

Wave Packets and Turbulent Jet Noise

Peter Jordan¹ and Tim Colonius²

¹Département Fluides, Thermique, Combustion, Institut PPRIME, CNRS – Université de Poitiers – ENSMA, UPR 3346, 86036 Poitiers, France

²Division of Engineering and Applied Science, California Institute of Technology, Pasadena, California 91125; email: peter.jordan@univ-poitiers.fr, colonius@caltech.edu

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Abstract

Turbulent jet noise is a controversial fluid mechanical puzzle that has amused and bewildered researchers for more than half a century. Whereas numerical simulations are now capable of simultaneously predicting turbulence and its radiated sound, the theoretical framework that would guide noise-control efforts is incomplete. Wave packets are intermittent, advecting disturbances that are correlated over distances far exceeding the integral scales of turbulence. Their signatures are readily distinguished in the vortical, turbulent region; the irrotational, evanescent near field; and the propagating far field. We review evidence of the existence, energetics, dynamics, and acoustic efficiency of wave packets. We highlight how extensive data available from simulations and modern measurement techniques can be used to distill acoustically relevant turbulent motions. The evidence supports theories that seek to represent wave packets as instability waves, or more general modal solutions of the governing equations, and confirms the acoustic importance of these structures in the aft-angle radiation of high subsonic and supersonic jets. The resulting unified view of wave packets provides insights that can help guide control strategies.

1. THE JET-NOISE PUZZLE

Jet-noise reduction is an important long-range goal for the commercial and military aviation communities. Compared with their early counterparts, modern, ultrahigh-bypass-ratio turbofans on commercial aircraft are quiet, but more stringent noise regulations dictate further reductions. Meanwhile, hearing loss by personnel and community noise issues are prompting the military to reduce jet noise on future tactical aircraft (Bowes et al. 2009).

For fluid mechanicians, jet noise is a daunting yet compelling and beautiful jigsaw puzzle. The intricate pieces derive from the disparate physics and scales of the energetic, turbulent jet flow and the weak, radiated acoustic field, and the way in which these two components of the solution of the compressible flow equations meet in the middle. Research on linear acoustics completed the outer edges of the puzzle more than a century ago. Turbulence theory, experiment, and simulations have completed large sections of the middle. But, despite 60 years of research in aeroacoustics, the middle is only connected to the edges by tenuous strands. Numerical simulations render the entire compressible flow solution, despite the missing theoretical pieces, but we press on with the knowledge that to achieve our objectives of understanding the mechanics (and then controlling them), we must understand how the puzzle is put together and how the pieces might be rearranged to produce a different picture.

Prior to the 1960s, the turbulent motions of a jet were supposed to be entirely stochastic, as were, consequently, the associated sound sources. The discovery of coherent structures changed this view, and it is the connection between these and the coherent acoustic waves in the far field that this review addresses. The focus of the article, i.e., wave packets, refers to a particular spatiotemporal structure observed in the jet and its radiated sound. Like many topics in fluid mechanics, there is an early hero in the story of wave packets, Erik Mollo-Christensen, whose astute observations from the 1960s of the structure of a jet's acoustic field underpin and anticipate nearly all the developments we report here. What is different now is the availability of superior diagnostics and simulation databases that provide far more detailed evidence connecting the advecting, coherent structures in the jet to the dominant far-field radiated sound (**Figure 1**). Moreover, relatively simple models, even linear ones, have been developed that predict certain aspects of the turbulence and the radiated sound, without recourse to equivalent source descriptions and acoustic analogy theories (Goldstein 2003, 2005; Lighthill 1952; Lilley 1974; Sinayoko et al. 2011), giving rise to the hope of a more complete causal theory connecting turbulence structure to far-field sound.

Figure 2 shows a large-eddy simulation (LES) of a turbulent jet and its radiated sound and serves to introduce the main features of the flow. The spherical and cylindrical coordinate systems used through this review are defined in the figure. At the nozzle exit, the jet has nearly uniform velocity characterized by a Reynolds number, $Re = \frac{\rho_j U_j D}{\mu_j}$, typically larger than approximately 10^5 in the laboratory and applications; temperature ratio, $\frac{T_j}{T_\infty}$; and Mach number, $M_j = \frac{U_j}{a_j}$ (or $M_\infty = \frac{U_\infty}{a_\infty}$). Depending on upstream conditions, there may be turbulent fluctuations in the core of



Figure 1

The jet-noise puzzle.

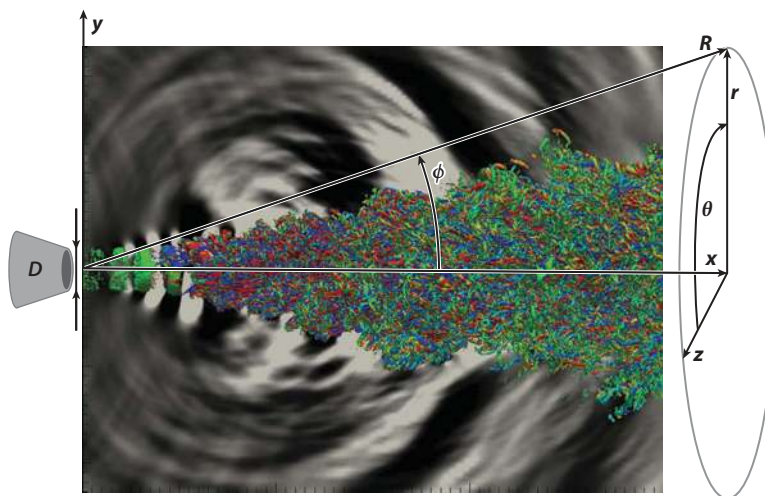


Figure 2

Large-eddy simulation of an $M_j = 0.9$, $T_j/T_\infty = 1$ turbulent jet at $Re = 400,000$. Figure reprinted from Cavalieri et al. (2011a) with permission from Elsevier.

the jet at the inlet. For typical laboratory experiments and applications, the nozzle wall boundary layers are thin compared to D but may be, even for the same value of Re , laminar, transitional (as in the figure), or turbulent (see the sidebar, Jet-Noise Sensitivity to Upstream Conditions).

2. SIGNATURES OF WAVE PACKETS

2.1. In the Far-Acoustic Field

The far-acoustic field is the region far from the vortical, turbulent motions where the flow can be approximated, via linearization, by the wave equation and, furthermore, where the wave field can be described as spherical waves emanating from a point. Once the radial decay ($1/R$) is accounted

JET-NOISE SENSITIVITY TO UPSTREAM CONDITIONS

Despite a large number of experimental campaigns over the years, the sensitivity of high-Reynolds number jets to upstream and ambient conditions is hotly debated. Questions persist as to the asymptotically high value beyond which the turbulence and radiated noise can be considered independent of Re . Viswanathan (2004) carefully analyzed data for cold and hot jets from nozzles with different diameters such that Re and T_j/T_∞ could be independently varied and called into question whether earlier studies that attributed differences in the far-field sound to heating were instead measuring a Reynolds number effect. He suggested a threshold of $Re > 4 \times 10^5$.

