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Wage Announcements with a Continuum of Worker Types

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ABSTRACT. – We present models of labor-market discrimination in which identical employers choose among job applicants according to a continuous characteristic such as skin color or worker height. The characteristic in question is assumed to be unrelated to worker productivity. Firms are required to announce wage offers that are not conditioned on the characteristic. Workers apply to firms on the basis of those announcements. Firms rank all applicants with respect to the characteristic and select the most desirable one. We show that in equilibrium all firms will offer the same wage, and workers will apply to each firm with equal probability. All employed workers will receive the same wage, but lower ranked workers will have a higher rate of unemployment than higher ranked workers and will thus have a lower expected income. These results differ from those of directed-search models characterized by a finite number of worker types.

L'affichage d'offres salariales avec continuum de types de travailleurs

RÉSUMÉ. – Nous développons un modèle de la discrimination dans lequel les employeurs identiques choisissent parmi des demandeurs du travail selon une caractéristique continue telle que la couleur de peau ou la taille d'ouvrier. On assume que la caractéristique en question soit indépendante de la productivité de l'ouvrier. Les entreprises ne peuvent pas annoncer des offres de salaire qui sont conditionnées sur la caractéristique. Les ouvriers font demande aux entreprises sur la base de ces annonces. Les entreprises mettent tous les demandeurs en rang selon la caractéristique et choisissent le plus souhaitable. Nous démontrons que dans l'équilibre toutes les entreprises offrent le même salaire, et les ouvriers font demande à chaque entreprise avec probabilité égale. Tous les ouvriers employés reçoivent le même salaire, mais les ouvriers qui sont rangés plus bas ont un taux du chômage plus élevé que celui des ouvriers plus fortement rangés et ont ainsi un revenu prévu inférieur. Ces résultats sont fort différents de ceux des modèles de « recherche dirigée » caractérisés par un nombre fini de types d'ouvrier.

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1 Introduction

Models of job search with wage posting provide a promising path for understanding differences in wages and unemployment rates among similar but not identical workers. For example, LANG, MANOVE and DICKENS ([2002, hereafter LMD) show that small differences in productivity or small discriminatory tastes can generate large differences in outcomes.

Previous research on job search with wage-posting has focussed on the case of discrete types (SHIMER [2001]) and much of the literature is limited to the case of two types (LANG and KAHN [1998], LMD, MOEN [2002]). All of these papers, while able to explain certain anomalous aspects of labor markets also make counterfactual predictions. In particular, under plausible scenarios, Shimer predicts that firms will offer higher wages to less productive workers while LMD and MOEN predict lower unemployment rates for less productive workers.

In this paper, we develop models of wage-posting when there is a continuum of worker types. We find that the results are very different from those obtained in other directed-search models. In particular, in the equilibrium of our model, all firms announce and pay the same wage. This means that all employed workers, regardless of type, earn the same amount, though more favored workers have lower unemployment rates and, thus, higher expected earnings. This finding contrasts sharply with the prediction of the closely related model of LMD, in which there are only two types of workers and in which more favored workers have higher wages and higher unemployment rates.

A fundamental difference between the current models and the LMD model is that in equilibrium LMD produces segregation and the current model, with a continuum of types, does not. This is not surprising, because it is more difficult to find wages that separate workers of many types than it is to find wages that separate workers of only two types. Given that wage offers cannot be conditional on worker type, wage differences between types are not likely to arise without segregation.

Finally, when complete segregation by worker type exists in equilibrium, there can be no competition for employment between different types, so there is no reason why the less preferred types should have higher equilibrium unemployment than do the more preferred types. In fact, given their lower wage, it is reasonable to expect that less preferred types would have lower unemployment rates than do more preferred types of similar productivity. Without segregation in equilibrium, however, different types are competing for the same jobs, and it is to be expected that the less-preferred will suffer more unemployment. Indeed, in LMD, if wages were sufficiently out of equilibrium that blacks and whites applied to the same firms, blacks would also have more unemployment.

In the next section, we review the literature on wage-posting with heterogeneous workers. We argue that all the principal papers, including our own, generate counterfactual predictions and/or are limited in their applicability.

In section 3, we begin our analysis of a continuum of types with the case of two identical firms and two workers whose types are drawn from a continuous distribution. We show that there is a unique equilibrium in which both firms offer the same wage and workers apply to the two firms with equal probability.

In section 4, we extend the model to the case of many firms and workers and show that there is a unique symmetric equilibrium with the same properties, although we are unable to eliminate the possibility of other, asymmetric equilibria.

In our view, neither a model with a small number of discrete types nor a model with continuous types provides a good representation of reality. Describing workers as "blacks and whites" or "men and women" misses important within-group heterogeneity. Yet, even when within-group differences are observed, workers cannot be perfectly ordered in terms of quality. Even for identical jobs, employers may disagree about which applicant is the best. Anyone who has served on a hiring committee will be aware that reaching agreement on who is the best candidate can be extremely difficult.

We conjecture that a more realistic model, either with a large number of discrete types or a continuum of types with imperfect observability of type, would combine elements of the discrete and continuous-type models. We believe that such a model would generate both higher wages and lower unemployment rates for more preferred workers. Unfortunately, we have not been able make this type of model tractable.

In Section 5 we present an example with three types of workers. The example does indeed combine elements of both the continuous and discrete type models. In equilibrium there is a wage that attracts all three types but there are also wages that attract only the middle type and only the low type. The wages and employment rate of the preferred type are always higher than those of the two other types. For some, but not all, parameter values both the wages and employment rates of the middle type are higher than those of the low type.

