

SEISMIC EXPLORATION

Volume 31

**WAVE FIELDS IN REAL MEDIA: WAVE
PROPAGATION IN ANISOTROPIC, ANELASTIC
AND POROUS MEDIA**

by

José M. CARCIONE

Istituto Nazionale di Oceanografia e di Geofisica Sperimentale (OSG)

Borgo Grotta Giganta 42 C

34010 Sgonico

Trieste, Italy



2001

PERGAMON

An Imprint of Elsevier Science

Amsterdam – London – New York – Oxford – Paris – Shannon – Tokyo

Contents

<i>Preface</i>	xv
<i>Acknowledgments</i>	xx
<i>About the author</i>	xxi
<i>Basic notation</i>	xxii
<i>Glossary of main symbols</i>	xxiii
1 Anisotropic elastic media	1
1.1 Strain-energy density and stress-strain relation	1
1.2 Dynamical equations	4
1.2.1 Symmetries and transformation properties	6
Symmetry plane of a monoclinic medium	7
Transformation of the stiffness matrix	9
1.3 Kelvin-Christoffel equation, phase velocity and slowness	10
1.3.1 Transversely isotropic media	11
1.3.2 Symmetry planes of an orthorhombic medium	13
1.3.3 Orthogonality of polarizations	14
1.4 Energy balance and energy velocity	15
1.4.1 Group velocity	17
1.4.2 Equivalence between the group and energy velocities	19
1.4.3 Envelope velocity	20
Transversely isotropic media	20
1.4.4 Elasticity constants from phase and group velocities	22
1.4.5 Relationship between the slowness and wave surfaces	24
SH-wave propagation	24
1.5 Finely layered media	25
1.6 Anomalous polarizations	29
1.7 Analytical solutions for transversely isotropic media	34
1.7.1 2-D Green's function	34
1.7.2 3-D Green's function	36
1.8 Reflection and transmission of plane waves	36
1.8.1 Cross-plane shear waves	38

2	Viscoelasticity and wave propagation	45
2.1	Energy densities and stress-strain relations	46
2.1.1	Fading memory and symmetries of the relaxation tensor	48
2.2	Stress-strain relation for 1-D viscoelastic media	49
2.2.1	Complex modulus and storage and loss moduli	49
2.2.2	Energy and significance of the storage and loss moduli	51
2.2.3	Non-negative work requirements and other conditions	51
2.2.4	Consequences of reality and causality	52
2.2.5	Summary of the main properties	54
	Relaxation function	54
	Complex modulus	54
2.3	Wave propagation concepts for 1-D viscoelastic media	55
2.4	Mechanical models and wave propagation	59
2.4.1	Maxwell model	61
2.4.2	Kelvin-Voigt model	64
2.4.3	Zener or standard linear solid model	65
2.4.4	Generalized Zener model	69
	Nearly constant Q	71
2.4.5	Nearly constant- Q model with a continuous spectrum	73
2.5	Constant- Q model and wave equation	73
2.5.1	Phase velocity and attenuation factor	74
2.5.2	Wave equation in differential form. Fractional derivatives.	75
	Propagation in Pierre shale	76
2.6	Memory variables and equation of motion	77
2.6.1	Maxwell model	77
2.6.2	Kelvin-Voigt model	79
2.6.3	Zener model	79
2.6.4	Generalized Zener model	80
3	Isotropic anelastic media	83
3.1	Stress-strain relation	84
3.2	Equations of motion and dispersion relations	84
3.3	Vector plane waves	86
3.3.1	Slowness, phase velocity and attenuation factor	86
3.3.2	Particle motion of the P wave	88
3.3.3	Particle motion of the S waves	90
3.3.4	Polarization and orthogonality	92
3.4	Energy balance, energy velocity and quality factor	93
3.4.1	P wave	94
3.4.2	S waves	100
3.5	Boundary conditions and Snell's law	100
3.6	The correspondence principle	102
3.7	Rayleigh waves	102
3.7.1	Dispersion relation	103
3.7.2	Displacement field	104
3.7.3	Phase velocity and attenuation factor	105

3.7.4	Special viscoelastic solids	106
	Incompressible solid	106
	Poisson's solid	106
	Hardtwig's solid	106
3.7.5	Two Rayleigh waves	106
3.8	Reflection and transmission of cross-plane shear waves	107
3.9	Memory variables and equation of motion	110
3.10	Analytical solutions	112
3.10.1	Viscoacoustic media	112
3.10.2	Constant- Q viscoacoustic media	113
3.10.3	Viscoelastic media	114
3.11	The elastodynamic of a non-ideal interface	115
3.11.1	The interface model	116
	Boundary conditions in differential form	117
3.11.2	Reflection and transmission coefficients of SH waves	117
	Energy loss	118
3.11.3	Reflection and transmission coefficients of P-SV waves	119
	Energy loss	121
4	Anisotropic anelastic media	125
4.1	Stress-strain relations	126
4.1.1	Model 1: Effective anisotropy	128
4.1.2	Model 2: Attenuation via eigenstrains	128
4.1.3	Model 3: Attenuation via mean and deviatoric stresses	130
4.2	Wave velocities, slowness and attenuation vector	131
4.3	Energy balance and fundamental relations	133
4.3.1	Plane waves. Energy velocity and quality factor	135
4.3.2	Polarizations	139
4.4	The physics of wave propagation for viscoelastic SH waves	140
4.4.1	Energy velocity	140
4.4.2	Group velocity	142
4.4.3	Envelope velocity	143
4.4.4	Perpendicularity properties	143
4.4.5	Numerical evaluation of the energy velocity	145
4.4.6	Forbidden directions of propagation	146
4.5	Memory variables and equation of motion in the time domain	147
4.5.1	Strain memory variables	148
4.5.2	Memory-variable equations	150
4.5.3	SH equation of motion	151
4.5.4	qP-qSV equation of motion	151
4.6	Analytical solution for SH waves in monoclinic media	153
5	The reciprocity principle	155
5.1	Sources, receivers and reciprocity	156
5.2	The reciprocity principle	156
5.3	Reciprocity of particle velocity. Monopoles	158
5.4	Reciprocity of strain	158

