CHAPTER FORTY SEVEN

WAVE GROUPS IN THE FREQUENCY AND TIME DOMAINS

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ABSTRACT. The identification of wave groups in wave records is sought in terms of the classical linear analysis techniques in the frequency and time domains. Unwrapping and detrending of the phase spectrum identifies apparent order where none is assumed in the Gaussian random wave model. Similarly unexpected order is observed in the tail of the correlogram of both the wave record and the Rice envelope function. These aspects are strongly suggestive of wave grouping.

INTRODUCTION

Sophisticated design and analysis in coastal and ocean engineering requires a detailed description of incident sea conditions. Wave groups, a finite run of higher than normal waves, are frequently observed at sea and in wave records. They have an important impact on a wide range of coastal and offshore activities and some measure of wave groupiness needs to be included among the standard analysis and synthesis techniques routinely adopted by data collection authorities and coastal and ocean design groups. Despite the growing literature, the true nature and extent of wave grouping remains unresolved. There is sufficient evidence that wave grouping exists but insufficient evidence to confirm the nature and extent of these groups. Neither theoretical investigations nor numerical simulations can resolve these issues. The information must be sought initially from field data. Considerable recent research effort has focussed on the development of new analysis techniques to accommodate wave grouping. In contrast, the present paper will concentrate on linear wave theory and classical time series analysis techniques in the frequency and time domains. There is evidence to suggest that wave grouping is reasonably well described by these classical linear techniques.

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DATA SERIES FOR ANALYSIS

The basis of the present research is an extensive data base of deep water wave records assembled from sites off the eastern, western and southern coastlines of Australia. These are good quality records in computer compatible form. Data set A, obtained from the Maritime Services Board of New South Wales, comprises about fourteen hundred twenty-minute records off Newcastle and Botany Bay under generally winter storm conditions in the Tasman Sea. Data set B, obtained from Woodside Offshore Petroleum, comprises about three hundred twenty-minute records during six separate hurricanes on the North West Shelf. Data set C was obtained through Esso Australia for a site on the continental shelf of the Great Australian Bight. This region is exposed to Southern Ocean swell, recognized (Chelton, et al., 1981) as the most extreme wave climate in any of the world's oceans and the data set is an almost continuous record for several days of big, long-period swell. Together, these three data sets provide excellent samples of the common wave climate extremes.

Although the typical 20-minute wave record is the basic data series, considerable attention has recently been focussed on the data series of individual wave heights in the record, identified by the zero up-crossing method. Statistical parameters commonly extracted are R_H, the correlation coefficient between consecutive wave heights, and j_1 and j_2, respectively the mean number of waves in a group and the mean number of waves between the start of one group and the start of the next group. Theoretical estimates consistent with the Gaussian random wave model have been developed by Goda (1970) for j_1 and j_2, and Arhan and Ezraty (1978) for R_H. Field observations for j_1 (Goda 1976, Su et al. 1982) and R_H (Arhan and Ezraty, Su et al.) are consistently above the theoretical values, although there is qualitative agreement in the trends. It is apparent that wave groups are rather longer and more coherent than predicted by the Gaussian random wave model. Battjes (1984) has, however, obtained adequate agreement between field records and a theory that is based on a finite lag one auto-correlation between consecutive wave heights.

As a sea state description, this data series of individual waves has considerable value, although the same cannot be said of its potential in the development of an appropriate sea state model. A predictive model is ultimately sought, in which the reversibility of analysis and synthesis is a required property. The data series of individual waves focusses attention on amplitude relationships and considers time relationships only in an averaged sense.

Much of the facility of the individual wave approach is implicit in the complex wave envelope A(t) introduced by Rice (1944, 1945). A carrier frequency, f_o, typically the peak frequency f_p, is identified and removed from the record $\eta(t)$, such that

$$\eta(t) = \text{Real}[A(t)\ e^{i2\pi f_o t}] \tag{1}$$

In principle, the complex envelope function is the natural vehicle for wave group studies. No information is lost from the original record and attention is focussed on the envelope modulations. For typically narrow-banded sea states, the Rice envelope function $|A(t)|$ is a good approximation to the wave envelope and the R_H, j_1 and j_2 statistics are readily extracted.

