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WAVE-FRONT TRACKING FOR THE EQUATIONS OF ISENTROPIC GAS DYNAMICS

By

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Dedicated to Professor Atsushi Yoshikawa on his sixtieth birthday

Abstract. We study the 2×2 system of conservation laws of the form $v_t - u_x = u_t + p(v)_x = 0$, $p = k^2 v^{-\gamma} (\gamma \geq 1)$, which are the model equations of isentropic gas dynamics. Weak global in time solutions are obtained by Nishida-Smoller (CPAM 1973) provided $(\gamma - 1)$ times the total variation of the initial data is sufficiently small. The aim of this paper is to give an alternative proof by using the Dafermos-Bressan-Risebro wave-front tracking scheme. We obtain new estimates of the total amount of interactions, which also imply the asymptotic decay of the solution. The main idea is to define appropriate amplitude to the path that is a continuation of shock fronts.

1. Introduction. The equations of one dimensional isentropic gas dynamics in Lagrangian coordinates are given by

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = 0, \ (x,t) \in \boldsymbol{R} \times \boldsymbol{R}_+. \end{cases} \tag{1.1}$$

Here u is the velocity, p the pressure, and v the specific volume satisfying $v > 0$. If the gas is *ideal*: $pv = R\Theta$ and *polytropic*: $e = C_v \Theta$ (Θ: temperature, e : internal energy), then it follows from the first and second laws of gas dynamics that the entropy η is expressed as

$$\eta = C_v \{\log p + (1 + \tfrac{R}{C_v}) \log v\} + \text{const}.$$

Setting $\eta =$ constant, we obtain the pressure in the form

$$p = k^2 v^{-\gamma}, \quad \gamma = 1 + \tfrac{R}{C_v} > 1. \tag{1.2}$$

Note that $p = k^2 v^{-1}$ for $\gamma = 1$, which coincides with the *isothermal* gas.

In this paper, we will be mainly concerned with the Cauchy problem for the equations (1.1) with pressure in the above form, but our study will certainly cover the case $\gamma = 1$.

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Since the equations (1.1) constitute a nonlinear hyperbolic system, jump discontinuities will appear in a solution even if the initial data are sufficiently smooth. Hence, the *weak solution* will be taken into account. These theories go as far back as Riemann [12] and various studies before 1948 are presented in Courant-Friedrichs [5]. By setting $\rho = v^{-1}$ (mass) and $U = {}^t(\rho, u)$, initial data are given by

$$U(x,0) = {}^t(\rho_0(x), u_0(x)), \tag{1.3}$$

where $\rho_0(x) \geq \underline{\rho} > 0$ and $\rho_0(x), u_0(x) \in BV(\boldsymbol{R})$: the space of functions having bounded total variation in $\boldsymbol{R}$. If the initial data have small total variation, Glimm [7] says that there exists a global in time weak solution. For the isothermal gas equations ($\gamma = 1$), Nishida [10] has proved the existence of global weak solutions for the initial data having arbitrarily large total variation. Nishida-Smoller [11] has shown that global weak solutions exist if $(\gamma - 1)$ times the total variation of the initial data is sufficiently small, which is a generalisation of [10]. These authors use Glimm's random choice scheme.

The aim of this paper is to show that Nishida-Smoller solutions are also constructed by the wave-front tracking method that has been developed by Dafermos [6] for scalar conservation laws, and by Bressan [3], [4] and Risebro [13] for systems. In order to control the large total variation, we define a *path* and its *strength*. A path is a continuation of shock fronts. A single shock front is a part of a finite number of paths and the strength of the shock front is the summation of the strength of these paths. The notion of the path has been introduced by Temple-Young [15] and the idea of this decomposition has already been used in this author's previous paper [1]. In sections 2, 3 and 4, we will summarise basic results on the Riemann problem, the wave-front tracking scheme following [4], and the interaction estimates obtained by [11]. In Sec. 5, we will define a path and its strength; the stability of the front tracking scheme will be proved in Sec. 7. Our estimates also imply the decay of the solution that will be discussed in the last section.

2. Riemann Problem. We find by direct computation that the characteristic speeds are the roots of the equation $\lambda^2 + p'(v) = 0$:

$$\lambda_1(U) = -k\sqrt{\gamma}\rho^{\epsilon-1}, \quad \lambda_2(U) = k\sqrt{\gamma}\rho^{\epsilon-1} \quad (\gamma = 1 + 2\epsilon) \tag{2.1}$$

and the corresponding characteristic fields are

$$R_1(U) = {}^t(1, \sqrt{-p'(v)}), \quad R_2(U) = {}^t(1, -\sqrt{-p'(v)}). \tag{2.2}$$

Following Nishida-Smoller [11], we define the Riemann invariants:

$$z = u + \frac{k\sqrt{\gamma}}{\epsilon}(\rho^\epsilon - 1) : \text{1-invariant}, \quad w = u - \frac{k\sqrt{\gamma}}{\epsilon}(\rho^\epsilon - 1) : \text{2-invariant}. \tag{2.3}$$

We note that if $\gamma = 1$ ($\epsilon \to 0$), then

$$z = u + \log\rho, \quad w = u - \log\rho$$

that coincide with the Riemann invariants defined in Nishida [10]. Moreover, the region $\{(\rho, u);\ \rho > 0\}$ corresponds to $\{(w, z);\ z - w > -\frac{2k\sqrt{\gamma}}{\epsilon}\}$ in wz coordinates.

