

## Wave Group Properties of Coastal Waves

Hajime Mase<sup>1</sup>, M. ASCE, Takao Yamashita<sup>2</sup> and Katsuyuki Hayashi<sup>3</sup>

### ABSTRACT

This paper examines the following properties of observed wave groups: (1) the change of wave grouping from offshore to shallow water region including the surf zone; (2) the relation between wave groupiness and wave height distributions; and (3) the distribution of run lengths and time series of wave heights of extreme wave groups which contain the maximum wave height in each wave record. Main results are summarized as follows: (1) wave groups become flattened as they propagate landward; (2) no clear maximum in mean length of runs is seen around the position where the significant wave height is the maximum, in contrast to the laboratory data, while the tendency of the change of  $GF$  is the same as the experimental data; (3) when wave grouping becomes pronounced, the distribution of wave heights becomes wider; (4) extreme wave groups accompany more large waves than ordinary wave groups; (5) for the case of narrow spectra, the time series of wave heights of extreme wave groups is represented fairly well by that of the wave group formed by the side-band instability, and the distribution of three high waves is also in qualitative agreement with that of envelope soliton, and (6) for wide spectra, which may occur during the peak of storms, wave heights in front and rear of the maximum wave are half as large as the maximum wave height, implying that a maximum wave tends to appear without accompanying other large waves.

### INTRODUCTION

Certain sequences of large waves or wave groups are important not only in engineering problems, such as the stability of rubble mound breakwaters, the fluctuation of wave overtopping, and the slow drift oscillation of moored and floating structures, but also in hydrodynamical problems, such as the surf beat, the modulational instability, and the evolution of finite amplitude waves.

The degree of wave grouping is described by run lengths of large waves, groupiness factors, correlation coefficients of successive wave heights, and parameters related to the wave spectra. Elgar et al. (1984) showed that the observed mean lengths

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<sup>1</sup> Research Assoc., Department of Civil Engrg., Kyoto University, Kyoto, 606, Japan.

<sup>2</sup> Research Assoc., Disaster Prevention Research Institute, Kyoto University, Uji, 611, Japan.

<sup>3</sup> Engineer, Aichi Prefectural Government, Nagoya, 460, Japan.

of runs and occurrence probabilities of run lengths in relatively deep water were statistically consistent with the numerical simulations based on linear wave theory using the measured spectra with assumed random phases. However, the observations in shallow water deviated from the linear simulations and the observed mean lengths of runs were larger than the simulated mean lengths. Lake and Yuen (1978) explained amplitude-modulation periods of laboratory wind waves from a hydrodynamical viewpoint such as the side-band instability. Su (1986) examined the time series of wave heights of extreme wave groups which contained the maximum wave height in each wave record and showed that the observed time series of wave heights was described by the wave height sequence of the wave group formed by the side-band instability. The fact that wave group properties can be explained in different ways may show that various phenomena or mechanisms exist in random sea waves. Therefore, further examinations of wave group properties from various aspects or by different methods are necessary so that the wave group properties may eventually become predictable.

This study is a part of large field observations to investigate wave transformations, directional spectra, nearshore currents, and sediment transports. This paper examines the following aspects of wave groups by analyzing the observed wave data: (1) the change of wave grouping from offshore to shallow water region as well as in constant shallow water region; (2) the relation between wave groupiness and wave height distributions; and (3) the distribution of run lengths and time series of wave heights of extreme wave groups. These properties are compared with theoretical and experimental results.

## FIELD OBSERVATION

Wave observations were carried out at the Ogata Wave Observatory, Disaster Prevention Research Institute, Kyoto University, located at the Ogata Coast facing the Japan Sea. A T-shaped observation pier, which is 256 m long in the cross-shore direction and 107 m long in the alongshore direction, was constructed with truss girders of five spans of 50 m supported by piers composed of three steel piles to minimize its effects on the nearshore hydrodynamics and sediment transport. The location of the Ogata Wave Observatory and the bottom topography in the vicinity of the T-shaped pier are shown in Fig.1.

