CHAPTER 10

WAVE BOUNDARY LAYERS AND FRICTION FACTORS

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ABSTRACT

In the last two decades the problem of wave height damping due to bottom friction has received increasing attention among near-shore oceanographers. This fact is reflected in the wealth of papers on the subject; the list of references given herein presents a minor selection only.

This paper is an attempt to re-evaluate and systematize the many observations and the rather few detailed measurements of the phenomenon. In nature the wave boundary layer will always be rough turbulent. This is not necessarily the case in a hydraulic model. The aim is therefore to make it possible to determine the proper flow regime for a pure short-period wave motion over a given bed. Values for the wave friction factor and the wave boundary layer thickness are also proposed.

The main results of the study are presented in three diagrams giving flow regimes, friction factors and boundary layer thicknesses. Flow parameters are a_{1m}/k and RE = $U_{1m}\, a_{1m}/\nu$, a_{1m} and U_{1m} being maximum bottom amplitude and velocity according to first order potential wave theory. k is the Nikuradse roughness parameter.

1. INTRODUCTION

It seems to be generally recognized to-day that the boundary layers developing at the sea bottom under gravity waves for all practical purposes can be regarded as turbulent, see for instance [6] and [14]. For many years it has been extensively discussed, however, whether turbulence could appear in laboratory studies of wave phenomena, see [6] and the long discussions in La Houille Blanche, [2], [4], [29] and [30]. The experiments by Miche [29], [30], Vincent [36], Lhermitte [23], Zhukovets [37] and Collins [6] demonstrate, on the other hand, that turbulent oscillatory boundary layers can be generated under laboratory conditions also.

The importance of a sound estimate of the wave friction factor for shallow water wave forecasting is obvious. In this context reference can be made to the pioneer works by Bagnold [1] and Johnson and Putnam [13]. Since measurements in a prototype scale are scarce, and difficult to perform, however, it is imperative to know to what extent model results in this field are applicable in nature, and vice versa. This calls for a detailed analysis of the behaviour of the wave boundary layer.

As long as the flow is entirely laminar, the problem is open for an analytical treatment. This is not the case for turbulent flow. No consistent theory dealing with turbulent oscillatory boundary layers exists. In this paper an approach by Lundgren, adjusted according to the experimental results of the present author, has been adopted for the rough turbulent case, see [14] and [15]. The experimental results of Bagnold a. o. will be shown to agree quite well with the proposed friction factors. Measurements of turbulent flow near a smooth wall seem to be missing entirely, so an analogy with rough flow has been introduced.

A preliminary report is given in [17]. The more complex problem of bottom friction and energy dissipation in a wave motion when superimposed by a current has been studied in [18].

2. NOTATION

			Eq.No.
D	(m)	Water depth	
D_e	(m)	"Equivalent depth"	(3.1)
E_w	(kgf/m s)	Specific energy loss per s	
H	(m)	Wave height	
L	(m)	Wave length	
RE	(dim.less)	Amplitude Reynolds number	(3.3)
Re	(dim.less)	Reynolds number	(3.2)
T	(s)	Wave period	
U	(m/s)	Wave particle velocity	
U_1	(m/s)	U at the bottom	(3.6) & (5.2)
U_c	(m/s)	Current velocity	(3.10)
U_f	(m/s)	Friction velocity	(3.10)
a_1	(m)	Wave particle amplitude at bottom	(5.2)
c	(m/s)	Wave celerity	
d	(m)	Diameter of cylindrical roughness (Kalkanis)	
f_e	(dim.less)	Wave energy loss factor	(3.9)
f_w	(dim.less)	Friction factor for τ_w	(3.4)
h	(m)	Ripple height (Bagnold)	
k	(m)	Nikuradse roughness parameter	(3.10)
p	(m)	Ripple pitch (crest to crest) (Bagnold)	

			Eq.No.
t	(s)	Time	
u	(m/s)	Velocity fluctuation in x-direction	
w	(m/s)	Velocity fluctuation in z-direction	
x	(m)	Coordinate in direction of wave travel	
z	(m)	Coordinate at right angles to bottom	
δ	(m)	Wave boundary layer thickness	(Fig. 2)
δ_{visc}	(m)	Thickness of viscous sublayer	(5.21)
ν	(m^2/s)	Kinematic viscosity	
ϱ	(kgf s^2/m^4)	Density	
τ	(kgf/m^2)	Instantaneous shear stress for a pure wave motion	
τ_w	(kgf/m^2)	τ at bottom	(3.4)
φ_o	(^{o})	Phase shift between τ_{wm} and U_{1m}	
ω	(1/s)	Angular frequency	
log		$\log_{10}$	
—		Mean value sign	

Suffix m denotes maximum.

3. DEFINITIONS

First order potential wave theory is applied outside the boundary layer, see Fig. 1. The wave boundary layer thickness δ is conveniently defined from the velocity profile shown in Fig. 2. As the thickness of the boundary layer for short-period waves is of the order of magnitude 1/100 of the water depth, it will not affect the motion of the body of water, and U_1 in Fig. 2 can be taken equal to the theoretical bed velocity for a frictionless fluid.

At $z = 2\delta$, τ_m is approximately 0.05 τ_{wm}, where τ_w is the bottom shear stress, and "m" denotes maximum. 2δ can therefore be said to be analogues to the depth of a steady flow in an open channel, and could be denoted "the equivalent depth", D_e, i.e.

$$D_e = 2\delta \tag{3.1}$$

This analogy will be made use of later. It will be shown to yield remarkably reliable results.

At $z = \delta$, τ_m equals 0.21 τ_{wm} for laminar motion, see (5.8), and was measured to be 0.35 τ_{wm} in Test No. 1 in the oscillating water tunnel (fully developed rough turbulence, see [14]). Thus it appears, that the boundary layer thickness here defined is only similar to the boundary layer thickness employed in steady flow, in the sense that it gives a measure of the thickness of the layer adjacent to the wall over which the velocities deviate significantly from the free-stream velocity. If the boundary layer is thought of as that part of the flow, where shear stresses play a rôle, (3.1) gives a more consistent measure.