Further studies (Harper-Bourne 2010, Zaman 2011) suggest that the situation is more complicated and that the far field is also sensitive to the contraction ratio of the nozzle, even when the nozzle boundary layer is nominally turbulent. Laboratory-type installations typically have high contraction ratios, thin boundary layers, and produce more high-frequency noise than their industrial counterparts. These questions are currently being addressed by a number of groups. An interesting question is whether such sensitivity could be understood in terms of the influence of jet parameters and inlet disturbances on wave packets and exploited for control.

for, the power spectrum depends only on ϕ . It is directive, with a maximum $20^\circ < \phi < 40^\circ$. At these shallow angles, the spectrum peaks around $St = \frac{fD}{U_j} \approx 0.2$ and has a steep spectral decay. As the polar angle is increased, the precipitous drop in amplitude is accompanied by a broadening of the spectrum and an upward shift in frequency to around $St = 0.4$ by 90° . For cold jets up through moderately supersonic M_j , the overall sound pressure level scales with U_j^n with $n = 8$ at 90° and $n > 8$ at shallower angles. The low-frequency ($St < 1$) sound may be decomposed, almost entirely, into just three azimuthal Fourier modes: $m = 0, 1$, and 2 (Cavalieri et al. 2011a,c; Juvé et al. 1979; Kopiev et al. 2010). A final important characteristic of the far field is its temporal intermittency. Noise is observed to recur at $St \approx 0.2$ in temporally localized bursts, a behavior that can be detected using time-local analysis (Cavalieri et al. 2011a, Guj et al. 2003, Hileman et al. 2005, Juvé et al. 1980) or characterized with a wavelet basis (Koenig et al. 2012).

These far-field characteristics are captured by simple sound-source models. The strong directivity and low-angle velocity component ($n > 8$) can be mimicked using an axially coherent, noncompact, wavy line-source model (Crow 1972; Michalke 1970, 1972; Michel 2009). Michalke & Fuchs (1975) provided an explanatory analysis for the azimuthal structure by invoking the azimuthal dependence of the integral solution to an inhomogeneous wave equation, the Bessel function, $J_m(2\pi St M \sin \phi)$. For any source whose radial extent is small compared to the acoustic wavelength, this term dictates that only the $m = 0, 1$, and 2 components of the source can make significant contributions to the radiated sound, regardless of the axial structure of the source. **Figure 3** illustrates this behavior and shows that at low emission angles, mode $m = 0$ will dominate, with modes $m = 1$ and $m = 2$ becoming increasingly important at higher emission angles and frequencies, in agreement with the measured characteristics. The intermittent character has

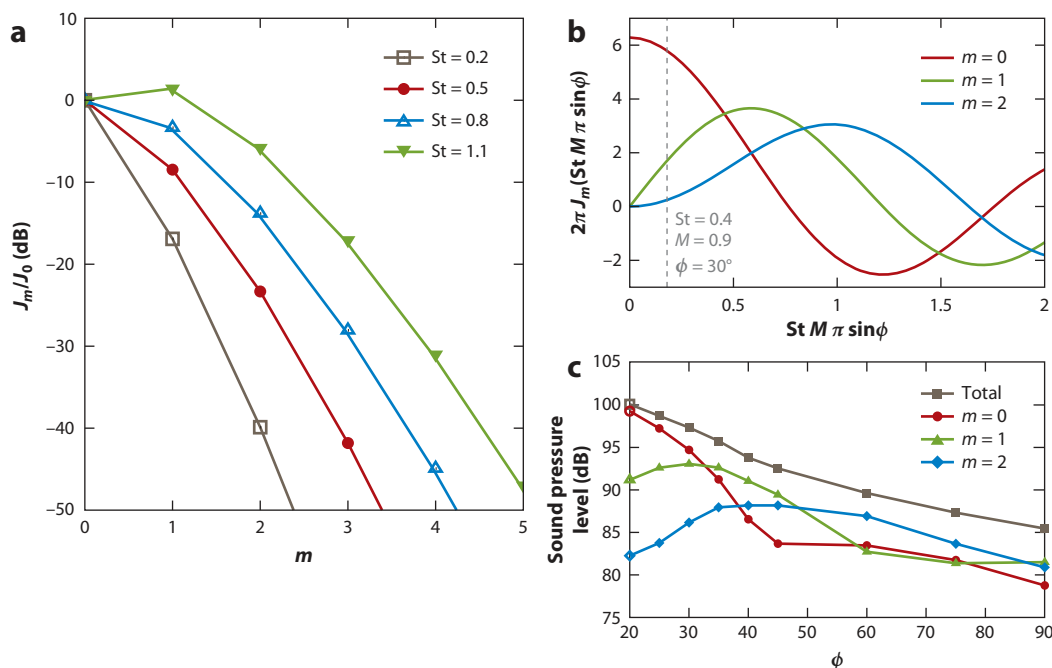


Figure 3

The azimuthal structure of the sound due to $J_m(2\pi St M \sin \phi)$ (a) as a function of frequency and azimuthal mode number for $\phi = 30^\circ$, (b) as a function of $St M \sin \phi$, and (c) as measured experimentally at $St = 0.2$ (Cavalieri et al. 2012a).

also been analyzed using simple models (Cavalieri et al. 2011b, Ffowcs Williams & Kempton 1978, Sandham et al. 2006) that confirm that intermittency in wave-packet sources can enhance sound radiation efficiency. Physical grounds for these models—wave packets in turbulence and the near-acoustic field—are discussed in the next sections.

Inferring the characteristics of a source based on its radiated sound (source localization) alone is an ill-posed problem. However, such techniques can be useful if constrained by physically appropriate source ansatzes. The traditional methods use monopole sources but can be extended to multipoles (see Suzuki 2010). When a wave-packet ansatz is used (Koenig et al. 2012, Morris 2009, Papamoschou 2011), one can obtain parameters such as envelope amplitude, wavelength, position, and convection velocity using far-field measurements. Cavalieri et al. (2012a) recently imposed radial structure on the wave-packet ansatz using eigenfunctions from linear stability analysis and used azimuthally decomposed far-field measurements to determine envelope parameters for higher-order azimuthal modes ($m = 1$ and $m = 2$). To understand which kind of source ansatz is most appropriate, one must explore the near field of the flow.

2.2. In the Near-Acoustic Field

The near-acoustic field is the region in which fluctuations are small enough to permit linearization but close enough to the jet so that they are not necessarily propagating acoustic waves. This region may be alternatively termed the irrotational hydrodynamic pressure field, the pseudosound region, or the entrainment region. Here local, nonpropagating solutions of the wave equation must be matched to an inner solution, the turbulence, and to an outer solution, the propagating acoustic field. In the jet, turbulence pressure fluctuations advect downstream with a phase speed, $U_c < U_j$. When $U_c < a_\infty$, matching involves evanescent waves, solutions of the wave equation with exponentially decaying (with r) amplitude.

The first observations of a wave-packet structure to this near-acoustic field were made by Mollo-Christensen (1963, 1967). The form of the two-point pressure correlation led him to suggest that the jet be modeled as a “semi-infinite antenna for sound.” Far more extensive mappings of the near-field pressure are now possible. **Figure 4** shows instantaneous and statistical renderings of wave packets from simulations and experiments.

The structure of this overlap region may be determined with simplified models (e.g., Crighton & Huerre 1990). The mathematics leads to the schematic shown in **Figure 5**. If, at a particular frequency, the advecting pressure fluctuations were homogeneous, they would produce a purely evanescent wave field (i.e., silence in the far field). If instead their amplitude varies, then a small portion of their energy is acoustically matched to a wave propagating to angle $\phi = \arccos(\frac{\alpha a_\infty}{\omega})$, where α is the axial wave number. In other words, the advecting turbulence pressure fluctuations leak a small amount of their energy to the far field. The amount of leakage depends on the envelope of the wave packet, with certain forms leading to a superdirective (exponential decay of pressure with ϕ) radiation (Crighton & Huerre 1990), a feature that is observed for the axisymmetric mode (Cavalieri et al. 2012a). When the phase speed of the wave packet becomes supersonic, the spectral peak of the turbulence pressure fluctuations is acoustically matched, and much stronger Mach waves are emitted from the jet. There are analogies to flow over a wavy wall (Tam 1995) and electromagnetic emitters and receivers (Crighton 1975), in the sense that noncompact interference between correlated regions of potential creates a propagating wave.

