JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 90, NO. C6, PAGES 11,939–11,944, NOVEMBER 20, 1985

Wave Energy Saturation on a Natural Beach of Variable Slope

ASBURY H. SALLENGER, JR.[1]

Branch of Pacific Marine Geology, U.S. Geological Survey, Menlo Park, California

ROBERT A. HOLMAN

College of Oceanography, Oregon State University, Corvallis, Oregon

Time series of flow were measured across the inner surf zone during a storm. These data were used to quantify the dependence of wave height (transformed from measured flow) and velocity on local slope and depth. Similar to previous studies, as incident waves broke and propagated into the surf zone, wave energy became saturated, and wave height was strongly dependent on depth. However, the ratio of rms wave height to local depth ($\gamma_{\rm rms}$) was found not to be constant but to vary between 0.29 and 0.55; $\gamma_{\rm rms}$ increased with local slope and was independent of deepwater wave steepness. Thus the surf zone similarity parameter (the ratio of slope to the square root of steepness) did not adequately parameterize $\gamma_{\rm rms}$.

INTRODUCTION

Wave energy in the inner surf zone is thought to become saturated so that the local wave height H is a linear function of the local depth h,

$$H = \gamma h \tag{1}$$

where γ, according to some physical model studies, varies with bottom slope and wave steepness. The significance of (1) is that wave height is described everywhere in the surf zone without having to determine the details of energy dissipation that results from breaking waves. Equation (1) is a critical element of theories of longshore currents [*Bowen*, 1969; *Longuet-Higgins*, 1970; *Thornton*, 1971], wave setup [*Bowen et al.*, 1968], and other surf zone processes.

Although the dependence of γ on slope and wave steepness has been extensively investigated for monochromatic waves, few data are available that quantify the variability of γ for random waves in the field. In the present study we measured time series of flow across the surf zone during a storm. We transformed the measurements of flow to wave heights and determined γ across a profile whose slope varied, both in time and space, nearly an order of magnitude. The data were collected under a wide range of incident wave conditions from steep sea waves during a storm to long-period swell waves. The data set was varied enough to allow the dependence of γ on bottom slope and wave steepness to be thoroughly examined.

In the following section we review theory and the previous model and field studies on wave energy saturation. Then, after discussing the design of the experiment and its setting, we present results on how γ varied with slope and wave steepness.

LITERATURE REVIEW

Using solitary wave theory, *McGowan* [1891] showed theoretically that γ should be a constant equal to 0.78. However, physical model studies employing monochromatic waves have shown that γ depends on beach slope and/or wave steepness, varying roughly between 0.7 and 1.2 [*Galvin and Eagleson*, 1965; *Iverson*, 1952]. For monochromatic waves, *Bowen et al.* [1968] and *Battjes* [1974] used a surf zone similarity parameter ξ_0 to show the dependence of γ on slope and steepness

$$\xi_0 = \frac{\tan\beta}{(H_0/L_0)^{1/2}} \tag{2}$$

where β is the slope of the bottom profile, H_0 is deepwater wave height, and L_0 is deepwater wavelength. For most model data, γ increased with increasing ξ_0.

Most field studies have determined γ by using films that follow individual wave crests across the surf zone [*Sverdrup and Munk*, 1946; *Suhayda and Pettigrew*, 1977; *Weishar and Byrne*, 1978]. *Weishar and Byrne* [1978] found γ to have a mean value close to the solitary wave theoretical result, although they found γ to vary with breaker type, breaker type being a function of β and H_0/L_0. Sverdrup and Munk reported similar results. However, these film results probably do not reflect the total distribution of waves. For random waves at any given mean depth in the surf zone there will be distributions of both breaking and unbroken wave heights [*Thornton and Guza*, 1983]. As a result of following single-breaking wave crests, presumably the best developed and largest breakers, the measured γ of a film study may be significantly larger than γ determined from the entire wave distribution, which includes both breaking and unbroken waves.