Two Reynolds numbers are introduced, one with the boundary layer thickness, the other with the maximum amplitude a_{1m} (half stroke length) in the free stream as length scale, i.e.

$$Re = \frac{U_{1m}\delta}{\nu} \tag{3.2}$$

$$RE = \frac{U_{1m}a_{1m}}{\nu} \tag{3.3}$$

The wave friction factor f_w is defined from

$$\tau_{wm} = f_w \frac{1}{2} \varrho U_{1m}^2 \tag{3.4}$$

although τ_{wm} and U_{1m} are not simultaneous.

In this connection it can be shown, that if we assume a constant friction factor f in the equation

$$\tau_w = f \frac{1}{2} \varrho |U_1)^2 \tag{3.5}$$

with U_1 given by

$$U_1 = U_{1m} \sin \omega t \tag{3.6}$$

and the specific energy loss E_w per s simply by

$$E_w = \tau_w U_1 \tag{3.7}$$

then the mean specific energy loss per s is

$$\bar{E}_w = \frac{2}{3\pi} \varrho f U_{1m}^3 \tag{3.8}$$

It is often implied, that f in (3.8) is identical with f_w. This is obviously not true, for the following reasons. Firstly a phase shift should be introduced in (3.5), and secondly the constancy of f during a wave cycle can be questioned. Finally, (3.7) is only a good guess. So it can be stated, that as a matter of principle, f_w cannot be determined correctly by a wave attenuation test. The side-wall and surface corrections, which are difficult to control, and the reflection, are other sources of error.

f in (3.8) will be denoted f_e, so that we obtain the following equation of definition for the "wave energy loss factor":

$$\bar{E}_w = \frac{2}{3\pi} \varrho f_e U_{1m}^3 \tag{3.9}$$

It should be mentioned here though, that while it will be shown, that $f_w \neq f_e$ in the laminar case, it was found in Test No. 1 (see [14]) for a rough turbulent boundary layer, that f_w was practically equal to f_e. For this reason no distinction will be made in the turbulent case between f_w and f_e for the very few neasurements available. Introducing the right phase shift (~25°) in (3.5) it was also found, that f was practically constant, using the bottom shear stresses determined from velocity profiles.

The roughness parameter (k) introduced for the (fixed) bed is the Nikuradse sand roughness, as defined from the expression for the turbulent velocity profile near a rough bottom:

$$\frac{U_c}{U_f} = 5.75 \log \frac{30\ z}{k} \qquad (3.10)$$

4. METHODS OF MEASURING THE WAVE FRICTION FACTOR

The wave friction factor can be found in a variety of ways. The "classical" procedure is to measure the wave height attenuation in a flume. In the preceding chapter certain disadvantages of this method were outlined. Hence, a short descriptive review of existing methods might be of interest here.

In general one can distinguish between three main principles: Measurement of energy loss, force or velocity. These can again be subdivided as shown below. Quantitative information will not be given. This can be found in chapters 5 and 6 and in the references cited.

MEASUREMENT OF ENERGY LOSS

The quantity measured hereby is really the wave energy loss factor, see (3.9) and the appurtenant discussion.

Direct measurement - Bagnold [1] used a technique which was simple and ingenious. A celluloid plate, to which fixed imitation ripples were attached, was hung vertically in a large tank of water. The plate was oscillated by a mechanism driven by a weight in a wire. The energy dissipation was simply found from the falling velocity of the weight, corrected for mechanical friction.

Measurement of wave height attenuation - This method is based upon the principle, that the reduction in wave power between two stations equals the energy loss per s over the same distance. The procedure has been adopted by Miche [30], Imman and Bowen [9], Iwagaki et al. [10], [12], Zhukovets [37], and many others.

In this context it must be mentioned that wave height attenuation in the presence of a laminar boundary layer always seems to exceed the theoretical value. Much discussion has been devoted to this problem. In the author's opinion one or more of the following three phenomena are mainly responsible: The side-wall correction for the zone around MWL is underestimated by standard methods. The flow regime is not fully laminar (see Fig. 3). Due to (invisible) contamination, a boundary layer is present at the surface, see van Dorn [35].

MEASUREMENT OF FORCE

Direct measurement - Eagleson [7], and Iwagaki et al. [12] have measured directly the force exerted on a smooth plate by progressive shallow water waves.

Measurement of the slope of mean water level - Using the concept of the wave thrust, introduced by Lundgren [28], it was shown in [8] and [18] how the wave energy loss factor can be found by measuring the slope of the mean water level. The method is based upon elimination of dH/dx from the energy equation mentioned above, and the equilibrium condition, stating that the reduction in wave thrust between two stations equals the difference in pressure force from the rise of the mean water level over the same distance. (The wave thrust is identical to the "radiation stress" obtained independently of Lundgren by Longuet-Higgins and Stewart, [25] and [26]).

MEASUREMENT OF VELOCITY

The velocity field can be measured either over an oscillating plate, Kalkanis [20] and [21], or in an oscillating fluid, Jonsson [14].

Equation of motion - A knowledge of the complete velocity field makes possible a determination of the bed shear stress through integration of the equation of motion, see [14].

The law of the wall - Very near the wall, the turbulent velocity, relative to the wall, will be logarithmic. Thus, the friction velocity and from that the friction factor can be calculated, see [14].

Velocity measurement at a fixed level - This method is analogous to the Preston tube technique, see Jonsson [16]. If the drag cofficient corresponding to a fixed level near the bed is found in a steady flow experiment, the maximum shear stress can be found directly from measuring the maximum velocity at this fixed level.

5. CHARACTERISTICS OF THE WAVE BOUNDARY LAYER

DIMENSIONAL CONSIDERATIONS

For a given "form" of the outer (potential) velocity (here sinusoidal), dimonsional analysis yields directly the following relationships for the wave boundary layer thickness and the wave friction factor:

	δ/a_{1m}	f_w
Laminar case	$f(U_{1m}a_{1m}/\nu)$	$f(U_{1m}a_{1m}/\nu)$
Rough turbulent case	$f(a_{1m}/k)$	$f(a_{1m}/k)$
Smooth turbulent case	$f(U_{1m}a_{1m}/\nu)$	$f(U_{1m}a_{1m}/\nu)$

"f" denoting "function of". Quantitative expressions will be given in the following.