In contrast to the inverse problem evoked above, determining the far field from the near field is well-posed. Variants of the Kirchhoff surface technique (Lyrantzis 1994) are routinely employed for this purpose in computations. One can also apply such methods to experimental data from microphone arrays (Reba et al. 2010), provided the microphone spacing is sufficiently fine and one is cognizant of potential errors associated with the (inevitably) missing data on portions of the bounding

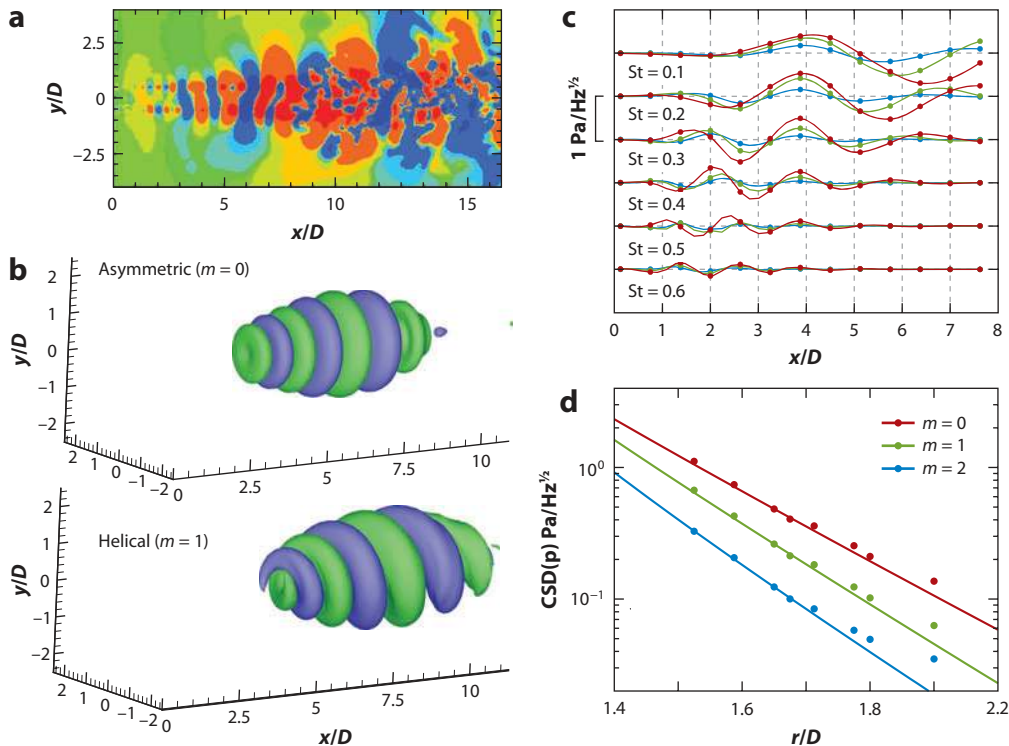


Figure 4

Wave packets: (a) instantaneous slice of the $M_j = 0.9$, $Re = 3,600$ DNS of Freund (2001); (b) the most energetic axisymmetric ($m = 0$) and helical ($m = 1$) pressure wave packets from the DNS; (c) the streamwise structure as a function of St and m based on the cross-spectral density (CSD) of the pressure from a near-field caged microphone array (Suzuki & Colonius 2006) at $M_j = 0.5$, $Re = 700,000$; and (d) radial decay at $x/D = 3.5$ from the same experiments. The lines represent the exponential radial decay associated with $J_m(i\beta r)$, where $\beta = \sqrt{\alpha^2 - (\frac{\omega}{a_\infty})^2}$, with α the measured local wave number.

surface (i.e., the end caps of the microphone array that pass through the turbulent jet). Reba et al. (2010) projected the near-field caged-array data of Suzuki & Colonius (2006) and found reasonable agreement with the far-field, aft-angle, low-frequency directivity, leading the authors to assert that it is the convecting wave-packet structure that leads to the peak aft-angle radiation, via the mechanism illustrated in **Figure 5**, and in agreement with the wave-packet source models discussed above.

The recent experiments of Viswanathan et al. (2011) utilized 294 microphones placed along a half-cone of angle 10° to study near-field pressure fluctuations. The microphone locations were not in the near field as it is defined here. The first microphone at the nozzle exit plane was three diameters from the jet centerline, meaning that for all but perhaps the very low frequencies, the array measures purely propagating waves (especially for the subsonic conditions considered). High levels of correlation between the microphone array and aft-angle, far-field measurements were interpreted to indicate source activity from 15 to 30 diameters downstream (which far exceeds all previous estimates based on source-localization techniques). An alternative, less controversial, interpretation follows from tracing their array position on **Figure 2**, which makes clear that the high level of correlation was associated with low-order azimuthal-mode acoustic waves propagating from the array to the far field.

Matching the near-acoustic field to the structure of the turbulent motions of the jet is a less straightforward exercise. But the situation is not so seemingly hopeless as trying to infer the

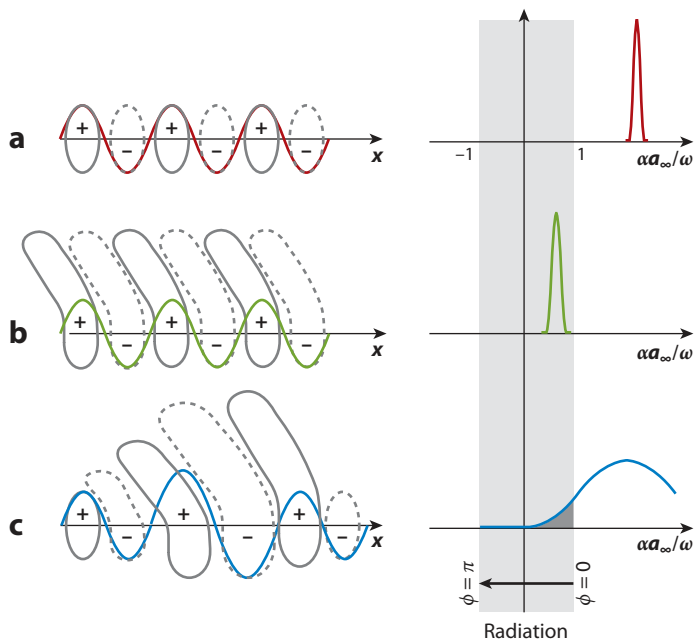


Figure 5

Spatial and spectral representations of wave packets: (a) subsonic advection; (b) silent supersonic advection, generating Mach waves; and (c) subsonic advection, with spatial modulation leading to sound leakage.

characteristics of the source based on the far-field spectral characteristics alone. Indeed, the evanescent wave structure is the footprint of the coherent part of the turbulent field, discussed in the next section, so it can be used to provide guidance for conditional averaging and other statistical techniques that attempt to decompose the turbulence by radiation efficiency.