The field data were collected from November 27 to December 8, 1987, and from January 11 to 18, 1989. Fig.2 shows the arrangement of sensors used in the observations. Nine capacitance-type and seven ultrasonic-type wave gauges were mounted on the pier, as indicated by the letter 'C' and 'US' in Fig.2 (a). Three ultrasonic current meters, indicated by the letter 'CM', were installed at the elevation of approximately 1 m above the seabed under the ultrasonic wave gauges. On the extension of the line of the pier (along the straight line normal to the shoreline), a waverider buoy ('WR' in short) was installed at the water depth of 30 m (in the first observation) or 20 m (in the second observation), and three sets of an ultrasonic wave

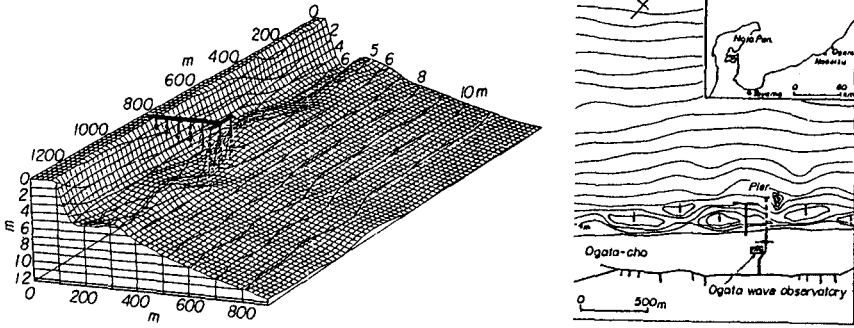


Figure 1 Location of Ogata Wave Observatory and bottom topography around T-shaped pier (from the pamphlet of Ogata Wave Observatory, Kyoto University).

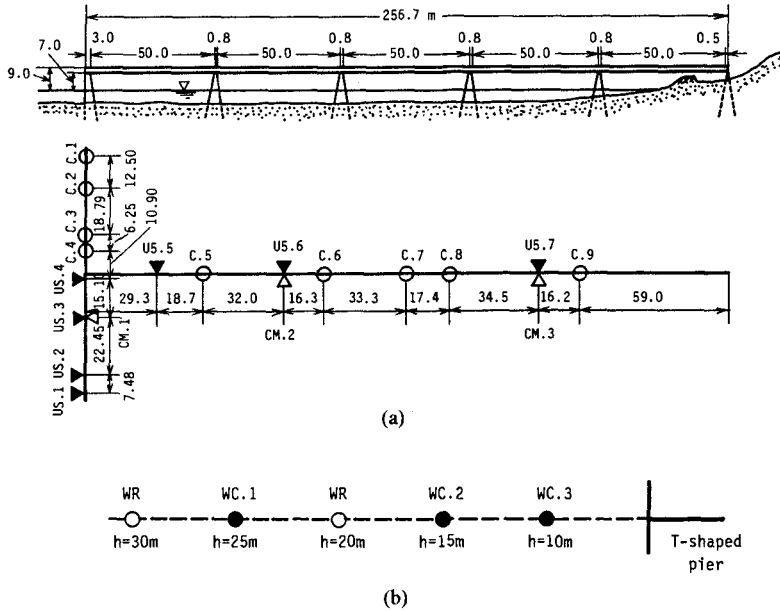


Figure 2 Arrangement of sensors at T-shaped pier in (a) and seaward of T-shaped pier in (b).

gauge and a two-components electromagnetic current meter ('WC' in short) were set on the seabed at the water depths of 25, 15 and 10 m, which are indicated by 'WC.1', 'WC.2' and 'WC.3' in Fig.2 (b). The water depth at the locations of 'C.1'~'C.8' is approximately constant and 5.1 m. The bottom slope seaward of the tip of the pier is about 1/100. For the first field data from November 27 to December 8, 1987, the time series from all sensors were recorded by two analog data recorders for 90 minutes every three hours. The recorded time series were digitized at a sampling interval  $\Delta t=0.256$  sec. For the second field data from January 11 to 18, the time series from the sensors 'C', 'US', 'WR' and 'CM' were recorded directly on digital magnetic tapes as follows: for 40 minutes every hour at  $\Delta t=0.1$  sec from 11th 19:00 to 15th 7:40; for 20 minutes every hour at  $\Delta t=0.2$  sec until 16th 8:20; and hereafter for 20 minutes every two hours at  $\Delta t=0.2$  sec. The data of 90 and 40 minutes long were divided into segments of 20 minutes. The time series of 'WC' were stored on an internal digital magnetic tape for 17 minutes every hour at  $\Delta t=0.5$  sec.