Thus we believe that the wage-posting model holds considerable promise for explaining wage and employment differentials among groups of workers with similar abilities.

2 Literature Review

The literature on directed search with heterogeneous workers is still in its infancy, and there is no clear consensus on critical modelling decisions. We focus on three models — SHIMER, MOEN, and LMD. In each of these models, play takes a broadly similar form. Firms simultaneously announce wages. Workers observe the wages and decide to which firm to apply. The firm chooses the most productive worker(s) from among its applicants. In all three models a worker's type is observable before hiring takes place.

In LMD, there are only two types and all workers of a given type are equally productive.

The paper focuses on the case where the productivity differential between the types is small. The key feature of LMD that distinguishes it from Shimer is that firms are constrained to offer a single wage that is independent of type. As a result, the less-preferred types know that they will lose out to the more preferred types if they apply for the same job. The less-preferred workers are so averse to competition with the more-preferred workers that firms can attract them most effectively by offering a wage just below what the more preferred workers would accept in equilibrium. Provided that the productivity differential is not too large, this equilibrium will be characterized by complete segregation. Furthermore, the equilibrium is inefficient. Relatively simple policies, such as the enforcement of random hiring from the pool of applicants or a minimum wage would increase efficiency.[1]

In MOEN, as in LMD there are only two types. However, in addition to average productivity differences across types, there are match-specific variations in productivity such that in a particular match a worker from a generally low productivity type may be more productive than a worker from a generally high productivity type. As in LMD, firms are restricted to announce a single wage they will pay to any worker they hire. Again, the equilibrium requires that the two types be segregated provided that the (average) productivity difference is not too large. In contrast with LMD, applicants from the low type may displace high types, but they lower expected productivity conditional on the worker being hired. Put differently, replacing a low type applicant with a high type applicant is profitable, on average, because the productivity of the high type is higher both unconditionally and conditional on being hired. As a result, some firms set a high wage that deters the low types from applying. The incentive compatibility constraint is binding on the low types in MOEN rather than on the high types as in LMD. Again the equilibrium is inefficient. Firms would benefit if they could commit to hiring only one type of worker.

In SHIMER as in LMD, all workers of a given type are equally productive. However, there can be many discrete types, and firms can condition their wage announcement on worker type. In contrast with LMD and MOEN, in the SHIMER model, there cannot be complete segregation and the equilibrium is constrained PARETO efficient.

From a modeling perspective, the fact that LMD and MOEN address only the case of two types of workers is unfortunate. It is also somewhat problematic that firms hire only one worker in LMD and SHIMER, but we find this much less objectionable than the assumption in MOEN that firms are very large. Many job openings are unique and the results in LMD and SHIMER appear to depend on the number of openings being small rather than equal to 1. Perhaps most importantly, MOEN suffers from a paucity of excess empirical content.

The models also have problematic predictions. In LMD, if the productivity of the two types does not differ by too much, the unemployment rate is higher for the preferred type than for the lower type. This problem also arises in MOEN for reasonable distributions of the match-specific productivity. Since the models do not address entry into unemployment, it is probably more reasonable to view them as predicting shorter unemployment duration for less preferred workers. The evidence on this relation is, at best, inconsistent.

1. Note, however, that the minimum wage must be close to the equilibrium wage for the high types. A minimum wage just slightly above the equilibrium wage for the low types is welfare decreasing in LMD (LANG and KAHN [1998]).

While SHIMER does predict longer unemployment duration for less preferred workers if the productivity differential is not too large, he also predicts that firms announce a *higher* wage for less productive workers. This prediction is particularly problematic.

To demonstrate the contrasting predictions of the models, we present the equilibria under broadly comparable parameters. We assume that firms have an entry cost of 2. There are two types of workers with productivity 3.1 (type 1) and 3 (type 2). For LMD and SHIMER, we assume that there is no match specific productivity. For MOEN, we assume that match specific productivity (revealed only after the worker applies for a job) is distributed uniform on the interval (– 3,3). For SHIMER, we assume that there are equal numbers of the two types of worker.

The table below shows the wages and employment rates of each type of worker in equilibrium for each of the three models.

	Wage offers		Employment Rate	
	Type 1	Type 2	Type 1	Type 2
LMD	0.85	0.34	40.4	53.9
SHIMER	0.58	1.62	60.1	19.6
MOEN	1.06	1.00	64.4	66.7

Both LMD and MOEN have equilibrium wage rates that are higher for the preferred type but have employment rates that are lower for this type. The opposite occurs in the SHIMER equilibrium.

The contrast between LMD and SHIMER may appear surprising. In the SHIMER model, there are three candidate equilibria that are at least superficially plausible:

1. All firms offer wages to both types of workers such that he wage differential equals the productivity differential and firms choose randomly among the two types.
2. Some firms offer a wage differential exceeding the productivity differential and all type 2 workers apply to those firms. Other firms attract only type 1 workers.
3. The wage differential is less than the productivity differential and firms hire type 1workers in preference to type 2 workers.

We take these possibilities in turn. To see that case 1 cannot be an equilibrium, note that by slightly raising the wage for type 2's, the firm could commit to hiring type 1's in preference to type 2's. This would attract as many type 1's as the firm attracts type 1's and 2's in the posited equilibrium. If the firm continued to attract some type 2's, the firm would be more likely to fill its vacancy and therefore would increase its profits.

