# Wavelength Converter Placement in Least-Load-Routing based Optical Networks using Genetic Algorithms 

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abstractIn this paper we study the routing and wavelength converter placement problems in optical networks with sparse wavelength conversion. We propose a new dynamic routing algorithm with two new path cost functions based on the concept of the Least-Load Routing (LLR) with sparse converter placement. Moreover, we discuss the application of Genetic Algorithms (GA) to determine the optimal location of wavelength converters so that the call blocking probability is minimized. Simulation results show that the proposed dynamic routing algorithms performs significantly better than Shortest-Path (SP) routing and Fixed-Alternative-Routing (FAR), in terms of the call blocking probability. The GA model is able to obtain a nearly optimal solution of the wavelength converter placement problem within a reasonable time and its performance is better than that of two other popular heuristic placement algorithms. abstract

## 1. Introduction

Wavelength-Division Multiplexing (WDM) technology has emerged as the multiplexing technique of choice to share a single optical fiber bandwidth among concurrent flows. Compared with traditional circuit switched networks, the call (or connection) blocking probability in optical WDM networks is much higher due to the so-called wavelength continuity constraint: a lightpath must utilize the same wavelength from the source to the destination. To mitigate the effect of the wavelength continuity constraint on the call blocking probability, wavelength conversion can be used [1]. A wavelength converter is an input/output device that, using a different wavelength without signal distortion, converts the wavelength of an optical signal from an input port to an output port. If wavelength converters are installed in all optical network nodes, the wavelength continuity constraint is relaxed and, hence, the call blocking probability can be decreased tremendously [2]. However, due to technical difficulties, wavelength converters are still very expensive devices. Therefore, the research community has focused its efforts on sparse wavelength conversion networks, where only some nodes in this network have the wavelength conversion capability while the rest do not. Previous research indicates that networks with sparse wavelength conversion can achieve a similar blocking performance as optical networks with full wavelength conversion, if the wavelength converters are placed appropriately [2], [3]. Because of the high cost of wavelength converters and the significant performance gain of wavelength converters in optical networks, we are interested here in the placement of wavelength converter in sparse wavelength conversion networks to minimize the call blocking probability.

The converter placement problem is coupled with the Routing and Wavelength Assignment problem (RWA). Both problems are NP-hard problems [4], [5] and, thus, different heuristics have been proposed to obtain approximate solutions to these two problems [5] - [16].

RWA in optical networks was introduced in [7], and analyzed first in [8]. To select an appropriate routing algorithm and wavelength assignment algorithm, the traffic assumption is an important issue. It generally falls into one of two categories: static or dynamic. For static traffic, all connections are fixed and known beforehand, and the objective is to minimize the number of used wavelengths to satisfy a given request set. For dynamic traffic, all connection requests arrive at, and depart from, the network randomly and the objective is to minimize the call blocking probability. Since it is very difficult to know the details of all connections beforehand, the dynamic traffic assumption is more realistic and thus we consider dynamic traffic in our research.

Routing in optical networks is simply inherited from traditional circuit switching. Among all the routing algorithms, the Least Load Routing (LLR) is the most popular one in traditional circuit switched networks [17] [18]. Naturally there were also attempts at studying LLR in the context of optical networks in [19]; however, it has only been applied in two types of optical networks: a network with no wavelength conversion and a network with full wavelength conversion (i.e. every node has a wavelength converter). An LLR-based routing algorithm in optical networks with sparse wavelength conversion neither has been formulated nor studied to the extent of our knowledge.

Among all wavelength assignment algorithms, the random wavelength assignment and the first-fit assignment [4] are the most popular algorithms to be used in wavelength assignment. In the random assignment algorithm, available wavelengths are selected randomly in all links along a path, while in the first-fit assignment algorithm the wavelengths are selected according to a predetermined order from the pool of available wavelengths along the path in ascending order. In general, the first-fit assignment algorithm achieves a lower call blocking probability than the random assignment at the cost of a very minor increase in the complexity [4].

In most previous research, the RWA problem and converter placement problem were studied separately. Although Li et al. [6] recently argued that the RWA problem and the wavelength converter placement need to be considered jointly for a better overall system performance, it is very difficult to formulate an optimization model for both issues together because of the complexity. Therefore in [6], Li et al. proposed greedy-type wavelength converter placement algorithms given a particular RWA algorithm. We shared this view and studied the converter placement problem together with the proposed LLR-based RWA algorithms.

To solve the wavelength converter placement problem, various placement heuristics have been proposed in the literature [5] [9] - [16] . Different heuristic algorithms, based on different parameters such as the number of channels in transit, node degrees, link loading, and fiber utilization, are employed [5] [12] - [16]. An exhaustive search method for large scale optical networks is proposed in [15]. Genetic Algorithms (GA), to determine the optimal solution, are also proposed in [9] - [11]. In [11], the method is only applicable in a ring topology rather than in a general topology. The routing algorithms discussed in [9] - [11] are normally applied in fixed or fixed alternate routing. Dynamic routing determines routes after a consideration of the network status at the time when the connection request arrives. Generally, the performance (in terms of call blocking probability) of dynamic routing algorithms is much better (lower) than that of static routing algorithms since dynamic routing algorithms make routing decisions based on the current network status and, thus, a path, in which congestion (if any) can be avoided, can be established.