5.4.1	Single couples	159
	Single couples without moment	160
	Single couples with moment	161
5.4.2	Double couples	161
	Double couple without moment. Dilatation.	161
	Double couple without moment and monopole force	162
	Double couple without moment and single couple	162
5.5	Reciprocity of stress	163
6	Reflection and transmission of plane waves	167
6.1	Reflection and transmission of SH waves	168
6.1.1	Symmetry plane of a homogeneous monoclinic medium	168
6.1.2	Complex stiffnesses of the incidence and transmission media	170
6.1.3	Reflection and transmission coefficients	171
6.1.4	Propagation, attenuation and energy directions	173
6.1.5	Brewster and critical angles	179
6.1.6	Phase velocities and attenuations	183
6.1.7	Energy-flux balance	185
6.1.8	Energy velocities and quality factors	187
6.2	Reflection and transmission of qP-qSV waves	189
6.2.1	Propagation characteristics	189
6.2.2	Properties of the homogeneous wave	191
6.2.3	Reflection and transmission coefficients	192
6.2.4	Propagation, attenuation and energy directions	193
6.2.5	Phase velocities and attenuations	194
6.2.6	Energy-flow balance	194
6.2.7	Umov-Poynting theorem, energy velocity and quality factor	196
6.2.8	Reflection of seismic waves	197
6.2.9	Incident inhomogeneous waves	207
	Generation of inhomogeneous waves	209
	Ocean bottom	210
6.3	Reflection and transmission at fluid/solid interfaces.	212
6.3.1	Solid/fluid interface	212
6.3.2	Fluid/solid interface	213
6.3.3	The Rayleigh window	214
6.4	Reflection and transmission coefficients of a set of layers	215
7	Biot's theory for porous media	219
7.1	Isotropic media. Strain energy and stress-strain relations	220
7.1.1	Jacketed compressibility test	221
7.1.2	Unjacketed compressibility test	222
7.2	The concept of effective stress	224
7.2.1	Effective stress in seismic exploration	226
	Pore-volume balance	227
	Acoustic properties	230
7.2.2	Analysis in terms of compressibilities	230
7.3	Anisotropic media. Strain energy and stress-strain relations	234

7.3.1	Effective-stress law for anisotropic media	237
7.3.2	Summary of equations	239
	Pore pressure	239
	Total stress	239
	Effective stress	239
	Skempton relation	239
	Undrained-modulus matrix	240
7.3.3	Brown and Korringa's equations	240
	Transversely isotropic medium	240
7.4	Kinetic energy	241
	7.4.1 Anisotropic media	243
7.5	Dissipation potential	246
	7.5.1 Anisotropic media	246
7.6	Lagrange's equations and equation of motion	247
	7.6.1 The viscodynamic operator	248
	7.6.2 Fluid flow in a plane slit	250
	7.6.3 Anisotropic media	254
7.7	Plane-wave analysis	255
	7.7.1 Compressional waves	255
	Relation with Terzaghi's law	258
	The diffusive slow mode	259
	7.7.2 The shear wave	259
7.8	Strain energy for inhomogeneous porosity	261
	7.8.1 Complementary energy theorem	262
	7.8.2 Volume-averaging method	263
7.9	Boundary conditions	266
	7.9.1 Interface between two porous media	267
	Deresiewicz and Skalak's derivation	267
	Gurevich and Schoenberg's derivation	269
	7.9.2 Interface between a porous medium and a viscoelastic medium	271
	7.9.3 Interface between a porous medium and a viscoacoustic medium	271
	7.9.4 Free surface of a porous medium	271
7.10	Green's function for poro-viscoacoustic media	272
	7.10.1 Field equations	272
	7.10.2 The solution	273
7.11	Poro-viscoelasticity	276
7.12	Anisotropic poro-viscoelasticity	279
	7.12.1 Stress-strain relations	281
	7.12.2 Biot-Euler's equation	283
	7.12.3 Time-harmonic fields	283
	7.12.4 Inhomogeneous plane waves	285
	7.12.5 Homogeneous plane waves	287
	7.12.6 Wave propagation in femoral bone	289

8 Numerical methods	295
8.1 Equation of motion	295
8.2 Time integration	296
8.2.1 Classical finite differences	298
8.2.2 Splitting methods	299
8.2.3 Predictor-corrector methods	300
8.2.4 Spectral methods	300
8.2.5 Algorithms for finite-element methods	301
8.3 Calculation of spatial derivatives	301
8.3.1 Finite differences	301
8.3.2 Pseudospectral methods	303
8.3.3 The finite-element method	305
8.4 Source implementation	306
8.5 Boundary conditions	307
8.6 Absorbing boundaries	308
8.7 Model and modeling design – Seismic modeling	310
8.8 Concluding remarks	312
8.9 Appendix	314
8.9.1 Finite-differences code for the SH-wave equation of motion	314
8.9.2 Pseudospectral Fourier Method	320
8.9.3 Pseudospectral Chebyshev Method	323
<i>Examinations</i>	327
<i>Chronology of main discoveries</i>	331
<i>A list of scientists</i>	339
Bibliography	345
Name index	371
Subject index	380