In the second case, some firms effectively commit to hire type 2's in preference to type 1's even though type 1's are more productive. However, as SHIMER proves, complete segregation is impossible. In essence, the firm hiring only type 1's will always find it profitable to deviate by offering type 2's a wage just slightly below their productivity if they are hired.[2]

Thus we are left with the third case in which at least some firms hire both type 1's and type 2's but hire the former in preference to the latter. We know from LMD that if firms were offering the same wage to both types, it would be profitable to deviate and make an offer that only attracts the less productive type. Therefore, if firms make an offer that attracts both types, it makes intuitive sense that the offer to the less productive type will be higher provided the productivity differential between the two types is not too large.

Given these limitations of the existing literature, we analyze a somewhat different model with a continuum of potential types.

3 Two Firms and Two Workers Drawn from a Continuum of Worker Types

We begin with the case where there are two firms and two workers whose types are selected randomly from a continuum. We treat the workers as equally productive regardless of their type. Formally, we can think of firms as having lexicographical preferences – they maximize profit but conditional on equal profit prefer a worker with a higher rating to one with a lower rating. Informally, we think of this as the limiting case when differences in productivity among workers are small. It may be helpful to think of the rating as height. Firms choose to hire taller workers either because worker height has a positive but trivial effect on productivity or because in the absence of any basis for selecting a worker, choosing the taller worker is as good a strategy as any other.

Consider two workers, each with productivity one. Each worker has a rating or index which is drawn from a uniform distribution on the interval (0 ,1), an assumption that is without loss of generality because the rating is an ordinal one. Each worker knows his own rating but not the rating of the other worker. (If each worker knew the type of the other, we would be in a two-worker-type framework.) When a worker applies to a firm, his rating is revealed to that firm.

We will use x and y to denote the ratings of the two workers. Given the assumption that the distribution is uniform, these correspond to the probability that the worker is the more desirable of the two.

Let H and L denote the wage offers of the two firms, with H the higher offer and L the lower one. If both firms offer the same wage, the assignment of H and L is arbitrary and will be inconsequential.

2. Let z_1 and z_2 denote the expected number of applicants at firms attracting type 1 and type 2 workers. Then from LMD, we know that the expected wage of type 2's is $v_2 e^{-z_2}$. Moreover, a single type 2 worker applyiing to a firm attracting type 1's but offering v_2 to a type 2 if hired would have an expected wage of $v_2 e^{-z_1}$. It is straightforward to show that $z_1 < z_2$, so that the deviating firm would attract at least some type 2 applicants.

Play proceeds as follows.

- The firms each announce the wage that they will pay to any worker they hire.
- Each of the two workers observes his own rating and the two announced wages and decides independently where to apply. Worker strategies are assumed anonymous in the sense that they are based only on the announced wages and not on the identity of the firms.
- Each firm with one or more applicants observes their ratings. Firms with no applicants leave the job vacant; firms with one applicant hire the applicant, and firms with two applicants hire the worker with the higher rating. Each firm can hire at most one worker. Each worker hired produces one unit of output and receives the announced wage.

3.1 Equilibrium of the Worker Application Game

First we show that workers have a unique equilibrium within the class of symmetric equilibria. In these equilibria, before their types are known, one worker is arbitrarily designated h and the other is designated ℓ.

PROPOSITION 1: *There is a unique symmetric equilibrium of the worker-application game. Each worker uses the following strategy:*

- *workers with rating* $r \geqslant \dfrac{L}{H}$ *apply to the higher-paying job;*
- *workers with* $0 \leqslant r < \dfrac{L}{H}$ *apply to the higher-paying job with probability* $\dfrac{L}{H+L}$ *and to the lower-paying job with probability* $\dfrac{H}{H+L}$.

PROOF: Let $E\ (w|H)$ be the expected wage at the higher-wage job and $E(w|L)$ be the expected wage at the lower-wage job. If $r \geqslant \frac{L}{H}$, $E(w|H) \geqslant \dfrac{L}{H}H = L$. So applying to the higher-paying job is a dominant strategy. If $0 \leqslant r < \dfrac{L}{H}$, then

$$\begin{aligned} E(w|H) &= H\left(r + \left(\frac{L}{H} - r\right)\left(\frac{H}{H+L}\right)\right) \\ &= (1+r)\frac{HL}{H+L} \\ &= L\left(r + \left(1 - \frac{L}{H}\right) + \left(\frac{L}{H} - r\right)\left(\frac{L}{H+L}\right)\right) = E(w|L) \end{aligned}$$

which proves existence.

We prove uniqueness. Let a symmetric-equilibrium mixed strategy be defined by $f(r)$, the probability that a worker of rating r applies to the higher-wage job. Because the r worker has probability r of being the higher rated of the two workers, applying to the higher-paying job is the unique best response when $r > \frac{L}{H}$. So $f(r) = 1$ for $r > \frac{L}{H}$.

Let r' denote the rating of the other worker. Worker r can obtain the higher-wage job if $r' < r$ or if $r' > r$ and worker r' has applied to the lower-wage job. Consequently the probability that r can obtain the higher-wage job is given by

$$\left(r + \int_r^1 (1 - f(r'))dr'\right).$$

We have

$$E(W|H) = H\left(r + \int_r^1 (1 - f(r'))dr'\right),$$

where $E(w|H)$ denotes the expected wage of r at the higher-wage job. Similarly,

$$E(w|L) = L\left(r + \int_r^1 f(r')dr'\right).$$

Let

$$g(r) \equiv \int_r^1 f(r')dr'.$$

Then

$$E(w|H) = H(1 - g(r))$$

and

$$E(w|L) = L(r + g(r)).$$

Let R_m denote the set of all r for which players follow a mixed strategy equilibrium, that is, $R_m = \{r|0 < f(r) < 1\}$.Then for $r \in R_m$, we must have $E(w|H) = E(w|L)$, or

$$H(1 - g(r)) = L(r + g(r)).$$

Solving for $g(r)$, we have

$$g(r) = \frac{H - Lr}{H + L}.$$

Also, from the definition of $g(r)$ we know that $f(r) = -g'(r)$, so that for $r \in R_m$ we have

$$f(r) = \frac{L}{H + L}.$$

Let r_0 be the minimum point with the property that $f(r) > 0$ almost everywhere for $r > r_0$. If $r_0 > 0$, then by construction, there is an open interval