Using an extensive array of instruments, *Thornton and Guza* [1982] obtained time series of flow across a surf zone. They transformed these flow data to surface elevation spectra and found $\gamma_{\rm rms}$ (γ based on rms wave height) for the inner surf zone to be 0.42, a factor of 2 or more less than the monochromatic wave results. Since the beach on which they worked was of nearly uniform slope, they were unable to determine whether there was a dependence of γ on β. *Wright et al.* [1982] and *Downing* [1983] also found γ for random waves of the field to be significantly lower than what was found for monochromatic waves.

EXPERIMENT

During October 1982, a field experiment was conducted at the Field Research Facility (FRF) of the U.S. Army Engineer's Waterways Experiment Station in Duck, North Carolina. The overall objective of the experiment was to determine the processes that cause nearshore morphology to change during

---

[1] Now at U.S. Geological Survey, National Center, Reston, Virginia.

Paper number 5C0601.
0148-0227/85/005C-0601$05.00

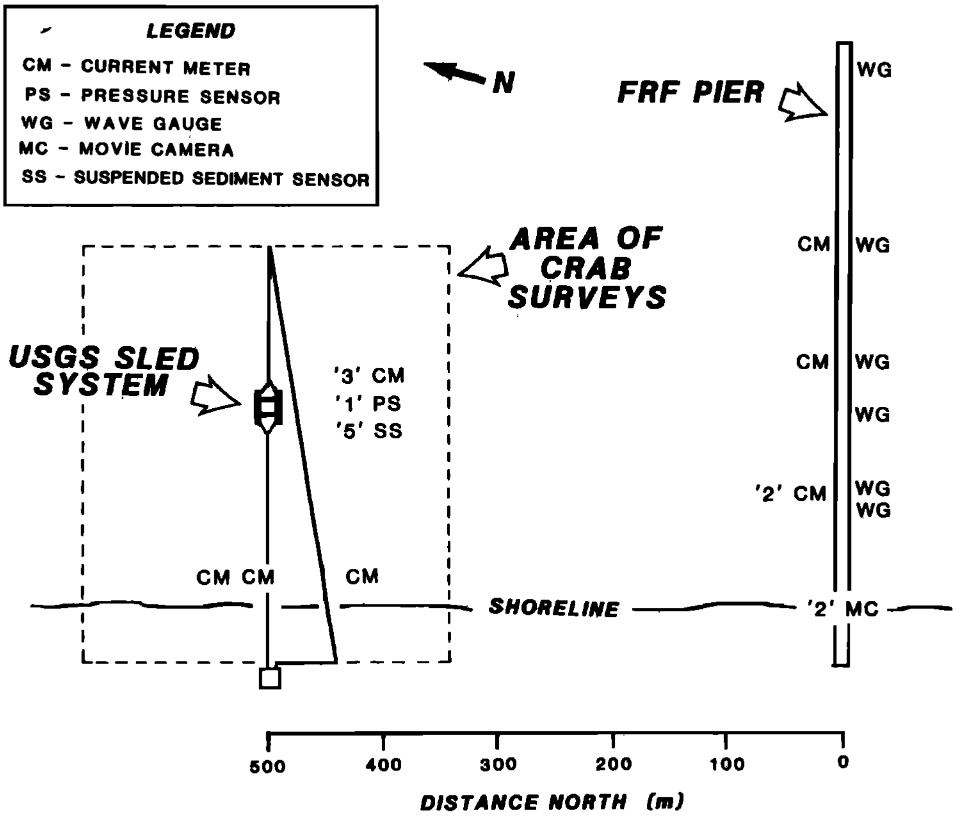


Fig. 1. Plane view of the USGS sled system and the FRF pier.

storms. Some initial results from this experiment are reported in *Sallenger et al.* [1985] and *Holman and Sallenger* [1985].

The FRF is located on a long straight beach of a barrier island that has a single well-developed nearshore bar. The foreshore is composed of coarse sand (mean diameter: 1–2 mm), whereas seaward of the trough of the bar, sand is relatively fine (mean: 0.18 mm).

