Journal of Advanced College of Engineering and Management

Water Hammer

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Abstract:

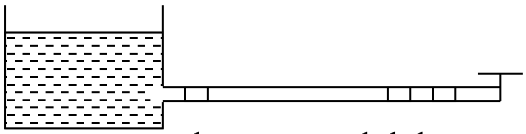
Dynamic Pressure rise or fall in all kinds of pipelines due to the operation of the valve which are transmitting fluids are very much essential part to consider in the design of pipeline. This paper presents how to calculate these dynamic pressure waves while designing pipelines.

Key Words: *Pressure wave, Water hammer, Elasticity of pipe*

Pipelines of all kinds, which are transmitting liquids of all types, must withstand transient pressures caused by opening or closing of valves in the system such as pumps and turbines. The pressures waves produced by these operations in more or less elastic pipelines conveying compressible liquids are called "Water Hammer". These dynamic pressure waves exerted in the system are in excess of the normal static pressure. The magnitude of these pressure waves are frequently much greater than that of any static pressure to which the pipe may ever be subjected. Therefore,

The possibility of the occurrences of such pressures must be calculated in connection with the design of pipelines.

In order to keep these pressure variations i.e. rise or fall within acceptable limits, we must introduce the pressure controlling device in the pipeline. If the device is more flexible then the conduit, which is introduced in the system the pressure waves of the water hammer quickly die out and a much slower



l_n l_3 l_2 l_1

Figure 1

mass oscillation of water column as a whole takes place.

This happens when e. g. an air vessel or surge tanks are provided to protect the part of the line from water hammer effects. Both water hammer and mass oscillation are hydraulic transients, i.e. time dependent phenomenon in closed conduit and in open channel waves are some time called surge.

The dynamic pressure is the result of the sudden transformation of the kinetic energy of the moving mass of water with in the pipe into pressure energy. Since force equals mass times acceleration,

$$F = M \frac{dv}{dt}$$

It follows that, if the velocity of the mass M could be reduced from V to zero instantaneously, this equation would become

$$F = M\frac{V}{0}$$

Or, in other words, the pressure resulting from the change would be infinite. However, such an instantaneous change is impossible.

The following Symbols are used in the derivation of the water hammer equation:

t = thickness of pipe wall in m

D = inside diameter of pipe in m

A = cross sectional area of pipe in m^2

L = Length of pipeline in m

K = Bulk Modulus of Elasticity of water in N/m^2

E = Youngs Modulus of Elasticity of pipe in N/m^2

H = total head measured at the point of valve in m

T = time of closing valve in $Sec.$ Sec.

V_v = mean velocity of water through valve in $m/Sec.$

V = mean velocity of water in the pipeline before closing the

Valve in $m/Sec.$

ϑ_ω = Velocity of pressure wave along pipeline in $m/Sec.$

g = Acceleration due to gravity in $m/Sec.^2$

Δh = Head due to water hammer (in excess of static head) in m

Consider the conditions with in the pipe immediately following the closure of the valve. Let l_1 l_2 l_3 ....l_n infinitesimally short sections of pipe, as shown in figure. The instant the valve is closed, the water in section l_1 is brought to rest, its kinetic energy is transformed into pressure energy, the water is some what compressed, and the pipe expands slightly as a result of the increased stress to which it is subjected. Because of the enlarged cross sectional area of l_1 and the compressed condition of the water within it, a greater mass of the water is now contained with in this section than before the closure. It is evident then that a small volume of water flowed into section l_1 after the valve was closed. An instant later a similar procedure takes place in l_2 and then in l_3, so that evidently a wave of increased pressure travels up the pipe to the reservoir. When this wave reaches the reservoir the entire pipe is expanded and the water within it is compressed by a pressure greater than that due to the normal static head.