2.3. In the Vortical Region

The instantaneous fog visualizations of Crow & Champagne (1971) (**Figure 6a**) give a qualitative sense of the coherent structures in the turbulent jet. Moore (1977) placed six microphones around

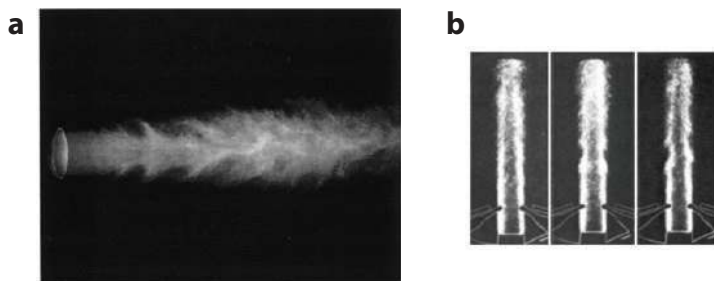


Figure 6

(a) Instantaneous CO_2 fog visualizations at $\text{Re} = 7.5 \times 10^4$ at low Mach numbers. Panel a reprinted from Crow & Champagne (1971) with permission from Cambridge University Press. (b) Flash schlieren images $\text{Re} = 5 \times 10^5$, $M = 0.83$: (left) random ensemble average, (middle) conditional average using a mode $m = 0$ near-field pressure signature as the trigger, and (right) conditional average using mode $m = 1$ as the trigger. Panel b reprinted from Moore (1977) with permission from Cambridge University Press.

the jet to conditionally average schlieren images based on high-amplitude spikes of modes $m = 0$ and 1 to produce the axisymmetric and helical patterns shown in **Figure 6b**. Although the trigger event was sampled at a single axial station, axially extensive waveforms are observed, showing, similar to what was found in a plane mixing layer (Dimotakis & Brown 1976), that a fixed-phase relation exists between fluid-particle motions separated by considerably more than an integral turbulence scale. That such conditional averages differ from the unconditional ensemble average is an indication that the flow repeatedly revisits this wavelike state; Keefe, as cited by Broze & Hussain (1994), suggested that this is proof of the existence of an underlying attractor, and in Section 4 we discuss reduced-order modeling techniques that attempt to identify this attractor.

Multipoint velocity and pressure measurements quantify these structures. Fuchs (1972), for instance, measured significant pressure correlations between probes separated (axially, radially, and circumferentially) by considerably more than the local integral scale. The axial correlation had a clear wave-packet structure. The convection velocity of this structure is found to vary weakly with position over an extensive region of the jet (Bradshaw et al. 1964, Davies et al. 1963, Franklin & Foxwell 1958, Lau et al. 1972). Furthermore, Michalke & Fuchs (1975) and Armstrong et al. (1977) showed that the fluctuation energy of the turbulence pressure field is contained in only three azimuthal Fourier modes (recall that the azimuthal structure of the sound field is similarly constituted). The azimuthal spectrum of the velocity field is broader, and even when techniques such as proper orthogonal decomposition (POD) are used to filter the fields, the dominant structure is not a wave packet (Bonnet et al. 1994; Freund & Colonius 2009; Glauser & George 1987; Jung et al. 2004; Sinha et al. 2010, 2011b; Tinney et al. 2008b).

Lau et al. (1972) used pressure-pressure, velocity-velocity, and pressure-velocity correlations to support a simple kinematic model for the wave-packet structure comprising a convected train of vortices. Subsequent conditional analysis, triggered by high-amplitude spikes of the axial velocity, confirmed this model (Lau & Fisher 1975).

An alternative to trigger-event-based conditional analysis (which is necessarily subjective) is to force the jet, typically with a loudspeaker. The latent order is thereby raised above background levels, and an objective phase reference is made available (Crow & Champagne 1971, Hussain & Zaman 1980, Moore 1977, Zaman & Hussain 1980). Hussain & Zaman (1981) studied the phase-averaged velocity field of a jet forced at $St_D = 0.3$ to reveal a train of structures spanning over six diameters in the axial direction. The authors obtained the structure's contribution to the fluctuation energy by triple decomposition and found it to be comparable with that of the background turbulence. Recent studies of forced jets using plasma actuators reveal similar structure at higher Mach numbers (Sinha et al. 2011b).

In unforced jets, however, flow visualizations show that such trains of vortices are intermittent and appear "puff-like," to quote Crow & Champagne (1971), rather than rolled up. Their contribution to the fluctuation energy is also reduced compared to the forced case: Michalke & Fuchs (1975) showed that the first three azimuthal velocity modes comprise only approximately 15% of the fluctuation energy at low frequencies, compared with 50% for just the axisymmetric forced mode of Hussain & Zaman (1981). More recent particle image velocimetry (PIV) measurements (Cavalieri et al. 2012b) further support this estimate of the energetic importance of coherent structures in unforced jets.

To eliminate the need for a fixed-phased reference (forced jets) or the dependence on an arbitrarily defined trigger threshold (classical conditional averaging), Adrian (1977) proposed the use of linear stochastic estimation (LSE). Extensions of LSE to distributed space-time sampling (Delville 1995, Kerhervé et al. 2012b, Picard & Delville 2000, Tinney et al. 2006, Vincendeau 1995) permit conditional analysis of a given field (e.g., the turbulent velocity field) with respect to the complete space-time structure of another (e.g., the near-field pressure). Picard & Delville

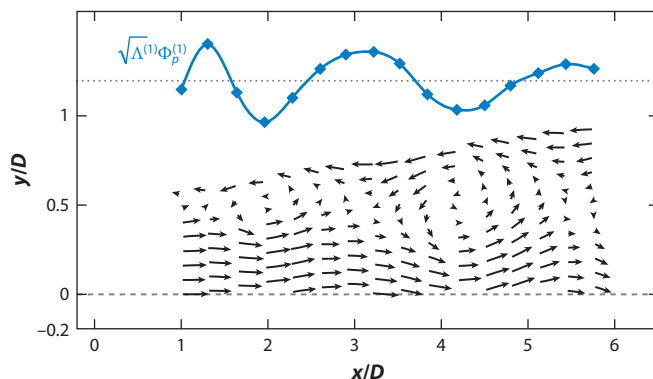


Figure 7

Proper orthogonal decomposition–filtered stochastic estimation of flow motions associated with near-field wave packets.

(2000) used LSE to educe the turbulent motions associated with the acoustic near-field pressure (**Figure 7**), providing the first definitive link, in unforced jets, between the previously observed trains of vortices and the wave packets measured in the near-acoustic field.

To summarize, the coherent part of the velocity field in unforced, turbulent jets is a wave packet characterized by its low azimuthal wave number, a nearly constant convection velocity, and coherence over radial and axial length scales far greater than the integral scales. Despite its relatively low contribution to the total fluctuation energy, it is associated with the observed near-field pressure and, as shown in Section 4, dominates the aft-angle sound radiation. Simply put, its high level of space-time coherence makes it considerably more acoustically efficient than the more disorganized, although more energetic, turbulence.

The proposition that sideline radiation is associated with an independent source mechanism, fine-scale turbulence, is frequently made. This follows the contention of Tam et al. (1996), who, from the analysis of a large number of jet-noise databases, identified two similarity spectra that fit downstream and sideline radiation. We argue that these empirical fits do not constitute evidence of different source mechanisms. The measured spectral shape in fact evolves smoothly with ϕ , and the most energetic parts of the aft-angle and sideline spectra span a similar bandwidth. This behavior may turn out to be associated with different radiation characteristics of the same wave-packet source (Papamoschou 2011).