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The Riemann problem is the Cauchy problem with initial data

$$U(x,0)=\begin{cases} U_L & \text{for} \quad x<0, \\ U_R & \text{for} \quad x>0 \end{cases} \tag{2.4}$$

where $U_L = {}^t(\rho_L, u_L)$, $U_R = {}^t(\rho_R, u_R)$ are constant vectors. A unique self-similar solution will be constructed. Self-similar C^1 solutions have the form $U(x,t) = \widehat{U}(\frac{x}{t})$ satisfying the equations if and only if

$$\widehat{U}' \propto R_j(\widehat{U}), \quad \frac{x}{t} = \lambda_j(\widehat{U}) \quad (j=1,2). \tag{2.5}$$

Hence $\widehat{U}(\xi)$ is an integral curve of $R_j(U)$. These solutions are called *centred rarefaction waves*. In our equations, the range of centred rarefaction waves is contained in the integral curves through (ρ_0, u_0), expressed as

$$\begin{aligned} u-u_0 &= -\tfrac{k\sqrt{\gamma}}{\epsilon}(\rho^\epsilon - \rho_0^\epsilon) \quad (\rho \le \rho_0): \text{ 1-rarefaction curve,} \\ u-u_0 &= \tfrac{k\sqrt{\gamma}}{\epsilon}(\rho^\epsilon - \rho_0^\epsilon) \quad (\rho \ge \rho_0): \text{ 2-rarefaction curve.} \end{aligned} \tag{2.6}$$

We note that the 1-rarefaction curve corresponds to the horizontal half-line $z = z_0$ ($w \ge w_0$) and the 2-rarefaction curve corresponds to the vertical half-line $w = w_0$ ($z \ge z_0$). On the other hand, self-similar jump discontinuities have the form

$$U(x,t)=\begin{cases} U_- & \text{for} \quad x<st, \\ U_+ & \text{for} \quad x>st. \end{cases} \tag{2.7}$$

This jump discontinuity is called a *shock wave,* if and only if it satisfies the *Rankine-Hugoniot condition*:

$$s(v_+ - v_-) = -(u_+ - u_-), \quad s(u_+ - u_-) = p(v_+) - p(v_-) \tag{2.8}$$

together with the *Lax entropy condition*:

$$\begin{aligned} &\lambda_1(U_+) < s < \lambda_1(U_-),\ s < \lambda_2(U_+): \text{ 1-shock wave,} \\ &\lambda_1(U_-) < s,\ \lambda_2(U_+) < s < \lambda_2(U_-): \text{ 2-shock wave.} \end{aligned} \tag{2.9}$$

For fixed $\rho_0 = \rho_-$, $u_0 = u_-$, quantities $\rho = \rho_+, u = u_+$ satisfying the Rankine-Hugoniot condition (2.8) and the Lax entropy condition (2.9) constitute pieces of smooth curves:

$$\begin{aligned} u-u_0 &= -k\sqrt{\tfrac{\rho^\gamma - \rho_0^\gamma}{\rho\rho_0(\rho-\rho_0)}}(\rho-\rho_0) \quad (\rho > \rho_0): \text{ 1-shock curve,} \\ u-u_0 &= k\sqrt{\tfrac{\rho^\gamma - \rho_0^\gamma}{\rho\rho_0(\rho-\rho_0)}}(\rho-\rho_0) \quad (\rho < \rho_0): \text{ 2-shock curve.} \end{aligned} \tag{2.10}$$

In order to solve the Riemann problem, we define the *forward* 1*-wave curve* $\mathcal{W}_1^F(U_L)$ and the *backward* 2*-wave curve* $\mathcal{W}_2^B(U_R)$ in the following way:

$$\begin{aligned} \mathcal{W}_1^F(U_L): \quad u-u_L &= \begin{cases} -\frac{k\sqrt{\gamma}}{\epsilon}(\rho^\epsilon - \rho_L^\epsilon) & (\rho \le \rho_L) \\ -k\sqrt{\frac{\rho^\gamma - \rho_L^\gamma}{\rho\rho_L(\rho-\rho_L)}}(\rho-\rho_L) & (\rho > \rho_L), \end{cases} \\ \mathcal{W}_2^B(U_R): \quad u-u_R &= \begin{cases} \frac{k\sqrt{\gamma}}{\epsilon}(\rho^\epsilon - \rho_R^\epsilon) & (\rho \le \rho_R) \\ k\sqrt{\frac{\rho^\gamma - \rho_R^\gamma}{\rho\rho_R(\rho-\rho_R)}}(\rho-\rho_R) & (\rho > \rho_R). \end{cases} \end{aligned} \tag{2.11}$$

Each wave curve constitutes a C^2-curve with Lipschitz continuous second derivative. If $(\rho, u) \in \mathcal{W}_1^F(U_L)$, then there is a 1-rarefaction wave or shock wave connecting (ρ_L, u_L)

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and (ρ, u). If, on the other hand, $(\rho, u) \in \mathcal{W}_2^F(U_R)$, then there is a 2-rarefaction wave or shock wave connecting (ρ, u) and (ρ_R, u_R).

The following theorem is essentially due to Riemann [12]; the proof is found in Courant-Friedrichs [5] and Smoller [14].

THEOREM 2.1. Suppose that $u_R - u_L < \frac{k\sqrt{\gamma}}{\epsilon}(\rho_R^\epsilon + \rho_L^\epsilon)$, or equivalently $z_L - w_R > -\frac{2k\sqrt{\gamma}}{\epsilon}$. Then there exists a unique solution composed of constant states U_L, U_M, U_R separated by centred rarefaction waves and shock waves. Moreover,

$$w \geq \text{Min}\ \{w_L, w_R\},\ z \leq \text{Max}\ \{z_L, z_R\}. \tag{2.12}$$

The *amplitude* of waves is defined by

$$\begin{aligned} \beta &= w_M - w_L &&: \text{amplitude of 1-waves,} \\ \gamma &= z_R - z_M &&: \text{amplitude of 2-waves.} \end{aligned} \tag{2.13}$$

Here $\beta, \gamma \geq 0$ for centred rarefaction waves and < 0 for shock waves; absolute values $|\beta|, |\gamma|$ are called their *strengths*. From now on, we also denote by β, β' the 1-waves and γ, γ' the 2-waves themselves.

3. Front Tracking Scheme. Let h be a (small) positive number. The approximate solution $U^h(x,t)$ is constructed by following Bressan [4].