FREQUENCY DOMAIN ANALYSIS OF DATA SERIES

The standard statistical summary in the frequency domain is the variance spectrum E(f) of the water surface time history $\eta(t)$. The common model of an irregular sea state, the Gaussian random wave model, was introduced by Rice (1944, 1945) as a model of random noise. The water surface time history is represented as the superposition of many linear waves of amplitude $a_n = [2E(f_n)\Delta f]^{1/2}$ and random phase $\Phi(f_n)$:

$$\eta(t) = \sum_n a_n \cos(2\pi f_n + \Phi_n) \tag{2}$$

The phase angle is randomly distributed over the range $(-\pi,\pi)$. This model has been the basis for analysis and design in coastal and ocean engineering for several decades.

There is a growing literature of detractors and defenders of the Gaussian random wave model. Rye (1983) and Goda (1983) argue that it is a sufficient description of wave grouping. Mollo-Christensen and Ramamonjiarisoa (1978), Hamilton et al. (1979) and Funke and Mansard (1980) propose supplementation to include wave grouping. Sobey and Colman (1982) have investigated a theoretical alternative, the nonlinear Schroedinger equation and the scattering transform. The field evidence is inconclusive, although the balance remains in favor of the Gaussian random wave model.

The intermediate step in the estimation of the variance spectrum is the complex Fourier transform $F(\omega)$ of the wave record $\eta(t)$, defined as

$$F(\omega) = \int_{-\infty}^{\infty} \eta(t)\ e^{-i\omega t}\ dt \tag{3}$$

$F(\omega)$ is a complex function of the angular frequency $\omega = 2\pi f$, and may be represented as

$$F(\omega) = |F(\omega)| e^{i\Phi(\omega)} \tag{4}$$

where $|F(\omega)|$ and $\Phi(\omega)$ are respectively the amplitude and phase spectrums. The Gaussian random wave model assumes the phase spectrum to be completely random but Funke and Mansard (1981) have questioned whether it may indeed contain some useful information. Two problems are identified in interpretation of the phase spectrum, respectively phase unwrapping and phase trend removal, which may contribute to the apparently random character of computed phase spectra. These aspects will be considered separately.

PHASE UNWRAPPING

Phase unwrapping refers to the modulo 2 operation on phase angles. The phase angle returned by the Fast Fourier Transform (FFT) algorithm is in the range $-\pi$ to π and is termed the principal phase $\Phi_p(\omega)$. Any principal phase angle may in fact be $\Phi_p(\omega) + 2n\pi$, where n is any signed integer, without changing either the complex Fourier transform or the variance spectrum. The "true" phase is obtained by "unwrapping" the principal phase through addition or subtraction of multiples of 2 ; this phase is called the unwrapped phase, $\Phi_u(\omega)$. The subscript u has been dropped but is implied in the subsequent discussion.

It remains to determine the signed integer n. The Schafer algorithm (Oppenheim and Schafer, 1975) assumes the unwrapped phase to be a continuous function of ω, the only discontinuities being those introduced by the modulo 2π operation in determining the principal phase. A discontinuity is defined to exist when the change in principal phase between adjacent values exceeds a given threshold. This process, however, is inconclusive and does not guarantee a unique result, as demonstrated by Tribolet (1977).

A unique result, however, does seem to be possible as the slope of phase spectrum is uniquely defined at all frequencies and uninfluenced by the modulo 2π operation on the principal phase (Oppenheim and Schafer, 1975). Taking the natural logarithm of both sides of Eq. 4 and differentiating with respect to ω gives

$$\frac{1}{F}\frac{dF}{d\omega} = \frac{d}{d\omega}(\ln|F|) + i\,\frac{d\Phi}{d\omega} \tag{5}$$

from which

$$\frac{d\Phi}{d\omega} = \mathrm{Imag}\left(\frac{1}{F}\frac{dF}{d\omega}\right) \tag{6}$$

Similarly differentiating Eq. 3 with respect to ω gives $dF/d\omega$ as

$$\frac{dF}{d\omega} = -i \int_{-\infty}^{\infty} t\eta(t)e^{-i\omega t}\, dt \qquad (7)$$

where the integral is recognized as the Fourier transform of $t\eta(t)$. The slope of the phase spectrum $d\Phi/d\omega$ is thus uniquely defined, at least in theory. Integration of Eq. 6 should yield the unwrapped phase spectrum but there are a number of problems in practice.

