CHAPTER 23

Wave Refraction Theory in a Convergence Zone

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ABSTRACT

An experimental investigation utilizing nonbreaking waves and one bottom topography with parallel circular contours symmetric about the center of the basin was performed to assess the limits of applicability of linear wave refraction theory in a convergence zone. Analytical computations of reflections from underwater topographic variations and of viscous dissipation of energy at the bottom agreed with experimental measurements. It was vividly illustrated that a significant amount of energy was involved in diffraction along the wave crest, thus the necessity of developing a theory to include the stepwise computation of diffraction processes simultaneously with the computation of wave refraction is now evident. Nonlinear transfer of energy from lower to higher frequency components was investigated by performing a harmonic analysis over one wave period. As expected, nonlinear effects increased with both wave period and wave height (for a given wave period). Only 20 percent of the energy remained in the fundamental component for the longest period largest amplitude wave tested. Higher frequency components ranged from being completely coupled to the fundamental frequency to almost completely uncoupled. It was clearly demonstrated that in some circumstances the consideration of nonlinear effects becomes extremely important.

INTRODUCTION

The objective of this investigation is to ascertain, as a function of the incident wave characteristics for a given bottom topography, the error incurred by utilizing the usual linear wave refraction theory in a convergence zone with energy conservation between orthogonals and without reflection or nonlinear effects. This study is limited to nonbreaking waves and one bottom topography; however, the variation of wave period and height allows for a wide range of refraction coefficients and relative steepness. The deviation of the experimental data from the usual theory is attributed to the effects reflection, diffraction, and nonlinearities according to their respective magnitudes.

One of the practical problems confronting the (coastal) physical oceanographer and design engineer is to determine the expected wave environment in an area of proposed coastal construction. Occasionally one must cope with the problem of determining a design wave for a structure in an area where a strong convergence of wave orthogonals can occur. The present method utilized is to compute a linear wave refraction coefficient assuming no reflection from underwater topographic variations, no diffraction of energy along the wave crest, and certainly no consideration of nonlinear effects. This investigation was initiated in order to obtain an experimental indication of the relative importance of these phenomena in more realistic theories of wave refraction.

The economic impact of the topics under investigation cannot be overemphasized. Each phenomenon affecting the refraction coefficient (diffraction, reflection, and nonlinear effects) tends to decrease the design wave for a particular structure (with some slight reservation relative to the nonlinear effects). Nonlinearities tend to peak the wave crest and redistribute the energy through the mechanism of energy transfer to higher frequency components. The data and analyses which follow indicate conclusively that a much better understanding of the dynamics of wave propagation in a strong convergence zone of wave orthogonals is necessary before analytical computations of the wave environment can be made with confidence.

MODEL DESIGN

The bottom topography utilized in the model consisted of circular bottom contours of the same radius, the center of the circles lying along the centerline of the test basin as illustrated in Figure 1. A crushed rock wave absorber with slope of 1:14 and a layer of rubberized horsehair were installed at the end of the model to eliminate reflections back into the measurement area.

Computations of the wave reflection coefficient from the sloping topography in the model were made from three different theories. The long wave theory of Cochran and Arthur (for which the Lamb theory is a special case for normal incidence) utilizing a vertical step with depths of 0.1524 m (0.5 ft) and 0.4572 m (1.5 ft) resulted in a reflection coefficient of 0.268 which is certainly too large. The Ogawa and Yoshida theory which also employs the long wave assumption was solved for the case of a weak reflection with a constant sloping bottom. The first order reflection coefficient, R_{r_1}, is 0.0045 (T = 1 sec), 0.0197 (T = 2 sec), and 0.0089 (T = 3 sec). Figure 2 illustrates the reflection coefficient as a function of wave period. The oscillatory behavior of the Ogawa and Yoshida theory appears to be a result of attempting to apply the theory where the basic assumption of a long wave is not satisfied. The primary point illustrated by the Ogawa and Yoshida theory is that reflections from the underwater topographic variations are indeed small (less than 0.02).

The most sophisticated linear reflection theory available for underwater topographic variations is due to Rosseau (1952). This theory provides an analytical solution which makes no assumption on the wave form other than linear small amplitude progressive waves emanating from $+\infty$ and the bottom profile must be described by a specified function which approaches a depth h_2 as $y \to -\infty$ and h_1 as $y \to +\infty$. This theory is quite complicated requiring the fitting of a bottom contour to model topography. Details of the application of this theory can be found in Rosseau (1952), Webb (1965), and Whalin (1971).

The reflection coefficient from the Rosseau theory is given by

$$R_r = \frac{\tanh \pi\lambda_1 - \tanh \pi\lambda_2}{\tanh \pi\lambda_1 + \tanh \pi\lambda_2}, \quad (1)$$