(1) Approximate the initial data by a step function $U_0^h(x)$ so that

$$|U_0^h - U_0|_\infty \leq h,\ T.V.U_0^h \leq T.V.U_0 \tag{3.1}$$

(2) Let $x_1 < \cdots < x_M$ be the points of discontinuity of $U_0^h(x)$. At each x_m, we solve the Riemann problem setting $U_L = U_0^h(x_m - 0),\ U_R = U_0^h(x_m + 0)$ and approximate the solution with piecewise constant function in the following way. If the solution is composed only of shock waves, we adopt this piecewise constant solution itself. If it contains a centred rarefaction wave, we approximate it by several small fans consisting of constant states and jump discontinuities separating them. These constant states are located on the rarefaction curve connecting two constant states in the solution to the Riemann problem and jump fronts propagate with speeds close to characteristic speeds; we may assume that the distance between the neighbouring state is less than h. We thus construct an approximate solution $U^h(x,t)$ composed of piecewise constant functions.

(3) $U^h(x,t)$ is constructed until a pair of neighbouring jump discontinuities interact. If they interact at $t = t_1$, we construct the approximate solution by solving the Riemann problems with initial data $U^h(x, t_1 - 0)$. Here, we may assume, changing the speed of shock waves by $O(1)h$, there are only two incoming waves at every interaction point.

(4) We can repeat the above construction as long as the number of jump discontinuities does not diverge within a finite time.

(5) To avoid the breakdown, we introduce a new approximate solution that is called a *simplified Riemann solver* (see [4] for details). At each interaction point, the amount of waves generated by the interaction is estimated by the product of the strengths of incoming waves $|\sigma\tau|$. We choose a threshold $\rho > 0$ so that: if

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$|\sigma\tau| \geq \rho$, then the usual approximate solution is constructed; if $|\sigma\tau| < \rho$, then the new approximate solution is constructed in the following way. Suppose that a 2-shock wave γ connects U_L and U_M, and a 1-shock wave β connects U_M and U_R. Then we can find two states U'_M, U'_R so that U_L and U'_M are connected by a 1-shock wave with strength $|\beta|$, and U'_M and U'_R are connected by a 2-shock wave with strength $|\gamma|$; the states U_R, U'_R are separated simply by a discontinuous front that propagates with a fixed speed $\widehat{\lambda} > \max|\lambda_j|$. This discontinuous front is called the *non-physical wave*.

In Sec. 7, we will prove that the approximate solution $U^h(x,t)$ is actually constructed for all $0 \leq t < \infty$. Estimates of *physical* waves are settled in [11]. The remaining problem is to choose the threshold ρ and estimate the total amount of non-physical waves, which will be carried out also in Sec. 7.

4. Interaction Estimates. Suppose that three constant states, denoted by U_L, U_M, U_R from left to right, are connected by two incoming waves. For example, let γ denote a 2-shock wave connecting U_L and U_M, and let β denote a 1-shock wave connecting U_M and U_R. These waves interact and generate an outgoing 1-shock wave β' and a 2-shock wave γ' that constitute the solution to the Riemann problem connecting U_L and U_R. This interaction is denoted, for simplicity, by $\gamma + \beta \to \beta' + \gamma'$. Let o denote the 1-rarefaction front and let π denote the 2-rarefaction front. Possible interactions of two waves are the following:

(1) $\gamma + \beta$, (2) $\gamma + o$ (or $\pi + \beta$), (3) $\gamma_1 + \gamma_2$ (or $\beta_1 + \beta_2$),
(4) $\gamma + \pi$ (or $o + \beta$), (5) $\pi + \gamma$ (or $\beta + o$), (6) $\pi + o$.

The local interaction estimates are obtained in the following way.

Lemma 4.1 (Nishida-Smoller [11]). Assume that $\underline{\rho} \leq \rho_L,\ \rho_R \leq \overline{\rho}$. Then there exist C and δ $(0 < \delta < 1)$ depending only on the equations and $\underline{\rho}, \overline{\rho}$, such that the following estimates hold.

(1) $\gamma + \beta \to \beta' + \gamma'$: one of the following holds:
  (a) $|\beta'| \leq |\beta| + C\epsilon|\beta\gamma|,\ |\gamma'| \leq |\gamma| + C\epsilon|\beta\gamma|$
  (b) $|\beta'| = |\beta| - \zeta,\ |\gamma'| \leq |\gamma| + C\epsilon|\beta\gamma| + \eta$
  (c) $|\gamma'| = |\gamma| - \zeta,\ |\beta'| \leq |\beta| + C\epsilon|\beta\gamma| + \eta, \quad 0 \leq \eta \leq \delta\zeta$.
(2) $\gamma + o \to o' + \gamma'$: $\quad |\gamma'| = |\gamma|, \quad |o'| \leq |o| + C\epsilon|o\gamma|$.
(3) $\gamma_1 + \gamma_2 \to o' + \gamma'$: $\quad |\gamma'| = |\gamma_1| + |\gamma_2|, \quad |o'| \leq C|\gamma_1\gamma_2|$.
(4) $\gamma + \pi \to \beta' + \gamma'$: there exist 1-shock wave β_0 and 2-shock wave γ_0 such that $|\gamma_0| = |\gamma| - \zeta$, $|\beta_0| = \eta$ $(0 < \eta \leq \delta\zeta)$ and $\beta_0 + \gamma_0 \to \beta' + \gamma'$.
(5) $\pi + \gamma \to \beta' + \gamma'$: $\quad |\gamma'| = |\gamma| - \zeta,\ |\beta'| = \eta$ $(0 < \eta \leq \delta\zeta)$.
(6) $\pi + o \to o' + \pi'$.

Remark 4.2. The estimates of rarefaction waves in Cases (2) and (3) are not contained in Lemma 4 of [11]. However, the first estimate is a direct consequence of the basic estimate and the second follows from the mean value theorem.

Let Q_j $(j = 1, 2)$ denote the difference of the amplitude of j-outgoing wave and j-incoming wave, that is: $Q_1 = \beta' - \beta,\ Q_2 = \gamma' - \gamma$ in Case (1) and $Q_1 = o,\ Q_2 =$

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$\gamma' - (\gamma_1 + \gamma_2) = 0$ in Case (3), etc. Q_j is the amount of j-wave generated by the interaction. Let σ, τ be the amplitude of incoming waves. Then Q_j is a C^2 function of σ and τ satisfying

$$Q_j(\sigma, 0) = Q_j(0, \tau) = 0 \quad \text{for all} \quad \sigma, \tau \quad (j = 1, 2).$$

Hence $Q_j(\sigma, \tau)$ has the basic estimate

$$|Q_j(\sigma, \tau)| \leq C|\sigma\tau|, \tag{4.1}$$

where the constant C depends only on the equations and $\underline{\rho}, \overline{\rho}$.