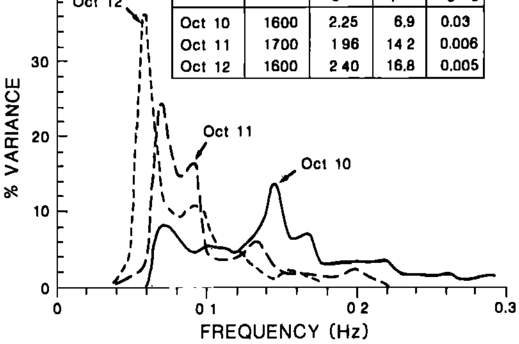
Most of the data discussed in this paper were obtained with the USGS sled system, which consists of an instrumented sled towed along the bottom, both offshore and onshore, with a double-drum winch and triangular line arrangement [*Sallenger et al.*, 1983]. On the sled, 4-cm-diameter electromagnetic current meters (Marsh-McBirney model 512) were mounted in a vertical array at 0.5 m, 1.0 m, and 1.75 m above the bottom. Flow data used in this report were measured with the 1.0-m sensor, although γ_{rms} was independent of depth of measurement. Also mounted on the sled was a pressure sensor, which was used to determine local depths. Data were telemetered to a shore receiving station where they were digitally recorded at a rate of 2 Hz. All record lengths were 34.1 min. As the sled moved along the shore-normal transect, the nearshore profile was measured by using an infrared rangefinder on the beach, sighting on optical prisms mounted on top of the sled's mast. The sled system was set up 500 m north of the FRF pier (Figure 1) to avoid effects the pier has on local flows and sediment transport [*Miller et al.*, 1983].

Additional wave data discussed in this report were measured with a pressure sensor located several hundred meters seaward of the sled transect and a wave-rider buoy moored in 20 m of water 3 km offshore of the FRF. The pressure sensor data were collected by the Coastal Data Information Program [*Scripps Institute of Oceanography*, 1982]. The wave-rider data were telemetered to shore and processed by the FRF [*Coastal Engineering Research Center*, 1982].

During October 1979, a similar, but smaller-scale, experiment was conducted on the shore of Monterey Bay, California, at Ford Ord. Flow and pressure data were obtained across this high-energy surf zone by using the USGS sled system.

RESULTS

Most of the data discussed here were obtained at the FRF during a 3-day storm, October 10–12, 1982. Representative spectra of surface elevation, using records from the offshore wave rider, are shown in Figure 2. During this high-energy period, wave steepness (calculated by using peak period and tabulated on Figure 2) and width of the incident spectrum varied considerably. On October 10, strong northeast winds, which reached 13 m/s, generated steep waves with a broad incident spectrum. By October 12 the storm had moved off-



DATE	TIME	H_0(m)	T_p(s)	H_0/L_0
Oct 10	1600	2.25	6.9	0.03
Oct 11	1700	1.96	14.2	0.006
Oct 12	1600	2.40	16.8	0.005

Fig. 2. Representative spectra for the 3 days of the storm, using data from the offshore wave-rider buoy. Inset tabulates significant wave heights (H_0), peak periods (T_p), and wave steepnesses (H_0/L_0).

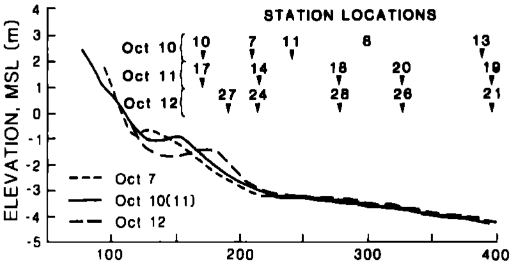


Fig. 3. Profiles and sled station locations for the experiment at Duck, North Carolina. The order in which the stations were occupied is indicated by each station number. Profiles for the 10th and 11th were very similar; only the profile of the 10th is shown.

shore, and local waves were light; waves were principally of the swell type and, relative to October 10, the incident spectrum had significantly decreased in width (Figure 2). Throughout the 3-day period, the sled transect was within the inner 50% of the surf zone. On October 12, significant breaker height of nearly 4 m resulted in a surf zone width of roughly 900 m, the breaker zone occurring 400 m seaward of the end of the pier (Figure 1).