In order to remove low and high frequency motions from each wave record and limit the following analyses to incident wind waves only, the Fourier components with frequencies smaller than half the peak frequency and larger than six times of the peak frequency were set zero. The modified Fourier components were then transformed into the time domain variations by a inverse FFT technique. The observed data included a wide range of incident wave conditions from narrow-banded to broad-banded spectra. The significant wave heights measured by 'WR' ranged from 0.5 m to 4.5 m, and the significant wave periods ranged from 4.0 sec to 10.0 sec. Dominant wave directions were normal to the shoreline.

## RESULTS AND DISCUSSION

### Change of Wave Grouping from Offshore to Shallow Water Region

In order to describe the magnitude of wave grouping, at least two parameters are needed (Mase and Iwagaki, 1986): One parameter for representing the length of sequence of high waves in a time series such as the mean length of runs,  $\overline{\lambda(H)}$ , exceeding the mean wave height,  $\bar{H}$ ; and the other for representing the magnitude of the variation of wave energy such as the groupiness factor,  $GF$ . The measured changes of the two parameters  $\overline{\lambda(H)}$  and  $GF$  will indicate whether wave grouping becomes pronounced or not. When wave grouping pronounced,  $GF$  increases but  $\overline{\lambda(H)}$  may not decrease. When wave grouping becomes weakened,  $GF$  decreases but  $\overline{\lambda(H)}$  may increase or decrease. Fig.3 shows the changes of  $\overline{\lambda(H)}$  and  $GF$  against the nondimensional water depth,  $k_m h$  with  $k_m$  being the wavenumber based on the mean wave period in the water depth  $h$ , where the data of 'WR', 'WC.3', 'C.4', and 'US.4' were plotted. It is noted that the classification by wave steepness did not show any clear tendency. The linear regression lines plotted in Fig.3 show that  $GF$  decreases and

$\overline{j(H)}$  increases as  $k_m h$  is decreased. This implies that wave groups of shoaling random waves tend to become flattened.

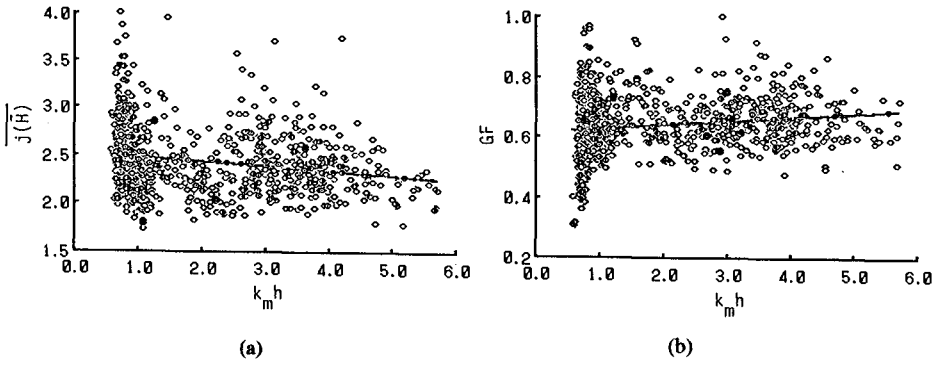


Figure 3 Change of wave grouping from offshore to shallow water region: (a) mean length of runs; (b) groupiness factor.

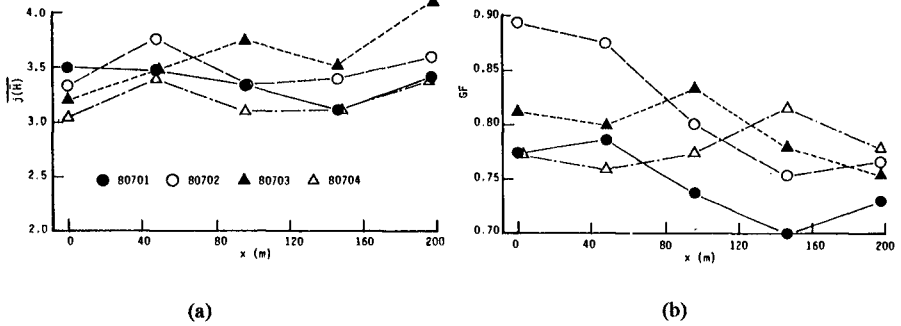


Figure 4 Change of grouping of non-breaking waves in shallow water of constant depth: (a) mean length of runs; (b) groupiness factor.