Suppose that the approximate solution $U^h(x, t)$ is constructed for $0 \leq t < T$. The global interaction estimate of the total amount of shock waves can be carried out in the same way as [11]. A Lipschitz curve J defined by $t = T(x), x \in \boldsymbol{R}$ is called an *I-curve*, if $|T'(x)| < \frac{1}{\lambda}$. We denote $J_2 > J_1$, if $T_1 \neq T_2$ and $T_2(x) \geq T_1(x)$ $(x \in \boldsymbol{R})$. Denoting by $S_j(J)$ the set of j-shock waves crossing J and $S(J) = S_1(J) \cup S_2(J)$, we define

$$L^-(J) = \sum_{\alpha \in S(J)} |\alpha|, \quad Q(J) = \sum_{\beta \in S_1(J), \gamma \in S_2(J)} |\beta\gamma| : \textit{approaching}. \tag{4.2}$$

We set $F(J) = L^-(J) + 4C\epsilon Q(J)$.

LEMMA 4.3 ([11]). If $C\epsilon F(O) \leq \min\{\frac{1}{2}, \frac{1-\delta}{4\delta}\}$, then it follows that $F(J_2) \leq F(J_1)$ for $J_2 > J_1$. Particularly, $L^-(J) \leq F(O)$.

Since $L^-(J)$ is the sum of the negative variation of w and z along J, their positive variation, the total amount of rarefaction waves, is also less than $F(O)$. We also notice that the above estimates are independent of T and valid as long as the approximate solution U^h is constructed.

5. Decomposition by Paths. Let us consider an approximate solution $U^h(x, t)$ for $0 \leq t < T$. A sequence of interaction points P_0, P_1, ..., P_n constitutes a *path*, if $\mathrm{P}_0 \in \{t = 0\}$ and each segment $\mathrm{P}_{j-1}\mathrm{P}_j$ is a shock front; this path is denoted by

$$\Gamma : \ \mathrm{P}_0 \to \mathrm{P}_1 \to \cdots \to \mathrm{P}_n.$$

As in Temple-Young [15], we define the *index* (c_j, k_j) of each segment $\mathrm{P}_{j-1}\mathrm{P}_j$ in the following way: by setting $k_1 = 1$,

$$c_j = \begin{cases} 1, & \text{if } \mathrm{P}_{j-1}\mathrm{P}_j \quad \text{is a 1-shock} \\ 2, & \text{if } \mathrm{P}_{j-1}\mathrm{P}_j \quad \text{is a 2-shock,} \end{cases}$$

$$k_j = \begin{cases} k_{j-1}, & \text{if} \quad c_j = c_{j-1} \\ k_{j-1} + 1, & \text{if} \quad c_j \neq c_{j-1}. \end{cases}$$

Each k_j is called the *generation order* of the segment and the sequence (c_1, k_1), (c_2, k_2), ..., (c_n, k_n) is called the *index of the path.*

We define the *strength* of the segment in a path in the following inductive way. Let t_1 be the first interaction time. Suppose that a shock wave β is issuing from a point $\mathrm{P}_0 \in \{t = 0\}$. The segment connecting P_0 and its first interaction point P_1 will belong to several paths. But at first, we regard the segment $\mathrm{P}_0\mathrm{P}_1$ as a single path $\Gamma : \ \mathrm{P}_0 \to \mathrm{P}_1$ and the strength is given by $|\beta|$. In this way, the strength is defined up to $t = t_1$ for each

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segment that constitutes a shock front issuing from $\{t=0\}$. Suppose that a $(1,1)$-wave β and a $(2,1)$-wave γ interact at P_1 and β' and γ' are generated. After the interaction, the strengths for β' and γ' are defined according to Case (1)-(a), (b), (c) of Lemma 4.1 in the following way.

Case (1)-(a): by Lemma 4.1, there exist positive constants C_1, C_2 such that

$$|\beta'| = (1 + C_1\epsilon|\gamma|)|\beta|, \quad |\gamma'| = (1 + C_2\epsilon|\beta|)|\gamma|. \tag{5.1}$$

The indices of β', γ', respectively, are defined to be $(1,1), (2,1)$, respectively and their strengths are defined to be $|\beta'|, |\gamma'|$, respectively.

Case (1)-(b): in this case, there exist positive constants C_2, δ_2 such that $C_2 \le C, \delta_2 \le \delta$ and

$$|\beta'| = |\beta| - \zeta, \quad |\gamma'| = (1 + C_2\epsilon|\beta|)|\gamma| + \delta_2\zeta. \tag{5.2}$$

The $(1,1)$-segment β is divided into two $(1,1)$-segments $\beta^{(1)}$ and $\beta^{(2)}$ so that $|\beta^{(1)}| = |\beta| - \zeta$ and $|\beta^{(2)}| = \zeta$. The index of β' is defined to be $(1,1)$. We define a $(2,1)$-segment $\gamma^{(1)\prime}$ and $(2,2)$-segment $\gamma^{(2)\prime}$ so that $|\gamma^{(2)\prime}| = \delta_2\zeta$ and $|\gamma'| = |\gamma^{(1)\prime}| + |\gamma^{(2)\prime}|$. Thus Γ is divided into two paths and extended beyond P_1 such that $\Gamma^{(1)} = \beta^{(1)} \cup \beta'$, $\Gamma^{(2)} = \beta^{(2)} \cup \gamma^{(2)\prime}$. Case (1)-(c) is treated in the same way.

For the general case, suppose that a 1-wave β and a 2-wave γ interact at P_n $(t = t_n)$ and β' and γ' are generated. We assume that β constitutes a segment of paths $\mathrm{B}_1, \mathrm{B}_2, \dots$, respectively, with strengths $|\beta_1|, |\beta_2|, \dots$, respectively and γ a segment of paths $\Gamma_1, \Gamma_2, \dots$, respectively, with strengths $|\gamma_1|, |\gamma_2|, \dots$, respectively so that $|\beta| = \sum_j |\beta_j|, |\gamma| = \sum_j |\gamma_j|$. We may assume that all shock fronts locating $t < t_n$ have such decomposition.