EQUATION OF MOTION

The linearized equation of motion in the boundary layer reads (for a fixed bed)

$$\frac{\partial U}{\partial t} = \frac{\partial U_1}{\partial t} + \nu \frac{\partial^2 U}{\partial z^2} - \frac{\partial \overline{u\,w}}{\partial z} \qquad (5.1)$$

with U_1 given by (3.6), u and w being the velocity fluctuations in the x- and z-directions, respectively. U_{1m} is given by

$$U_{1m} = \frac{\pi H}{T} \frac{1}{\sinh \frac{2\pi D}{L}} \left(= \frac{2\pi\ a_{1m}}{T}\right) \qquad (5.2)$$

A solution to (5.1) in the case of turbulent flow has not been found yet.*) It is known, however, that a logarithmic velocity distribution is found in the vicinity of the boundary, see [14]. The shear stress gradient at the boundary is found from (5.1):

$$\left.\frac{\partial \tau}{\partial z}\right|_{z=0} = -\ \rho\ \frac{\partial U_1}{\partial t} \qquad (5.3)$$

It is interesting to note, that this gradient is determined exclusively by the outer (potential) flow.

LAMINAR CASE

The solution to (5.1) with $\overline{u\,w} = 0$ reads ([22] p.622):

$$U = U_{1m} \left[\sin \omega t - \exp\left(-\frac{\pi}{2}\frac{z}{\delta}\right) \sin\left(\omega t - \frac{\pi}{2}\frac{z}{\delta}\right)\right] \qquad (5.4)$$

with

$$\delta = \sqrt{\frac{\pi}{4}} \cdot \sqrt{\nu\ T} \qquad (5.5)$$

From (3.3) and (5.5) we find

$$\frac{\delta}{a_{1m}} = \frac{\pi}{\sqrt{2\ RE}} \qquad (5.6)$$

where the important "amplitude Reynolds number" RE formally makes its first appearance. It can be interpreted as a measure of the square of the ratio between amplitude and theoretical laminar boundary layer thickness. (Note that the relationship

$$\frac{\delta}{a_{1m}} = \frac{Re}{RE} \qquad (5.7)$$

*) See note after refs.

is always valid, see (3.2) and (3.3)). (5.6) gives a straight line in Fig. 5.

The shear stress distribution is

$$\frac{\tau}{\varrho} = \frac{\pi}{\sqrt{2}} \frac{\nu U_{1m}}{\delta} \exp\left(-\frac{\pi}{2}\frac{z}{\delta}\right) \cos\left(\omega t - \frac{\pi}{2}\frac{z}{\delta} - \frac{\pi}{4}\right) \quad (5.8)$$

i.e. at the bottom

$$\frac{\tau_{wm}}{\varrho} = \frac{\pi}{\sqrt{2}} \frac{\nu U_{1m}}{\delta} \quad (5.9)$$

From (3.3), (3.4), (5.6) and (5.9) the wave friction factor is found

$$f_w = \frac{2}{\sqrt{RE}} \quad (5.10)$$

shown in Fig. 6.

On the other hand it can be shown, that

$$\overline{E}_w = \frac{\sqrt{\pi}}{2} \varrho U_{1m}^2 \sqrt{\frac{\nu}{T}} \quad (5.11)$$

so from (3.9)

$$f_e = \frac{3\sqrt{2}\pi}{8} \cdot \frac{1}{\sqrt{RE}} = \frac{1.67}{\sqrt{RE}} \quad (5.12)$$

i.e. different from f_w. The variation of f_e is shown in Fig. 6.

The "small" Reynolds number is here found to be:

$$Re = \frac{\pi}{\sqrt{2}} \sqrt{RE} \quad (5.13)$$

Direct measurements of the shear stress exerted on a smooth horizontal bottom by progressive, shallow water waves were made by Iwagaki et al. [12]; the results agree well with (5.10). This also applies to the energy dissipation measurements by Lukasik and Grosch [27]. The rather high values found by Eagleson [7] are presumably due to some instrumentation error. Iwagaki also found that the wave attenuation coefficients (= - (dH/dx)/(H/L)) were about 1.4 times the values as predicted by theory. The deficiency may be due to the development of a boundary layer at the free surface. (This effect has been studied both experimentally and theoretically by van Dorn [35]. Good agreement was found between theory and measurement. Although the present author does not agree entirely with the analytical treatment given in the above mentioned reference, there can be little doubt of the importance of the phenomenon). Capillary effects at the side walls may play a rôle, also.

According to [23] the roughness can be "felt" for $k/\delta > 0.25$, so the "start" of the laminar - rough turbulent regime is given by

$$\frac{a_{1m}}{k} = \frac{4\sqrt{2}}{\pi} \cdot \sqrt{RE} \qquad (5.14)$$

corresponding to line "LR" in Fig. 4. (Note that in the classical experiments by Nikuradse [32] a direct transition from laminar to rough turbulent flow was also found, for the larger ratios between roughness and pipe radius. From [34] p. 483 it appears, that the limiting ratio was close to $r/k = 15$, r being pipe radius. r is twice the "hydraulic radius" i.e. corresponds to 2 D_e or 4 δ according to (3.1). The criterion $r/k = 15$ is therefore transformed to $k/\delta = 4/15$ which comes very close to the value obtained by Lhermitte).

In the open channel experiments of Reinius [33], the transition between laminar and turbulent flow was found to occur for $\bar{U}_c D/\nu$ between 500 and 1000, with the most abrupt change of f for $\bar{U}_c D/\nu$ about 575. It is therefore proposed - using (3.1) and (3.2) - that smooth turbulence starts for Re = 250, corresponding to

$$RE = 1.26 \cdot 10^4 \qquad (5.15)$$

using (5.13) (line "LS" in Fig. 4). This guess was confirmed surprisingly well by Collins [6], who found the value $RE = 1.28 \cdot 10^4$ from measurements of mass transport velocities at the edge of the boundary layer. (Li [24] found $1.60 \cdot 10^5$, and Vincent [36] $6.2 \cdot 10^5$. These limits are without any doubt very subjective because of the method employed (visual observation of the stability of dye streaks)).

ROUGH TURBULENT CASE

The only velocity measurements known to the author are those reported in [14], [20] and [21]. However, the measurements of Kalkanis [20], [21] are made too far from the (oscillating) wall to allow a determination of the friction factor.