R_0 immediately to its left such that $f(r) = 0$ for all $r \in R_0$. Worker r_0 must be indifferent between the low-wage and high-wage job. But any worker $r \in R_0$ has the same expected wage as r_0 in the high-paying job (his competition is the same as that of r_0 there) but a lower expected wage in the low paying job (he has more competition than r_0 does there). Thus that worker would want to deviate and apply to the high-wage firm. It follows that in equilibrium $r_0 = 0$ and that $f(r) > 0$ almost everywhere. Therefore, the final result for workers is that

$$f(r) = \begin{cases} 1 & \text{for} \quad r \geqslant \frac{L}{H} \\ \frac{L}{H+L} & \text{for} \quad r < \frac{L}{H} \end{cases} \tag{1}$$

almost everywhere, and uniqueness is proven. ■

The following proposition states that there is, in fact, a continuum of asymmetric equilibria. The proof is relegated to the appendix.

PROPOSITION 2: *Each real number in the interval* $\left(0, \frac{L}{H}\right)$ *defines a distinct asymmetric equilibrium of the worker-application game. For each* $r^* \in \left(0, \frac{L}{H}\right)$, *the equilibrium strategy profile* $\langle \sigma_{hr^*}, \sigma_{\ell r^*} \rangle$ *for the two workers is as follows:*

- *For both* σ_{hr^*} *and* $\sigma_{\ell r^*}$, *workers with rating* $r \geqslant \frac{L}{H}$ *apply to the higher-paying job with probability 1;*
- *For both* σ_{hr^*} *and* $\sigma_{\ell r^*}$, *workers with* $r^* < r < \frac{L}{H}$ *apply to the higher-paying job with probability* $\frac{L}{H+L}$ *and to the lower-paying job with probability* $\frac{H}{H+L}$ *;*
- *For ratings* $r \leqslant r^*$, *the worker with strategy* σ_{hr^*} *applies to the higher-paying job and the worker with strategy* $\sigma_{\ell r^*}$ *applies to the lower-paying job.*

The unique symmetric equilibrium seems more plausible than the asymmetric equilibria. The asymmetric equilibria require a high degree of coordination so that both workers avoid applying to the same firm. Such coordination strikes us as inconsistent with a large economy. In the two-worker/two-firm game, we think of each worker and each firm as being a random draw from a large population. None of the players can know *ex ante* the type of the worker with whom they will be playing. Such coordination would be particularly unlikely in the large economy game we will present in the next section and for which the 2×2 game is intended to provide intuition. Therefore in the remainder of this section, we assume that the equilibrium of the worker-application stage is as follows: all workers with $r > \frac{L}{H}$ apply to the higher-paying job, while all workers with $r \leqslant \frac{L}{H}$ apply to the higher-paying job with probability $\frac{L}{L+H}$ and to the lower-paying job with probability $\frac{H}{L+H}$.

3.2 Wage-posting equilibrium

PROPOSITION 3: *When the equilibrium of the worker-application subgame is the symmetric equilibrium described in the previous sub-section, the equilibrium of the wage-posting game is that both firms announce a wage of .25.*

PROOF: From (1) we know that the probability p_h that a random worker will apply to the high-wage firm is

$$p_h = \frac{H}{H+L}, \tag{2}$$

The probability that a high-paying firm will get at least one applicant is

$$1-(1-p_h)^2 = 1 - \frac{L}{H+L}^2,$$

so its expected profits are

$$\left(1 - \frac{L}{H+L}^2\right)(1-H). \tag{3}$$

Likewise, the expected profits of the low wage firm are

$$\left(1 - \frac{H}{H+L}^2\right)(1-L). \tag{4}$$

Suppose a firm is facing a competitor who offers the wage $\widehat{w}$. If the firm decides to offer $w > \widehat{w}$ then it becomes the high-paying firm, and, substituting w for H and $\widehat{w}$ for L in (3),we see that its profits are given by

$$\pi(w|\widehat{w}) = \left(1 - \frac{\widehat{w}}{w+\widehat{w}}^2\right)(1-w). \tag{5}$$

If the firm decides to offer $w < \widehat{w}$, it becomes the low wage firm, but substituting w for L and $\widehat{w}$ for H in (4) yields (5) as well. Furthermore, when $w = \widehat{w}$ each of the two applicants must apply to each firm with probability $1/2$, so the probability that the designated firm will have at least one applicant is $3/4$, and once again (5) is valid. To find the best response to $\widehat{w}$, we maximize $\pi(w|\widehat{w})$. The first-order condition is

$$-2\widehat{w}^2 + 4\widehat{w}^2 w + w^3 + 3w^2\widehat{w} = 0. \tag{6}$$

Likewise, the best response of $\widehat{w}$ to w is the solution of

$$-2w^2 + 4w^2\widehat{w} + \widehat{w}^3 + 3\widehat{w}^2 w = 0. \tag{7}$$

Equations (6) and (7) have one positive real solution at $w = \widehat{w} = .25$. ■

In sum, when there are only two firms and two workers, each of whose type is not known to the other worker, then the equilibrium is one in which both firms offer the same wage and workers apply randomly. Strikingly, although the ratings provide the potential for differentiation between the firms, with firms trading off higher probabilities of having an applicant with lower wage costs, this does not occur in equilibrium. Indeed, the equilibrium wage announcements are identical to those that would occur if there were no ratings and in both cases workers apply to the two firms with equal probability. The ratings only effect on the equilibrium is to determine the relative probability of unemployment of the two workers.