Case (1)-(a), $|\beta'| \ge |\beta|, |\gamma'| \ge |\gamma|$: By Lemma 4.1, (5.1) holds. Then we extend the path B_j to the next interaction point without changing the index; its strength is defined by $(1 + C_1\epsilon|\gamma|)|\beta_j|$. In the same manner, the path Γ_j is extended without changing the index and with strength $(1 + C_1\epsilon|\beta|)|\gamma_j|$.

Case (1)-(b), $|\beta'| < |\beta|$: In this case, there exist positive constants C_2, δ_2 such that equation (5.2) holds. Moreover, we find an integer l and a constant $\overline{\beta}_l$ such that

$$\zeta = |\overline{\beta}_l| + \sum_{j \ge l+1} |\beta_j|, \quad 0 \le |\overline{\beta}_l| < |\beta_l|.$$

For $1 \le j \le l-1$, we extend the path B_j up to the next interaction point changing neither the index nor its strength. For $j = l$ we first divide B_l into two paths $\mathrm{B}_l^{(1)}$ and $\mathrm{B}_l^{(2)}$ so that the indices are not changed and strengths satisfy $|\beta^{(1)}| : |\beta^{(2)}| = |\beta_l| - |\overline{\beta}_l| : |\overline{\beta}_l|$ on every segment constituting the path. Then we extend $\mathrm{B}_l^{(1)}$ up to the next interaction point changing neither the index nor its strength. Let $(1, k_l)$ be the index of B_l on β. We extend $\mathrm{B}_l^{(2)}$ in the opposite direction with the generation order $k_l + 1$ and the strength $\delta_2|\overline{\beta}_l|$ so its index is $(2, k_l + 1)$ up to the next interaction point. For $j \ge l+1$, we extend the path B_j up to the next interaction point changing the direction; its index and strength are $(2, k_j + 1)$ and $\delta_2|\beta_j|$, respectively. Γ_j are extended up to the next interaction point without changing the index; its strength is $(1 + C_2|\beta|)|\gamma_j|$.

Case (1)-(c), $(|\gamma'| < |\gamma|)$ can be treated in the same manner as Case (1)-(b). If $\epsilon > 0$, the above case is the most complicated one and Cases (2) to (5) are easier. If $\epsilon = 0$, Case

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(1) is harmless and other cases can be treated as Case (1)-(b). However, Case (4) needs some accounting: here, we divide the interaction as $\gamma + o \to \beta_0 + \gamma_0 \to \beta' + \gamma'$. The first interaction is treated in the same manner as Case (1)-(b), but we do not increase the generation order here; the second interaction is Case (1) itself and generation order may change.

We have thus defined a collection of a finite number of paths $\mathbf{\Gamma} = \{\Gamma_j\}$ in the approximation. A path Γ is considered to be a Lipschitz curve $x = \Gamma(t)$. For all t different from interaction times, we define $\alpha_\Gamma(t)$: the strength of Γ at t and $k_\Gamma(t)$: the generation order of Γ at t. It follows from our definition:

LEMMA 5.1. For every approximate solution, we have a collection of a finite number of paths $\mathbf{\Gamma} = \{\Gamma_j\}$ such that

(1) $L^-(t) = \sum_{\Gamma \in \mathbf{\Gamma}} \alpha_\Gamma(t)$

(2) Let $\Gamma: \mathrm{P}_0 \to \mathrm{P}_1 \to \cdots \to \mathrm{P}_n$ and $(c_j, k_j), \alpha_j$ be the index and the strength, respectively, of $\mathrm{P}_{j-1}\mathrm{P}_j$. Then

$$\begin{array}{ll} k_{j+1} = k_j & \Rightarrow \quad \alpha_{j+1} \le (1 + C\epsilon|\beta_j|)\alpha_j, \\ k_{j+1} = k_j + 1 & \Rightarrow \quad \alpha_{j+1} \le \delta\alpha_j \end{array}$$

where β_j is the interacting shock wave.

Next, we consider the strength of a path Γ at t such that $k_\Gamma(t) = j$. We will use a simple inequality:

$$\prod_{j\ge1}(1 + |\beta_j|) \le 1 + \frac{3}{2}\sum_{j\ge1}|\beta_j|, \quad \text{if} \quad \sum_{j\ge1}|\beta_j| \le \frac{1}{2} \tag{5.3}$$

that comes from the inequality $1 + x \le e^x \le 1 + \frac{3}{2}x$ for $0 \le x \le \frac{1}{2}$.

LEMMA 5.2. Assume that $C\epsilon F(O) \le \min\{\frac{1}{2}, \frac{1-\delta}{4\delta}\}$. Then there exists positive constant κ depending only on δ and satisfying $0 < \kappa < 1$ such that

$$\begin{array}{lll} \alpha_\Gamma(t) \le 2\alpha_\Gamma(0) & \text{if} & k_\Gamma(t) = 1, \\ \alpha_\Gamma(t) \le \kappa^{j-1}\alpha_\Gamma(0) & \text{if} & k_\Gamma(t) = j \ge 2. \end{array} \tag{5.4}$$

Proof. We first claim that for all s, t different from interaction times and satisfying $s < t$, we have

$$k_\Gamma(t) = k_\Gamma(s) + 1 \ \Rightarrow\ \alpha_\Gamma(t) \le \kappa\alpha_\Gamma(s). \tag{5.5}$$