In [14] and [15] the following expressions for the boundary layer thickness and the wave friction factor were found

$$(30\,\frac{\delta}{k}) \cdot \log\,(30\,\frac{\delta}{k}) = 1.2\,\frac{a_{1m}}{k} \qquad (5.16)$$

$$\frac{1}{4\sqrt{f_w}} + \log\frac{1}{4\sqrt{f_w}} = -\,0.08 + \log\frac{a_{1m}}{k} \qquad (5.17)$$

corresponding to the horizontal lines in Figs. 5 and 6. (log is $\log_{10}$).

(5.16) can also be written

$$\frac{\delta}{a_{1m}} \cdot \log\,(\frac{\delta}{a_{1m}}\,\frac{30\,a_{1m}}{k}) = 0.04 \qquad (5.18)$$

Typical values of δ/a_{1m} and f_w are given in the table below.

$\frac{a_{1m}}{k}$	1	2	5	10	20	50	100	200	500	1000	2000
$\frac{\delta}{a_{1m}} \cdot 10^2$	9.15	6.65	4.72	3.80	3.14	2.54	2.20	1.94	1.67	1.51	1.38
$f_w \cdot 10^2$	47.8	23.8	11.2	7.00	4.65	2.93	2.19	1.67	1.22	0.985	0.810

It should be mentioned, that (5.16) and (5.17) are based upon a very simple theory, assuming that the logarithmic velocity profile in the turbulent wave boundary layer extends uninterrupted to the potential velocity. This gives the general "shape" of the formulae, see [27]. Furthermore the numerical coefficients or terms were determined from one test only in the oscillating water tunnel [14]. Consequently, checks on the validity of the two above expressions are naturally called for.

From the measurements of Kalkanis [21] values of δ can be deduced. For the two-dimensional roughnesses ($10 < a_{1m}/k < 64$, see Fig. 4), values were found being approximately 20% smaller than obtained by (5.18). Considering the many sources of error in the measurements this discrepancy can be accepted.

More information can be found from the literature on f_w. Bagnold's measurements have been re-analysed, and it was found that his friction factor "k" equals $f_e/3 \approx f_w/3$. The pitch/height ratio p/h of the ripples was 6.7/1 and the ripple trough sections consisted of circular arcs meeting to form sharp crests at an angle of 114°. Similar ripples have been analysed by Motzfeld [31], who found k = 4 h. The results are plotted in Fig. 3. (In the three test series not shown, 2 a_{1m} is smaller than p, and they are therefore without interest). It appears from Fig. 4 that the tests are all in the fully developed rough turbulent regime.

Eliasson et al. have measured the slope of the mean water level due to the reduction in wave thrust, see chapter 4 and [8]. Two test series are shown in Fig. 3. After the completion of the tests with k = 2.3 cm, the measuring system was highly improved, and the last series (with k = 1 cm) shows reasonable agreement with the theoretical curve. The measurements are very near the limit "RL", see Fig. 4. (Because of the very low a_{1m}/k ratio, no side-wall correction was introduced. It will be of the order of magnitude of 20%). A wave flume measurement of wave height attenuation (corrected for side-wall effects) by Inman and Bowen [9] (Test No. 1 A) is also shown.

It can be concluded that the number of reliable measurements is very scarce. In the author's opinion most importance should probably be attached to Bagnold's experiments and to the measurements in the oscillating water tunnel. The interpretation of the former is a little difficult, however. There is information, which indicates, that Motzfeld has overestimated k a little. If k was 3 (instead of 4) times h, then Bagnold's results would in fact coincide with the curve in Fig. 3.

So the expressions for δ and f_w given by (5.16) and (5.17) are preserved for the present. It could be mentioned here, that the tendency in Fig. 3 - f_w decreasing with increasing amplitude - was also found by Iwagaki and Kakinuma [11] from analysis of prototype observations.

In the preceding discussion we have anticipated the existence of limits for the turbulent regime. These are found as follows.

It is assumed that complete turbulence is developed for $U_{1m}D_e/\nu = 1000$. So we find from (3.1), (3.2) and (3.3)

$$Re = 500 \tag{5.19}$$

or

$$RE = 500 \, \frac{a_{1m}}{k} \cdot \frac{1}{\delta/k} \tag{5.20}$$

corresponding to the lines "RL" in Figs. 4, 5 and 6. Zhukovets' observation [37], that "the quadratic region exists for Reynolds numbers from $1.5 \cdot 10^4$ to $3.3 \cdot 10^4$" agrees well with these values. The transition curves in Figs. 5 and 6 are estimated. It is improbable that they will not be "smooth", since the main motion is unsteady.

Colorimetric investigations by Miche [29], [30] are plotted in Fig. 4. In all 7 tests turbulence was (visually) present.

The limit between the rough turbulent and the smooth turbulent- rough turbulent transition regime is determined by the ratio between roughness and thickness of the viscous sublayer. This quantity is here defined by

$$\delta_{visc} = \frac{11.6\ \nu}{\bar{U}_f} = \frac{11.6\ \nu}{\frac{2}{\pi} \cdot U_{fm}} = \frac{18.2\ \nu}{U_{fm}} \tag{5.21}$$

supposing τ_w to vary as $\sin^2(\omega t + \varphi_o)$.

Assuming

$$\frac{k}{\delta_{visc}} = 3 \tag{5.22}$$

by analogy with steady flow conditions, (5.22) can also be written as

$$RE = 77.2 \frac{1}{\sqrt{f_w}} \frac{a_{1m}}{k} \qquad (5.23)$$

This corresponds to lines "RS" in Figs. 4, 5 and 6.

It is worth while to look a little closer at the friction factor curve in Fig. 3. Firstly it is seen, that in the neighbourhood of a_{1m}/k equal to one we find $f_w \sim (a_{1m}/k)^{-1}$, which can be shown to yield an exponential wave height variation. It is sometimes stated, that this variation is a "sign of laminar damping"; it is interesting to note, that this well may be a false interpretation.

Secondly it can be shown, that the relationship between f_w and δ/k, as given by (5.16) and (5.17), is very closely fitted by

$$f_w = \frac{0.0604}{\log^2 \frac{22\ \delta}{k}} \qquad (5.24)$$

which is identical to the friction factor in a steady, uniform flow, if D is put equal to 2 δ (cf. (3.1)).

From certain compatibility relations ([19]) the phase shift φ_o (between τ_{wm} and U_{1m}) is found to decrease with increasing a_{1m}/k. The following values are proposed: $a_{1m}/k = 100 \Rightarrow \varphi_o = 29^o$, $a_{1m}/k = 1000 \Rightarrow \varphi_o = 11^o$.