4 Many Firms and Many Workers Drawn from a Continuum of Worker Types

We now extend the case of two workers and two firms to the case where the number of workers and firms is large.We assume that there is a fixed number N of identical firms. Workers randomly enter the job market from a large population, so that the number of workers searching for jobs is given by a Poisson-distributed random variable, which we denote by $\widetilde{Z}$ with mean $Z \equiv E(\widetilde{Z})$.

Each worker who enters the job market receives a quality or preference rating drawn from a uniform distribution on the interval [0 ,1]. As before, the assumption of a uniform distribution is without loss of generality. All workers produce output 1.

4.1 The Worker Application Subgame

We denote wages, $w_1, w_2, \ldots$ and ratings $y_1, y_2, \ldots$ listed from the highest to the lowest. Then we have the following proposition:

PROPOSITION 4: *Suppose there are N firms and suppose* $W = \langle w_1, \ldots w_N \rangle$ *is their vector of wage offers listed with* $w_1 \geqslant w_2 \geqslant \ldots \geqslant w_N$. *Let rating* y_i *for* $i = 1, \ldots, N$ *be defined by*

$$y_1 = 1$$

and

$$y_i = y_{i-1} - \frac{i-1}{Z} \log \frac{w_i - 1}{w_i} \tag{8}$$

for $2 \leqslant i \leqslant N$. Let $\bar{\imath} \leqslant N$ denote the greatest index i with y_i positive ($\bar{\imath} = N$ if all y_i are positive). Then, the following strategy constitutes a unique[3] *symmetric equilibrium: for $i \leqslant \bar{\imath}$, workers in $(y_i,\ y_i - 1]$ apply to firms 1 to $i - 1$ with equal probability; and workers in $[0, y_{\bar{\imath}}]$ apply to firms 1 to $\bar{\imath}$ with equal probability. If $\bar{\imath} < N$, then firms $\bar{\imath} + 1$ to N receive no applications.*

Note that this proposition covers the case in which subsets of firms post equal wages. If $w_i = w_{i-1}$ then $y_i = y_{i-1}$ and the interval $(y_i, y_{i-1}]$ reduces to the empty set. Therefore if, say, $w_4 = w_5$, no type would apply only to jobs 4 and above. All workers who would apply to either job 4 or 5 would apply to the other job with equal probability. No tie-breaking rule is necessary. This feature of the equilibrium conforms to our notion that firms that offer the same wage should receive applications with the same probabilities.

We illustrate the proof of this proposition for the case of three firms with wages $w_1 \geqslant w_2 \geqslant w_3$. From (8), we have

$$y_2 = 1 - \frac{1}{Z} \log \frac{w_1}{w_2}. \tag{9}$$

We show first that if $y_2 < 0$, no workers will apply to firms 2 or 3. We need only examine the choice of (a) worker (of type) 0, because if worker 0's optimal action is to apply only to firm 1, the same would be true for all other workers. So assume all workers of with types $y > 0$ would apply only to firm 1. Worker 0 would be hired at firm 1 if and only if no other workers enter the labor market, an event that happens with probability e^{-Z}. Consequently, if $w_2 < e^{-Z} w_1$, application to firm 1 provides worker 0 with a higher expected income than application to firm 2 (or 3), which means that no workers would apply to 2 and 3. But $w_2 < e^{-Z} w_1$ is equivalent to $y_2 < 0$.

Suppose then that $y_2 \geqslant 0$. Let $(x, 1]$ designate the interval of workers that apply only to firm 1 (which reduces to the empty set if $x = 1$). If worker x applied to firm 2, he would be the highest rated worker there, so he would receive the wage w_2 with probability 1. But he would be hired at firm 1 and receive w_1 with only the probability $e^{-Z(1-x)}$. Inasmuch as he is the borderline worker in a continuum of types, his expected income must be the same in both jobs, so we have

$$w_2 = e^{-Z(1-x)} w_1,$$

which implies that $x = y_2$.

Suppose, further, that workers in $(y_3, y_2]$ apply to firms 1 and 2 with equal probability. In general, the probability that no workers will materialize with ratings in the interval $(y_i, y_{i-1}]$ is $e^{-Z(y_i-1-y_i)}$.This implies that worker y_3 has probability $e^{-Z(y_2-y_3)/2}$ of getting a job at firm 2 and $e^{-Z((1-y_2)+(y_2-y_3)/2)}$ of getting a job at firm 1, so we have

$$e^{-Z(y_2-y_3)/2} w_2 \equiv e^{-Z((1-y_2)+(y_2-y_3)/2)} w_1.$$

3. up to a set of measure zero.

Applying (9), we see that this turns out to be true. We also know that worker y_3 is sure of getting a job at w_3, so that

$$w_3 = e^{-Z(y_2 - y_3)/2} w_2,$$

and this yields

$$y_3 = 1 - \frac{1}{Z} \log \frac{w_1}{w_2} - \frac{2}{Z} \log \frac{w_2}{w_3},$$

which completes our illustration. The formal proof of the proposition applies mathematical induction to this procedure.