3. WAVE PACKETS AS MODAL SOLUTIONS

Observations of large-scale coherent structures discussed in the last section have been accompanied by attempts to compare their characteristics (amplitude, wavelength, phase speed) to linear and nonlinear stability analyses or, more precisely, modal solutions for disturbances to a time-invariant base flow, typically an idealized profile or the measured or modeled mean of the turbulent flow. For the latter, the idea is that “the turbulence establishes an equivalent laminar flow profile as far as large-scale modes are concerned” (Crighton & Gaster 1976). A rigorous scale-separation analysis that would prove this hypothesis is still lacking, but the extensive spatiotemporal coherence of the observed wave-packet structure, and the optimistic comparisons with the theory that we discuss here, empirically supports Crighton & Gaster’s assertion.

No matter which ansatz is chosen for stability analysis, an important consideration is how much amplification a real disturbance suffers; if it is too great, then nonlinear effects (saturation,

pairing, and so on) will render moot the original choice of base flow. This is the case for initially laminar shear layers (Bradshaw 1966), and for strongly forced jets (Crow & Champagne 1971), for which nonlinear analyses are therefore required. Compared to a laminar shear layer, however, the much faster spread rate of the turbulent mean flow gives rise, at a fixed frequency, to a much smaller overall amplification. With nonlinear effects implicit in the mean flow, they might then be negligible in the disturbance evolution.

In what follows, we restrict our attention to jets that would be labeled convectively unstable according to parallel flow analysis, omitting discussion of absolute instabilities that are observed in highly heated, low-speed jets (Huerre & Monkewitz 1990, Lesshafft et al. 2006). By necessity, the discussion here also omits the rich mathematical and computational detail that stability theory deserves.

3.1. Ansatz

We adopt the Reynolds decomposition $\mathbf{q}(\mathbf{x}, t) = \bar{\mathbf{q}}(\mathbf{x}) + \mathbf{q}'(\mathbf{x}, t)$, where $\bar{\mathbf{q}}$ is the long-time average of the vector of dependent flow variables, \mathbf{q} . For statistically stationary turbulence, homogeneous in the jet's azimuthal direction, we decompose into modes

$$\mathbf{q}'(x, r, \theta, t) = \sum_m \sum_\omega \tilde{\mathbf{q}}_{m,\omega}(x, r) e^{im\theta} e^{-i\omega t}. \quad (1)$$

We insert these into the compressible Navier-Stokes (plus energy and continuity equations), which we represent $\mathcal{N}(\mathbf{q}) = 0$, to obtain

$$\mathcal{L}(\bar{\mathbf{q}}) = R_{0,0}, \quad (2)$$

$$\mathcal{L}(\tilde{\mathbf{q}}_{m,\omega}) = R_{m,\omega} \quad \forall m \quad \text{and} \quad \omega \neq 0, m, \quad (3)$$

where \mathcal{L} is \mathcal{N} linearized about the mean flow, and $R_{m,\omega}$ represents a generalized mode-dependent Reynolds stress. Equation 2 presents the standard Reynolds-averaged equations. We assume in what follows that the basic flow satisfies these equations, at least approximately. This would be the case for laminar boundary-layer solutions (where $R_{0,0} = 0$) and turbulent mean flows measured in the laboratory or computed via Reynolds-averaged Navier-Stokes modeling (where $R_{0,0} \neq 0$).

Equation 3 is exact and represents the evolution of disturbances to the mean flow. As written, this equation is deterministic and depends on suitable boundary conditions for the disturbance field at each frequency, including phase and amplitude. If, as we envision, the inlet perturbations are stochastic and we want to compare to averaged data from experiments (such as the power spectral density), we may ensemble-average Equation 3 with no change to the left-hand side.

If the Reynolds stresses in Equation 3 can be neglected, then linear solutions can be computed at that frequency. For the coherent wave-packet structures, we are generally interested in low frequencies and low azimuthal wave numbers. We really only need invoke the assumption that $R_{m,\omega}$ be negligible for those modes, it being irrelevant to their evolution whether they be negligible for the higher modes. This is essentially the linear theory pursued for jets by Crighton & Gaster (1976).

Even with $R_{m,\omega}$ set to zero, Equation 3 is still challenging to solve numerically. If the flow is assumed to be locally parallel (on the scale of the disturbance wavelength), then it reduces to the Orr-Sommerfeld equations for which modal solutions in a complex streamwise wave number, α , may be found (see Michalke 1984). For typical velocity profiles of a subsonic unheated round jet, at most one family of unstable modes exists for each azimuthal mode number (Mattingly & Chang 1974). Heuristic energy-integral formulations (Chan 1974, Liu 1974, Mankbadi & Liu 1981) or a rigorous multiple-scales analysis (Crighton & Gaster 1976, Tam & Morris 1980) extends the parallel flow theory to weakly diverging jet mean flow. A computationally simpler approach that

delivers quantitatively similar results (Chang et al. 1993) is the parabolized stability equation (PSE) framework (summarized in Herbert 1997). Finally, one may attack the global stability problem for \mathcal{L} directly, either as a multidimensional eigenvalue problem for (presumably damped) modes with a complex frequency (Nichols & Lele 2011) or as a boundary-value problem (Mohseni et al. 2002) with inlet perturbations specified.

It is also possible to retain a subset of resolved nonlinear interactions in $R_{m,\omega}$ using the nonlinear PSE framework and/or to specify an eddy-viscosity model for unresolved interactions. Aspects of nonlinearity may also be included in the aforementioned energy-integral formulations (Mankbadi & Liu 1984, Morris et al. 1990).

3.2. Comparison with Experiments

Early comparisons (1970s) of stability theory with experimental data generally showed only qualitative agreement with the most amplified frequencies, wavelengths, and growth/decay rates predicted by the theory. Crighton & Gaster (1976) reviewed most of the relevant comparisons and pointed out several dangers related to the differing apparent wave numbers and growth rates found by measuring the field in different locations and/or using different variables.

For unforced jets, recent multipoint measurements enable the extraction of correlated fluctuations over an extensive region of the jet and provide more detailed (and indeed favorable) comparison with theory. Suzuki & Colonius (2006) employed a caged microphone array capable of measuring azimuthally resolved pressure on a conical surface in the near-acoustic field and found reasonable agreement for the wavelength, phase speed, and growth rates predicted by quasi-parallel linear analysis using the measured mean flow field. This particular measurement location is advantageous for such comparisons because, as reflected by the data, the pressure signal in this region is primarily associated with the evanescent pressure field and naturally filters out smaller-scale turbulent fluctuations.

Gudmundsson & Colonius (2011) extended the analysis to consider PSE eigenfunctions; their results for an $M = 0.5$ cold jet are summarized in **Figure 8**. The most energetic POD mode of the cross-spectral density matrix agrees with theory for the entire envelope of the pressure fluctuations, even beyond the close of the potential core. Favorable agreement was likewise obtained for subsonic jets at higher speeds and with heating.

Further corroborating the linear theory, Cavalieri et al. (2012b) compared PSE solutions to time-resolved, stereoscopic PIV in cross-stream planes. The authors used a hot-wire traverse along the jet centerline within the potential core to calibrate the amplitude of the $m = 0$ mode. As shown in **Figure 8**, the calibrated PSE solutions are in striking quantitative agreement with the most energetic POD mode of the time-resolved PIV data. The good agreement extended, for a range of low frequencies, up to the close of the potential core, beyond which there are discrepancies that may imply nonlinearity or simply that the modal amplitude has decayed to the point at which the POD is ineffective at segregating the less energetic structure (or both).

3.3. Comparisons with Simulations

The extensive data available from numerical simulations are also useful for validating stability theories, with the caveat that LES is only now reaching the resolution required to represent thin, initially turbulent boundary layers (e.g., Bogey et al. 2011). The work discussed here considers initially laminar jets and other idealized, transitional flows.