SMOOTH TURBULENT CASE

Apparantly only Kalkanis [20], [21] has measured velocities at a smooth wall. The measurements do not allow a determination of f_w, however. And the boundary layer thicknesses, which can be deduced from the measurements are strangely enough equal to or smaller than the laminar thicknesses corresponding to the same values of RE. We are therefore compelled to make a reasonable guess, which will be to use the formulae for the rough turbulent case, with a formal roughness parameter, defined by

$$k = \frac{\nu}{0.3\ U_f} = 0.287\ \delta_{visc} \qquad (5.25)$$

(This relation corresponds to a von Kármán number Φ equal to one).

Using (5.25) together with (5.16) and (5.17) we obtain

$$\frac{\delta}{a_{1m}} = \frac{0.0465}{\sqrt[10]{RE}} \qquad (5.26)$$

and

$$\frac{1}{4\sqrt{f_w}} + 2 \log \frac{1}{4\sqrt{f_w}} = \log RE - 1.55 \qquad (5.27)$$

shown in Figs. 5 and 6. ((5.26) is a very close approximation to a complicated expression). Some typical values are listed below.

RE	$3 \cdot 10^4$	10^5	10^6	10^7
$\frac{\delta}{a_{1m}} \cdot 10^2$	1.70	1.45	1.13	0.92
$f_w \cdot 10^2$	1.26	0.916	0.544	0.354

If we again use (5.19) as a criterion for fully developed turbulenoe, (5.26) yields

$$RE = 3.00 \cdot 10^4 \tag{5.28}$$

corresponding to line "SL" in Fig. 4. Figs. 5 and 6 suggest a transition from the laminar regime which is much more "gentle" than in steady flow, as would be expected.

The limit between the smooth turbulent and the smooth turbulent - rough turbulent transition regime is supposed to be determined from

$$\frac{k}{\delta_{visc}} = 0.287 \tag{5.29}$$

which seems to be verified by Lhermitte [23]. This condition is identical with (5.25), so line "SR" in Fig. 4 can be found simply from the intersection of the smooth turbulent curve in Fig. 5 (and 6) with the horizontal lines from the rough regime.

The values of the friction factor in the region between the smooth turbulent and the rough turbulent regimes may be affected by the type of roughness. Note the difference in steady flow between uniform roughness elements (Nikuradse [32]) and non-uniform roughness elements (Colebrook [5]). The real transition lines can presumably only be determined by means of experiment.

A good approximation to (5.27) is

$$f_w = 0.09 \cdot RE^{-0.2} \tag{5.30}$$

The exponent is seen to be the same as in the expression for the friction factor for a smooth plate boundary layer (with zero pressure gradient), see [34] p. 500.

6. NUMERICAL EXAMPLES

COMPARISON BETWEEN PROTOTYPE AND MODEL

<u>Prototype</u> - D = 12 m, H = 2.3 m, T = 8 s, k = 0.1 m ⇒ L = 76 m, a_{1m} = 1.0 m, a_{1m}/k = 10, RE = $7.9 \cdot 10^5$. Fig. 4 shows that we are well within the rough turbulent regime, and Fig. 6 yields $f_w = 7.0 \cdot 10^{-2}$.

Model (scale 1:100, Froude) - D = 12 cm, H = 2.3 cm, T = 0.8 s, k = 0.1 cm ⇒ L = 76 cm, a_{1m} = 1.0 cm, a_{1m}/k = 10, RE = $7.9 \cdot 10^2$. Fig. 4 shows that we are in the laminar - rough turbulent transition regime, and Fig. 6 yields $f_w \approx 10 \cdot 10^{-2}$. (Incidentally, had the model bottom been perfectly smooth, the flow would have been laminar, and the same friction factor as in nature would have been obtained).

The example shows, that by Froude scaling the shear stresses may easily become 40% too high in the model.

SMOOTHNESS OF A MODEL BED

A necessary condition for the bed to act hydraulically smooth (in the turbulent regime) is that RE > $3.00 \cdot 10^4$, see Fig. 4. Even for the rather high value H/D = 0.3, it is found that waves of steepness 1.5%, 3% and 6% require depths larger than 27 cm, 47 cm and 102 cm, respectively, to reach this limit.

The bottom amplitudes corresponding to the above wave data are approximately 10 cm. Since from Fig. 4, a_{1m}/k must at least exceed 475 for the bed to be smooth, it is required, that k should be smaller than 0.2 mm. This corresponds to the hydraulic roughness of a smooth plaster finish. So it will be realized, that pure smooth turbulent flow will hardly ever be met.

7. CONCLUSIONS

(a) For a simple harmonic motion over a fixed bed, the proper flow regime can be found from Fig. 4, when the ratio between maximum amplitude and bottom roughness (a_{1m}/k) and the Reynolds number (as given by (3.3)) are known. Figs. 5 and 6 then yield the boundary layer thickness (defined by Fig. 2), and the friction factor (defined by (3.4)).

(b) In the laminar case experimental results of Iwagaki et al. [12] for the wave friction factor agree well with linear theory. In the rough turbulent case, the proposed friction factors are probably not wrong by more than 20%, see Fig. 3. In the case of smooth turbulent flow no measurements are available, so future revisions in the diagrams are not excluded. More measurements of the wave friction factor are earnestly needed, especially for the laminar - rough turbulent transition regime.

(c) In the laboratory the laminar - rough turbulent transition regime will often be found. Pure smooth flow is exceptional.

(d) In nature the boundary layer is always rough turbulent. The friction factor here will often exceed the value of $2 \cdot 10^{-2}$ adopted by Bretschneider [3]. This is also confirmed by observations of Iwagaki and Kakinuma [11].

(e) It may be difficult to attain the same friction factor in a wave model study as in nature.

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Added in proof: In a private communication ("On the Bottom Frictional Stress in a Turbulent Oscillatory Flow" March 8, 1965, received Aug. 1966), Professor Kinjiro Kajiura, Earthq. Res. Inst., Univ. of Tokyo, has made an important contribution to the analytical treatment of oscillatory turbulent boundary layers. Due to the time limit for the completion of this paper, the author unfortunately was unable to incorporate Professor Kajiura's valuable viewpoints and results.