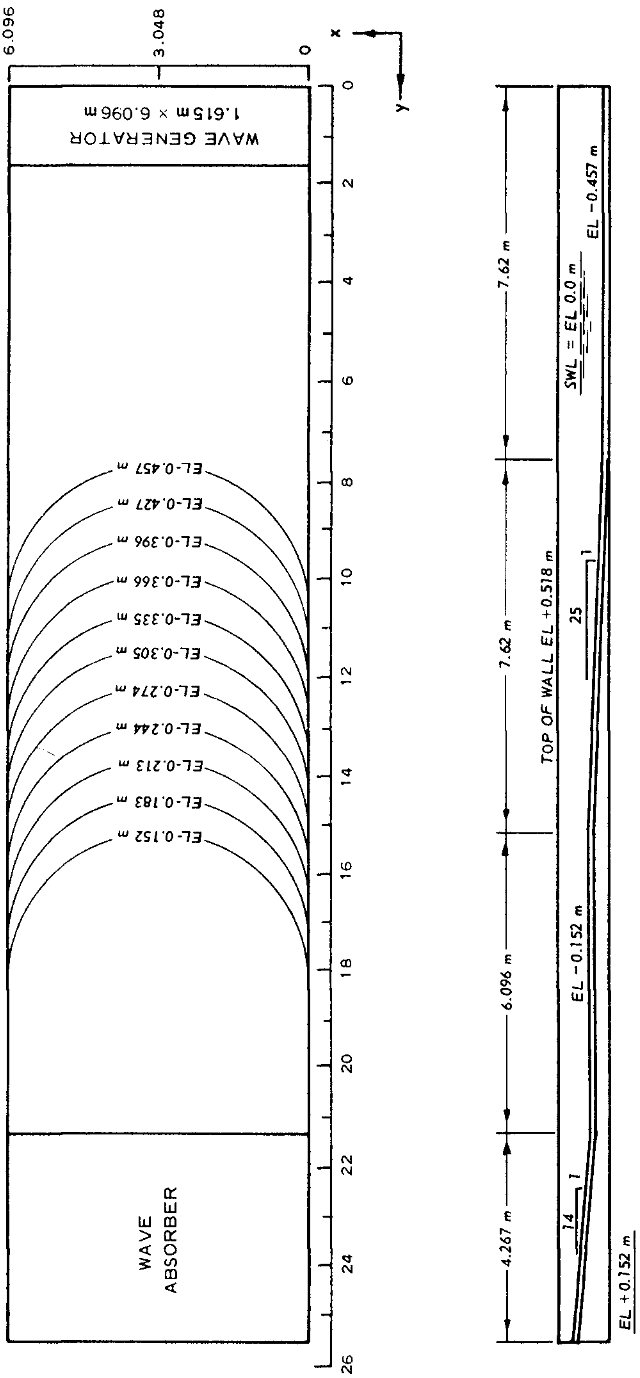

Figure 1. Model Topography

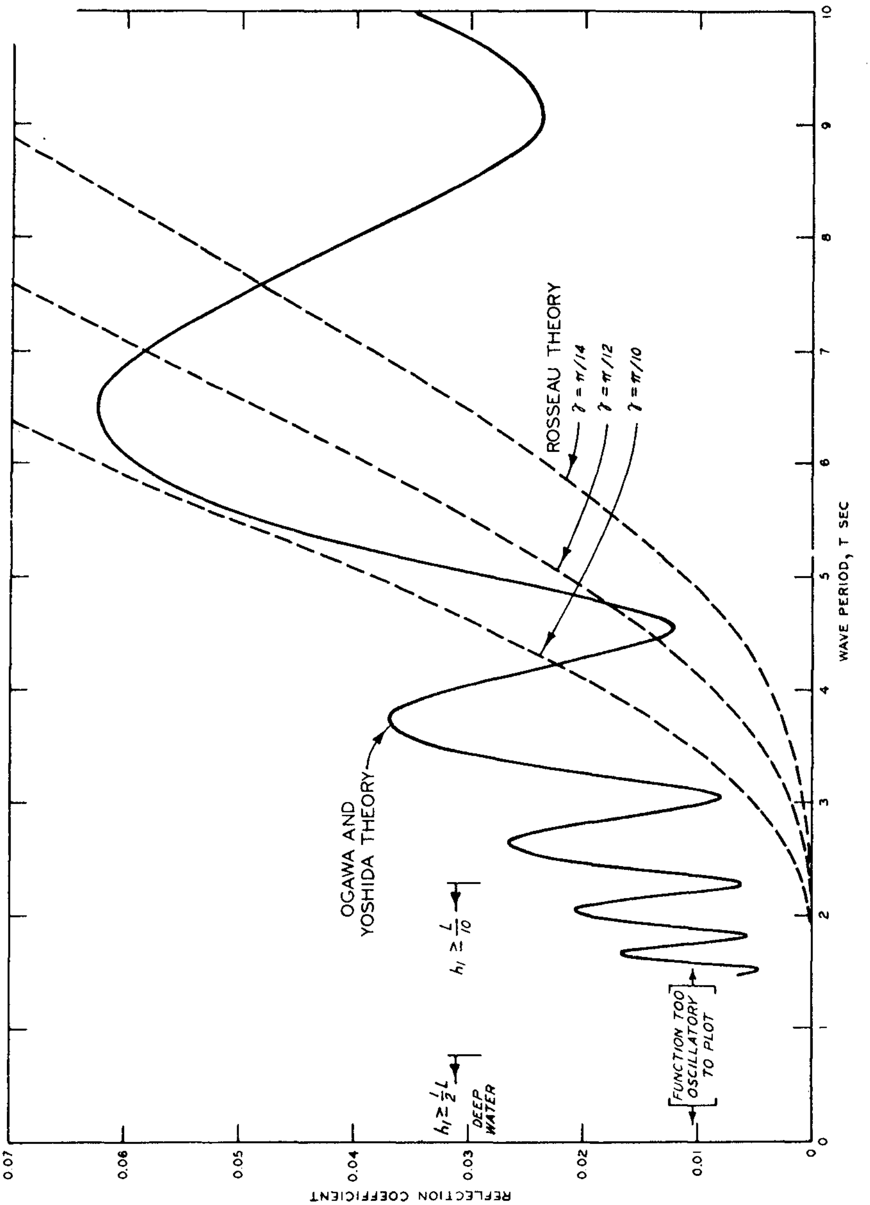

Figure 2. A Comparison of Rosseau and Ogawa and Yoshida Reflection Coefficients, T<10 sec

where λ_1 and λ_2 are solutions of

$$\lambda_i \tanh \gamma\lambda_i - \frac{\omega^2 h_i}{g\gamma} = 0; \; i = 1,2; \; \gamma = \pi/N \qquad (2)$$

In order to compare the Ogawa and Yoshida (1959) reflection theory with the Rosseau (1952) theory, a Rosseau bottom contour was selected to obtain the best fit to the centerline of the wave basin. Figure 3 illustrates the best fit which was obtained for $\gamma \doteq \pi/14$ (N = 14); however, the curve for $\gamma = \pi/12$ also is shown. The reflection coefficient from the Rosseau (1952) theory is obtained from (1) as a function wave period upon solution of (2) for λ_1 and λ_2. Figure 2 shows a comparison of the reflection coefficient from the Rosseau (1952) theory and the Ogawa and Yoshida (1959) theory for three values of γ. The oscillatory behavior of R_{r_1} exhibited by the Ogawa and Yoshida (1959) theory is absent from the Rosseau (1952) theory. This is interpreted as being a consequence of attempting to apply the theory of Ogawa and Yoshida (1959) to a situation where the basic assumption (a long wave) is not valid. The reflection coefficients computed from the Rosseau (1952) theory (Figure 2) were 2.5×10^{-11}, 1.3×10^{-5}, and 6.1×10^{-4} for wave periods of 1, 2, and 3 sec, respectively, and $\gamma = \pi/14$. Figure 2 illustrates the sensitivity of the reflection coefficient to the parameter γ. Both reflection theories approach the Lamb (1932) value for normal incidence as $T \to \infty$.

A theory has been developed by Keulegan (1950) to compute the wave damping due to viscous dissipation of energy. This theory applies to slowly damped periodic progressive water waves in the absence of refraction, diffraction, or shoaling. Computations indicated that the wave height attenuation over the entire measurement area was between 0.01 and 0.03 which was considered sufficiently small to be neglected.

A 20-channel wave-height measuring system was used to secure data at 60 points in the wave basin for the basic test series and at 40 other points for an alternate test series. Actual gage locations are shown in Figure 4. The wave generator spanned the width of the test basin with approximately a 1 cm clearance at each of the sidewalls. The generator moves in simple harmonic motion in the vertical. Data were acquired at the 60 basic wave gage locations shown in Figure 4 for wave periods of 1, 2, and 3 sec and for three wave amplitudes for each period. These data formed the basic set of measurements from which the refraction coefficients were computed and compared with linear theory. In order to ascertain if there were any measurable reflections from either the bottom topography or the end of the test basin, 40 wave gages were installed at the alternate positions shown in Figure 4. Since only 20 wave gages were available, the locations delineated by A, B, and C (for the basic test series) and A and B (for the alternate test series) represent positions to which the gages were moved and the same series of tests repeated. Two permanent monitor gages were set in the positions denoted on Figure 4 (M-1 and M-2) and were not moved during the entire test series in order to eliminate any errors in resetting the wave generator stroke for the alternate gage locations.