### Change of Grouping of Non-Breaking Coastal Waves in Shallow Water of Constant Depth

Fig.4 shows the values of  $\overline{\lambda(H)}$  and  $GF$  at the locations of 'C.4', 'C.5', 'C.6', 'C.8' and 'C.9' (the horizontal coordinate  $x$  is positive landward with  $x=0$  at the location of 'C.4') under the condition of  $k_m h < 1.0$ , narrow-banded wave spectra, and no wave breaking. For the cases of B0701 (20:30 of November 30, 1987) and B0702 (23:30 of November 30, 1987),  $\overline{\lambda(H)}$  is nearly constant and  $GF$  decreases in the shoreward direction. For the case of B0703 (2:30 of December 1, 1987),  $GF$  decreases and  $\overline{\lambda(H)}$  increases. These tendencies mean that wave groups of random waves become flattened with the propagating distance as was the case with the experimental results of Mase and Iwagaki (1987) in which the flattening of a single wave group was verified in laboratory experiment and by numerical simulations using the nonlinear Schrödinger equation.

### Change of Wave Grouping in Shallow Water Including Surf Zone

Fig.5 shows the changes of  $\overline{\lambda(H)}$  and  $GF$  against the normalized water depth  $h/H_0'$  ( $H_0'$  is the deep water significant wave height estimated from the significant wave at 'WR' using linear wave theory). The curves in Fig.5 (a) and (b) are the fifth and fourth polynomial regression curves, respectively, which are found to give the best fit among the first to fifth polynomial equations. The change of the significant wave height  $H_{1/3}/H_0'$  against  $h/H_0'$  is shown in Fig.6, where the curves are the results calculated by the random wave model of Mase et al. (1986) for wave steepness of 0.01 and 0.04, approximately corresponding to the observed maximum and minimum wave steepness. The experimental data on  $\overline{\lambda(H)}$  by Mase (1989) became the maximum around the point where  $H_{1/3}/H_0'$  was the maximum. This point for the present field data is roughly at  $h/H_0'=2.5$ ; however, no clear maximum in  $\overline{\lambda(H)}$  is seen around  $h/H_0'=2.5$  in Fig.5 (a). It is seen in Fig.5 (a) that the range of  $\overline{\lambda(H)}$  in the region of  $h/H_0' > 2.5$  is larger than that in the region of  $h/H_0' < 2.5$ .  $GF$  is almost constant in the region of  $h/H_0' > 3.0$ , and decreases with the decrease of the water depth for  $h/H_0' < 3.0$ . This is the same tendency as the experimental data by Mase (1989). The mean length of runs seems to be more unstable statistically than the groupiness factor, since the number of runs is small for twenty-minute-long records.

### Wave Grouping and Wave Height Distribution

The observed wave height distributions were presented by the Weibull distribution whose the shape factor,  $m$ , was estimated by the maximum likelihood method (Cohen, 1965). The Rayleigh distribution is a special case with  $m=2$  of the Weibull distribu-

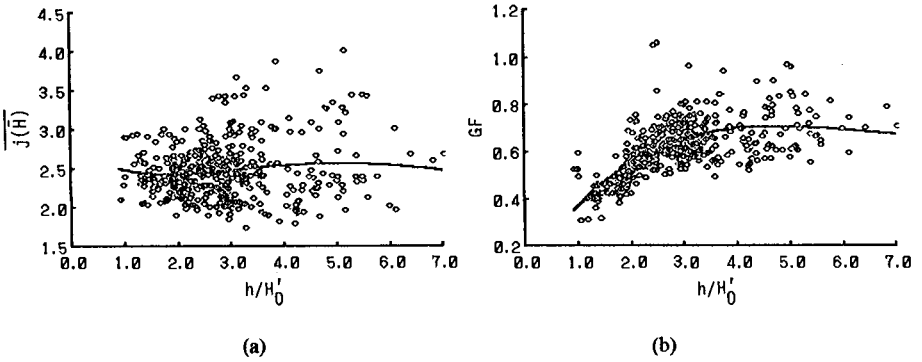


Figure 5 Change of wave grouping in shallow water including surf zone: (a) mean length of runs; (b) groupiness factor.

