresponding to the second stage. That is, suppose that the decision concerning $b$ is taken behind a "veil of ignorance" about the outcome of the second stage. We can think of a constitutional amendment on the size of budget deficits as being chosen in this way. 12 Under these hypothesis, the optimal level of $b$ for agent is determined as the solution to the following problem:

$$
\begin{equation*}
\underset{b}{\operatorname{Max}} E\left\{a^{i}\left[u\left(g\left(a_{1}^{m}, b\right)+u\left(G\left(a_{2}^{m}, b\right)\right)\right]+\left(1-a^{i}\right)\left[u\left(f\left(a_{1}^{m}, b\right)\right)+u\left(F\left(a_{2}^{m}, b\right)\right)\right]\right\}\right. \tag{14}
\end{equation*}
$$

where $E$ is the expectations operator with respect to the random variables $\alpha_{1}^{m}$ and $\alpha_{2}^{m}$; and where $g(\cdot), f(\cdot), G(\cdot)$ are defined implicitly by (6) and (3) of the previous section. If $\alpha_{1}^{m}$ and $\alpha_{2}^{m}$ are drawn from the same prior distribution, then it is easy to show that the only solution to (14) is $b=0$, for any value of $a^{i} .^{13}$ Thus, using the terminology of Holmstrom-Myerson (1983), we can conclude that a balanced budget rule is "ex-ante efficient": before knowing the identity of the current majority, the group is unanimous in favoring a balanced budget amendment. ${ }^{14}$

If however the value of $\alpha_{1}^{m}$ corresponding to the second stage is known when choosing $b$, then we are back in the equilibrium examined in the previous section, where a majority might support a deficit and oppose the balanced
budget amendment. In other words, each current majority generally does not want to be bound by the amendment, even though it wants such an amendment for all future majorities. Thus, such an amendment can be approved only if it does not bind the current majority. However, a budget amendment taking effect at some prespecified future date would be irrelevant: if one needs only a simple majority to abrogate the rule, then any future majority would follow the policy described in Section 3 and would abrogate the amendment. Using again the terminology of Holmstrom-Myerson (1983), we can conclude that a balanced budget amendment, though ex-ante efficient, is not "durable" under majority rule.

This problem could be overcome by requiring a qualified majority to abrogate the amendment. But this requirement would eliminate the flexibility that may be needed to respond to mexpected and exceptional events. Obviously, a budget rule could be contingent on prespecified events, such as cyclical fluctuations of tax revenues or "wars." However, since it is very difficult, or even impossible, to list all the relevant contingencies, it might be desirable to retain some degree of flexibility. Thus, requiring a very large majority to abandon (even semporarily) the budget balance constraint may be counterproductive.

These normative results may contribute to explain the empirical observation that the majority of voters seems to generally favor an abstract notion of balanced budgets, even though when choosing specific policies it votes in favor of budget deficits (see the literature quoted in footnote 1). Balance budgets are ex-ante efficient. Therefore, the majority of voters, asked in a poll if they would like a balanced budget constitutional amendment, would answer "yes." However the same majority of voters may choose to run a budget deficit in the current peiiod, if uncertain about the preferences of future majorities.

More generally, these results suggest the desirability of institutions that would enable society to separate its intertemporal choices from decisions concerning the allocation of resources within any given period. In evaluating such institutions, there seems to be an inescapable conflict between the goal of preserving sufficient flexibility to meet unexpected contingencies, and the constraints imposed by the requirement of enforcing this separation.

## 5. SUMMARY AND EXTENSIONS

This paper shows that disagreement between current and future voters about the composition of public expenditure generates a suboptimal path of public debt. Public debt becomes the legacy left by today's voters to the future, so as to influence the choices of future voters. This legacy tends to increase with the likelihood of disagreement between current and future voters. Thus, this paper establishes a precise link between political polarization and budget deficits. Political polarization can be interpreted as a situation where the preferences of future majorities can be very different from the preferences of today's majority. This can occur if a government with extreme preferences (relative to the historical average) wins the temporary support of a majority of the voters. Alternatively, it can occur in political systems where parties with very different preferences are equally likely to obtain a majority. Thus, the implications of this paper are in principle testable against either time series or cross sectional data. Perhaps the most natural empirical work along these lines would be a cross sectional comparison of the deficit policies of countries governed by different political institutions and with different degrees of political conflict.