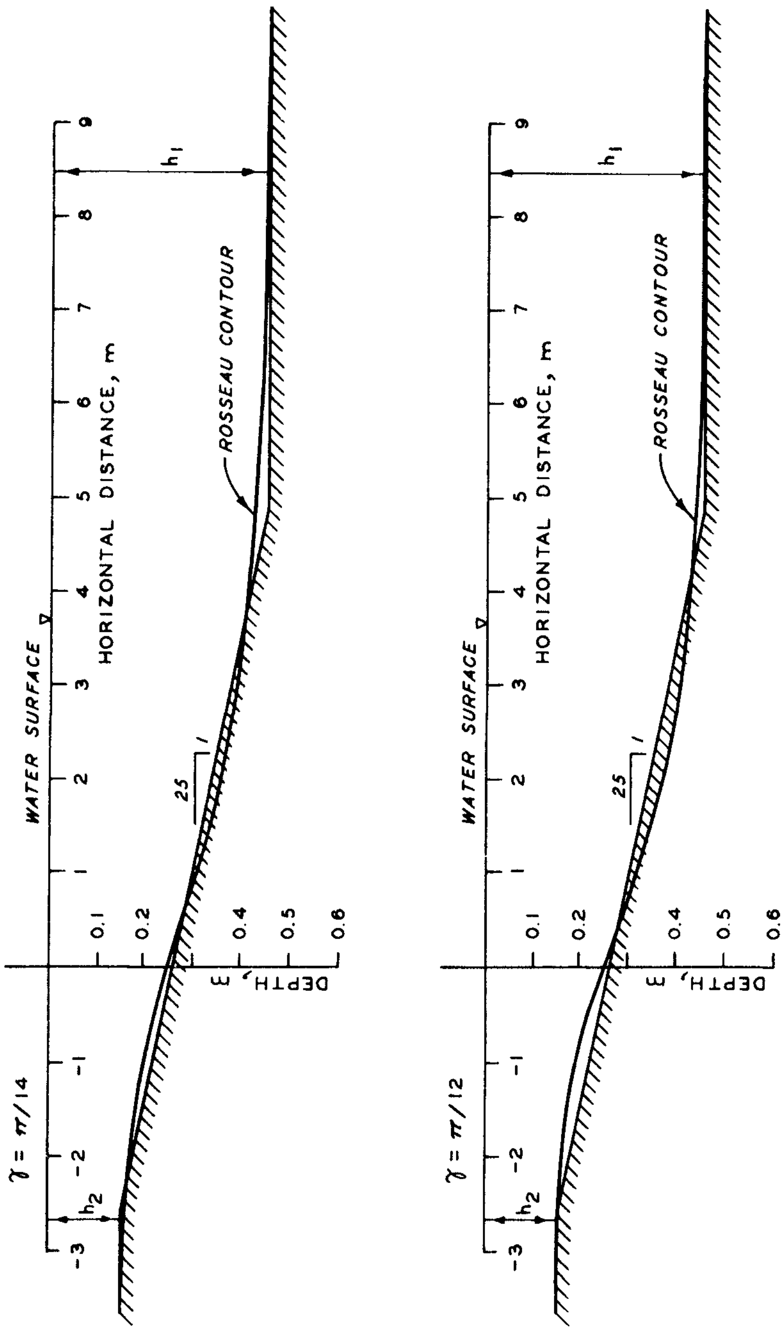

Figure 3. Rosseau Contours for Topography Along the Centerline of the Wave Basin

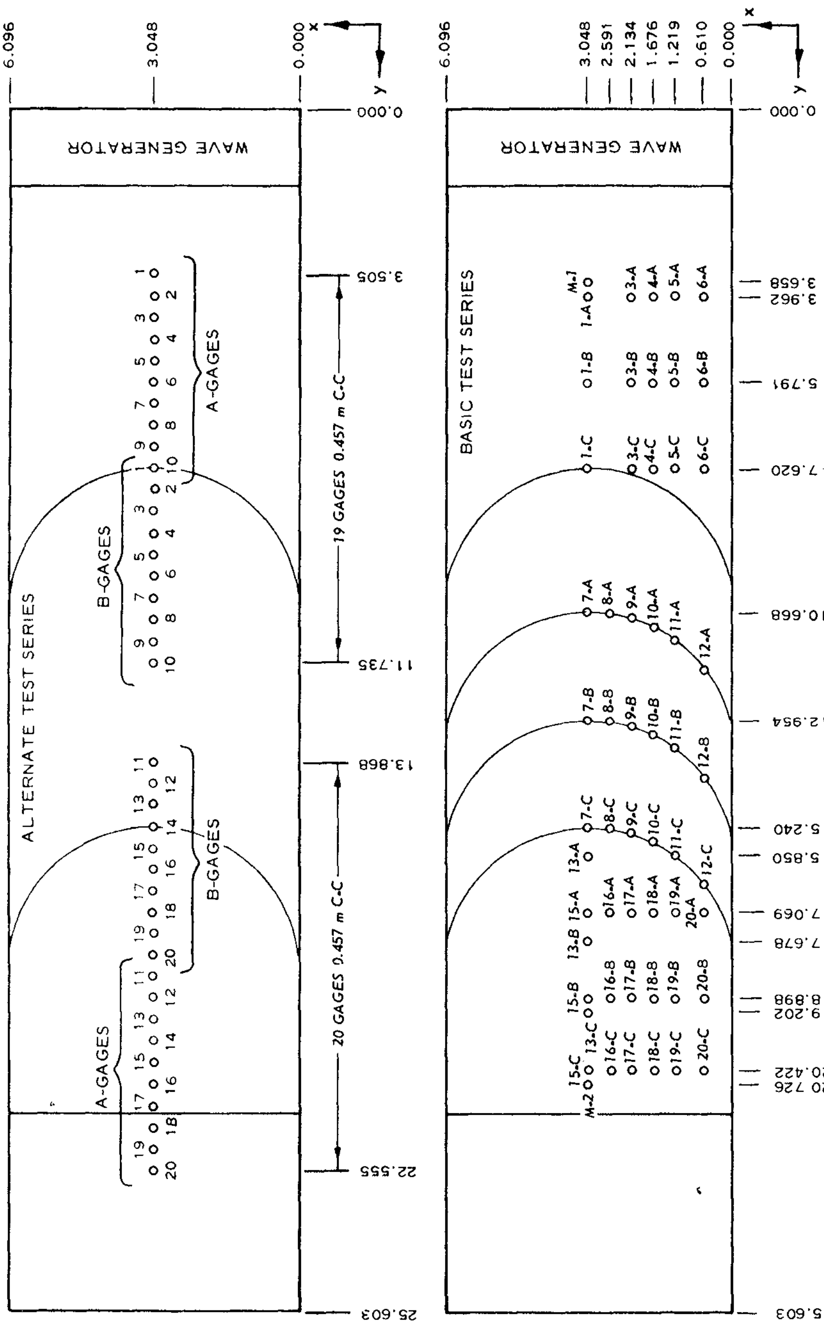

Figure 4. Wave Gage Locations

DATA ANALYSIS

The analysis of these data is predicated on the assumption that the total energy or wave power is equally divided between potential and kinetic energy as is the case for the Airy wave theory. The departure from an equal partitioning of kinetic and potential energy is small and the effect of this departure on the analysis will be even smaller. The important point relative to the data analysis is that the same ratio between potential and kinetic energy be preserved over the measurement area.