Mohseni et al. (2002) used direct numerical simulation (DNS) data for an $M = 1.92$ jet at $Re \approx 2,000$ and directly compared linear and nonlinear computations that prescribed the same

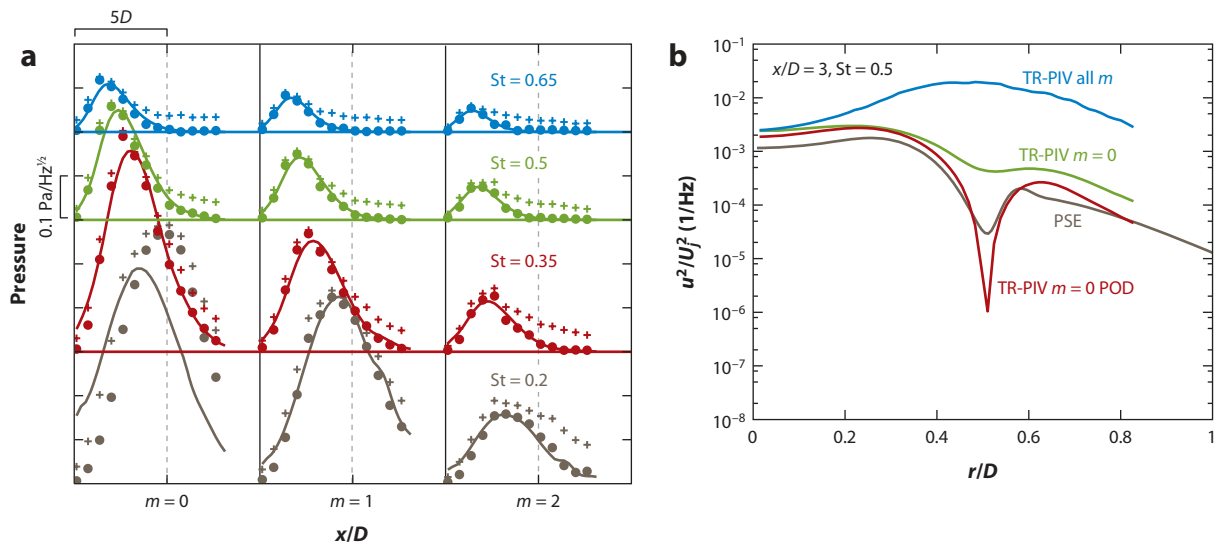


Figure 8

Comparison of linear parabolized stability equation (PSE) solutions with (a) pressure from a near-field microphone array at $M_j = 0.5$ (Gudmundsson & Colonius 2011) and (b) velocity from time-resolved particle image velocimetry (TR-PIV) in a cross-stream plane at $M_j = 0.4$. Abbreviation: POD, proper orthogonal decomposition.

inlet perturbations. Although there was reasonable quantitative agreement at the most amplified frequency and azimuthal mode, amplification rates of other frequencies and modes were substantially weaker in the linear case. Piot et al. (2006) also found good agreement around the most amplified frequencies when comparing linear PSE to LES at $M = 0.7$ up to the close of the potential core.

Cheung & Lele (2009) analyzed a forced two-dimensional shear layer and compared DNS to both linear and nonlinear PSE models. They concluded that nonlinear effects were needed for both mean flow correction and proper resolution of subharmonic pairing, which in this flow was the dominant source of sound. Suponitsky et al. (2011, 2010) compared linear and nonlinear Navier-Stokes solutions for perturbations to a base flow corresponding to the low-Reynolds number jet of Stromberg et al. (1980). To maintain the base flow in nonlinear calculations, in the absence of broadband turbulence, the authors added an artificial forcing term to the equations. By limiting the jet excitation to just a few frequencies, they showed that sound produced by nonlinear interactions (difference modes) dominated that directly radiated by a linear mode at the difference frequency.

Although simulations provide the most detailed data possible for confrontation with theory, it is not yet clear whether the conclusions of the studies performed to date apply to high-Reynolds number, fully turbulent jets. There is room for doubt because, as discussed above, growth rates are much higher in laminar shear layers, and when examining the turbulent case, one must be cognizant that the large coherent wave packet represents only a small fraction of the fluctuation energy.

3.4. Acoustic Prediction

There are three techniques by which models can predict the acoustic far field: directly, via a Kirchhoff surface (Section 2), or by evaluating an equivalent source. Mankbadi & Liu (1984)

appear to be the first to directly predict the acoustic field from stability theory using an energy-integral formulation to capture nonlinear effects, and an equivalent source to compute the sound. Although a great deal of empirical input was required, their model reproduced certain features of experimental data (Lush 1971), including the aft-angle directivity, the shift in spectral peak with emission angle, and scaling with jet velocity. It is a pity they did not have access to modern diagnostics to more completely evaluate their approach.

For linearized solutions of the full compressible equations (and global stability analyses of such), the acoustic field can be directly resolved or extended using a Kirchhoff surface. For models employing PSE, especially for subsonic jets, neither a Kirchhoff surface nor an equivalent source evaluation is straightforward because they are sensitive to the error introduced by parabolization. The near-acoustic evanescent wave field, however, is approximately accounted for and may provide sufficient fidelity for direct projection to the far field. Recently, Colonius et al. (2010) employed a tailored Green's function (Reba et al. 2010) to project linear PSE solutions from a conical surface in the near-acoustic field to the far field. They found reasonable agreement with the measured axisymmetric component of the far-field, low-frequency, aft-angle sound of a heated $M_\infty = 0.9$ jet, but it remains to assess the sensitivity of the prediction to the Kirchhoff surface location and to consider other flow conditions. Alternatively, Cheung & Lele (2009) used an equivalent source to evaluate the far field from linear and nonlinear PSE solutions and found a close match with DNS solutions, provided the nonlinearity inherent in their transitional, two-dimensional mixing layers was properly treated.

The problem of predicting noise from PSE is not as severe for modes with supersonic phase speed. PSE in this case can be viewed as an approximate solution of the general matched asymptotic expansion procedure developed by Tam and coworkers (see Tam 1995) to predict Mach wave radiation from instability waves in slowly diverging flows. Once calibrated by an initial disturbance spectrum, these linear solutions are capable of describing the aft-angle directivity at low frequency (Balakumar 1998, Tam & Chen 1979, Zaitsev et al. 2009). Piot et al. (2006) compared PSE solutions to full solutions of the linearized Euler equations for the modes of a supersonic jet with generally good agreement. Finally, recent computations (Nichols & Lele 2011) show the power of global mode analysis at simultaneously predicting the coherent structures and their associated radiation under either subsonic or supersonic conditions.

3.5. Moving Forward

Renewed interest in stability-based approaches to jet noise has followed from advances in experimental diagnostics that permit a more extensive evaluation of models and faster computers that enable more complex and nonlinear ansatzes to be considered. For fully turbulent jets, it appears that stability models, even linear ones, offer sufficiently detailed agreement with experimental data to regard them as a promising avenue for further study. It is important to recognize that the success of linear analysis in this case does not suggest that the generation of turbulence in the jet is a linear process. Indeed, the (required) use of the turbulent mean flow as the basic state for these models shows that nonlinearity is essential. Future studies should clarify the scale-separation hypothesis and the role of nonlinear wave interactions directly and/or through the contribution of a modeled, frequency-dependent Reynolds stress. Also, as we discuss below, dynamical models (especially ones for control) will have to confront the intermittency of the large-scale coherent structures, rather than just predict the average fluctuations; it is not yet clear that a linear model will suffice.