Prior to the storm, a relatively small longshore bar was surveyed (Figure 3). Additional surveys to the north and south of the profile showed the bar at this time to be reasonably linear and shore parallel [*Sallenger et al.*, 1985]. During the 3 days of the storm, the bar became better developed, and it migrated offshore about 57 m (Figure 3). Unfortunately, during this high-energy period, we were unable to adequately determine the three-dimensional morphology of the bar. Observations by *Short* [1979] and others suggest that bars are linear when incident waves are large.

For each day of the storm, flow records were obtained across the nearshore profile (Figure 3). (Since one would not expect waves to be saturated in the trough of the bar, we do not discuss any trough data). Following *Thornton and Guza* [1982], we transformed flow spectra to surface elevation spectra by using linear theory, and we calculated H_{rms} as

$$H_{\text{rms}} = 2\sqrt{2}\left\{\int_{0.05}^{0.33} |H(f)|^2\,[G_u(f) + G_v(f)]\,df\right\}^{0.5} \tag{3}$$

where $G_u(f)$ is the cross-shore flow spectrum, $G_v(f)$ is the longshore flow spectrum, and $H(f)$ is the linear transformation function given by

$$H(f) = \frac{\sinh|k|h}{\sigma\cosh\left(|k|(h+z)\right)} \tag{4}$$

where k is wave number and z is the instrument depth measured positively upward from the still water surface. H_{rms} was calculated over the incident wave band of 0.05–0.33 Hz. The cutoff of 0.05 Hz was chosen to remove low-frequency waves, such as surf beat, that would not be expected to break in the surf zone. Thornton and Guza showed that (3) was a reasonable approximation of H_{rms}, even in shallow water. *Guza and Thornton* [1980] showed that H_{rms} calculated by using (3) and (4) compared within 20% with H_{rms} data calculated from both pressure sensor and surface-piercing wave staff records. For the present data we found H_{rms} calculated from flow data was roughly 10% below that calculated from pressure. It should be emphasized that H_{rms} of (3) incorporates both breaking and unbroken waves.

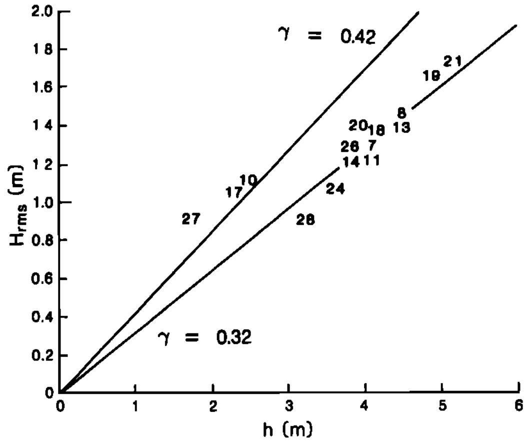


Fig. 4. Sled data from the storm (October 10–12, 1982). Station numbers are used as data points. Refer to Figure 3 for station locations.

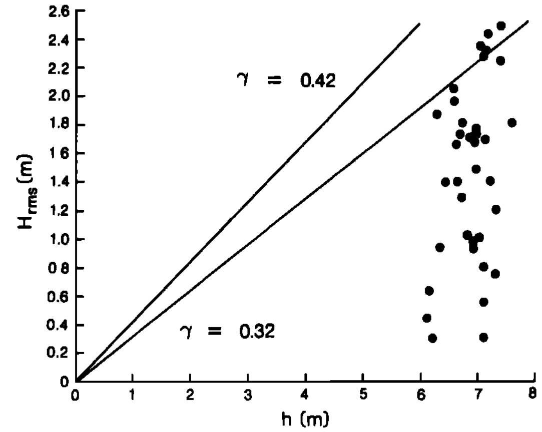


Fig. 5. Pressure data measured with Sxy gauge 580-m offshore (Figure 1). The data were obtained from the Coastal Data Information Program. Data were taken at 6-hour intervals for the periods October 10–13 and October 23–25, 1982.