One of the primary assumptions of the linear wave refraction theory being investigated is that no energy is diffracted laterally along the wave crest; that is, for steady conditions of wave generation the same wave power exists at all positions between any two orthogonals. Moreover, since the property of interest is the energy transmitted per unit time one must deal with the group velocity rather than the phase velocity. The actual quantity to be computed from the data is the energy transmitted per unit crest width over one wave period; the units of this quantity being energy per unit width (joule m^{-1}). The potential energy per unit crest width over one wave length is given by

$$\frac{PE}{W} = \frac{\rho g}{2} \int_0^L \eta_1^2 \, (y,t) \, dy \qquad (3)$$

where W is the width along the wave crest, ρ the water density, g the acceleration due to gravity, η_1 the water level anomaly above equilibrium level, and L the wave length. After appropriate transformation of equation (3) to correspond to a wave record obtained at a fixed point, a computer program was written to perform the integration given a number of values η at equally spaced intervals over one wave period.

In addition the program also performed a harmonic analysis of the wave form to determine the amplitude and phase of the fundamental (that corresponding to the period set on the wave generator) and the higher order harmonics. These data were then utilized to interpret the magnitude of the nonlinear effects during propagation of the waves over the topography in terms of the amount of deviation of the generated wave form from purely sinusoidal. The output of the harmonic analysis routine (the mean adjusted to zero) was expressed in the form

$$\eta_1(t) = \sum_{k=1}^{n} (a_k \cos k\omega t + b_k \sin k\omega t) \qquad (4)$$

or

$$\eta_1(t) = \sum_{k=1}^{n} c_k \cos (k\omega t - \theta_k) \qquad (5)$$

where

$$c_k = \left[a_k^2 + b_k^2\right]^{\frac{1}{2}} \text{ and } \theta_k = \tan^{-1} \frac{b_k}{a_k} \quad (6)$$

For k = 1, the wave crest occurs at $\omega t_o = \theta_1$. Letting $t' = t - t_o$

$$k\omega t - \theta_k = k\omega t' + \theta_k' \quad (7)$$

where $\theta_k' = k\theta_1 - \theta_k$ represents the phase lead of the higher order harmonics relative to the phase of fundamental (k = 1) frequency. If θ_k' is positive, the higher order harmonics are leading the fundamental; if negative, they are lagging the fundamental. If θ_k' increases as a function of propagation distance, the higher order harmonics are catching up with or are propagating faster than the fundamental (a situation for which there is no easy explanation) and if θ_k' decreases as a function of propagation distance, the higher order harmonics are continually lagging the fundamental.

If θ_k' remains constant, this can be interpreted as a verification that the higher order frequencies are completely coupled to the fundamental frequency and hence the wave acts as a Stokes type of nonlinear wave.

In performing the analytical computation of the refraction coefficient from linear theory, expressions are needed for the phase velocity, its first and second order derivatives with respect to x and y, the bottom topography, and its first and second derivatives with respect to x and y. The equation of the sloping portion of the bottom topography is given by

$$h(x,y) = h_1 + \frac{1}{25}\left\{y_o + \frac{1}{2}w - y - \left[x(w - x)\right]^{\frac{1}{2}}\right\} \quad (8)$$

where y is measured along the wave basin, x across the wave basin, y_o is the coordinate of the first circular bottom contour along the centerline of the wave basin, h_1 is depth in the deep portion of the basin (0.4572 m) and w is the width of the basin (6.096 m). Note that any line parallel to the y axis has a slope of 1:25 over the variable portion of the bottom. This expression for the bottom topography makes it possible analytically to compute the phase velocity and its derivatives at any location (x,y) in the basin.

Figure 5 illustrates the computed and measured refraction coefficient for a wave period of 3 sec. Similar comparisons for periods of 1 and 2 sec are given in Whalin (1971). The dotted lines represent a caustic. The computed refraction coefficient is merely the square root of the ratio of the potential energy transmitted per unit crest width over one wave period at any particular wave gage to the average of the potential energy transmitted per unit crest width over one wave period at either gages 1A to 6A, 1B to 6B, or 1C to 6C depending upon whether A, B, or C gage is being analyzed (remembering that A, B, and C locations represent locations to which the gages were moved and the same test repeated).