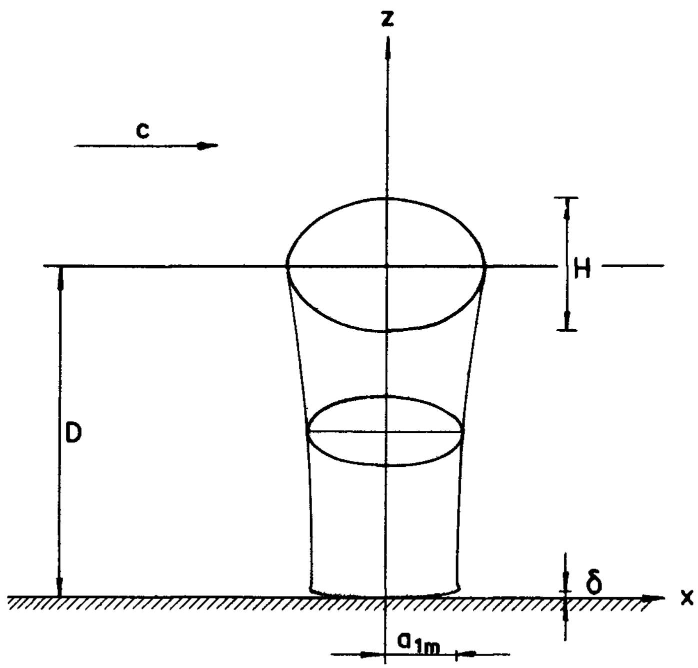

Fig. 1. Wave particle motion at different levels.

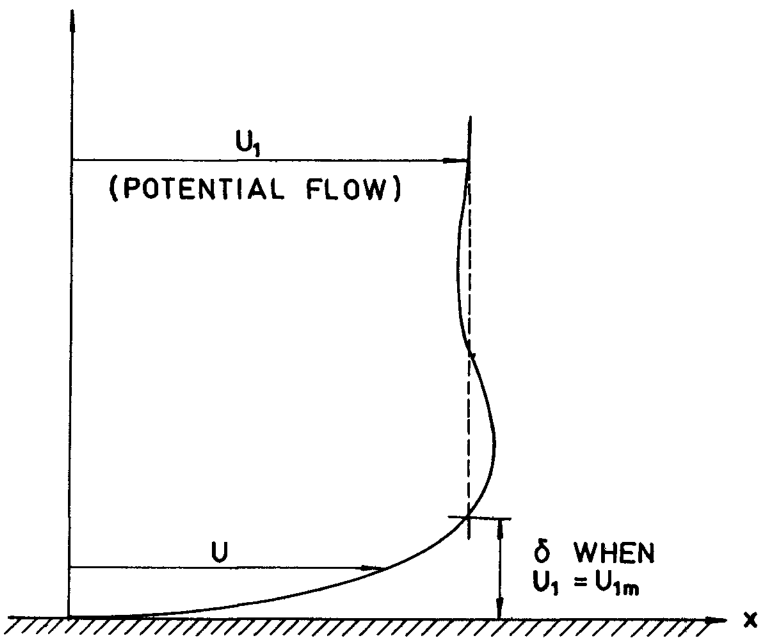

Fig. 2. Typical velocity profile in the boundary layer.

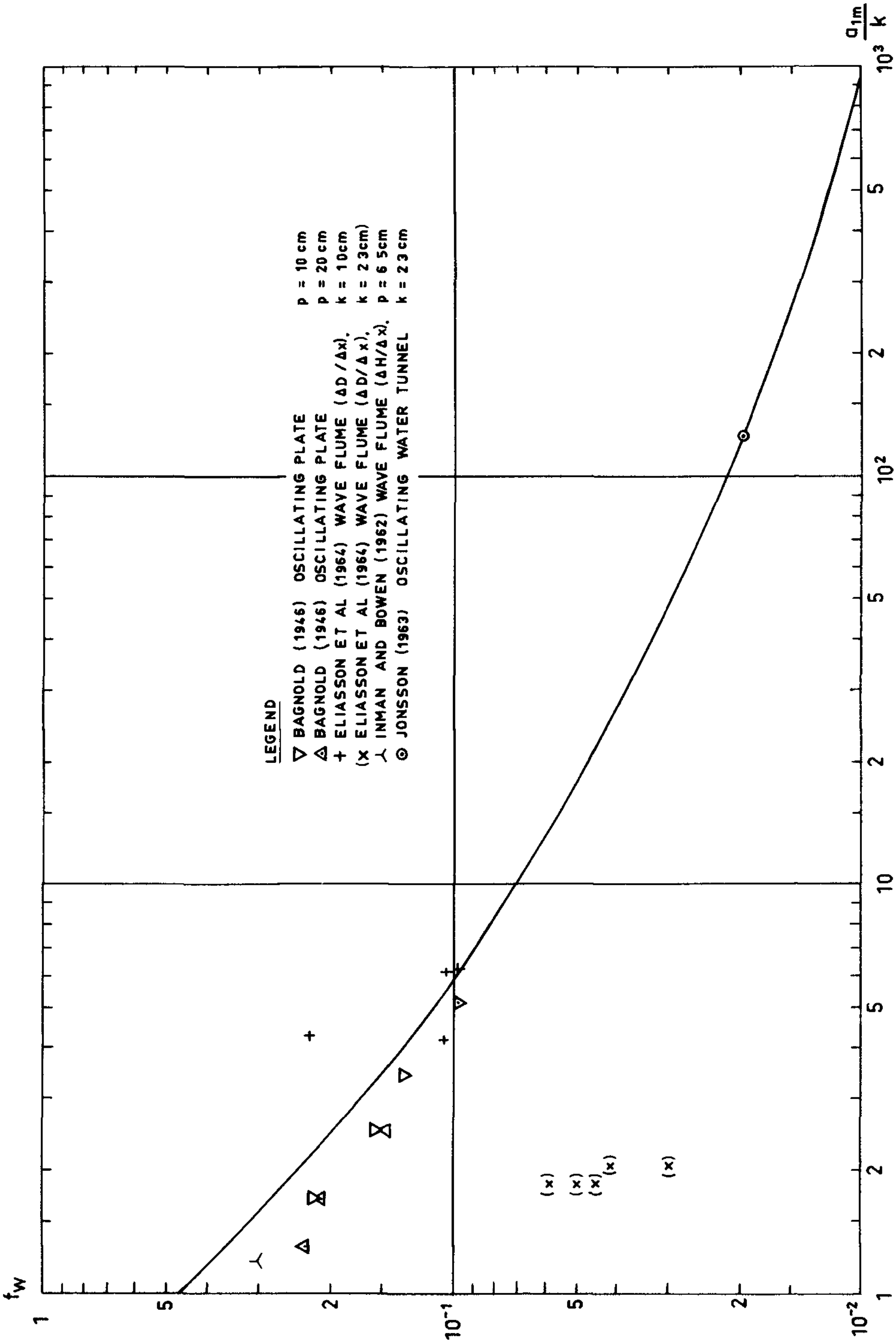

Fig. 3. Measured wave friction factors in the rough turbulent regime.