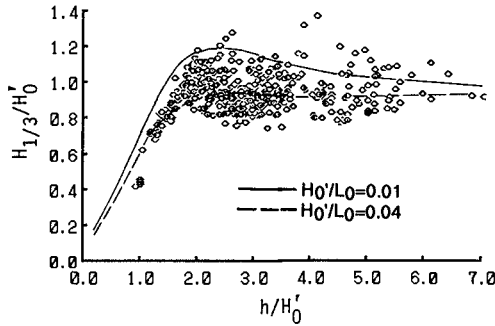


Figure 6 Transformation of significant wave heights in shallow water.

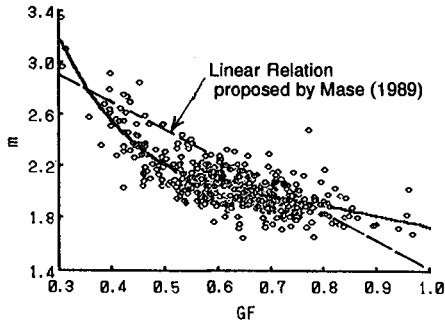


Figure 7 Relation between groupiness factor  $GF$  and shape factor  $m$  of wave height distribution.

tion. Fig.7 shows the relationship between  $GF$  and  $m$ . The tendency of the data points may be represented simply by the broken line proposed by Mase (1989), whereas the solid line is the following fifth polynomial equation fitted to the data points:

$$m = 9.45 - 37.83 GF + 76.33 GF^2 - 75.34 GF^3 + 35.30 GF^4 - 6.17 GF^5 \quad (1)$$

Fig.7 shows that as wave grouping increases ( $GF$  becomes larger), the distribution of wave height becomes wider ( $m$  becomes smaller). The Rayleigh distribution would underestimate the occurrence probability of extreme wave heights. The scatter in the data suggests that the relationship between the two parameters is not simple because of neglected effects of other wave characteristics.

### Distribution of Run Lengths of Extreme Wave Groups

A phenomenon that the maximum wave in a wave record seldom appears alone but usually accompanies with several large waves was reported by Goda (1976). Fig.8 (a) shows that the occurrence probability of  $j(\bar{H})=1$  is the largest for the case of ordinary runs of wave heights, as indicated by the symbols of  $\circ$  and  $\square$  corresponding to the data of 'WR' and 'C.4', respectively, while the occurrence probability of  $j(\bar{H})=4$  is the

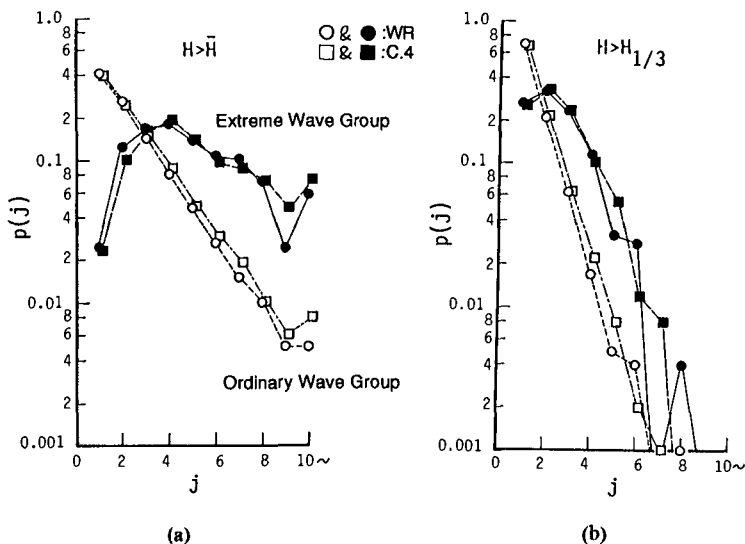


Figure 8 Occurrence probabilities of run lengths of ordinary and extreme wave groups where the threshold is the mean wave height in (a) and the significant wave height in (b).



largest for the conditional runs containing the maximum wave height in each wave record (extreme wave groups), as indicated by the symbols of ● and ■. The occurrence probabilities of run lengths with the threshold of  $H_{1/3}$  are shown in Fig.8 (b), which also shows that extreme wave groups contain more large waves than ordinary wave groups. However, this analysis of runs is based on the number of large waves exceeding a threshold and does not account for the relative magnitude of the waves around the maximum wave.