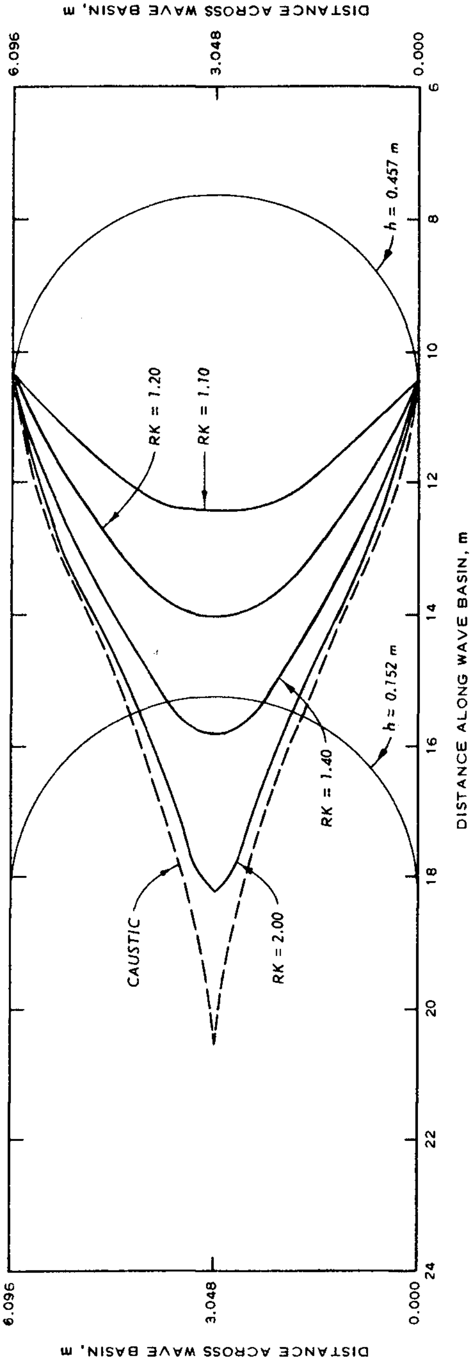

Figure 5. Theoretical Refraction Pattern, T = 3 sec

A word of caution should be interjected at this point relevant to the interpretation of the computed refraction coefficients. The theoretical refraction coefficient implies energy conservation between orthogonals, a condition which is in the process of being investigated. On the other hand, what we are computing from the data is merely the square root of the ratio of the energy transmitted since, as will be illustrated shortly, a significant amount of energy has been diffracted. In short, a refraction coefficient alone is insufficient in this case except in areas on the sloping portion of the topography where little diffraction has occurred. More properly one should have a refraction coefficient, a diffraction coefficient, a friction coefficient, a shoaling coefficient, and a reflection coefficient.

Caution must be exercised in analyzing the amount of transmitted energy involved in diffraction processes. However, one very meaningful estimate of this quantity can be obtained from the three gage alignments in the shallow portion of the wave basin. Gages 15-20A, 15-20B, and 15-20C can be analyzed to ascertain the amount of diffraction of transmitted potential energy (or total energy transmitted through the assumption that transmitted energy is equally partitioned between kinetic and potential energy). Since the caustic boundary intersects each of these gage alignments, all potential energy transmitted outside the caustic line must be diffracted with the exception of the energy refracted from the opposite side of the centerline of the wave basin. Therefore, a computation of the percent energy measured outside the caustic line minus the theoretical amount refracted into this region from the opposite side of the wave basin was made for each condition tested and for each gage alignment. Figure 6 illustrates the distribution of measured potential energy transmitted per unit width relative to the distribution obtained by linear wave refraction theory. The percent energy diffracted represents 100 times the ratio of the intergrated potential energy (from the caustic to the edge of the basin) transmitted per unit width minus the amount refracted to the left of the caustic from the opposite side of the wave basin to the total potential energy transmitted (integrated from the centerline to the edge of the basin). Table 1 summarizes the data for all test conditions.

Table 1

Percent of Transmitted Energy Diffracted Over One Wave Period

Period sec	S cm	15-20A %	15-20B %	15-20C %
3	15.24	59.58	84.73	48.20
	10.16	59.98	85.38	48.22
	7.62	63.16	83.98	48.21
2	10.16	44.84	80.57	70.87
	7.62	44.51	79.06	69.57
	5.08	46.83	75.24	68.68
1	5.08	26.20	36.04	54.24
	2.54	24.84	37.48	54.87
	1.27	22.49	26.95	32.88

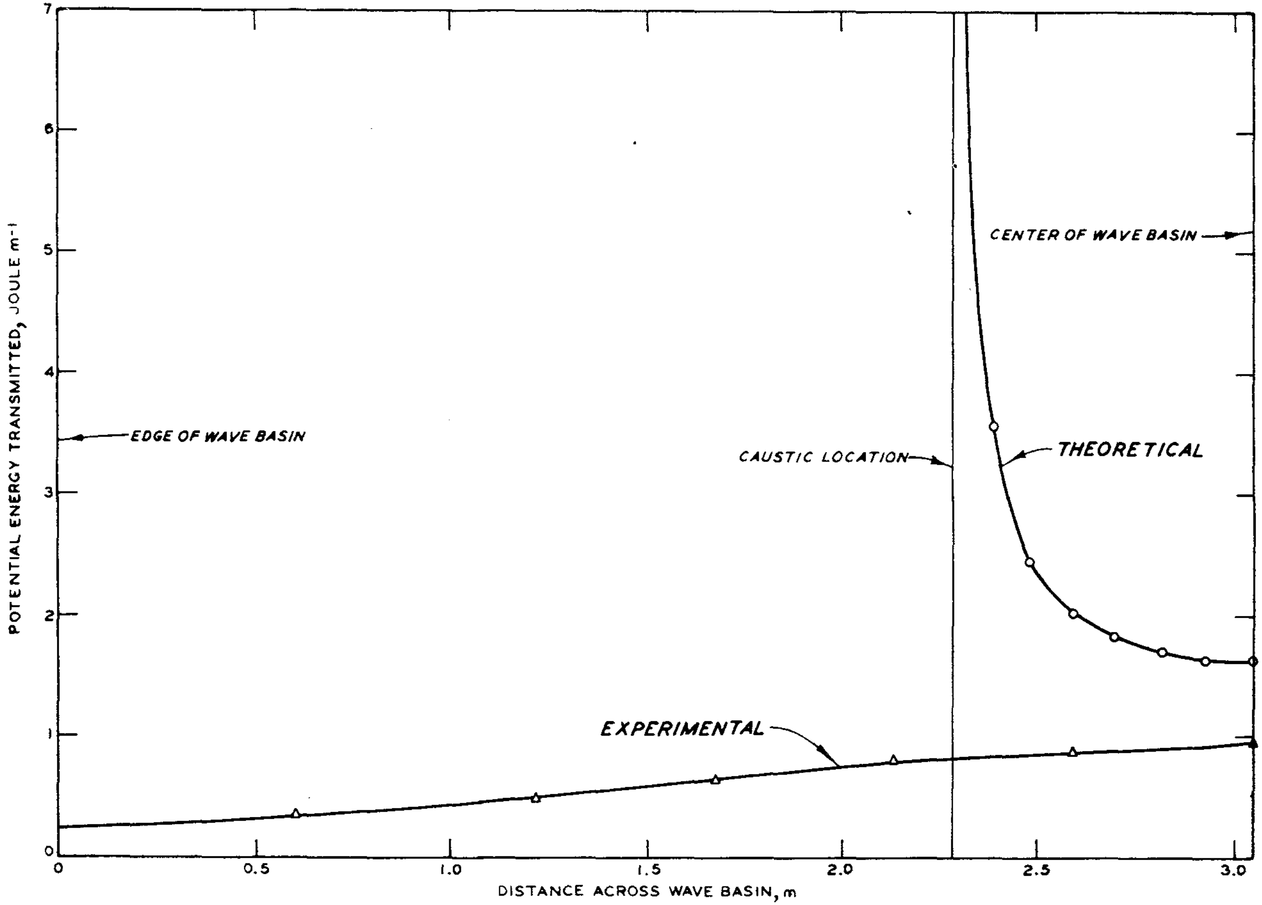

Figure 6. Diffraction of Transmitted Potential Energy, T = 3 sec, S = 7.62 cm, Gages 15-20A

The lower values in the right hand column of Table 1 (gage alignment 15-20C) for the 2- and 3-sec waves arise because there is a significant amount of energy refracted across the caustic from the opposite side of the centerline of the wave basin. Gage alignment 15-20C for wave periods of 2 and 3 sec was the only situation where a significant (greater than 2 percent) amount of energy was refracted (according to the linear theory) across the centerline into the shadow zone. It is obvious that diffraction of energy must be considered in order to obtain an accurate estimate of the wave height; the assumption of energy conservation between orthogonals is certainly not valid at any location in the shallow end of the wave basin.