Let us denote $\Gamma: \mathrm{P}_0 \to \mathrm{P}_1 \to \cdots \to \mathrm{P}_n$. Suppose that $\Gamma(s) \in \mathrm{P}_{j_0-1}\mathrm{P}_{j_0}, \Gamma(t) \in \mathrm{P}_{j_2-1}\mathrm{P}_{j_2}$, and the generation order changes at P_{j_1}. It follows from Lemma 5.1 that

$$\begin{aligned} \alpha_\Gamma(t) &\le \alpha_\Gamma(s) \prod_{j_0 < j \le j_1}(1 + C\epsilon|\beta_j|)\delta \prod_{j_1 < j \le j_2}(1 + C\epsilon|\beta_j|) \\ &\le \delta\alpha_\Gamma(s)\prod_{j\ge1}(1 + C\epsilon|\beta_j|) \\ &\le \delta(1 + \tfrac{3}{2}C\epsilon\sum_{j\ge1}|\beta_j|)\alpha_\Gamma(s) \\ &\le \delta\{1 + \tfrac{3}{2}C\epsilon F(O)\}\alpha_\Gamma(s) \le \tfrac{1}{4}(3 + \delta)\alpha_\Gamma(s). \end{aligned}$$

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Here we use the global interaction estimate $C\epsilon\sum_{j\geq 1}|\beta_j| \leq C\epsilon F(O) \leq \min\{\frac{1}{2}, \frac{1-\delta}{4\delta}\}$. Thus by setting $\kappa = \frac{1}{4}(3+\delta)$, we have proved the claim. We note that κ is independent of the choice of a path. Repeating this argument, we get (5.5) for $j \geq 1$. If $j = 1$, we have obviously

$$\alpha_\Gamma(t) \leq \{1 + \tfrac{3}{2}C\epsilon F(O)\}\alpha_\Gamma(0) \leq 2\alpha_\Gamma(0),$$

which completes the proof of the lemma. □

Using this lemma, we have

$$\sum_{\Gamma:k_\Gamma(t)=j} \alpha_\Gamma(t) \leq \kappa^{j-1} \sum_{\Gamma:k_\Gamma(t)=1} \alpha_\Gamma(0) = \kappa^{j-1}L^-(0).$$

Denoting $L_j^-(t) = \sum_{\Gamma:k_\Gamma(t)=j} \alpha_\Gamma(t)$, we obtain

PROPOSITION 5.3. Assume that $C\epsilon F(O) \leq \min\{\frac{1}{2}, \frac{1-\delta}{4\delta}\}$. Then there exists positive constant κ depending only on δ and satisfying $0 < \kappa < 1$ such that

$$L_1^-(t) \leq 2L^-(0), \quad L_j^-(t) \leq \kappa^{j-1}L^-(0) \ (j \geq 2). \tag{5.6}$$

This proposition will provide estimates of the total amount of non-physical waves generated by the interaction of waves whose generation orders are larger than k and the threshold parameter will be chosen according to the above estimate.

6. Estimates of Total Amount of Interactions. Now we carry out the estimates of total amount of waves generated by all of the interactions between $t = 0$ and T. From now on, we suppose that $C\epsilon F(O) \leq \min\{\frac{1}{2}, \frac{1-\delta}{4\delta}\}$. It follows from Lemma 4.1 that

(1) A new rarefaction wave is generated only by the interaction of two shock waves of the same family.
(2) The amplitude of rarefaction waves decreases by the interaction with shock waves of the same family and increases by those with shock waves of the opposite family.
(3) The above interactions generate no rarefaction wave of the opposite family.

Let $\{\mathrm{P}_m\}$ denote the collection of all points where the interactions of two shock waves of the same family occur. The strength of interacting (incoming) shock waves at P_m are denoted by $\alpha_j(\mathrm{P}_m)$ $(j = 1, 2)$.

LEMMA 6.1. The total amount of new rarefaction waves generated by the interaction of two shock waves of the same family is estimated by

$$\sum_{\mathrm{P}_m} |\alpha_1(\mathrm{P}_m)\alpha_2(\mathrm{P}_m)| \leq \frac{2}{1-\kappa}L^-(O)^2. \tag{6.1}$$

Proof. Let $\boldsymbol{\Gamma}_{\alpha_j}(\mathrm{P}_m)$ denote the collection of paths composing $\alpha_j(\mathrm{P}_m)$ and let $\alpha_{\Gamma_j}(\mathrm{P}_m)$ denote the strength of the path $\Gamma_j \in \boldsymbol{\Gamma}_{\alpha_j}(\mathrm{P}_m)$ $(j = 1, 2)$. Then we have

$$\begin{aligned}\sum_{\mathrm{P}_m} |\alpha_1(\mathrm{P}_m)\alpha_2(\mathrm{P}_m)| &\leq \sum_{\mathrm{P}_m} \sum_{\Gamma_1\in\boldsymbol{\Gamma}_{\alpha_1}(\mathrm{P}_m)} \sum_{\Gamma_2\in\boldsymbol{\Gamma}_{\alpha_2}(\mathrm{P}_m)} |\alpha_{\Gamma_1}(\mathrm{P}_m)\alpha_{\Gamma_2}(\mathrm{P}_m)| \\ &\leq \frac{1}{2}\sum_{\Gamma\in\boldsymbol{\Gamma}} \sum_{\mathrm{P}_m\in\Gamma} |\alpha_\Gamma(\mathrm{P}_m)| \sum_{\Gamma^*\in\boldsymbol{\Gamma}^*(\Gamma,\mathrm{P}_m)} |\alpha_{\Gamma^*}(\mathrm{P}_m)|,\end{aligned}$$

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where $\mathbf{\Gamma}^*(\Gamma, \mathrm{P}_m)$ denotes the set of paths interacting with Γ at P_m and $\mathbf{\Gamma}$ is defined in Lemma 5.1. Since $\cup_{\mathrm{P}_m\in\Gamma}\mathbf{\Gamma}^*(\Gamma,\mathrm{P}_m) \subset \mathbf{\Gamma} - \{\Gamma\}$, we find by interchanging the order of summations that

$$\sum_{\mathrm{P}_m\in\Gamma}|\alpha_\Gamma(\mathrm{P}_m)| \sum_{\Gamma^*\in\mathbf{\Gamma}^*(\Gamma,\mathrm{P}_m)}|\alpha_{\Gamma^*}(\mathrm{P}_m)| \leq \sum_{\Gamma^*\in\mathbf{\Gamma}-\{\Gamma\}}\ \sum_{\mathrm{P}_m\in\Gamma\cap\Gamma^*}|\alpha_\Gamma(\mathrm{P}_m)\alpha_{\Gamma^*}(\mathrm{P}_m)|.$$