**Time Series of Wave Heights in Extreme Wave Groups**

Following the analysis of Su (1986), we describe the time series of wave heights in a extreme wave groups as  $H_{-3}, H_{-2}, H_{-1}, H_0, H_1, H_2, H_3$ , where  $H_0$  is the maximum wave height. Su (1986) found that the extreme wave group consisted of three high waves whose heights were greater than the significant wave height. On the basis of the experimental observations of the nonlinear side-band instability of finite amplitude waves, Su (1986) showed that the extreme wave group could be explained as the manifestation of the maximum modulation of waves. Time series of the surface displacement,  $\eta(t)$ , under the maximum modulation due to the side-band instability, was expressed as:

$$\eta(t) = a_0 \sin 2\pi f_0 t + \frac{a_0}{\sqrt{2}} \left[ \sin\{2\pi(1-a_0k_0)f_0t\} + \sin\{2\pi(1+a_0k_0)f_0t\} \right] \tag{2}$$

which can be simplified as

$$\eta(t) = a_0 \left\{ 1 + \sqrt{2} \cos(2\pi a_0 k_0 f_0 t) \sin(2\pi f_0 t) \right\} \tag{3}$$

where  $a_0, k_0$  and  $f_0$  are the amplitude, wavenumber and frequency of a carrier wave. Under the maximum modulation, the amplitudes of disturbances were equal to  $a_0/\sqrt{2}$  and the frequency differences from a carrier wave were  $\pm a_0 k_0 f_0$ . For  $a_0 k_0 = 0.1$  as a typical value, the phase of wave envelope changes  $36^\circ$  during one cycle of the carrier wave. If the maximum wave height is taken as

$$H_0 = 2a_0(1 + \sqrt{2} \cos 0^\circ) = 4.828a_0 \tag{4}$$

the wave heights around the  $H_0$  are expressed by

$$\left. \begin{aligned} H_{-1}, H_1 &= 2a_0(1 + \sqrt{2} \cos 36^\circ) = 0.888H_0 \\ H_{-2}, H_2 &= 2a_0(1 + \sqrt{2} \cos 72^\circ) = 0.595H_0 \\ H_{-3}, H_3 &= 2a_0(1 + \sqrt{2} \cos 108^\circ) = 0.233H_0 \end{aligned} \right\} \tag{5}$$

Zakharov and Shabat (1972) showed that finite disturbances could evolve into a number of envelope solitons. Extreme wave group may be attributed to envelope solitons formed by some mechanism even in usual wind wave fields. As an attempt, time series of wave heights of extreme wave groups are analyzed using the envelope soliton model as well as the instability model by Su (1986), although these models may not be really applicable to the field data.

The temporal variation of amplitude of the envelope soliton in deep water region was described by

$$A = a_0 \operatorname{sech}\{(a_0^2 \beta / 2\alpha)^{1/2} C_g t\} \quad (6)$$

$$\text{with } \left. \begin{aligned} \alpha &= \omega_0 / 8k_0^2 \\ \beta &= \omega_0 k_0^2 / 2 \\ C_g &= \omega_0 / 2k_0 \end{aligned} \right\} \quad (7)$$

Substitution of Eq.(7) into Eq.(6) yields

$$A = a_0 \operatorname{sech}(a_0 k_0 \omega_0 t) \quad (8)$$

For  $a_0 k_0 = 0.1$  as a typical value, the maximum wave height may be taken as

$$H_0 = 2a_0 \operatorname{sech}(0) = 2a_0 \quad (9)$$

The wave heights around  $H_0$  may be estimated by

$$\left. \begin{aligned} H_{-1}, H_1 &= 2a_0 \operatorname{sech}(0.2\pi) = 0.831H_0 \\ H_{-2}, H_2 &= 2a_0 \operatorname{sech}(0.4\pi) = 0.527H_0 \\ H_{-3}, H_3 &= 2a_0 \operatorname{sech}(0.6\pi) = 0.297H_0 \end{aligned} \right\} \quad (10)$$