Finally, only a few of the studies mentioned in this section have tackled the issue of the appropriate processing of turbulent experimental or numerical data in order to extract the coherent

motion for comparison with models. Some clues on the kinds of techniques that will be useful are given in the next section.

4. DYNAMICS AND CONTROL

4.1. Acoustic Distillation

By acoustic distillation, we mean a reduction of the complexity of the flow field to a minimal description that generates the same sound, the aim being to analyze the underlying mechanism and providing links to theory. The idea is to exploit modern capabilities in experimental measurement, data processing, and numerical simulation to identify the sound-producing motions without the artifice of an equivalent source model. In this pursuit, one wishes to go beyond two-point flow-acoustic correlations (Bogey & Bailly 2007, Juvé et al. 1980, Lee & Ribner 1972, Panda et al. 2005, Seiner & Reethof 1974, Tam et al. 2008), which provide limited insight because wave-packet structures are spatially extensive. Furthermore, a causal interpretation of such measurements requires a source model that can be represented by a (locally) measurable quantity such as the velocity fluctuation (e.g., the shear-noise component of Lighthill's source).

A step in this direction is provided by Hileman et al. (2005), who conditionally sampled high-speed images of condensation in an $M_j = 1.28$ jet based on far-field sound to produce visualizations of flow structures during loud and quiet time windows. High-amplitude sound radiation was associated with a rapid breakdown of wave-packet structures near the close of the potential core. This is consistent with high levels of flow-acoustic correlation observed in this region in the above studies, as well as other experimental (Morrison & McLaughlin 1979) and numerical (Bogey et al. 2003, Cavalieri et al. 2011a, Freund 2001) studies that also highlight the dynamics near the close of the potential core as an important noise source.

Extensions of POD have been developed with an aim toward weighting the acoustically efficient fluctuations more heavily in the averaging. Freund & Colonius (2009) applied POD to DNS data from an $M = 0.9$ jet. When pressure data were used (with the standard L2 norm), the most energetic modes had a clear wave-packet structure. However, a large number of such modes were required to resolve a significant fraction of the far-field data. An alternative inner product that gave higher weight to the far-field data was defined to resolve the far field with fewer modes. This also produced wave-packet structures, but they showed a more rapid decay near the close of the potential core. Similar acoustically optimized POD modes have been computed using different approaches (Kerhervé et al. 2012b, Schlegel et al. 2012).

Kerhervé et al. (2012b) used LSE based on multipoint flow-acoustic correlations to educe, from an LES database for an $M_j = 0.9$ jet (Bogey & Bailly 2006), the conditional space-time velocity and pressure fields associated with low-angle sound radiation. The educed field, displayed in **Figure 9**, comprises an axially extensive, intermittent wave packet whose amplification rate and radial structure closely match those obtained in quasi-parallel linear stability analysis (based on the mean flow field) over a range of low frequencies and axial locations. This study is a compelling demonstration that wave packets underpin low-angle sound emission from turbulent jets and furthermore that their intermittent temporal dynamics seems to be a key aspect of their radiation efficiency. A simplified source model (Cavalieri et al. 2011b), based on a fitted wave packet whose amplitude and spatial extent are modulated in accordance with a short-time Fourier transform analysis of the educed field (**Figure 9a**, panels iii and iv), produced aft-angle radiation in good agreement with the LES. Tinney et al. (2008a) used a similar methodology to educe coherent motions from time-resolved PIV data based on near-field pressure measurements near the nozzle exit (rather than the aft-angle far field).

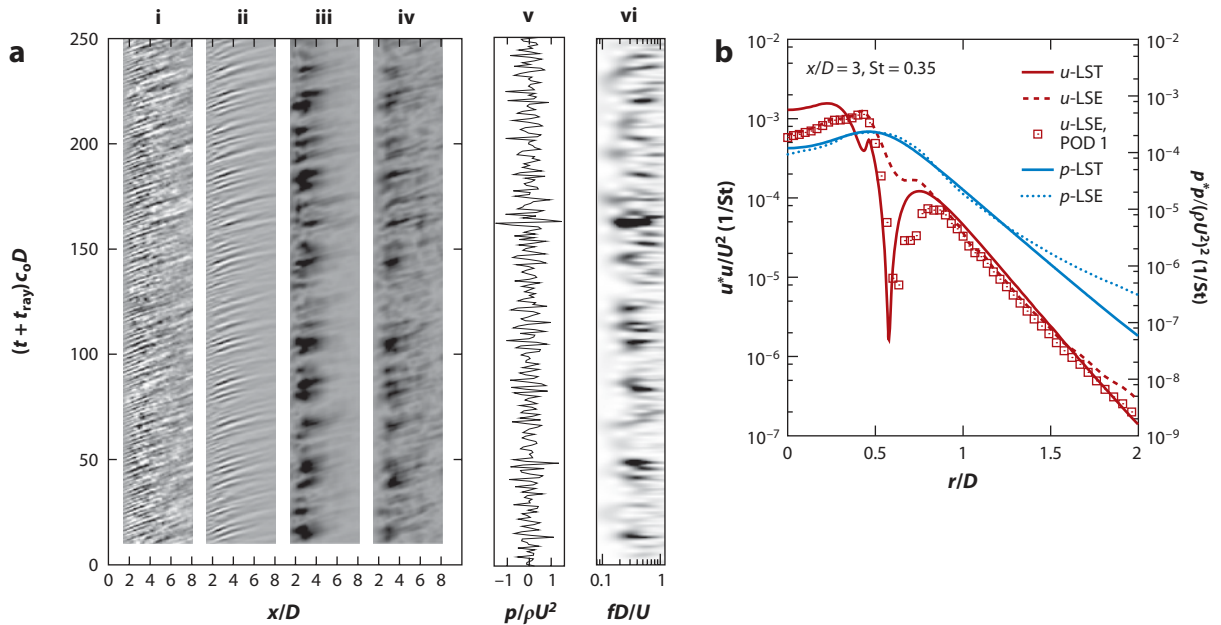


Figure 9

(a) Time histories of (i) full large-eddy simulation (LES) pressure on the jet centerline, (ii) educed (acoustically distilled) pressure on centerline, (iii) short-time Fourier transform of educed pressure (showing time-dependent wave-packet envelope), (iv) short-time Fourier transform of radially integrated linear source term (computed using distilled velocity field), (v) time-delayed acoustic far field at $\phi = 30^\circ$, and (vi) wavelet transform of panel v. (b) Comparison of the educed velocity [and proper orthogonal decomposition (POD)-filtered educed velocity] and pressure at $x/D = 3$ with eigenfunctions from parallel linear theory based on the LES mean flow field. Abbreviation: LSE, linear stochastic estimation.

4.2. Intermittency: Loud and Quiet Flows

Wave packets are observed instantaneously, but they are intermittent: “Three or four puffs form and induct themselves downstream, an interval of confused flow ensues, several more puffs form, and so on” (Crow & Champagne 1971). This wave-packet behavior (**Figure 9**) is correlated with temporally localized bursts of sound in the far field (Cavalieri et al. 2011a, Guj et al. 2003, Hileman et al. 2005, Juvé et al. 1980, Kerhervé et al. 2012b, Koenig et al. 2012). Recent optimal-control studies provide further confirmation. In their simulations of two-dimensional mixing layers, Wei & Freund (2006) used adjoint-based optimal-control theory to minimize the far-field sound by adjusting the spatiotemporal behavior of an imposed body force in the initial region of the mixing layer. They showed how the wave packets in the controlled flow are more regular, and thus quieter, than their uncontrolled counterparts. **Figure 5** demonstrates the basic mechanism at play. Cavalieri et al. (2010) analyzed the same data and showed that the optimizer achieved its order-of-magnitude reduction in the radiated sound by eliminating a particular vortex merger associated with a high-amplitude acoustic emission. Early results show that the application of optimal control to three-dimensional LES of turbulent jets may be successful via the same mechanism educed for the two-dimensional mixing layer (Freund 2011, Kim et al. 2011). Sandham et al. (2006) and Cavalieri et al. (2011b) provided additional model problems that show how the temporal modulation of wave packets increases their acoustic efficiency, particularly at high subsonic Mach numbers.