H_{rms} is plotted against depth in Figure 4. Again, all of the data were obtained from the inner surf zone where depth was shoal enough for most waves in the incident spectrum to break. Thus energy saturation would be expected. The $\gamma_{\text{rms}} = 0.42$ relationship for the inner surf zone [*Thornton and Guza*, 1982] does not fit the majority of data. For most of the data, $\gamma_{\text{rms}} = 0.32$ is indicated. Note that the data that seem to agree with the 0.32 trend were all obtained on the relatively gentle slope, well seaward of the bar crest (compare run numbers of Figure 4 to station locations of Figure 3). The data that are closer to the 0.42 trend were obtained from the much more steeply sloping part of the profile immediately seaward of the bar crest.

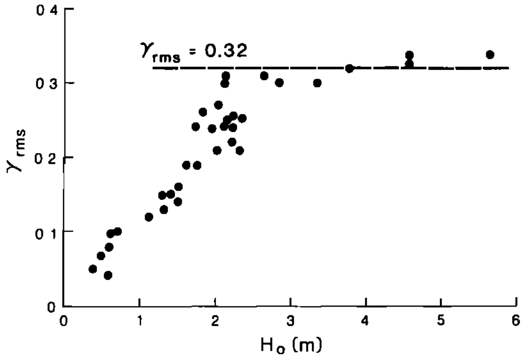


Fig. 6. γ_{rms} versus H_0 for pressure sensor data shown in Figure 5.

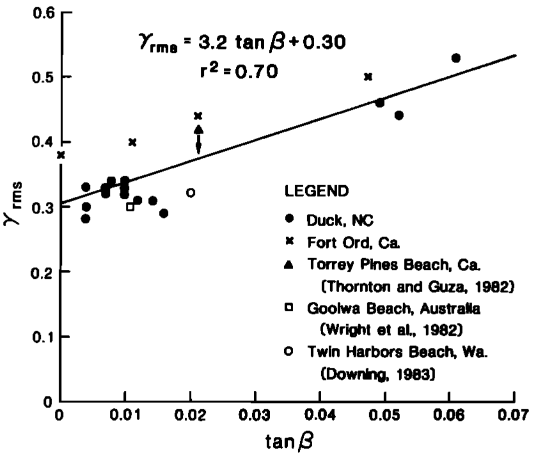


Fig. 7. γ_{rms} versus tan β, where β is the slope of the bottom profile at the appropriate sled station. The data point from *Thornton and Guza* [1982] represents a maximum γ_{rms}. Thus this datum is not directly comparable to the other data and is not used in calculating the trend.

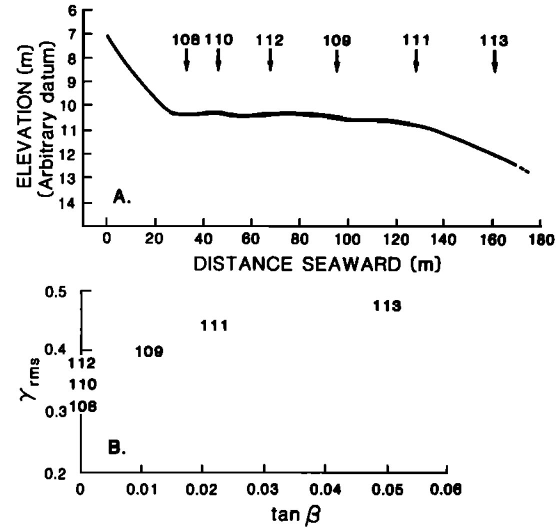


Fig. 8. (*a*) Station locations for Fort Ord, California. (*b*) γ_{rms} versus β for the Fort Ord data.

Well seaward of the sled line where the slope was about the same as the gentle slope discussed above, wave energy also appeared to saturate with $\gamma_{rms} = 0.32$. Plotted in Figure 5 are data from the pressure sensor located 560 m offshore. Low-frequency waves were included in calculating these published wave heights. However, the pressure sensor was far enough offshore so that the wave height data were not seriously contaminated. Included in the plot are data obtained every 6 hours through two storms. One was the storm discussed above in regard to the sled data, and the other was a storm 2 weeks later, during which offshore significant wave heights reached nearly 6 m. Since the pressure sensor was in much deeper water than depths occupied by the sled and since we included low-energy data preceding and following each storm, not all of the data plotted in Figure 5 would be expected to be saturated. Thus for the roughly constant depth of the pressure sensor (varying somewhat with tides), wave height ranged from relatively small to a maximum saturated value governed by equation (1) with γ_{rms} equal to about 0.32 (Figure 5). This apparent saturation is confirmed in Figure 6, where using the same data, we plot γ_{rms} against H_0, deepwater significant wave height (approximated by significant height at the wave-rider buoy). For H_0 less than about 2.3 m, γ_{rms} simply increasd with H_0, indicating the waves were not saturated. However, for H_0 greater than about 2.3 m, wave energy appeared to saturate, with γ_{rms} becoming roughly a constant at about 0.32, irrespective of H_0.