The observed time series of wave heights of extreme wave groups normalized by the square root of the zero moment  $m_0$  of the spectral density  $S(f)$  were analyzed statistically according to the classification of spectral width based the following parameter:

$$\kappa(T_p) = \left[ \left\{ \int_0^\infty S(f) \cos(2\pi f T_p) df \right\}^2 + \left\{ \int_0^\infty S(f) \sin(2\pi f T_p) df \right\}^2 \right]^{1/2} / \left[ \int_0^\infty S(f) df \right] \quad (11)$$

where  $T_p$  is the peak period and  $f$  is the frequency. The parameter  $\kappa$  represents the correlation between two points of wave envelope separated by the time lag  $T_p$  and is more stable than the spectral peakedness parameter  $Q_p$  in terms of the degree of

freedom and resolution frequency of wave spectrum. In the calculation of  $S(f)$ , the number of data points was 4096, the Nyquist frequency 0.014Hz, and the degree of freedom was 30. The range of integration in Eq.(10) was taken to be  $0.5f_p$  to  $1.8f_p$  with  $f_p$  being the peak frequency instead of 0 to  $\infty$ . As a whole,  $\kappa$  was found to be linearly correlated to  $Q_p$ , seen in Fig.9.

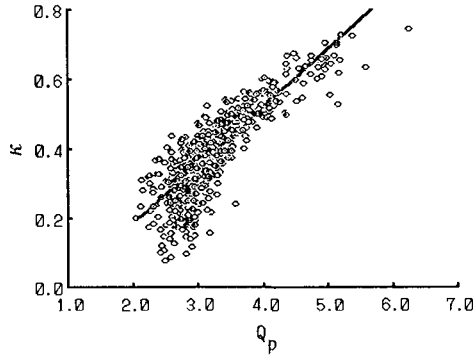


Figure 9 Relation between correlation coefficient  $\kappa$  and spectral peakedness parameter  $Q_p$ .

Fig.10 shows the time series of wave heights, in which the data of 'WR', 'C.4' and of Su (1986) are denoted by  $\circ$ ,  $\bullet$ , and  $\Delta$ , respectively. The solid line and dotted line bars denote the ranges of wave heights when  $a_0k_0$  is taken as 0.1 and 0.2 in Eqs.(3) and (8), respectively. For the case of narrow spectra shown in Fig.10 (a), the wave height sequence is represented fairly well by that of wave groups formed by the nonlinear side-band instability as well as by the envelope soliton, although the agreement depends on the selected value of  $a_0k_0$ . Even in shallow water (the data of 'C.4') where wave breaking may have occurred and the side-band instability cannot occur theoretically, the distribution of wave heights of extreme wave group is almost the same as that in deep water.

For the case of wide spectra of Fig.10 (b), usually observed during the peak of a storm, the wave heights in front and rear of the maximum wave are half as large as the maximum wave height. This implies that the maximum wave tends to appear without accompanying other large waves comparable to the maximum wave.