There is now no longer any moving mass of water within the pipe, the conservation of whose kinetic energy into pressure energy serves to maintain this high pressure, and therefore the pipe begins to contract and the water to expand with a consequent return to normal static pressure. This process starts at the reservoir and travels as a wave to the lower end. During this second period some of the water stored within the pipe flows back into the reservoir, but on account of the inertia of this moving mass an amount flows back greater than the excess amount stored at the end of the first period so that the instant this second period wave reaches the valve the pressure at that point drops not only to the normal static pressure but below it.

A third period now follows during which a wave of pressure less than static sweeps up the pipe to the reservoir. When it reaches the reservoir the entire pipe is under pressure less than static, but since all the water is again at rest the pressure in l_n immediately returns to the normal static pressure due to the head of water in the reservoir. This starts a fourth period marked by a wave of normal static pressure moving down the pipe. When the valve is reached, the pressure there is normal and for an instant the conditions throughout the pipe are similar to what they were when the valve was first closed. The velocity of the water (and the resultant water hammer) is now, however, somewhat less than it was at the time of closure because of friction and imperfect elasticity of the pipe and the water.

Instantly another cycle begins similar to the one just described and then another, and so on, each set of waves successfully diminishing, until finally the waves die out from the influences mentioned.

The above equation no shows that for instantaneous closure of valve the pressure created would be infinite. If the water were incompressible and the pipe inelastic. Since it is impossible to close a valve instantaneously, it is apparent that a series of pressure waves is created similar to the one just described, causing an increasing pressure at the valve. If the valve is completely closed before the first pressure wave has come time to return to the valve as a low pressure, or in other words, if T is less than $\frac{2L}{\vartheta_\omega}$, it is evident that the pressure has been continually increasing up to the time of complete closure and that the resulting pressure is same as if the valve had been instantaneously closed. Bit if T is greater than $\frac{2L}{\vartheta_\omega}$, then before the valve is completely closed the earlier pressure waves have returned as wave of low pressure and tend to reduced the rise of pressure resulting from the final stages of valve closure.

Hence, if T is equal to or less than $\frac{2L}{\vartheta_\omega}$, h will be the same as for instantaneous closure, but if T is greater than $\frac{2L}{\vartheta_\omega}$, then h will be diminished as T increases.