4.2 The Wage-Posting Game

Let z be the expected number of workers per firm, so that $z = Z/N$. We now show the following:

PROPOSITION 5: *If the expected number of workers per firm z is held constant as the number of workers and firms is parametrically increased, then in the limit, there is a unique symmetric equilibrium strategy*[4] *in which all firms offer the wage*

$$w^* = e^{-z}. \tag{10}$$

In this equilibrium, profits per firm are given by

$$\pi^* = (1 - e^{-z})^2 \tag{11}$$

PROOF: Suppose all other firms are offering $\widehat{w}$ and consider the optimal strategy of a single, possibly deviating firm. If that firm offers $w > \widehat{w}$, then

$$y_2 = 1 - \frac{1}{zN} \log \frac{w}{\widehat{w}}. \tag{12}$$

For $w > \widehat{w}$ applicants in the interval $(y_2, 1]$ apply only to the designated firm. Thus the expected number of applicants from that group is $zN(1 - y_2)$ or $\log w/\widehat{w}$. Applicants from $[0, y_2]$ apply to all N firms with equal probability, so the expected number of applicants from that group is Zy_2/N or

$$z - \frac{1}{N} \log \frac{w}{\widehat{w}}.$$

4. We state the limiting result more precisely as follows: Suppose all firms but a designated firm offer the wage $w^* = e^{-z}$. Then the best response of the designated firm converges to w^* as N and Z increase, provided that $z = Z/N$ remains constant.

The total number of expected applicants to the designated firm is then

$$z + \frac{N-1}{N} \log \frac{w}{\widehat{w}}$$

or, if the expected number of workers per firm is held constant as the number of firms is parametrically increased, the expected number of applicants to the designated firm goes to

$$z + \log \frac{w}{\widehat{w}} \tag{13}$$

as the number of firms becomes very large.

For $w < \widehat{w}$ the firm's expected numbers of applicants is the fraction $1/N$ of those in the interval $[0, y_N]$,where

$$y_N = 1 - \frac{1}{z} \log \frac{\widehat{w}}{w}. \tag{14}$$

Thus the expected number of applicants is Zy_N/N, which yields (13) as well. Finally, for $w = \widehat{w}$, all firms would receive exactly Z/N or z expected applicants, which is precisely the value of (13) when $w = \widehat{w}$.

It follows that given an offer of $\widehat{w}$ by all other firms, the probability that the designated firm will receive at least one applicant is

$$1 - e^{-z - \log \frac{w}{\hat{w}}}$$

or

$$1 - e^{-z} \frac{\widehat{w}}{w}$$

so that expected profits are

$$\pi = \left(1 - e^{-z} \frac{\widehat{w}}{w}\right)(1 - w) . \tag{15}$$

Given $\widehat{w}$, expected profits are maximized when

$$w = \sqrt{e^{-z} \widehat{w}} .$$

This implies that in the limit w^* is the best response to offers of w^* by all other firms if and only if $w^* = \sqrt{e^{-z} w^*}$, so that

$$w^* = e^{-z}.$$

From (15), we see that equilibrium profits per firm are given by (11). ∎

4.3 Comparison with the Nondiscriminatory Equilibrium

In an analogy with Marxist theory, LMD argue that capitalists (firms) benefit as a group if they individually make arbitrary distinctions among workers. In that model, if firms hire whites in preference to almost equally productive blacks, expected profits are higher and wages for both blacks and whites are lower than if firms treat blacks and whites equally.

The same phenomenon arises with a continuum of types. If firms hire applicants randomly rather than on the basis of y, the equilibrium wage offer yields an expected income to the workers of e^{-z} which is the wage offer times the probability that the worker gets employment (see LMD). Thus the expected income in the nondiscriminatory case is the same as the wage offer in the discriminatory case. With the exception of a worker who draws a rating of 1 (a set of measure zero), all workers have a positive probability of unemployment and so have a lower expected income when firms use the rating to distinguish among them.

Expected profit in the nondiscriminatory case is

$$\pi = 1 - e^{-z} - ze^{-z}$$

which is less than the expected profit in (11).

As in LMD, the equilibrium with distinctions among workers is more favorable to capitalists and hurts workers, a result with a Marxian flavor but which does not rely on theories of capitalist collusion.

4.4 Equilibrium with Free Entry

We assume that there is a continuum of firms that can enter the market at a cost of d. Free entry implies that all firms make zero profit, and thus that

$$(1 - e^{-z})^2 = d.$$

Solving for z gives

$$z = -\ln\left(1 - \sqrt{d}\right). \tag{16}$$

Combining (10) and (16) gives

$$w^* = 1 - \sqrt{d}.$$

5 Equilibrium with Three Types of Workers

When there are two discrete types of workers as in LMD, there tends to be complete segregation in equilibrium, so that the favored type earns higher wages but also has a higher unemployment rate. The increased probability of employment does not offset their lower wage, and the less desirable type is worse off. In contrast, with a continuum of workers, complete segregation does not arise, the equilibrium wage is independent of type, but the probability of finding a job is much higher for more highly rated workers. These results are unsatisfying. In general, factors that are associated with increased probability of employment are also associated with higher wages (see for example, MURPHY and TOPEL [1987]).

We believe that the most appropriate model would yield an intermediate degree of segregation with both higher wages and lower unemployment for the more preferred groups. Neither the assumption of two discrete types nor the assumption of a continuum of types is intuitively appealing. On the one hand, there are more than two types of workers. On the other hand, for all practical purposes, many job applicants look identical to prospective employers. Certainly when choosing between two applicants for similar jobs, two different employers need not select the same applicant. Although it is tempting to rely on match-specific productivity to explain the different choices, employers often choose among workers on an almost random basis (who applied first, trivial differences in interview performance).