Nonlinear effects were analyzed by performing a harmonic analysis over one period of all data acquired during the test series. It was observed during the course of data acquisition that the wave form changed quite rapidly along the centerline of the wave basin at the top of the sloping portion of the topography. Over a propagation distance of less than 2 m, the wave form became very steep at the crest with a long trough. Therefore, an alternate series of tests were designed with two purposes in mind, one to verify the computation of a negligible wave reflection coefficient and more importantly, to investigate (through a harmonic analysis) the nonlinear effects occurring, especially along the centerline of the wave basin. Gage locations for this test series are illustrated in Figure 4 (alternate test series).

Figure 7 shows the amplitude of the first three components from the harmonic analysis. Data for the other test conditions can be found in Whalin (1971). The amplitudes are represented by c_1, c_2, and c_3 and correspond to frequencies of ω, 2ω, and 3ω where ω is the frequency of the fundamental component. The limits of the sloping topography are shown in each figure and it is easily seen that a considerable amount of energy is transferred to the higher frequency components. Figure 8 shows the percent of potential energy transmitted by the fundamental frequency component, c_1 as a function of distance along the centerline of the wave basin for the 3-sec waves. The total energy per unit width available to be transmitted (along the centerline) is of course increasing with distance due to refraction. Relative to the 3-sec data (Figure 8) the ratio of energy transfer to the higher frequency components always seems to be greater than the rate of increase due to refraction. The rate of energy loss by diffraction in the fundamental frequency component must also be considered, hence the interpretation is not so simple. In order for the percent energy in the fundamental frequency component to increase, the rate of transmitted energy increase (per unit width) by refraction must be greater than the rate of transmitted energy loss (per unit width) by diffraction and nonlinear transfer to higher frequency components.

A further insight into the nonlinear processes occurring is gained from Figure 9 which shows the relative phase (θ_2^1 and θ_3^1) of the 2ω and 3ω frequency components for the 3-sec wave. These relative phases are defined by (7).

The trend was essentially identical for all conditions tested, the phase of the 2ω and 3ω frequency components relative to the ω component

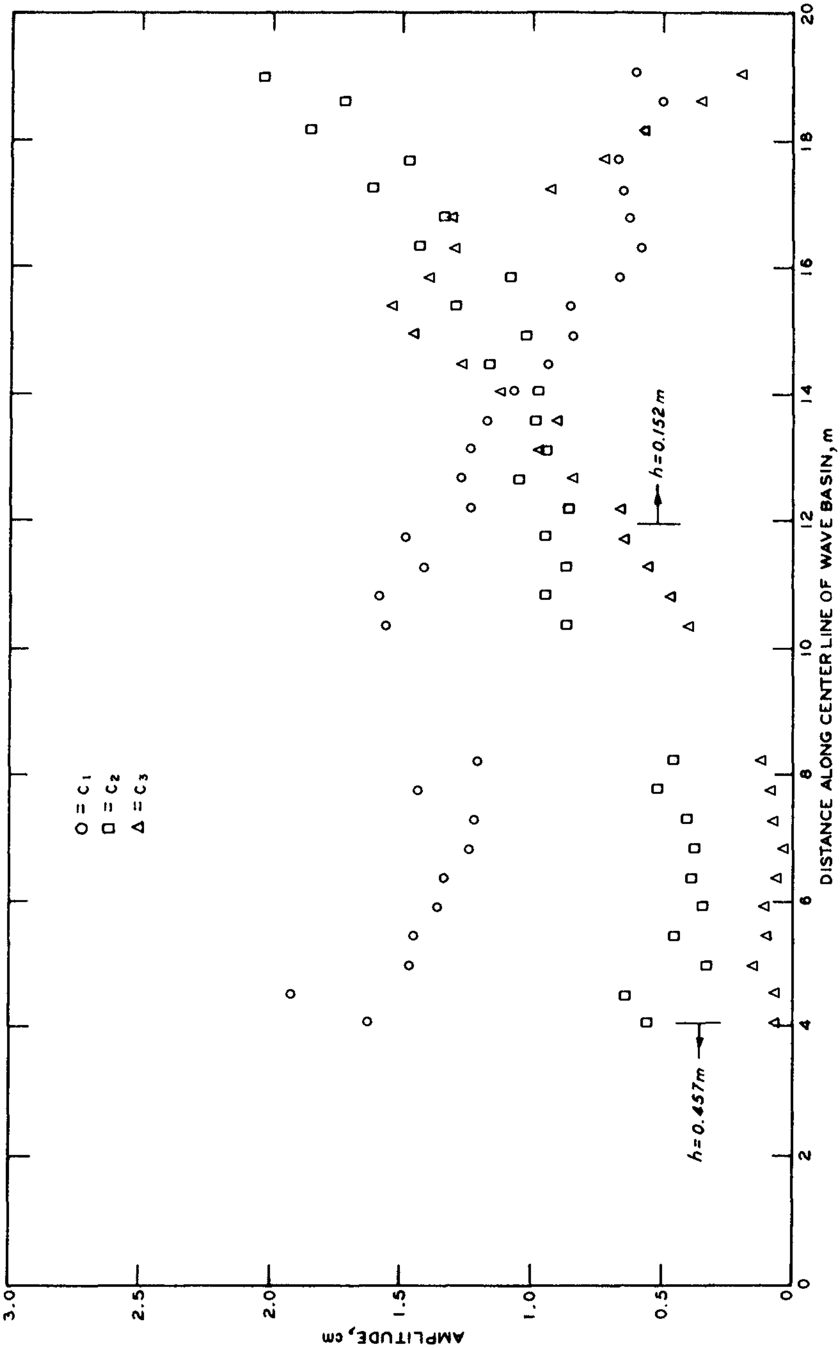

Figure 7. Values of c_1, c_2, and c_3 as Functions of Distance, T = 3 sec, S = 15.24 cm