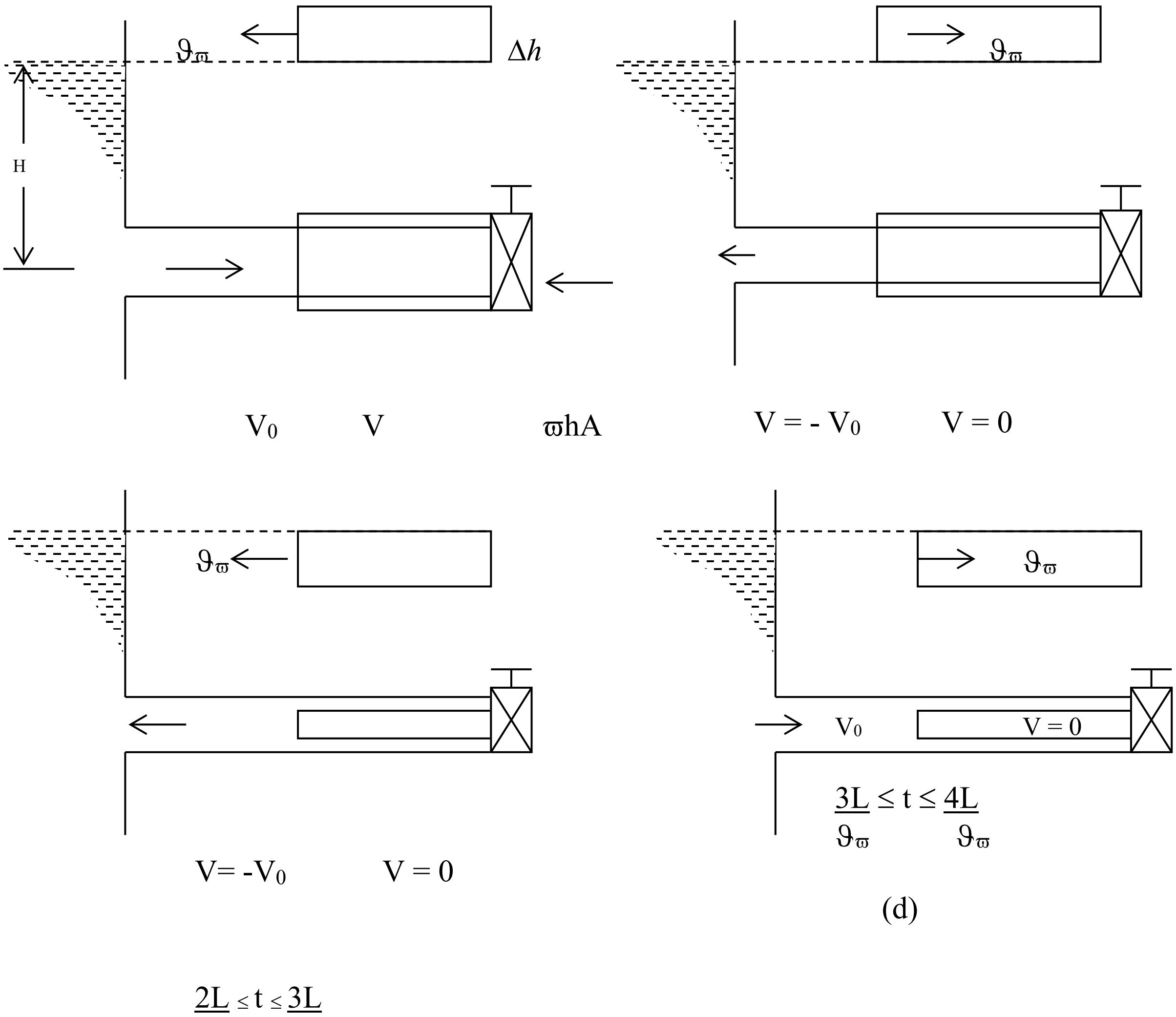


Figure 2

1. Pressure Rise at Sudden closure when $T \leq \dfrac{2L}{\vartheta_{\omega}}$

The theory of water hammer is based upon the law of conservation of energy. The amount of kinetic energy contained in the moving column of water with in the pipe is $\dfrac{MV^2}{2} = \dfrac{\omega ALV^2}{2g}$ This energy is used up in doing work in compressing the water and in stretching the pipe walls.

If the resulting pressure head is h in m above normal the compression of the water column absorbs $\dfrac{\omega h^2 AL}{2}$ $KN - m$ of energy, since the final intensity of pressure is ωh, the average total pressure is $\dfrac{\omega hA}{2}$, unit compression is $\dfrac{\omega h}{K}$ and the total compression or distance through which the average total pressure acts is $\dfrac{\omega hL}{K}$

In similar manner the total work done in stretching the pipe walls is $\frac{(\omega h)^2 LAD}{2Et}$.

Since the unit stress in the pipe walls is $\frac{\omega hD}{2t}$, the average total stress in the pipe wall is $\frac{\omega hDL}{4}$ the unit elongation is $\frac{\omega hD}{2tE}$ and the total elongation or distance through which the force acts is $\frac{\omega h\pi D^2}{2tE}$.

The conservation of Energy equation therefore becomes,

$$\omega AL\frac{V^2}{2g} = \frac{(\omega h)^2 AL}{2K} + \frac{(\omega h)^2 ALD}{2Et} \quad ....1$$

By solving the above equation we get,

$$h = \frac{V}{g\sqrt{\frac{\omega}{g}(\frac{1}{K} + \frac{D}{Et})}}$$

$$h = \frac{V}{g}\sqrt{\frac{Kg}{\omega}}\ \frac{1}{\sqrt{(1 + \frac{KD}{Et})}}$$

Where $\sqrt{\frac{Kg}{\omega}}$ = ύώ is the velocity of sound in water medium

ύώ = 1440 m/ sec.

$$h = \frac{V}{g}\ \frac{1440}{\sqrt{(1 + \frac{KD}{Et})}} \quad ....2$$

Now let us see the travelling speed of wave velocity ϑ_ω which will be inside the pipe line.