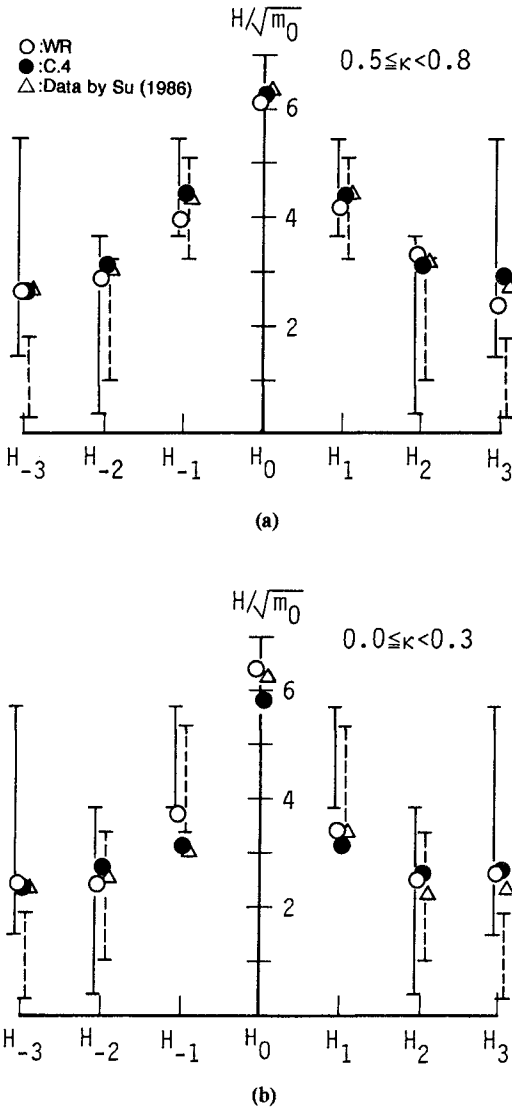


Figure 10 Time series of wave heights of extreme wave groups: (a) narrow band spectra; (b) wide band spectra.

## CONCLUSIONS

The wave group properties observed in the field and analyzed in this paper are summarized as follows:

1. The observed tendencies of increase in  $\overline{H}$  and decrease in  $GF$  with the decrease of  $k_m h$  from offshore to shallow water as well as with the increase of propagating distance in shallow water of constant depth imply that wave groups become flattened as they propagate landward.

2. No clear maximum in  $\overline{H}$  was seen around the position where the significant wave height was the maximum, in contrast to the laboratory data, while the tendency of the change of  $GF$  is the same as the experimental data. The mean length of runs was more unstable statistically than the groupiness factor, since the number of runs was small for the twenty minutes records.

3. When wave grouping became pronounced, the distribution of wave heights became wider. The Rayleigh distribution would underestimate the occurrence probabilities of higher waves.

4. Extreme wave groups accompanied more large waves than ordinary wave groups. However, the analysis of runs yields the number of waves only and does not account for the relative magnitude of the waves around the maximum wave.

5. For the case of narrow spectra, the time series of wave heights of extreme wave groups was represented fairly well by that of the wave group formed by the side-band instability (Su, 1986). The distribution of three high waves was also in qualitative agreement with that of envelope soliton. For wide spectra, which may occur during the peak of storms, wave heights in front and rear of the maximum wave were half as large as the maximum wave height, implying that a maximum wave tends to appear without accompanying other large waves.

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## REFERENCES

- Cohen, A. C., Jr. (1965): Maximum likelihood estimation in Weibull distribution based on complete and on censored samples, *Technometrics*, Vol.7, No.4, pp.579-588.
- Elgar, S., Guza, R. T. and Seymour, R. J. (1984): Groups of waves in shallow water, *J. Geophys. Res.*, Vol.89, No.C3, pp.3623-3634.
- Goda, Y. (1976): On wave groups, *Proc. Int. Conf. on Behavior of Offshore Structures*, pp.115-128.
- Lake, B. M. and Yuen, H. C. (1978): A new model for nonlinear wind waves, *J. Fluid Mech.*, Vol.88, pp.33-62.
- Mase, H. (1989): Groupiness factor and wave height distribution, *J. Waterway, Port, Coastal and Ocean Eng.*, Vol.115, No.1, pp.105-121.
- Mase, H. and Iwagaki, Y. (1986): Wave group analysis from statistical viewpoint, *Proc. Ocean Structural Dynamics Symp. '86*, ASCE, Oregon State Univ., pp.145-157.
- Mase, H. and Iwagaki, Y. (1987): Evolution of wave groups in shallow water and wave group properties of random waves, *Coastal Engrg. in Japan*, JSCE, Vol.30, No.1, pp.19-32.
- Mase, H., Matsumoto, A. and Iwagaki, Y. (1986): Calculation model of random wave transformation in shallow water, *Proc. JSCE*, Vol.375, No.11-6, pp.221-230 (in Japanese).
- Su, M. Y. (1986): Extreme wave group in storm seas near coastal water, *Proc. 20th Int. Conf. on Coastal Engrg.*, ASCE, pp.767-779.
- Zakharov, V. E. and Shabat A. B. (1972): Exact theory of two-dimensional self-focusing and one-dimensional self-modulating waves in nonlinear media, *Sov. Phys. -JETP (Engl. Transl.)* 65, pp.997-1011.