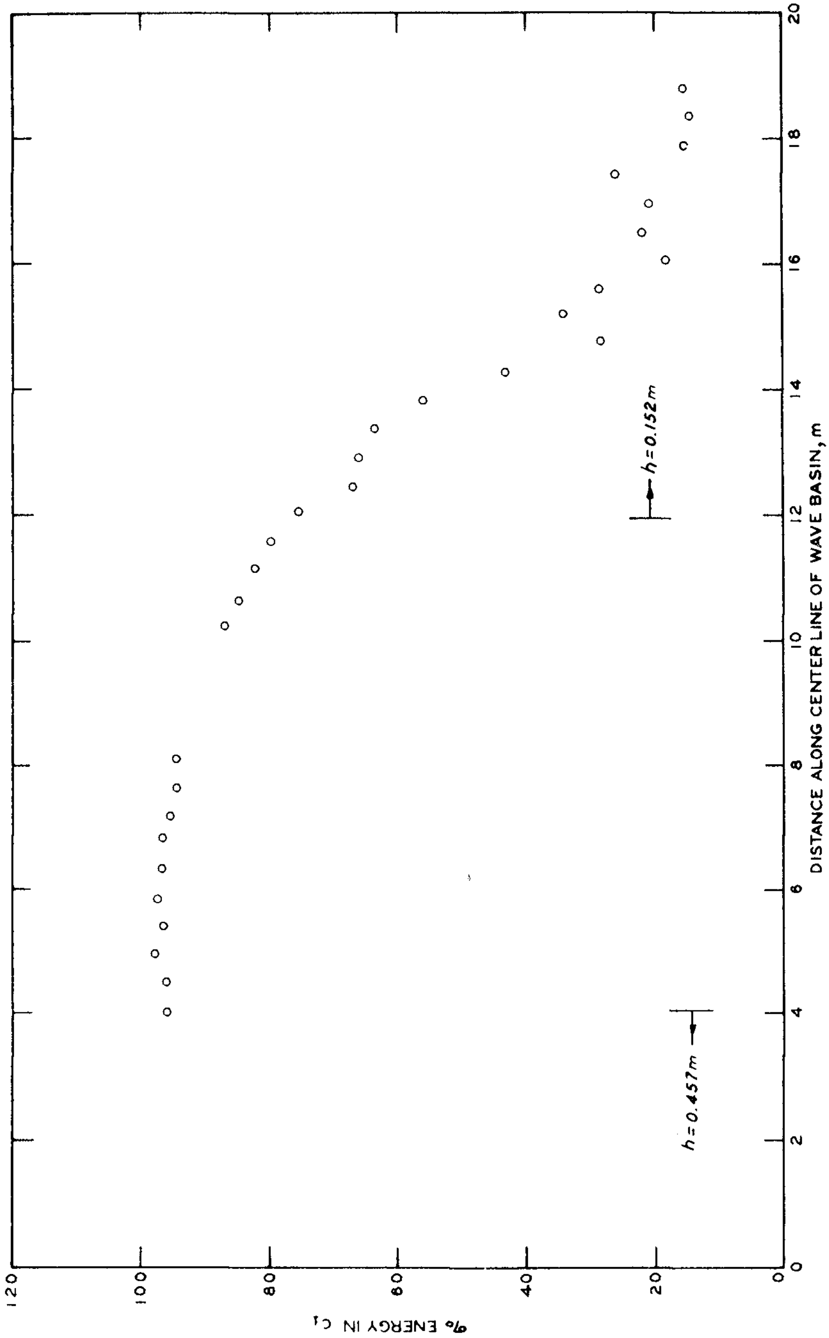

Figure 8. Percent Energy in Fundamental Frequency Component, T = 3 sec, S = 15.24 cm

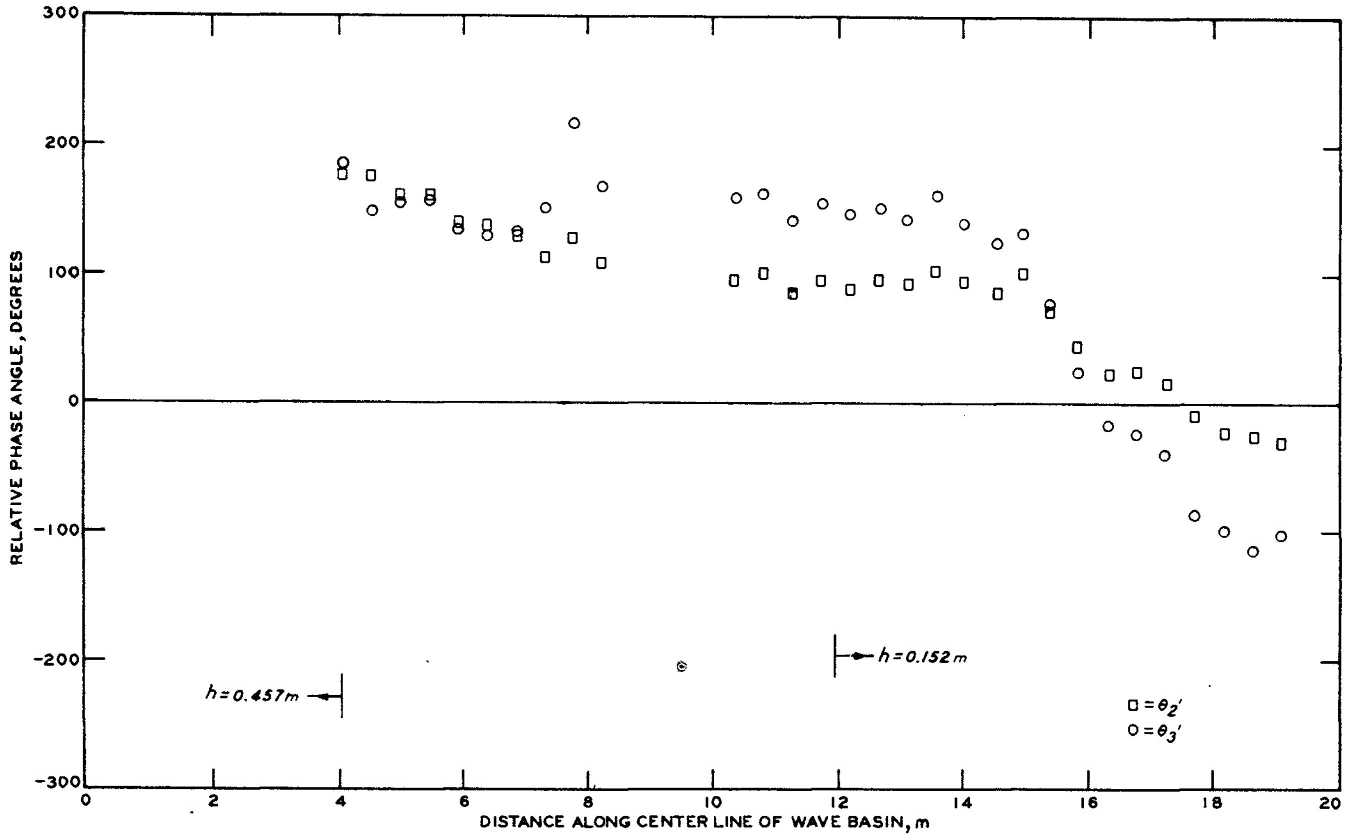

Figure 9. Phase of 2ω and 3ω Relative to the ω Component, T = 3 sec, S = 15.24 cm

decreases with distance propagated down the flume. The interpretation of this is very clear; if the relative phase does not change with distance then the higher order harmonics are completely coupled to the fundamental frequency and they propagate at its velocity. Since the relative phase of the higher order harmonics is decreasing, it means that they are continually lagging the fundamental frequency or are propagating slower. In order to investigate the possibility that the harmonics are completely uncoupled, one can compute the change in the relative phase over a given propagation distance. This computation revealed that the higher order harmonics were (1) completely coupled for the 1-sec waves, (2) quasi-coupled for the 2-sec waves, and (3) completely uncoupled for the 3-sec waves.