Let $\mathrm{P}_m, \mathrm{P}'_m \in \Gamma\cap\Gamma^*$ ($\mathrm{P}_m \neq \mathrm{P}'_m$) and suppose that there is no point of $\Gamma\cap\Gamma^*$ between P_m and P'_m. Since it is impossible that both $k_\Gamma(\mathrm{P}_m) = k_\Gamma(\mathrm{P}'_m)$ and $k_{\Gamma^*}(\mathrm{P}_m) = k_{\Gamma^*}(\mathrm{P}'_m)$ occur, the generation order of Γ or Γ^* increases by at least one as $\mathrm{P}_m \to \mathrm{P}_{m'}$ and we have

$$\begin{aligned}\sum_{\mathrm{P}_m\in\Gamma\cap\Gamma^*}|\alpha_\Gamma(\mathrm{P}_m)\alpha_{\Gamma^*}(\mathrm{P}_m)| &\leq 4\sum_{j\geq 1}\kappa^{j-1}|\alpha_\Gamma(\mathrm{P}_0)\alpha_{\Gamma^*}(\mathrm{P}_0^*)| \\ &\leq \frac{4}{1-\kappa}|\alpha_\Gamma(\mathrm{P}_0)\alpha_{\Gamma^*}(\mathrm{P}_0^*)|,\end{aligned}$$

where $\mathrm{P}_0 \in \Gamma, \mathrm{P}_0^* \in \Gamma^*$ are initial points at $t = 0$. Using this, we obtain

$$\begin{aligned}\sum_{\mathrm{P}_m}|\alpha_1(\mathrm{P}_m)\alpha_2(\mathrm{P}_m)| &\leq \frac{2}{1-\kappa}\sum_{\Gamma\in\mathbf{\Gamma}}\sum_{\Gamma^*\in\mathbf{\Gamma}}|\alpha_\Gamma(\mathrm{P}_0)\alpha_{\Gamma^*}(\mathrm{P}_0^*)| \\ &\leq \frac{2}{1-\kappa}L^-(O)^2.\end{aligned}$$

Thus the lemma follows. □

Let $L^+(J)$ denote the total amount of rarefaction waves crossing J and O, a space-like curve lying between the initial line and the first interaction point. We have proved that the total amount of rarefaction waves generated by the interaction of two shock waves of the same family is estimated by $\frac{2C}{1-\kappa}L^-(O)^2$. Moreover, the amplitude of rarefaction waves increases only by the interaction with shock waves of the opposite family and obviously a path composing a shock wave of that family does not interact again with the same rarefaction waves from the opposite direction. Thus by Lemma 4.1, Case (2), we have

$$\begin{aligned}L^+(J) &\leq \{1 + C\epsilon F(O)\}\Big\{L^+(O) + \frac{2C}{1-\kappa}L^-(O)^2\Big\} \\ &\leq 2L^+(O) + \frac{4C}{1-\kappa}L^-(O)^2. \qquad (6.2)\end{aligned}$$

Lemma 6.2. The total amount of waves generated by the interaction of shock waves α and rarefaction waves θ is estimated by

$$\sum_{\mathrm{P}_m}|\alpha(\mathrm{P}_m)\theta(\mathrm{P}_m)| \leq \Big\{F(O) + \frac{2L^-(O)}{1-\kappa}\Big\}\Big\{L^+(O) + \frac{2C}{1-\kappa}L^-(O)^2\Big\}. \qquad (6.3)$$

Proof. It follows from the above observation that the total amount of waves generated by the interaction of rarefaction waves and shock waves of the opposite family is at most

$$F(O)\Big\{L^+(O) + \frac{2C}{1-\kappa}L^-(O)^2\Big\}.$$

When a rarefaction wave and a shock wave of the same family interact, the cancellation occurs and either the rarefaction wave or the shock wave remains in this family. Hence

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each rarefaction wave will not interact again with the same shock (path) with its generation order unchanged and the amount of interactions is enumerated over all generation orders. Thus the total amount of waves generated by these interactions is estimated by

$$\begin{aligned} & 2\sum_{j\geq 1}\kappa^{j-1}L^-(O)\Big\{L^+(O)+\frac{2C}{1-\kappa}L^-(O)^2\Big\} \\ \leq\ & \frac{2L^-(O)}{1-\kappa}\Big\{L^+(O)+\frac{2C}{1-\kappa}L^-(O)^2\Big\}, \end{aligned}$$

which proves the lemma. □

Let σ,τ denote the incoming waves at P_m. We set $Q(\mathrm{P}_m)=|\sigma\tau|$ that estimates the amount of waves generated by the interaction at P_m (see (4.1)). Combining the above propositions, we have

Proposition 6.3. Assume that $C\epsilon F(O)\leq \min\{\frac{1}{2},\frac{1-\delta}{4\delta}\}$. Then the total amount of waves generated by the interaction is uniformly bounded: there exists a constant C depending only on the equations, $\underline{\rho},\overline{\rho}$, and $T.V.U_0$ such that

$$\sum_{\mathrm{P}_m}Q(\mathrm{P}_m)\leq C. \tag{6.4}$$

7. Estimates of Non-Physical Waves. First we prove that the approximate solution is constructed for all $0\leq t<\infty$. Let us assume the contrary. Suppose that there is a sequence of interaction time T_m such that $\lim_{m\to\infty}T_m=T_\infty<\infty$. Since the estimates in Sec. 6 are all true for $0<t<T_\infty$, there exists a uniform constant C_∞ such that

$$\sum_{0<t_m<T_\infty}Q(\mathrm{P}_m)\leq C_\infty \tag{7.1}$$

where t_m denotes the interaction time at P_m and the summation runs over all of the interaction points between $t=0$ and T_∞. Let ρ be a threshold introduced in Sec. 3. The above estimate says that there are less than C_∞/ρ interaction points such that the strengths of incoming waves satisfy $Q(\mathrm{P}_m)\geq\rho$: Since new *physical* fronts are generated only at such points, the number of physical fronts is thus finite. A new non-physical front is generated through the interaction of two physical fronts and any two physical fronts can interact only once. Hence the number of non-physical fronts is also finite. Consequently, we conclude that the total number of fronts is finite; this is the contradiction.