The first is a familiar one in discrete frequency domain analysis. The $d\Phi/d\omega$ estimates from the discrete Fourier transform of $t\eta(t)$ are raw estimates and their erratic nature significantly complicates the numerical integration. Some theoretical assistance is available in that the principal phase prediction should be identical with that available from the Fourier transform of $\eta(t)$, but these discrete estimates are also raw estimates. Advantage can be taken of both estimates. The second problem is the specification of the phase and the phase gradient at zero or near zero magnitude points for the Fourier transform. This problem arises in the specification of the initial conditions for numerical integration of Eq. 6 and at any other frequency where $|F(\omega)| \sim 0$ and $1/|F(\omega)| \sim \infty$ in Eq. 6, a situation that is not uncommon given raw estimates of $F(\omega)$.

Tribolet (1977) considers the former problem in the context of the complex cepstrum, proposing an algorithm based on adaptive stepsize integration using the trapezoidal rule. If the difference between the unwrapped phase estimate and the principal phase estimate at each ω is not sufficiently close to an integer multiple of 2π, then the frequency interval $\Delta\omega$ is continually halved until the specified accuracy is achieved. The discrete Fourier transform is used to estimate the Fourier transform by interpolation at intermediate points between those on the FFT raster. The initial phase was assumed to be zero, without discussion. The coding for this algorithm has been published and initial experiments utilized this code. Results were rather erratic for wave records and it was apparent that detailed consideration of the singular or near-singular points was essential.

These singular points can be accommodated by classical limit theory. L'Hospital's rule confirms that the principal phase and the phase derivative both exist and are given by

$$\Phi_p(\omega) = \tan^{-1} \frac{F_I^{(k+1)}}{F_R^{(k+1)}} \tag{8}$$

$$\frac{d\Phi}{d\omega} = \frac{1}{k+2} \frac{F_I^{(k+2)} F_R^{(k+1)} - F_I^{(k+1)} F_R^{(k+2)}}{F_R^{(k+1)} F_R^{(k+1)} + F_I^{(k+1)} F_I^{(k+1)}} \tag{9}$$

where $|F^{(k)}(\omega)| = 0$, $j = 0,1, \ldots k$ and $|F^{(k+1)}(\omega)| \neq 0$ and the bracketed superscripts represent differentiation with respect to ω:

$$F_R^{(j)} + i\, F_I^{(j)} = \frac{d^j F(\omega)}{d\omega^j}$$

These higher derivatives can be calculated in a similar manner to Eq. 7, from the discrete Fourier transforms of $t^j \eta(t)$ respectively. A further enhancement of the algorithm has been the adoption of Simpson's rule for numerical integration. Fourth order Runge-Kutta integration was initially utilized but Simpson's rule is numerically equivalent and has some coding advantages.

PHASE TREND REMOVAL

If the time origin of the record $\eta(t)$ is shifted from $t = 0$ to $t = t_o$, the Fourier transform of the origin-shifted record becomes $F(\omega)e^{i\omega t_o}$. The amplitude spectrum and hence the variance spectrum is not changed but the phase spectrum is changed by the addition of a linear trend ωt_o to $\Phi(\omega) + \omega t_o$. For an origin shift even as small as a few discrete record time steps Δt, this would impose a saw tooth variation on the principal phase, which potentially contributes to the random appearance of the principal phase spectrum. Although any linear trend is accommodated by phase unwrapping, the question of the "true" phase spectrum again arises. Interpretation of the phase spectrum will certainly be facilitated by the removal of any non-physical influences.

Funke and Mánsard (1981) observe that a phase spectrum including a linear trend component will approach $\pm\infty$ as ω becomes very large and suggest that the correct choice of time origin (and this choice is arbitrary) will have zero slope, $d\Phi/d\omega \to 0$, as $\omega \to \infty$. This is a compelling argument but there are difficulties in implementation. The phase spectrum is known only as far as the Nyquist frequency $\omega_N = \pi/\Delta t$ and not to infinity. Also, energy levels beyond two to three times the peak frequency are very small for typical records and raw phase estimates are increasingly unreliable.