The dependence of γ_{rms} on tan β is shown in Figure 7. Slopes plotted on Figure 7 and presented in Table 1 are local slopes at the measurement location evaluated over horizontal distances of 10–15 m. The solid circles indicate the sled data discussed thus far; a rough trend is apparent with γ_{rms} increasing with tan β.

As discussed above, additional sled data were gathered at Ford Ord, California. These data were analyzed in the same manner as described above. Unlike the Duck data, for which

TABLE 1. Wave Height and Profile Parameters

Date	Station Number	Location	H_{rms}, m	u_{rms}, m/s	h, m	γ_{rms}	tan β
October 10, 1982	7	Duck, North Carolina	1.29	0.85	4.11	0.31	0.014
	8		1.44	0.92	4.51	0.32	0.007
	10		1.10	1.02	2.51	0.44	0.052
	11		1.20	0.81	4.03	0.30	0.004
	13		1.43	0.91	4.51	0.32	0.010
October 11, 1982	14	Duck, North Carolina	1.20	0.85	3.83	0.31	0.012
	17		1.05	1.03	2.29	0.46	0.049
	18		1.36	0.92	4.14	0.33	0.004
	19		1.64	1.02	4.87	0.34	0.010
	20		1.39	0.96	4.03	0.34	0.008
October 12, 1982	21	Duck, North Carolina	1.71	1.04	5.19	0.33	0.010
	24		1.05	0.78	3.61	0.29	0.016
	26		1.26	0.91	3.81	0.33	0.007
	27		0.92	1.06	1.73	0.53	0.061
	28		0.89	0.71	3.23	0.28	0.004
October 18, 1979	108	Fort Ord California	0.79	0.73	2.6	0.30*	0.0
	109		1.00	0.95	2.5	0.40	0.011
	110		0.74	0.77	2.1	0.35*	0.0
	111		1.09	1.04	2.5	0.44	0.021
	112		0.64	0.74	1.7	0.38	0.0
	113		1.37	1.17	2.9	0.47	0.05

*Not included in trend calculations of Figure 7 (see text).

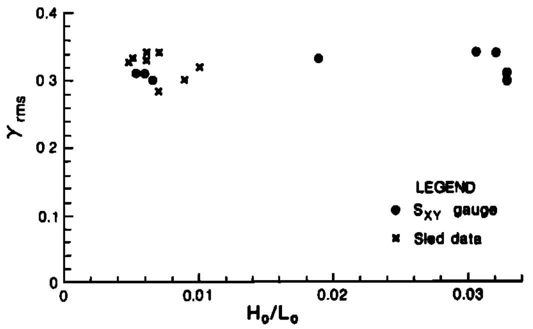


Fig. 9. γ_{rms} versus H_0/L_0 for the sled data obtained on the gentle slopes (Figure 3) and the eight apparently saturated data points from the offshore pressure sensor (see Figure 6) that were obtained on about the same slope as the sled data.

γ_{rms} increased with decreasing depth (compare Figures 3 and 4), γ_{rms} for the Ford Ord data decreased with decreasing depth (Figure 8). Note that the part of the Fort Ord profile of interest was convex up (Figure 8*a*), whereas the part of the Duck profile of interest was concave up (Figure 3). Because of these differences in profile shape, γ_{rms} for both the Fort Ord and Duck data increased with tan β (Figure 8*b*). Most of the Fort Ord data (Figure 8*b*) are plotted along with the Duck data in Figure 7. Not plotted are data from stations 110 and 108 (Figure 8), which are on the part of the profile with zero slope and are apparently not saturated. On this part of the profile, wave height decreased as a result of continued breaking, even though depth was constant (Figure 8). The Ford Ord and Duck data, along with some additional data from the literature, show a reasonable trend, although there is significant scatter with γ_{rms} for the Ford Ord data plotting higher than γ_{rms} for the Duck data. Linear regression gives

$$\gamma_{rms} = 3.2 \tan \beta + 0.30 \tag{5}$$

with $r^2 = 0.70$. Standard errors for slope and offset are 0.5 and 0.01, respectively. The 99% confidence level that γ_{rms} is dependent on β is $r^2 > 0.26$.