The model presented in this paper can be generalized in several directions. First of all, the restriction to only two types of public goods is made only for simplicity. Enlarging the set of public goods presents only one difficulty: it implies an increase of the dimensionality of the parameter space over which different voters disagree. This complicates the description of the collective decision problem. However, this difficulty can be overcome by either imposing special assumptions on the distribution of voters over this larger dimensional space -- for instance by assuming the existence of a "median-in-all-directions", as in Davis-Degroot-Hinich (1972); or by requiring a super-majority vote to change the status-quo, as in Caplin-Nalebuff (1988). Under either of these assumptions, the results of the previous sections could be extended to the case of more than two public goods.

Secondly, the model could be extended to the infinite horizon, by applying the dynamic programming solution procedure presented in AlesinaTabellini (1987a). In an infinite horizon model one could also study the possibility of cooperation between current and future median voters, for
instance based upon trigger strategy equilibria. In these equilibria the path of the public debt could be brought arbitrarily close to the socially efficient value. However, in order to obtain the socially efficient solution one needs cooperation between successive median voters: cooperation amongst different voters within the same time period would not solve the intertemporal distortions that are the focus of this paper. As such, the reputation mechanism that would be needed to enforce cooperation might require substantial amounts of coordination. In addition, with discounting of the future, the qualitative implications of seputational equilibria may be similar to those of the equilibrium studied in the present paper. ${ }^{15}$

Finaliy, a natural and yet difficult extension of the basic model would be to allow the voters to choose mether or not to repudiate the debt. Indeed, the results of this paper are driven by a fundamental asymmetry in the commitment techologies available to the voters: even though current voters cannot bind the spending choices of future majorities, nonetheless they are assumed to be able to force future majorities to honor their debt obligations. This asymmetry seems to faithfully reflect a feature of the real world, at least in industrialized economies during the recent decades. But still, the puzzle remains of what is the source of this asymmetry. Some recent interesting literature has investigated the idea that reputational mechanisms create incentives in favor of honoring the internal debt obligations of previous governments (see in particular Grossman-Van Huyck (1987a)). 16 The political economy approach of this paper suggests a second line of attack: domestic debt repudiation may not be politically viable, because it would be strenuously opposed by the private sector holders of the debt. Recent accounts of historical episodes of debt repayments in Europe during the interwar period lend support to this view (see for instance Alesina (i988b)). Further investigation of this line of thought sets an exciting task for future research.

## FOOTNOTES

1. Both recent polls (New York Times, November 1987) and polls taken in the early eighties (Blinder-Holtz Eakin (1983)) show that a large majority of American voters is in favor of budget balance amendments. A much lower fraction of voters is in favor of any specific measure to reduce budget deficits and there is disagreement on which expenditures (taxes) to reduce (increase), if any.
2. For recent arguments explaining the deficit as the result of "fiscal illusion", see Buchanan et. al. (1987) and the references quoted therein. Rogoff-Sibert (1988) and Rogoff (1987) have shown that suboptimal budget deficits may be observed before elections if voters are rational but imperfectly informed. This mechanism can explain short budget cycles around elections, but it cannot explain long lasting and large budget deficits which go well beyond the electoral cycle.
3. One interpretation of this model is that each member of the group is taxed identically and the total amount of tax revenues is exogenously. given. For a model with similar features, in which distortionary taxes and private consumption are endogenously determined, see Alesina Tabellini (1987a). By following the same procedures used in that paper, the present model can be extended to incorporate endogenous taxation with no qualitative changes in the results.
4. Any expected utility function that is linear in a vector of parameters belongs to this class. Even though linearity was not essential in Grandmont (1978), it is essential here, since we deal with an expected utility function. There are families of preferences which do not admit linear representation and yet are intermediate preferences. However, in the absence of linearity in the vector of parameters, the property of intermediate preferences would not necessarily be preserved by the expectations operator. The essential property of intermediate preferences is that supporters of distinct proposals are divided by a hyperplane in the space of most preferred points. See Grandmont (1978) and also Caplin-Nalebuff (1988).
5. This setting is reminiscent of that analyzed in Strotz (1956), where a consumer with time inconsistent preferences solves a dynamic optimization problem. See also Peleg and Yaari (1973) and the references
quoted therein. In those papers, like here, the time consistent solution is described as the non cooperative equilibritm of a game played by successive decision makers.
6. This second order sufficient condition can be stated as follows:

$$
\begin{aligned}
& R\left(f_{2}\right)^{3} R\left(g_{2}\right)+R\left(g_{2}\right)^{2} R\left(f_{2}\right)^{2}+(i-\gamma) R^{\prime}\left(g_{2}\right) R\left(f_{2}\right)^{2}+ \\
& +\gamma R\left(g_{2}\right)^{3} R\left(f_{2}\right)+\gamma R\left(g_{2}\right)^{2} R\left(f_{2}\right)^{2}+(\gamma-1) R^{\prime}\left(f_{2}\right) R\left(g_{2}\right)^{2}>0
\end{aligned}
$$

where $\quad y=\frac{1-\alpha_{1}^{m}}{\alpha_{1}^{m}} \frac{1-\alpha_{2}^{m}}{\alpha_{2}^{m}}$ and where $R(\cdot)=-u^{\prime \prime}(\cdot) / u^{\prime}(\cdot)$ is the coefficient of absolute risk aversion of $u(\cdot)$. In turn, a sufficient (but not necessary) condition for ( $F, 1$ ) to hold is that:

$$
R\left(f_{2}\right) R\left(g_{2}\right)+R\left(g_{2}\right)^{2}+R^{\prime}\left(g_{2}\right)>0
$$

and

$$
R\left(f_{2}\right) R\left(g_{2}\right)+R\left(f_{2}\right)^{2}+R^{\prime}\left(f_{2}\right)>0
$$

7. In a more general framework, the socially optimal policy might imply running a deficit or a surplus (for instance, to smooth tax distortions over time, as in Barro (1979) and Lucas-Stokey (1983)). Here, for simplicity, we abstract from these complications.
8. This condition can also be stated as:

$$
u^{\prime \prime \prime}(x)>2\left[u^{\prime \prime}(x)\right]^{2 / u '}(x), \quad i>x>0
$$

or

$$
R^{\prime}(x)+R^{2}(x)<0, \quad 1>x>0
$$

9. The same results would go through if the definition of "more polarized" in (13) was stated with respect to other measures of distance between $a_{2}^{T}$ and $\alpha_{1}^{m}$, such as euclidean norm or $\left(\alpha_{1}^{m}-\alpha_{2}^{m}\right)^{2}$.
10. This implication of decreasing absolute risk aversion would remain true even if points $A$ and $B$ lied on the same half of the budget line (that is, if either $\alpha_{1}^{m}, \alpha_{2}^{m}>\frac{1}{2}$, or $\alpha_{1}^{m}, \alpha_{2}^{m}<\frac{1}{2}$ ), as long as $\alpha_{1}^{m} \neq \alpha_{2}^{m}$.
11. These generalizations however would introduce an additional complication. Namely, the probability that the future decisionmaker will be at a corner could now be endogenous, and in particular depend on the borrowing policies of previous governments. This would add another
dimension to the problem of choosing the optimal debt policy.
12. The notion that optimal social contracts may be thought of as bfing chosen under a "veil of ignorance" concerning how the policy game is played in subsequent stages is due to Rawls (1971) and Buchanan-Tullock (1962).
13. If $a_{1}^{m}$ and $a_{2}^{m}$ have the same probability distribution, say $H(\cdot)$, then the first order condition of (14) with respect to $b$ can be written as:

$$
\begin{aligned}
& a^{i} \int_{0}^{1}\left[u^{\prime}(g(a, b)) g_{b}(a, b)+u^{\prime}(G(a, b)) g_{b}(a, b)\right] d H(a)+ \\
& +\left(1-a^{i}\right) \int_{0}^{1}\left[u^{\prime}\left(f(a, b) f_{b}(a, b)+u^{\prime}(F(a, b)) F_{b}(a, b)\right] d H(a)=0\right.
\end{aligned}
$$