A reasonable compromise would be to take advantage of the phase estimates where they are expected to be most reliable, which is in the vicinity of the spectral peak. Accordingly the origin shift has been estimated from a least-squares curve fit of

$$\Phi(\omega) = \Phi(\omega = 0) + \omega t_o \tag{10}$$

to the raw, unwrapped phase spectrum, weighted by the raw variance spectrum. This requires minimizing the sum

$$S(t_o) = \sum_i E(\omega_i)\ [\Phi(\omega_i) - \Phi(\omega = 0) - \omega_i t_o]^2 \tag{11}$$

where the ω_i are the FFT raster points. The trend ωt_o is then substracted from the unwrapped phase spectrum. A number of variations on this approach and that suggested by Funke and Mansard (1981) were investigated but the Eq. 11 approach appeared to be the most consistent. This is recognized, however, as a subjective judgement.

Figures 1 and 2 are typical results from the phase unwrapping and detrending algorithms, showing the raw but unwrapped and detrended phase spectrum together with the raw variance spectrum. The results are presented non-dimensionally in terms of the peak frequency f_p and truncated at $3f_p$ along the frequency axis. The oscillations in the tail of the phase spectrum are a computational effect in the phase unwrapping. A tolerance level must be set to define a near-singular point and to invoke the L'Hospital rule algorithm. These oscillations can be damped by appropriate choice of the tolerance level.

Similar results are consistently achieved, strongly indicating the existence of some coherent structure in the phase spectrum, against a background that has a distinctly random character.

TIME DOMAIN ANALYSIS OF WAVE RECORD

The standard statistical summary in the time domain is the correlogram $R(\tau)$, which forms a Fourier transform pair with the variance spectrum $E(f)$. In principle, both summaries describe the same information but from different perspectives. In practice, however, this equivalence becomes confused. The FFT algorithm yields the raw variance spectrum (Figures 1 and 2) which is then subjected to frequency domain smoothing, but the nature and extent of that smoothing is an especially subjective operation. Variations from a visually smooth result are commonly regarded as noise and given little attention; it remains possible that these variations from the smooth and expected result in fact describe wave grouping. This possibility is indeed sugges-

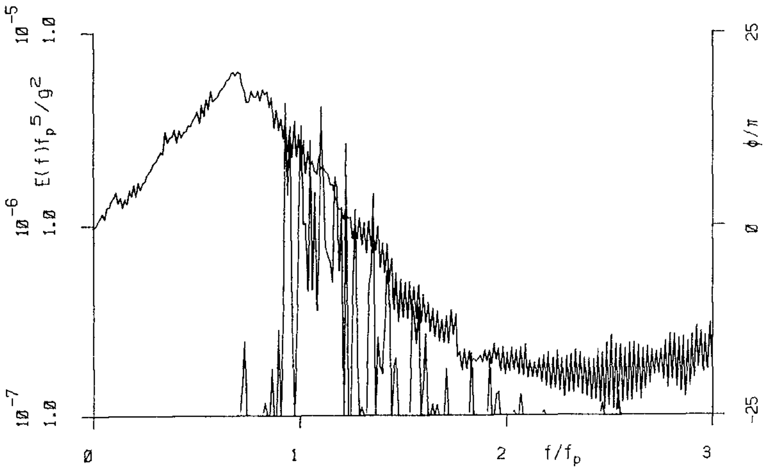

Fig. 1 Raw Variance and Unwrapped Phase Spectrum in Deep Water off Botany Bay, 5:20 hrs on 9 July 1981