The parameter γ_{rms} does not appear to depend on wave steepness. Sled data, obtained on the seaward part of the profile where bottom slope was roughly constant (Figure 3), are plotted against H_0/L_0 in Figure 9. Also plotted are the offshore pressure sensor data, which appear to be saturated (Figure 6) and were obtained on about the same slope as the sled data. Figure 9 shows no dependence of γ_{rms} on H_0/L_0.

Discussion

Based on early monochromatic studies, equation (1) has commonly been used with γ equal to a constant of 1. In this study, γ_{rms} was found not to be constant but to vary significantly with slope. Furthermore, similar to results of some earlier studies, we found that $\gamma = 1$ for random waves overestimates γ_{rms} by a factor of 2 or more for most natural slopes.

For laboratory data the surf similarity parameter ξ_0 (equation 2) appears to parameterize many different surf zone processes, including setup, runup, breaker type, and saturation [*Bowen et al.*, 1968; *Battjes*, 1974]. Furthermore, runup data obtained in the field can be parameterized by ξ_0 [*Holman and Sallenger*, 1985]. However, in the present field study we find γ to be independent of H_0/L_0 and thus ξ_0.

In the field there is a distribution of wave heights, and the largest waves will break farther offshore than the smaller waves, causing a landward increase in γ; γ would continue increasing landward until all waves were broken and energy was saturated [*Thornton and Guza*, 1983]. The data from Duck indicated a landward increase in γ that we have related to an increase in β. Another interpretation would be that the landward increase in γ resulted from the distribution of wave heights in the incident wave field. However, this interpretation seems unlikely, since the Duck data were obtained within the inner surf zone, where we assume most waves would have broken. In any case the importance of slope is confirmed by the Ft. Ord data, which showed γ decreasing landward with β, opposite to what would be predicted by distribution of wave heights. In contrast to the inner surf zone data discussed here, distribution of wave heights will likely have a significant control of γ in the outer surf zone, where some waves are unbroken.

For sediment transport applications it is of interest to predict the wave-induced velocities directly. Using linear theory and shallow-water approximations, wave-induced oscillatory velocity is given by

$$u = 0.5(gh)^{0.5}\gamma \tag{6}$$

where g is acceleration of gravity. Substituting from (5) yields

$$\frac{u_{rms}}{(gh)^{0.5}} = 1.6 \tan \beta + 0.15 \tag{7}$$

Using measured U_{rms}, calculated from measured flow spectra, results in essentially the same relationship as (7).

Conclusions

Flow measurements, which were transformed into measures of wave height, were obtained across the storm surf zone. The data set was appropriate to quantify the dependence of wave height on local slope and depth. As incident waves broke and propagated into the inner surf zone, wave heights became strongly depth dependent and could be described by equation (1); γ_{rms} increased with increasing slope and was independent of wave steepness. Thus the surf similarity parameter–the ratio of tan β to the square root of steepness–does not adequately parameterize saturation.

Acknowledgments. We thank Jeff List, Tom Reiss, Bruce Richmond, Bruce Jaffe, and Peter Howd for setting up and operating the sled system and Curt Mason and the FRF crew for general support of the experiment. Jeff List did the programming for most of the calculations used in this paper. This work was funded jointly by the U.S. Geological Survey and the U.S. Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, Vicksburg, Mississippi. Additional support for R. A. H. was provided by Office of Naval Research, Coastal Sciences Branch contract NR 388-168.

References

Battjes, J. A., Surf similarity, in *Proceedings of the 14th Conference on Coastal Engineering*, vol. 1, pp. 466–480, American Society of Civil Engineers, New York, 1974.