The data indicate that the magnitude of nonlinear effects increases with the wave period for a given topographic variation. Further, the energy transfer to higher frequency components becomes quite significant (as much as 80 percent for the largest amplitude and longest period wave) and these higher frequency components may or may not be coupled to the fundamental component.

One phenomenon not analyzed but observed during the course of this investigation was the effect of wave height on the phase velocity. At the top of the sloping portion of the topography, the wave crest in the middle of the basin was lagging slightly behind that toward the basin edges; however, by the time the wave reached the absorber material, the middle portion of the wave crest was leading. The effect of wave height on propagation speed is always neglected in refraction computations. The change in propagation speed across the wave front may alter the refraction coefficient by a significant amount in instances of a strong convergence of orthogonals. Furthermore, if the relative position of the wave front is important in any particular design problem, then this effect should be included in the refraction computation. It is an effect which can be relatively easily accounted for, although it certainly will necessitate a slightly longer running time on the computer.

CONCLUSIONS

A number of conclusions have arisen as a result of the rather extensive data acquisition and analysis phases of this investigation. Any conclusions drawn from the investigation must be interpreted with a view toward the effects of a more general type of bottom topography since the experimental data were acquired for only one topography. The conclusions of this investigation are enumerated below.

In the tests conducted, linear refraction theory was completely inadequate for predicting the refraction coefficient. Even for values of the refraction coefficient as low as 1.1 to 1.2, diffraction of energy along the wave crest completely altered the form of the isolines of equal refraction coefficient. Actually one cannot speak of a refraction coefficient alone whenever diffraction is significant since by definition refraction means conservation of energy between orthogonals thus implying no diffraction. In situations where a convergence zone exists it is imperative to include diffraction effects and speak of a design wave height of a design wave profile which includes the effects of diffraction, convergence of energy by the bottom topography (rather than refraction), reflection, frictional energy dissipation and nonlinear effects. No analyses are available at the present time which combine the computation of refraction and diffraction simultaneously. Such a combination of linear theories must be developed into a working numerical scheme in the form of a computer program which can routinely handle this type of problem. In the absence of a refraction theory which includes diffraction effects, the best approach to determining a design wave is to conduct a model study.

The magnitude of nonlinear effects, in particular the transfer of energy to higher frequency components, was found to be quite significant for the longest wave period (3 sec) and the largest wave height tested. In this case, less than 20 percent of the energy remained in the fundamental frequency component. Once the energy transfer to higher frequency components was initiated, it continued over the extent of the measurement area. In the case of the 1-sec wave period, practically all energy remained in the fundamental frequency component over the entire measurement area. The percent energy remaining in the fundamental frequency component for the 2-sec period exhibited an unusual behavior. It decreased and reached a minimum of approximately 80 percent and then increased again to the end of the measurement area. This is interpreted as meaning that the rate of energy being transferred to the higher frequency components is initially greater than the rate at which energy is being supplied by refraction but later becomes less.

The higher frequency components produced by the nonlinear transfer of energy from the fundamental component were not completely coupled to the primary wave except for the 1-sec period. The phase of the higher frequency components relative to the phase of the fundamental component indicated that a moderate coupling existed for the 2-sec period tested and a very weak coupling existed for the 3-sec waves tested. The amount of coupling decreased as the wave period increased and also as the wave height increased (for a given period) as one would expect. The significance of these findings is that it is clearly exhibited that an extremely complicated process is occurring; therefore to accurately predict the wave form (which may be important for the design of certain coastal structures) at a given location within a strong convergence zone of orthogonals, it is necessary to develop a nonlinear theory capable of describing not only the development of a nonlinear wave profile, but also the transition from coupled to uncoupled higher order frequency components. It seems apparent that the small amplitude assumption should be abandoned when there is a marked increase (for the 3-sec waves) in the nonlinear energy transfer to higher frequency components as the initial wave height increases. Values of the relative wave height vary from approximately 0.15 to 0.4 at the top of the sloping topography for the 3-sec waves. The maximum wave steepness was approximately 0.03, 0.022, and 0.015 for periods of 1, 2, and 3 sec, respectively, indicating that wave steepness is relatively unimportant with regard to the nonlinear effects analyzed.

An interesting point which was vividly illustrated in the present investigation was the effect of wave height on the phase velocity of the wave profile. Along with the inclusion of wave diffraction, one should also include effect of a finite wave height on the phase velocity of wave since this can alter the refraction coefficient obtained (especially in a zone of large convergence of wave orthogonals).

The computation of a wave reflection coefficient from underwater topographic variations requires some careful consideration of both the Ogawa and Yoshida (1959) long wave theory and the more general (in terms of assumptions relative to the wave form) theory of Rosseau (1952). The oscillatory behavior of the Ogawa and Yoshida (1959) theory seems to be unwarranted and evidently is a consequence of the error introduced by using the long wave theory in deep to intermediate water depths. Furthermore, the Ogawa and Yoshida (1959) theory usually overestimates the reflection coefficient. On the other hand, the difficulty of the Rosseau (1952) theory is that it probably underestimates the reflection coefficient due to the nature of the bottom profile (asymptotically approaching constant water depths on both sides of the topographic variation). A computation by both methods is recommended in order to bracket the actual value of the reflection coefficient with the preference toward one theory or the other depending on the characteristics of the actual bottom contour (how well this can be matched by a Rosseau bottom contour) and the relative water depth involved (the validity of the long wave assumption).

Viscous dissipation of energy at the bottom and sides of the wave basin was shown to result in a wave height attenuation of approximately 3 percent over the entire measurement area. The theory of Keulegan (1950) seems to be very satisfactory as long as the basic assumptions of small amplitude waves and small damping are satisfied as was the case in the present investigation.

The economic impact of the phenomena studied in this investigation cannot be overemphasized. The point is that diffraction acts to decrease the wave height computed by linear refraction theory, thus decreasing the design wave for any coastal structure located in such an environment. The nonlinear effects observed may act to either increase or decrease the design wave height and more importantly they need to be predictable where significant in order to evaluate such practical design considerations as wave forces, breaking wave heights, and longshore components of velocity. Therefore, future analytical investigations should be aimed first at including the effects of a linear diffraction theory and secondly toward describing the nonlinear effects adequately. The fact that the phenomena studied tend to decrease the design wave height can produce a very significant savings in the cost of breakwaters and jetties alone if they are situated shoreward of a convergence zone.

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