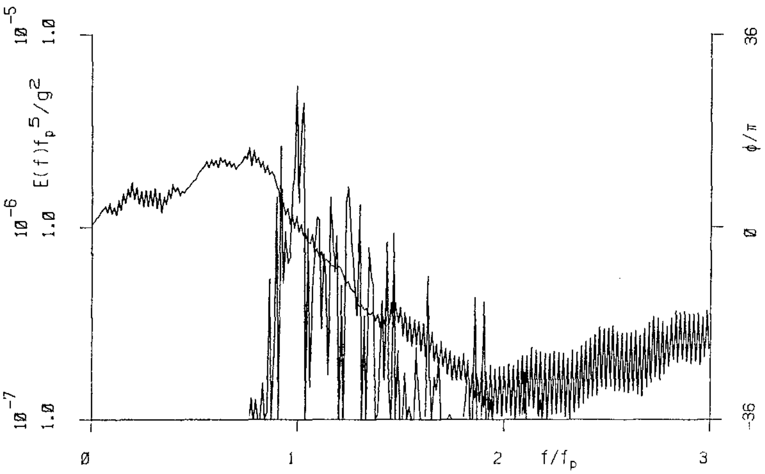

Fig. 2. Raw Variance and Unwrapped Phase Spectrum in Deep Water off Botany Bay, 7:20 hrs on 9 July 1981

ted by the correlograms in Figures 3 and 4, corresponding to the same records as Figures 1 and 2, where τ is the time lag and σ^2 the record variance. The time lag scale is non-dimensionalized by the peak frequency, as were the frequency domain results. If the phase spectrum is indeed random, the envelope of the correlogram should go asymptotically to zero in about five wave periods. This is not observed in Figures 3 and 4 (solid line), where the dominant feature is the ordered tail. The correlogram is inherently smoothed in a reasonably unbiased manner and the ordered tail is strongly suggestive of wave groups. These are typical results for moderate sea states and were not specially selected for presentation.

It remains posible that the oscillations in the correlogram tail are a computational effect. It is well known (Kendall and Stuart, 1966) that the correlogram for short series can be unreliable and may not damp out as rapidly as expected; as a rule of thumb, only the first twenty percent might be given any credence. Wave records are moderately long series, typically 2048 points over seventeen minutes. Figures 3 and 4 present the correlogram for non-dimensional lags f_p up to twenty. This corresponds to approximately twenty waves, about one-fifth of a typical wave record of one hundred waves. Further Kendall and Stuart give the 95% confidence limits of the correlogram for a random process as

$$CL = -1/N \pm 2/(N)^{1/2} \tag{12}$$

where N is the number of points in the record. For N = 2048, the 95% confidence limits are -0.00 ± 0.04. The amplitude of the oscillations in the tail of Figures 3 and 4 is of order 0.15, well outside the 95% confidence limits for a random process. This conclusion is not influenced by the adoption of the "unbiased estimator" for the autocorrelation, which uses the factor 1/(N-k) in place of 1/N in the estimation of $R(\tau = k\Delta t)$. In this case both the correlogram tail and the confidence limits are scaled up by the factor N/(N-k).

TIME DOMAIN ANALYSIS OF RICE ENVELOPE FUNCTION

In the initial discussion of data series for analysis it was observed that the complex envelope function A(t) defined by Eq. 1 appeared to be the natural vehicle for wave group analyses. A procedure for extracting the complex envelope function from the wave record is described by Sobey and Colman (1983). It involves the computation of the discrete Hilbert transform $\hat{\eta}(t)$ from the wave record $\eta(t)$ using the FFT and inverse FFT algorithms. Then follows the definition of the pre-envelope function

$$\eta(t) + i\,\hat{\eta}(t) = A(t)e^{i\omega_o t} \tag{13}$$

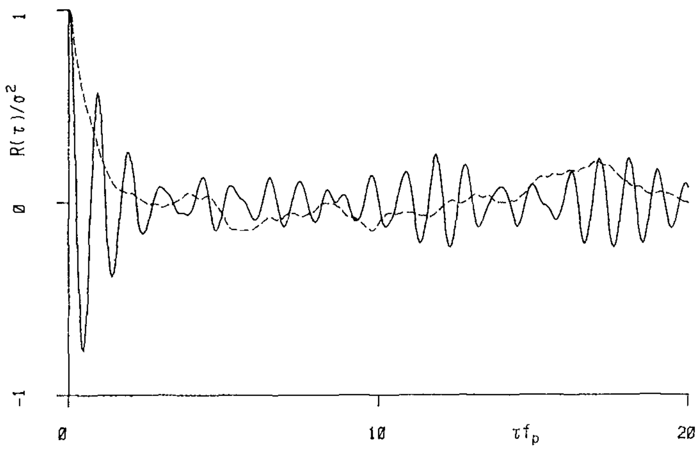

Fig. 3 Correlogram of Record and Rice Envelope Function in Deep Water off Botany Bay, 5:20 hrs on 9 July 1981