The temporal behavior of the wave packets described here is not necessarily incompatible with the (linear) frequency-space models based on stability theory. The question, not yet resolved, is whether a deterministic, nonlinear mechanism underpins the intermittency, or whether it is a consequence of the stochastic broadband inlet perturbations. For example, even in a linear context, two waves of similar frequencies can occasionally be excited with a phase that produces either mutual reinforcement or cancellation.

4.3. From Statistics to Dynamics

The modeling challenge presented by intermittent wave-packet behavior is considerable. Although the favorable comparisons with linear PSE (Gudmundsson & Colonius 2011) show that linear models are useful for a time-averaged rendering of a wave packet, this does not imply that the deterministic, time-local processes are linear. A time-domain evolution operator would, in any event, be more useful for real-time sensing and control.

There are two essential elements in the real-time reduced-order models discussed in the literature: a suitably truncated basis and an ansatz for the dynamics. The basis could be eigenfunctions from stability theory, POD modes from data, or other empirical modes. Projection of the governing equations onto this basis and truncation of the resulting system of ordinary differential equations give a reduced-order model. Application of the POD/Galerkin (orthogonal projection) combination (Holmes et al. 1998) can lead to fragile or even unstable models, but there are now many variants being developed in the flow control community to overcome these limitations (see, e.g., Noack et al. 2011).

Sinha et al. (2010) developed a 35-dimensional, stable, reduced-order model for the jet flow structures (without regard to optimization for acoustics) derived from experimental data of transonic jets (see also Sinha et al. 2011b). In addition to modeling the unforced jet, their model also incorporates forcing with localized plasma actuators. In a parallel effort, the reduced-order model was validated for real-time estimation (Kalman filter) needed for feedback control applications (Sinha et al. 2011a).

Kerhervé et al. (2012a) used the acoustically optimized POD modes discussed above to develop a 20 degree-of-freedom reduced-order model, which was found to closely reproduce the intermittent time evolution of the wave packets. Furthermore, when used with the line-source model of Cavalieri et al. (2011b), the model produced aft-angle radiation similar to that of the LES. Schlegel et al. (2009) have used a similar strategy, involving acoustically optimized modes, in which actuation was included in the reduced-order model to demonstrate the potential for model-based noise reduction.

4.4. Control of Jet Noise

Attempts to reduce jet noise by steady alteration of the flow field include nozzle modifications such as tabs and chevrons (e.g., Zaman et al. 2011), noncircular nozzles (e.g., Gutmark & Grinstein 1999), and steady fluidic injection (e.g., Henderson 2010). We suggest that the success of these studies has been limited by the lack of a conceptual framework that connects flow perturbations near the nozzle exit to the far field.

As elsewhere in the world of flow control, attention has lately shifted toward unsteady perturbations, such as those produced by fluidic actuators (Koenig 2011, Low et al. 2010, Maury et al. 2011), plasma discharge actuators (Samimy et al. 2007), and piezoelectric flappers (Butler & Calkins 2003). Kearney-Fischer et al. (2011) employed pulsed plasma actuators to force subsonic and supersonic jets at frequencies up to approximately $St = 3$. By phasing eight actuators around the periphery of the nozzle, they excited (approximations of) azimuthal modes up to $m = 3$. The

most effective noise reductions generally occurred for the higher-frequency and higher-azimuthal mode forcing.

The wave-packet dynamics discussed in the previous section may yet provide the framework necessary to understand the basic mechanisms at play in noise control. One possibility is that the initial spectrum of disturbances near the nozzle exit is modified, for example, such that energy in the azimuthal spectrum is shifted away from the acoustically efficient low-order modes. Another possibility is that small-scale or high-frequency perturbations in the near-nozzle region lead to Reynolds stresses that spread the mean flow faster, thereby reducing the amplification of wave packets.

For example, Gudmundsson (2010) showed that the mean flow field of chevron jets led to substantial reductions in the amplification and phase speed of these low-order azimuthal modes compared to round nozzles at similar conditions. An analysis confirming this mechanism of noise reduction was developed by Koenig (2011) based on experiments of steady fluidic injection from a spinning center body that primarily excites disturbances with $m = 2$ (Koenig et al. 2011). They considered actuator frequencies for which the baseline mean flow was found to be either stable or unstable according to a parallel stability analysis for $m = 2$. When the jet was driven at unstable frequencies, noise was increased; actuation led to enhancement of the coherent part of the Reynolds stress (computed via the triple decomposition), and quadratic wave-packet interactions were evident from a bicoherence analysis. When the flow was driven at stable frequencies, conversely, noise reduction was observed. Moreover, the incoherent part of the Reynolds stresses was found to be enhanced (turbulent response of the flow to the excitation of stable frequencies) by actuation, leading to a faster spreading mean flow, and consequently a lower amplification of mode $m = 0$. The attenuation of the $m = 0$ mode was confirmed in velocity measurements, corroborating the hypothesized mechanism for noise reduction.

SUMMARY POINTS

1. Coherent structures in the form of wave packets have been observed using a variety of measurement techniques in both natural and forced jets. These structures, which appear in flow visualizations as a train of puffs, are spatiotemporally coherent over distances far exceeding the integral scales of the turbulence, with slowly evolving wavelength and phase speed.
2. Depending on the magnitude of the convection velocity, compared to the ambient sound speed, the wave-packet structures are associated with a predominantly evanescent near-pressure field (subsonic) or Mach wave radiation (supersonic).
3. In natural jets, the wave packets constitute a relatively small fraction of the disturbance energy but are nonetheless acoustically important, on account of their high space-time coherence and intermittency. Indeed, wave packets dominate the low-frequency (at least up to $St = 1$), aft-angle radiation. (Their contribution to sideline radiation remains to be quantified.)
4. In unforced, turbulent jets, the measured wave-packet characteristics agree well with predictions of linearized stability analyses of the turbulent mean flow field, at least up to the close of the potential core, and possibly beyond. In forced jets and in jets with initially laminar boundary layers, nonlinear interactions of wave packets are significant.
5. Recent studies suggest that the wave-packet solution to the jet-noise puzzle may enable noise-reduction mechanisms to be developed and explained in a self-consistent manner.

FUTURE ISSUES

1. Rich databases provided by simulations and advanced measurement technology, particularly real-time PIV and simultaneous flow-acoustic observation, will enable a more complete identification of wave-packet structure, dynamics, and acoustic relevance. Improved diagnostics and novel postprocessing techniques are needed to fully exploit these advances.
2. Direct measurement and simulation of the upstream flow and near-nozzle dynamics are needed to characterize (among other things) the sensitivity of wave-packet evolution in the near and far field to inlet perturbations.
3. Real-time modeling and control are in their infancy; efforts should be guided as much as possible by theory and historical observations if substantial noise reduction is to be achieved.
4. Direct jet-noise prediction models based on linear and nonlinear disturbance evolution equations, and low-order dynamical models informed by acoustically efficient empirical modes, would aid in control efforts and provide closure on a complete theory of wave packets.

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The authors are not aware of any biases that might be perceived as affecting the objectivity of this review.

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