To develop a more realistic model, we would need to consider a case with a moderate number of discrete types (but more than two) or one with a continuum of types in which firms observe rank with error. We conjecture that by combining elements of the discrete and continuous models, such a hybrid model would also combine the key aspects of the equilibria and yield more realistic degrees of segregation and wage and unemployment differences. Unfortunately, we have not found such models to be mathematically tractable.

However, in support of our conjecture, we solve a numerical example with three types of workers in which, in equilibrium, both wages and employment are increasing in worker quality. In this example there are three types of equally productive workers ranked, from most to least preferred, 1, 2, 3. As previously, all workers produce one unit of output. The order of play is as before. Firms decide whether or not to pay an entry cost d. Once all firms have entered, firms simultaneously announce wage offers not conditioned on worker type. Workers observe the wage offers and decide where to apply. Firms hire at random from the most preferred type that applies.

We conjecture that there are parameter values for which in this case the equilibrium takes the following form. There are three wages in equilibrium:

1. a high wage that attracts all three types,
2. a wage equal to the expected wage of type-1 workers that attracts only type 2's, and

3. a wage equal to the expected wage of type-2 workers that attracts only type 3's.

Below, we outline the nature of the conjectured equilibrium. We demonstrate numerically that this is, in fact, the equilibrium for some parameter values.

We denote the expected number of applicants of each type at high wage firms by z_1,z_2, and z_3. We let y equal the expected number of type 2 applicants at the middle wage firm and x equal the expected number of type 3 applicants at low-wage firms.

In equilibrium, we require that all three offers generate expected profit equal to zero:

$$(1 - e^{-z_1 - z_2 - z_3})(1 - w_H) - d = 0 \tag{17}$$

$$(1 - e^{-y})(1 - w_M) - d = 0 \tag{18}$$

$$(1 - e^{-x})(1 - w_L) - d = 0. \tag{19}$$

The probability that a worker of type 1 is hired is given by

$$e_1 = \frac{1 - e^{-z_1}}{z_1}$$

and the expected wage for type 1 workers is

$$E(w_1) = w_H \frac{1 - e^{-z_1}}{z_1}. \tag{20}$$

For type 2 and 3 workers applying to the high-wage firm, the corresponding expressions are

$$e_2 = e^{-z_1} \frac{1 - e^{-z_2}}{z_2}$$

$$e_3 = e^{-(z_1 + z_2)} \frac{1 - e^{-z_3}}{z_3}$$

for employment and

$$E(w_2) = w_H e^{-z_1} \frac{1 - e^{-z_2}}{z_2} \tag{21}$$

$$E(w_3) = w_H e^{-(z_1 + z_2)} \frac{1 - e^{-z_3}}{z_3} \tag{22}$$

for expected wages.

Equilibrium also requires that type 2 workers be indifferent between applying to high-wage and middle-wage firms and that type 3 workers be indifferent between applying to high-wage and low-wage firms:

$$E(w_2) = w_M \frac{1 - e^{-y}}{y} = w_H e^{-z_1} \frac{1 - e^{-z_2}}{z_2} \tag{23}$$

$$E(w_3) = w_L \frac{1 - e^{-x}}{x} = w_H e^{-(z_1+z_2)} \frac{1 - e^{-z_3}}{z_3}. \tag{24}$$

To fully determine w_H, w_M and w_L, we require that

$$w_M = E(w_1) \tag{25}$$

$$w_L = E(w_2) \tag{26}$$

and that

$$w_H = \arg\max_{w_H} (1 - e^{-z_1 - z_2 - z_3})(1 - w_H) - d \quad \text{s.t. (20), (21), (22).} \tag{27}$$

For our numerical estimation, we set $d = .26424$ and let 30% of workers be type 1, 30% type 2 and 40% type 3. Given d, equations (17)-(19), (23)-(27) fully determine the unknowns, w_H, w_M, w_L, z_1, z_2, z_3, x and y. If all the equations hold, then no worker wishes to deviate. We solve the equations and then verify numerically that no firm wishes to deviate.[5]

Given that type 1's apply only to the highest wage jobs while at least some type 2's and 3's apply to lower wage jobs, type 1's have the highest average wage. Whether type 2's or type 3's have higher wages on average depends on the relative numbers applying to different jobs and thus on their proportions in the population. Similarly, type 1's have the highest employment rate at type 1 jobs, followed by type 2's. However, type 3's applying to the low-wage job have a higher employment rate than type 2's applying to the middle-wage job. Again, overall employment rates will depend on the relative numbers of different types in the population.

5. Solving the equations numerically gives $w_H = 0.58198$, $w_M = 0.45705$, $w_L = 0.33356$, $z_1 = 0.50445$, $z_2 = 0.10523$, $z_3 = 0.39032$, $y = 0.66684$, $x = 0.505$. This equilibrium is feasible if the ratio of type 2 workers to type 1 workers is at least $\frac{.10523}{.50445}$ or about .21 and the ratio of type 3 workers is at least $\frac{.39032}{.50445}$ or about .77 .We have chosen the proportions of the three types in the population to ensure feasibility of this equilibrum.

Given these numerical values, we can compute average wages for employed workers and overall employment rates for each type. The parameter values ensure that type 1's always have the highest employment rate. We can readily establish that if the numbers of type 1's and type 2's are equal, the employment rate of type 2's will be higher than the employment rate of type 3's unless there are almost three times as many type 3's as type 2's in the economy. Under the same condition, the average wage of employed type 2's will exceed the average wage of employed type 3's if there are at least 10% more type 3's than type 2's. Thus there is a fairly wide range of parameters for which the ranking of employment rates and average wages of the three types correspond to their preference ranking including the case where 30% are type 1's, 30% are type 2's and 40% are type 3's, the example given in the text.