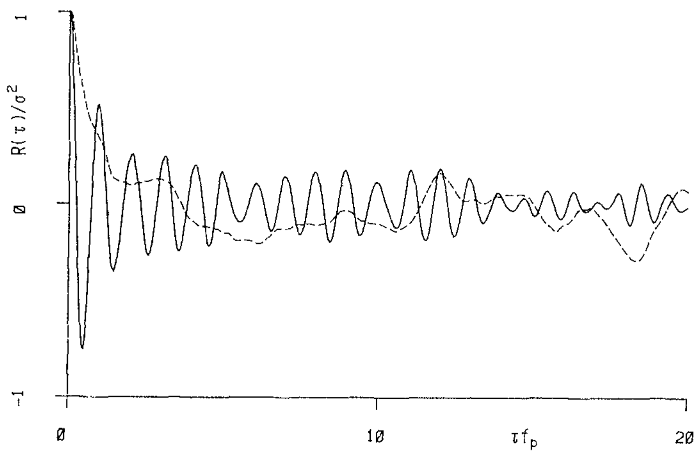

Fig. 4 Correlogram of Record and Rice Envelope Function in Deep Water off Botany Bay, 7:20 hrs on 9 July 1981

where $\omega_o = 2\pi f_o$ is the carrier frequency. The carrier frequency is identified as the peak frequency $2\pi f_p$ and estimated as

$$f_p = \frac{\int f E^8(f)df}{\int E^8(f)df} \tag{14}$$

Multiplying both sides of Eq. 14 by $\exp(-i\omega_o t)$ recovers the complex envelope function A(t). The Rice envelope is simply the modulus of A(t).

Sample computations of the Rice envelope function and comparison wit Rice envelope is simply the modulus of A(t).

Sample computations of the Rice envelope function and comparison with the wave record confirm that it is indeed an excellent representation of the wave envelope. It is this property of the Rice envelope function that was used by Rice (1944, 1945) and later Longuet-Higgins (1952) to establish the Rayleigh distribution as an excellent approximation to the probability distribution for wave heights. In the context of wave grouping, the Rice envelope function very conveniently concentrates attention on the wave envlope and hence the wave groups. A similar identification of wave groups was the rationale for the definition of the SIWEH function by Funke and Mansard (1980).

The considerable attention recently given to the data series of individual wave heights might also be considered in the context of the Rice envelope function. Amplitude domain arguments led Rice and Longuet-Higgins to the Rayleigh distribution. Time domain or correlogram arguments appear to lead to R_H (the correlation coefficient between consecutive wave heights) and j_2 (the average number of waves between the start of one wave group and the start of the next group). The correlogram of the Rice envelope function is shown as the dashed lines on Figures 3 and 4.

The initial exponential decay is closely related to typical statistics extracted from the data series of individual wave heights. In particular R_H might be defined as

$$R_H = R(\tau f_p = 1)/\sigma^2 \tag{15}$$

and j_2 from the first zero crossing of the correlogram as

$$R(\tau f_p = 1/2\ j_2) = 0 \tag{16}$$

Alternately the zero-crossing frequency rather than the peak frequency could be used in these definitions. The difference is typically small; the peak frequency has rather more dynamic significance while the zero-crossing frequency occurs naturally in the theory of random noise. Note that R_H defined in terms of the zero crossing frequency is the parameter proposed by Battjes (1984) as the sole spectral shape parameter determining the group statistics.

The long period persistence in the tail of the Rice envelope correlograms is again suggestive of wave grouping.

CONCLUSIONS

Evidence suggestive of wave grouping has been sought in the frequency and time domain. Ordered structures have been identified in the phase spectrum and in the tail of the correlogram, both for wave records and for the Rice envelope function. There is a reasonable expectation that wave grouping will be adequately described by these classical linear techniques of time series analysis.

APPENDIX I - REFERENCES

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