In the time t a column of water of length $\vartheta_\omega t$ is brought to rest. The rate of change of momentum is therefore,

$$\frac{MV}{t} = \frac{\omega At\vartheta_\omega V}{gt} = \frac{\omega A\vartheta_\omega V}{g} \quad ....3$$

and this must be equal to the force exerted by the increased pressure which is ωhA, therefore,

$$\omega hA = \frac{\omega A \vartheta_\omega V}{g}$$

from which $h = \frac{\vartheta_\omega V}{g}$ ………4

Comparing the above equations it is apparent that compressive wave velocity of water is found to be ϑ_ω, which is as shown below:

$$\vartheta_\omega = \frac{1440}{\sqrt{(1+\frac{KD}{Et}}} \text{ ....5}$$

Since the sound is transmitting by means of pressure waves, ϑ_ω is the velocity of sound through water in that particular pipe.

The foregoing theory was first derived and experimentally verified by professor N. Joukovsky, of Moscow Russia in 1898.

2. Pressure Rise at Slow closure when $T \geq \frac{2L}{\vartheta_\omega}$

Numerous formulas have been derived for the determination of the rise in pressure in a pipeline resulting from the slow closure of valve or turbine gates, but most of them are unreliable or else are true only under special conditions.

The method that will be followed here is that of arithmetic integration, as proposed by Norman R. Gibson and published in vol. 83 of Transactions of the American Society of Civil Engineers for 1919. It is based on the foregoing theory of pressure waves and consists of a method of tracing the action of these waves instant by instants.

Assume that the valve, instead of being closed in a continuous motion, is closed by a series of small instantaneous movements. Each small movement of the valve will destroy a small portion, Δv of the velocity V : and since the destruction is instantaneous the resulting increase in pressure head will be,

from above equation $h = \frac{\vartheta_\omega V}{g}$

but $\Delta h = \vartheta_\omega \frac{\Delta v}{g}$

There will thus be transmitted up the pipeline a series of pressure waves which, when added together, will give the total excess pressure produced.

If it is assumed, as usual, that each small movement of valve produces the some reduction in area of valve opening. It will be necessary to determine the resulting reduction in velocity in the pipe, for obviously, the instant the valve starts closing, the pressure behind it starts rising, and this rise in pressure increase the velocity through the opening and diminishes the rate of retardation of the velocity in the pipe. This reduced retardation of the velocity has an important bearing on the problem and must be taken into account in determining the valve opening may be expressed by the formula for discharge through orifices.

$V_v = C_v \sqrt{(2gH)}$ ……………..6

This formula may be written in the form as,

$Vv = \varphi' \sqrt{H}$

in which φ' may be determined in any given problem in which the velocity in the pipe V, the total head H, and the ratio β of the area of the valve opening to the area of the pipe are all known, since

$V_v = \frac{V}{\beta} = \varphi' \sqrt{H}$ ………7

again,

$V = V = \beta\varphi' \sqrt{H}$ OR

$V = \varphi \sqrt{H}$ 8

$\varphi = \beta C_v \sqrt{2g}$

As the valve is closed the value of β or of K, will become smaller and smaller, with corresponding increments, Δh, in the head producing discharge through the valve and with simultaneous decrements Δv, in the velocity n the pipe, so that the equation may become

$V - \Delta v = \varphi \sqrt{(H + \Delta h}$ 9

The solution of the problem consists in finding those values of Δv and Δh that will satisfy the above equation for different value of φ .