Using our numerical values, we can see that type 1's have the highest average wage and employment rate and type 3's the lowest. On average, conditional on being employed, type 1's earn about one-third more than do type 3's and have an unemployment rate that is about 60% of that of type 3's.

	Type 1	Type 2	Type 3
	Employment Rate at Job		
$w_H = .582$	.786	.573	.450
$w_M = .457$	–	.730	–
$w_L = .334$	–	–	.785
Mean Wage if Employed	.582	.478	.443
Overall Employment Rate	.786	.697	.591

The three-type equilibrium with these parameters is an interesting mixture of the two-types and continuum models. The equilibrium has a distribution of wages as in the two worker equilibrium. However, we do not get perfect sorting. In the equilibrium of the worker application sub-game with a continuum of types, the best workers apply only to the highest wage jobs. Less good workers apply to the best jobs but also to lower wage jobs. The equilibrium in the three-type game has somewhat similar features. The critical difference between the equilibria with two and three types is that with three types pooling can occur. With two types, if a wage attracts both types, it is always profitable to lower the wage because the loss of the preferred type is more than offset by increased applications from the less-preferred type. However, if the wage attracts all three types, it need not be profitable to lower the wage. Lowering the wage will attract fewer type 1's and more 1's and 2's combined but fewer 1's, 2's and 3's combined. With three types, the firm faces a trade-off between the benefit of lowering the wage and the cost in terms of fewer applicants.

6 Conclusion

In standard labor-market search models, workers and firms meet randomly and workers have no information about a firm's wage offer, until worker and firm meet. After they meet, the parties may determine the wage by bargaining, or the firm may make a take-it-or-leave-it offer. This assumption stands in sharp contrast with evidence that high-wage firms attract more applicants (HOLZER, KATZ and KRUEGER [1991]) and that even young low-skill workers have considerable awareness about differences in compensation across employers (WIAL [1991]).

It is more natural to model search as being directed, thereby recognizing that wage-offer strategies of firms are designed to affect the number of applicants. However, there is no consensus as yet about the best way to model directed search. The principal papers in this area use very different modelling strategies, and each generates problematic predictions. In particular, provided that the productivity difference is sufficiently small, LMD and MOEN predict-shorter unemployment durations for less productive workers while SHIMER predicts higher wage offers for these workers.

We have shown that with a continuum of worker types with similar productivity, in equilibrium all workers receive the same wage but lower quality workers experience more unemployment. This prediction still falls short of what we believe to be the empirically correct outcome: that lower quality workers receive lower wages, on average, and have higher unemployment rates.

However, neither a continuum of types nor two types provides a good approximation to reality. Employers are frequently unsure which is the best applicant. Members of a hiring committee, sharing the same objectives, may nevertheless rank applicants differently. Thus, on the one hand, in contrast to the models presented here, workers are not completely ordered. On the other hand, the gradation of types is far finer than only two.

We conjecture that a model with many more types would predict that lower types have both lower wages and higher unemployment rates. We confirm that for some parameter values, the equilibrium with three discrete types of workers has this property.

The equilibrium of the worker application subgame is also of interest in its own right. If wages are not determined optimally or if firms are innately heterogeneous, then wage differentials may persist in equilibrium. Our model predicts that higher types restrict their search to high-wage firms while lower types randomize among firms. The model casts light on choice in any setting in which individuals are limited to a single application per period as in some school choice programs and as with marriage proposals. ▼

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APPENDIX

PROOF. OF THEOREM (2) If a type-r is the only worker to apply to a job, he obtains the job for certain. In the event that the other worker has applied to the same job, the type-r worker has probability r of being the higher rated player and so gets the job with probability r. For $r \geqslant \frac{L}{H}$, applying to the higher-paying job is the best response even when he has to compete with the other worker, because $rH \geqslant L$.

For $r^* < r < \frac{L}{H}$, the probability that the r-type can get a job at the higher-paying firm is the probability r that he is the higher-type worker plus the joint probability $\left[\frac{L}{H} - r\right]\frac{H}{H+L}$ that he is not the higher-type and that the other worker applies to the lower-paying firm. Therefore, his expected wage at the H-firm is

$$E(w|H) = \left(r + \left[\frac{L}{H} - r\right]\frac{H}{H+L}\right)H$$

$$= \frac{LH}{H+L}(1+r)$$

Analogously, his expected wage at the L-firm is

$$E(w|L) = \left(1 - \frac{L}{H} + \left[\frac{L}{H} - r\right]\frac{L}{H+L} + r\right)L \, .$$

$$= \frac{LH}{H+L}(1+r) = E(w|H)$$

For $r \leqslant r^*$, a worker whose strategy requires application to the L-firm is sure of a job there, unless the other worker has a type in the range $[r^*, L/H]$ and applies to the L-firm, which happens with probability $(L/H - r^*)\frac{H}{L+H}$.Thus, his expected wage is

$$E(w|L) = \left(1 - (L/H - r^*)\frac{H}{L+H}\right)L = \frac{LH}{H+L}(1+r^*)\, .$$

If he applies to the H-firm, he will get the job only if either he has the higher rating (probability r) or if the other worker has the higher rating but applies to the L-firm (probability $(L/H - r^*)\frac{H}{L+H}$). Therefore, his expected income is

$$E(w|H) = \left(r + (L/H - r^*)\frac{H}{L+H}\right)H = E(w|L) - (r^* - r)H,$$

so he will not deviate. In the same manner, we can show that a worker whose strategy requires him to apply to the H-firm will likewise have no incentive to deviate. ■