Let α be a shock wave at t. This shock wave contains several paths which can be arranged so that

$$\Gamma_1(t),\ \Gamma_2(t),\ \Gamma_3(t),\ \ldots\quad (k_{\Gamma_1}(t)\leq k_{\Gamma_2}(t)\leq k_{\Gamma_3}(t)\leq\cdots). \tag{7.2}$$

The *generation order* of α is defined by $k_{\Gamma_1}(t)$ and denoted by k_α that accords with the definition of Bressan [4]. Let $V_j^-(t)$ be the total amount of shock waves at t whose generation orders are larger than j. Then it follows that $V_j^-(t)=\sum_{l\geq j}L_l^-(t)$ and from Proposition 5.3,

$$\sup_{t\geq 0}V_j^-(t)\leq L^-(0)\sum_{l\geq j}\kappa^{l-1}=\frac{\kappa^{j-1}L^-(0)}{1-\kappa}. \tag{7.3}$$

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Now we shall carry out the estimates of non-physical waves. Note that only the simplified Riemann solver generates a non-physical wave; in particular, one or two shock waves have to be involved in the interaction. Since $|\beta'| = |\beta|,\ |\gamma'| = |\gamma|$ in the simplified Riemann solver, the total amount of shock waves $L^-(t)$ does not increase and $L^-(t)$ is estimated by Lemma 4.3. Suppose that the shock waves β and γ interact. Then the generation order of the generated non-physical wave is defined to be $\max\{k_\beta, k_\gamma\} + 1$; if a shock wave β and a rarefaction wave interact, then it is defined to be $k_\beta + 1$. Let us denote by $\mathcal{NP}$ the set of all non-physical waves. Note that non-physical waves do not interact with each other.

LEMMA 7.1. Let ϵ denote an arbitrary non-physical wave. There exists a constant depending only on the equations and $\underline{\rho}, \overline{\rho}$ such that the following estimates hold.

$$(1)\ |\epsilon| \le \rho, \quad (2) \sum_{\substack{\epsilon\in\mathcal{NP}\\ k_\epsilon \ge j}} |\epsilon| \le C \sup_{t\ge 0} V_j^-(t). \tag{7.4}$$

Proof. (1) is obvious by definition. (2) can be easily seen from

$$\sum_{\substack{\epsilon\in\mathcal{NP}\\ k_\epsilon \ge j}} |\epsilon| \le 2C\epsilon F(O) \sup_{t\ge 0} V_j^-(t).$$

□

We have by the above lemma

PROPOSITION 7.2. For given $h > 0$, there exists a threshold $\rho > 0$ so that the approximate solution constructed by the front tracking scheme satisfies

$$\sum_{\epsilon\in\mathcal{NP}} |\epsilon| \le h. \tag{7.5}$$

The proof is carried out in the same way as [4]. Let N_0 be the number of shock waves at $t = 0$. Then there exists a certain polynomial $P(\xi, \eta)$ such that

$$\begin{aligned}
\sum_{\epsilon\in\mathcal{NP}} |\epsilon| &= \sum_{\substack{\epsilon\in\mathcal{NP}\\ k_\epsilon \le j}} |\epsilon| + \sum_{\substack{\epsilon\in\mathcal{NP}\\ k_\epsilon \ge j+1}} |\epsilon| \\
&= O(1)P(N_0, h^{-1})\rho + O(1) \sum_{k\ge j+1} V_j^- \\
&= O(1)P(N_0, h^{-1})\rho + O(1)\kappa^j.
\end{aligned}$$

Hence, we choose j such that $O(1)\kappa^j \le \frac{h}{2}$ and then ρ so that (7.5) holds.

In this way, we have obtained a uniform bound of non-physical waves and hence $T.V.U^h(*, t)$. The existence of a global solution is proved by the usual argument in [4] and Smoller [14].

THEOREM 7.3. Assume that $(\gamma - 1)T.V.U_0$ is sufficiently small; then the front tracking scheme is stable and provides a global in time solution.

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8. Asymptotic Behaviour. As Risebro [13] shows, our approximate solutions indicate the asymptotic behaviour of solutions. Since the total number of wave-fronts is finite in an approximate solution, we can see that the approximate solution eventually consists of a finite number of *non-interacting* wave-fronts. More precisely, there exist certain intermediate states $\overline{U}_M$ and U'_∞ such that $U^h(-\infty, t) = U_\infty$ and $\overline{U}_M$ are connected by a single 1-shock front or a collection of 1-fronts that come from 1-rarefaction waves, $\overline{U}_M$ and U'_∞ by similar 2-fronts, and finally U'_∞ and $U^h(\infty, t) = U_\infty$ are connected by a collection of non-physical fronts. We find by Proposition 7.2 that the total amount of non-physical waves is less than h. Hence there exists a certain T^h such that

$$T.V.U^h(*, t) \leq 3h \quad \text{for} \quad t \geq T^h. \tag{8.1}$$

Then it follows that for every bounded interval I

$$\lim_{t \to \infty} \|U(*, t) - U_\infty\|_{L^1(I)} = 0. \tag{8.2}$$

Employing the Glimm-Lax theory [8], we can say more about the asymptotic behaviour. Since the approximate solutions are constructed globally in time, (6.4) is true for all $0 < t < \infty$. This estimate shows that the above theory can be built up for our large amplitude solutions in the framework of the wave-front tracking method (see Bressan [4], chap. 10 and also Asakura [2]). We can also verify that our solutions fulfill all requirements of Theorem 5.7 in Liu [9], which implies

THEOREM 8.1. Assume that $(\gamma - 1)T.V.U_0$ is sufficiently small; then

$$\lim_{t \to \infty} T.V.U(*, t) = 0.$$

REMARK 8.2. If we assume further that there is a constant $M > 0$ such that the initial value satisfies $U_0(x) = U_\infty$ for $|x| \geq M$, then the argument in Asakura [1] shows that $T.V.U(*, t)$ approaches zero at the rate $t^{-1/2}$.

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