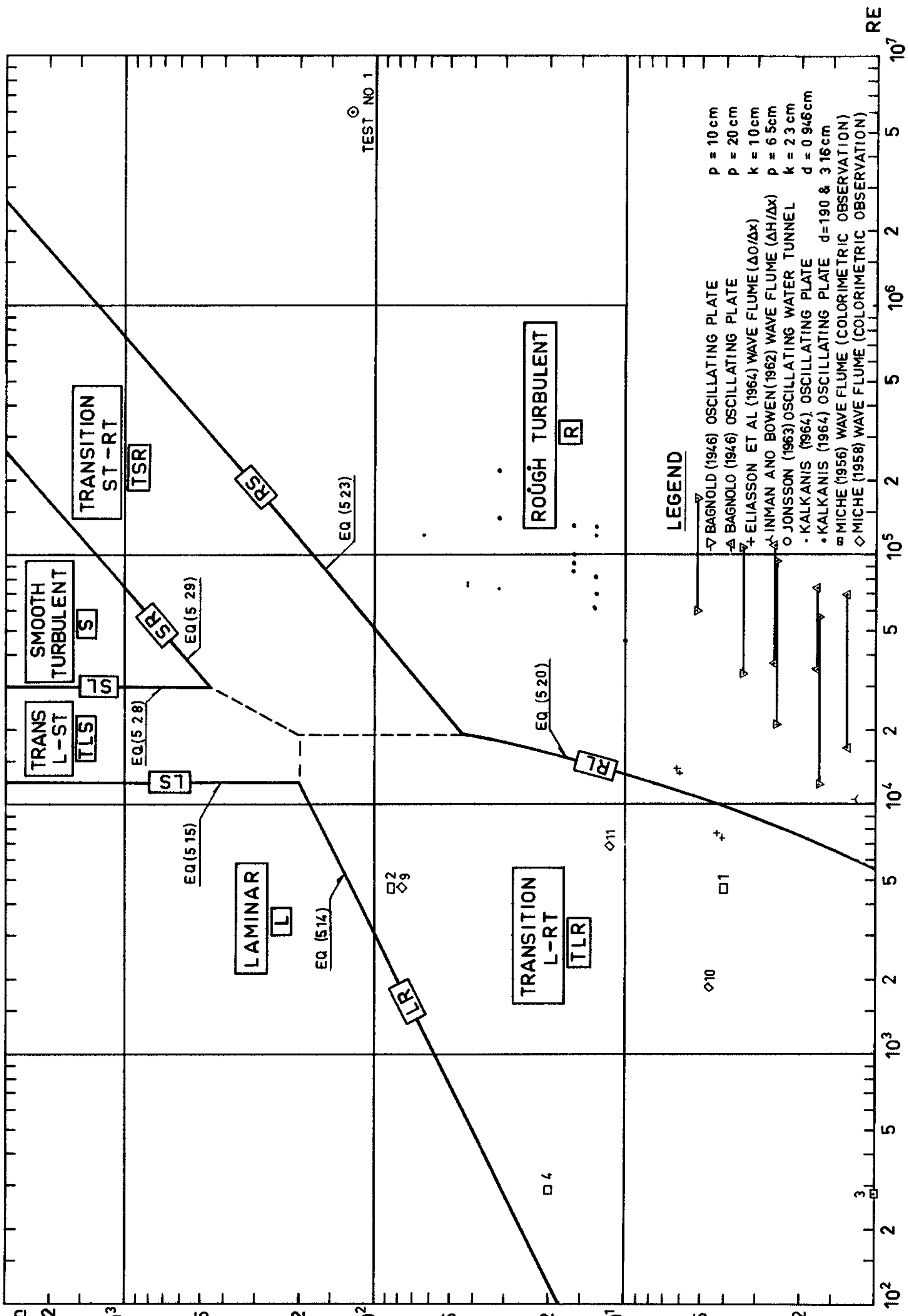

Fig. 4. Flow regimes.