The method may be best explained from the solution of the given problem.

Problem:

Determine the rise in pressure that will occur in a penstock leading to a power plant if the turbine gates are closed in 15 sec. H = 107m, L = 1800m, V = 3 m/sec,

ϑ_ω = 1440 m/ sec.

Solution:

For convenience, T has been taken as an even multiple of $\frac{2L}{\vartheta\omega}$,

Therefore, $\frac{2L}{\vartheta\omega}$, = 2*1800/1440 = 2.5 sec.

To simplify the problem as much as possible, friction will be neglected. Also, for convenience, it will be assumed that the gates are closed in 15/2.5 = 6, successive instantaneous movements. Each intervening interval will therefore be just long enough to allow the pressure wave resulting from one movement to travel up the penstock and return as a low pressure wave down surge) at the instant that the next movement takes place.

Using above equation,

$$\varphi = \frac{V}{\sqrt{H}} = 3/\sqrt{107} = .29$$

For each successive movement φ will be reduced by one-sixth of its original value, or by .048, since the movement are assumed to be of uniform magnitude.

1	2	3	4	5	6	7	8	9	10
Interval	**Time**	**φ**	**Head**	**Velocity V**	**ΔV**	**Δh**	**Σ(Δh)**	**Max.up Surge**	**Max.down Surge**
0	0	.29	107	3	.19	27.8			
1	2.5	.24	134.8	2.81	.46	67.5	27.8		
2	5	.19	146.6	2.35	.56	82.2	39.6		
3	7.5	.14	149.5	1.79	.58	85.1	42.5		
4	10	.09	149.6	1.21	.59	86.6	42.6		-86.6
5	12.5	.05	150.9	.62	.62	91	43.9	+91	
6	15	0	154	0			47		

The first three columns of the table following may be now be filled in, and also the initial values of columns 4 & 5.

The remaining values are obtained in the following manner.

Assume a value for Δv caused by the first movement of the gate, from above equation,

$$\Delta h = \frac{1440 * \Delta v}{9.81} = 146.7 * \Delta v$$

Substituting the assumed value for Δv in above equation, a trial value for Δh is found. It is now necessary to determine whether or not these trial values satisfy the equation..**9** This may be done by substituting this trial value for dh and solving for Δv .If this value for Δv does not check the assumed value, a new value for Δv must be assumed and the computations repeated until the two equations are satisfied. The correct value for Δv will always be found to lie between the assumed value and the computed value and will usually be found to be much nearer the former than the latter.

For the initial gate movement, Δv is found to be 0.19m/sec. and Δh = 27.8m, Hence, during the first interval of 2.5sec. the total head acting is 134.8m and the velocity in the penstock is 3 m/sec.

The computation will now be carried through for the second interval.

First assume that Δv = 0.3 then from equation 4 Δh = 146.7*.3= 44.01 substituting this in above equation,

2.81- Δv = .19 √(107+44.01- 27.8) = 0.701

from which Δv = 0.701

In substituting for (H + Δh) in equation 9, it must be remembered that this quantity represents the total effective head producing discharge through the gates during that interval.

The effect of the preceding pressure waves must therefore be taken in to account. At the beginning of the second interval the first pressure wave will have returned to the gates as a wave of low pressure and will reduce the effective head during the second interval by the same amount that it increased it during the first interval.

The correct value of dv is now known to lie between 0.3 and 0.701

Next assume Δv =0.5,then Δh = 146.7*.5= 73.35 and substituting in equation 9 and solving, Δv = 0463 Finally, assuming Δv = .46, Δh = 67.5 These values are found to satisfy equation.

During the third interval the first pressure wave will have returned as a wave of high pressure, while the second wave will be one of low pressure.

Therefore, $H + \Delta h$ = 107+27.8- 67.5+ Δh

and , during the fourth interval,

$H + \Delta h$ = 107-27.8+67.5-82.2+ Δh and so on