It can be shown that if $b=0$, then the terms inside each integral sum to zero. Hence, by the second order conditions, $b=0$ is the solution to (14).
14. Unanimity would be lost if the distributions of $a_{1}^{m}$ and $a_{2}^{m}$ in (14) were different from each other.
15. Alesina (1987), (1988a) investigates the properties of these reputational equilibria in a repeated static game played by two randonly alternating policymakers.
16. A much larger literature has investigated the problem of external det: repudiation, for instance Sachs (1985), Bulow-Rogoff (1987), GrossmariVan Huyck (1987b).

## AFPENDIX

Consider the function $\forall\left(\alpha_{\mathcal{L}}^{m}\right\}$ for a given value of $b$. This function is continuous in $1>a_{2}$ e (sinot w? ? was assumed to be twice continuously differentiabie). After sme algebra, $v^{\prime}\left(a_{2}^{m}\right)$ simplifies to:

whers $\frac{d g_{2}^{*}}{d a_{2}^{m}}>0$ and

$$
\Delta=R\left(g_{2}^{*}\left(R\left(f_{2}^{*}\right)^{2}+g^{\prime}\left(f_{2}^{*}\right)\right]+R\left(f_{2}^{*}\right)\left[R\left(g_{c}^{*}{ }^{*}+R^{\prime}\left(g_{2}^{*}\right)\right]\right.\right.
$$

If $u^{\prime}\left(x^{\prime} R(x)\right.$ is increasing in $x$ for $1>x>0$, (see aiso footnote 8 ), then $\Delta<0$. Hence, for any $b$ :

$$
\begin{equation*}
v^{\prime}\left(a_{2}^{m}\right) \quad \frac{>}{<} 0 \text { as } a_{2}^{m}<a_{1}^{m} \tag{A.2}
\end{equation*}
$$

if (c) hoids. These properties imply that, under (c), $v\left(\alpha_{2}^{m}\right)$ reaches a maximum at the point $\varepsilon_{2}^{m}=a_{1}^{m}$, and is strintly decreasing in $\left|a_{2}^{m}-a_{1}^{m}\right|$ if $a_{2}^{m} \neq a_{1}^{m}$. Hence, for given $a_{1}^{m}$ and given $b$, the expression $\left[\alpha_{1}^{m} u^{\prime}\left(g_{1}\right)-v\left(\alpha_{2}^{m}\right)\right]$ reaches a minimum at $\alpha_{2}^{\frac{\pi}{2}}=\alpha_{1}^{m}$ and is strictiy increasing in $\left|\alpha_{2}^{m}-\alpha_{1}^{m}\right|$ if $a_{2}^{m} \neq \alpha_{1}^{m}$.

Consider now this expression at the point $b=0$. The discussion on $p$. 8 of the text implies that, at $b=0, \alpha_{1}^{m_{1}^{\prime}}\left(g_{1}\right)-v\left(a_{1}^{m}\right)=0$. Since, as shown above, under (c) $a_{1}^{m}=\operatorname{argmin} v\left(\alpha_{2}^{m}\right)$, we have that, if $b=0$ :

$$
\alpha_{1}^{m} u^{\prime}\left(g_{1}\right)-v\left(a_{2}^{m}\right) \geq 0
$$

with strict inequality if $a_{2}^{m} \neq \alpha_{1}^{m}$. Thus, under condition (c) if $u^{\prime}(x) / R(x)$ is increasing in $x, v\left(\alpha_{2}^{\mathrm{m}}\right)$ can be drawn as in the diagram of figure (4).


Figure 4