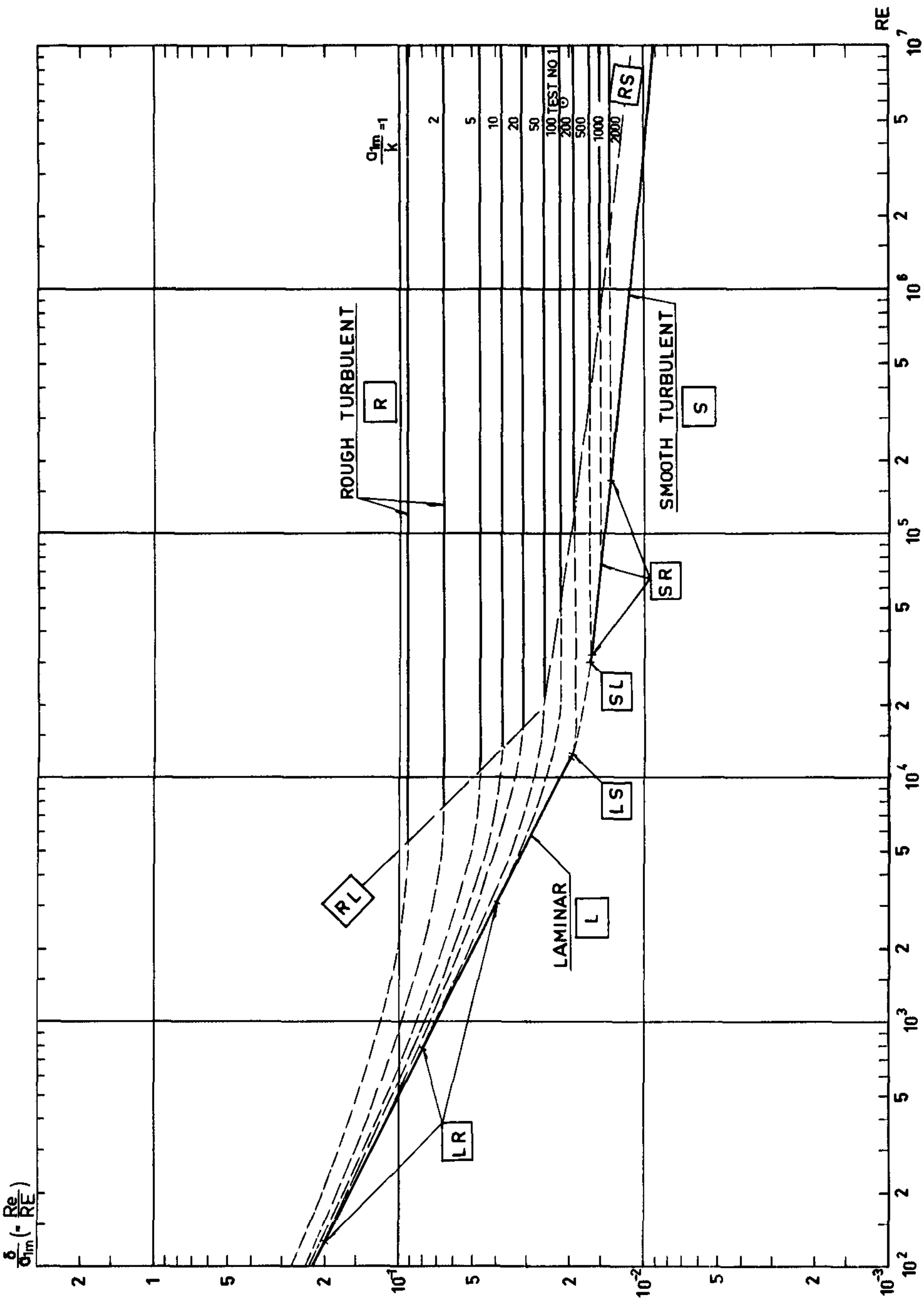

Fig. 5. Wave boundary layer thicknesses.

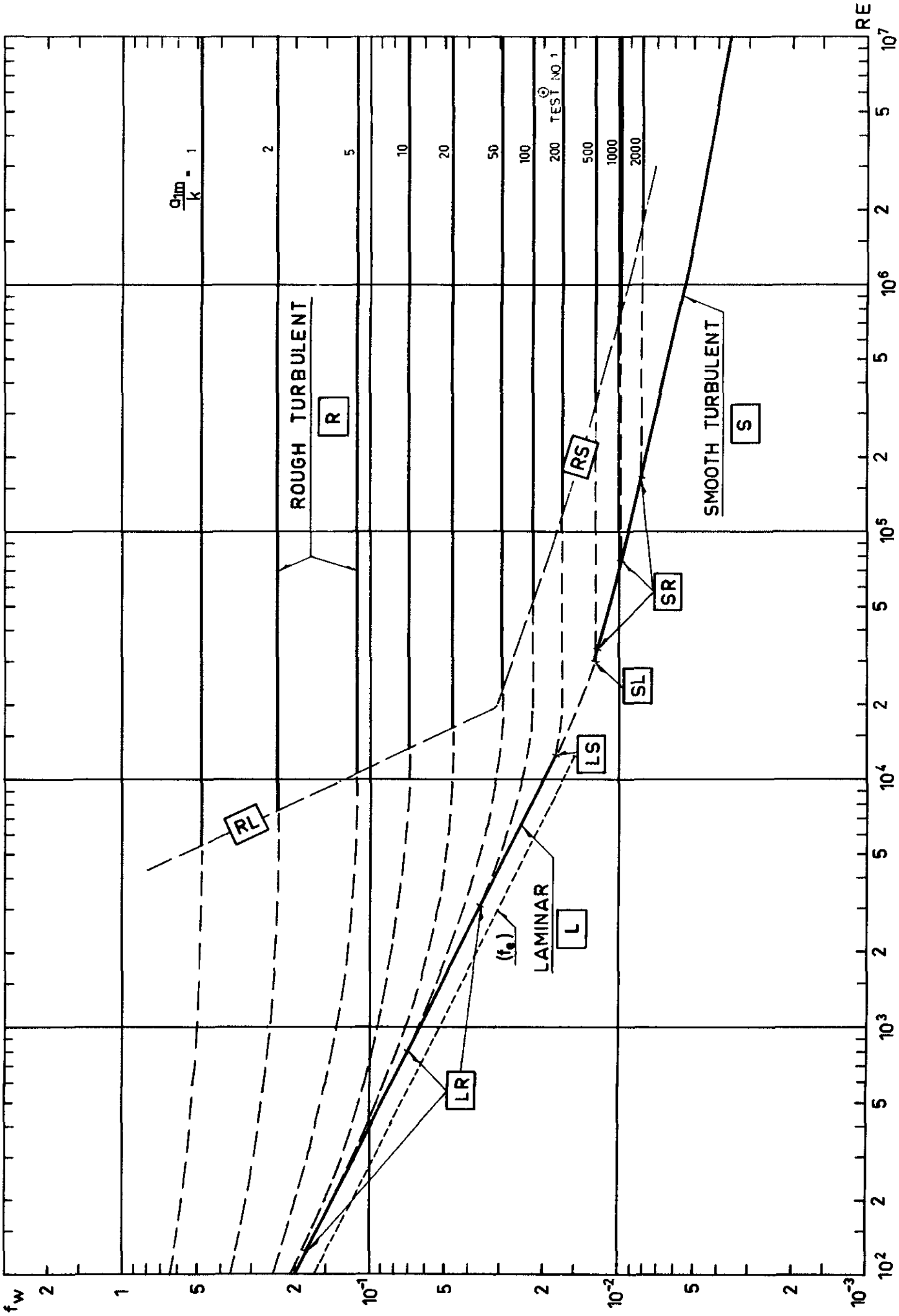

Fig. 6. Wave friction factors.