Bowen, A. J., The generation of longshore currents on a plane beach, *J. Mar. Res.*, *37*, 206–215, 1969.

Bowen, A. J., D. L. Inman, and V. P. Simmons, Wave setdown and setup, *J. Geophys. Res.*, *73*, 2569–2577, 1968.

Coastal Engineering Research Center, Preliminary Data Summary, Field Res. Facil., Kitty Hawk, N. C., 1982.

Downing, J. P., Field studies of suspended sand transport, Twin Harbors Beach, Washington, Ph. D. dissertation, 121 pp., Univ. Wash., Seattle, 1983.

Galvin, C. J., and P. S. Eagleson, Experimental study of longshore currents on a plane beach, *Tech. Memo. 10*, U.S. Army Coastal Eng. Res. Center, Fort Belvoir, Va., 1965.

Guza, R. T., and E. B. Thornton, Local and shoaled comparison of sea surface elevations, pressures, and velocities, *J. Geophys. Res.*, *85*, 1524–1530, 1980.

Holman, R. A., and A. H. Sallenger, Setup and swash on a natural beach, *J. Geophys. Res.*, *90*(C1), 945–953, 1985.

Iversen, H. W., Laboratory study of breakers, Gravity Waves, *Circ. 52*, U.S. Bur. Stand., Washington, D. C., 1952.

Longuet-Higgins, M. S., Longshore currents generated by obliquely incident sea waves, 1, *J. Geophys. Res.*, *75*, 6778–6789, 1970.

McGowan, J., On the solitary wave, *Philos. Mag.*, *32*, 45–58, 1891.

Miller, H. C., W. A. Birkemeier, A. E. DeWall, Effects of CERC research pier on nearshore processes, in *Coastal Structures '83, Proceedings of the American Society of Civil Engineers*, pp. 769–784, ASCE, New York, 1983.

Sallenger, A. H., P. C. Howard, C. H. Fletcher, and P. A. Howd, A system for measuring bottom profile, waves and currents in the high-energy nearshore environment, *Mar. Geol.*, *51*, 63–76, 1983.

Sallenger, A. H., R. A. Holman, and W. A. Birkemeier, Storm induced response of a nearshore-bar system, *Mar. Geol.*, *64*, 237–257, 1985.

Scripps Institution of Oceanography, Monthly Report, Coastal Data Inf. Program, La Jolla, Calif., 1982.

Short, A. D., Three dimensional beach-stage model, *J. Geol.*, *87*, 553–571, 1979.

Sonu, C. J., Three-dimensional beach changes, *J. Geol.*, *81*, 42–46, 1973.

Suhayda, I. N., and N. R. Pettigrew, Observations of wave height and wave celerity in the surf zone, *J. Geophys. Res.*, *82*, 1419–1424, 1977.

Sverdrup, H. V., and W. H. Munk, Theoretical and empirical relations in forecasting breakers and surf, *Eos Trans.* AGU, *27*, 828, 1946.

Thornton, E. B., Variations of longshore current across the surf zone, in *Proceedings of the 12th Conference on Coastal Engineering*, pp. 291–308, American Society of Civil Engineers, New York, 1971.

Thornton, E. B., and R. T. Guza, Energy saturation and phase speeds measured on a natural beach, *J. Geophys. Res.*, *87*, 9499–9508, 1982.

Thornton, E. B., and R. T. Guza, Transformation of wave height distribution, *J. Geophys. Res.*, *88*, 5925–5938, 1983.

Weisher, L. L., and R. J. Byrne, Field study of breaking wave characteristics, in *Proceedings of the 16th Conference on Coastal Engineering*, American Society of Civil Engineers, New York, 1978.

Wright, L. D., R. T. Guza and A. D. Short, Dynamics of a high-energy dissipative surf zone, Mar. Geol., *45*, 41–62, 1982.

R. A. Holman, College of Oceanography, Oregon State University, Corvallis, OR 97331.

A. H. Sallenger, U.S. Geological Survey, 914 National Center, Reston, VA 22092.

(Received February 26, 1985;
accepted April 22, 1985.)