In this paper, we first propose a new dynamic routing algorithm based on the concept
of the Least Load Routing (LLR) for optical networks with sparse wavelength conversion. Then define two new path cost functions that are related to the overall link loading of a path and the locations of the wavelength converters in a path. Then we discussed the use of GA to minimize the overall call blocking probability by finding the optimal wavelength converter placement. We will start by presenting the two new path cost functions in Section 2. The network model and our proposed routing algorithms will also be illustrated using an example. In Section 3, we discussed the employment of a GA-based optimization model to search for the optimal converter locations. Numerical results of the proposed dynamic routing algorithms and the GA model are presented in Section 4 A performance evaluation has also been conducted for other RWA algorithms and wavelength converter heuristic placement algorithms. Finally, in Section 5, concluding remarks are drawn, and future research directions are outlined.

## 2. LLR for networks with sparse wavelength conversion

## 2.A. Preliminaries

Different levels of wavelength conversion capability are possible in optical nodes [1]. Fullrange wavelength converters can convert an incoming wavelength to any outgoing wavelength, while limited-range wavelength converters can convert an incoming wavelength to only a subset of outgoing wavelengths. For simplicity, all wavelength converters in our network model are full-range wavelength converters.

Define a path $p$ as an ordered set of links $l=(u, v)$ starting at the source node and ending at the destination node. In an optical network with sparse wavelength conversion, we define a segment $s$ on a path $p$, as an ordered subset of $p \mathrm{i}$ ) starting at the source node and ending at the first converter if any; or ii) starting at a converter and ending at the next converter if any; or iii) starting at a converter if any, and ending at the destination; or, iv) if there is no converter on the path, the path itself. The wavelength continuity constraint can be relaxed at the frontier between segments. If there is no wavelength converter in a path, a segment of a path is the path itself. An example of paths segmented by a wavelength converter is shown in Fig. 1. The node $W C$ is a node equipped with a wavelength converter. The first segment of the path from node $S$ to node $D$ consists of two links $(S, a)$ and $(a, W C)$. The second segment consists of two links $(W C, b)$ and $(b, D)$. In the first segment, the same wavelength (i.e., $\lambda_{1}$ ) has to be used to establish a lightpath because of the wavelength continuity constraint. With the help of the wavelength converter at node $W C$, in the second segment another wavelength $\left(\lambda_{2}\right)$ may be used to establish an end-to-end lightpath from $S$ to $D$.


Fig. 1. Segments.

## 2.B. Network Model and Proposed Path Cost Functions

The LLR algorithm has been used in circuit switched networks since the early 1980s. In [19], to migrate it to optical networks, two path cost functions based on the concept of
the LLR were proposed for two types of optical networks. For optical networks with no wavelength conversion, a path $p$ is chosen if it achieves

$$
\begin{equation*}
\operatorname{maxmin}_{p, j} M_{l \in p}-A_{l j}, \tag{1}
\end{equation*}
$$

where $M_{l}$ is the number of fibers on link $l$ and $A_{l j}$ is the number of fibers for which wavelength $j$ is utilized on link $l$. For an optical network with full wavelength conversion capability, a path $p$ is chosen if it achieves

$$
\begin{equation*}
\operatorname{maxmin}_{p} K M_{l}-\sum_{j} A_{l j} \tag{2}
\end{equation*}
$$

where $K$ is the number of wavelengths in a single fiber. However, neither path cost functions can be directly applied to optical networks with sparse wavelength conversion. In this section, two path cost functions, able to be utilized in two LLR-based routing algorithms in sparse convertible networks, are proposed.

For each source-destination node pair to reduce the state space, we will only consider the $k$ edge-disjoint shortest paths and sort them by the hop count (total number of links in a path) in an ascending order and, then, by the number of wavelengths of all links in a path in a descending order. These paths are edge disjoint to ensure that the blocking along these paths is independent.

A channel of a path (segment) is defined as an available wavelength for end-to-end communication along the path (segment). Note that there may be more than one available channel using the same wavelength if multiple fibers are allowed within a link. Two new path cost functions are proposed as extensions of (1) and (2) when the network supports sparse conversion. The first, the so-called Least Load Routing using Min-Max-Min (LLRMMM) is given by

$$
\begin{equation*}
C(p(R))=\min _{s \in p(R)} \max _{j} \min _{l \in s} M_{l}-A_{l j} \tag{3}
\end{equation*}
$$

where $p(R)$ is a path for connection request $R$. If there are some nodes with wavelength converters in the path, the path is decomposed into segments and the cost of a segment is defined as the maximum number of available channels of all the wavelengths of the segment as in (1). Then the cost of a path is the minimum cost of all the segments in the path. Note that the cost function of this path involves both routing and wavelength assignment. The second cost function considered here is the so called Least Load Routing using Min-SumMin (LLR-MSM) and is,

$$
\begin{equation*}
C(p(R))=\min _{s \in p(R)} \sum_{j} \min _{l \in s} M_{l}-A_{l j} \tag{4}
\end{equation*}
$$

This path cost function takes into consideration the case where some nodes in the path do not have any wavelength converters. If some nodes in the path are not equipped with wavelength converters, the path is decomposed into segments and the cost of a segment is defined as the sum of the available channels of all the wavelengths through the segment. The cost of a path is the minimum cost of all the segments in the path. Compared with the LLRMMM, this path cost function is more aggressive with respect to the total available channels of a segment since the least load concept is applied to the total available channels rather than the maximum number of available channels in a single wavelength. As a tradeoff, this is possible because this path cost function does not involve any wavelength assignment algorithm in the mathematical model. Various wavelength assignment algorithms can work together with the LLR-MSM.

```
Algorithm 1 Least Load Routing in optical network with Sparse conversion
    Notations.
    \(p\) : a path.
    \(s\) : a segment.
    \(l\) : a link.
    \(j\) : a wavelength.
    \(N_{c}^{p}\) : the number of converters in the path \(p\).
    \(N_{s}^{p}\) : the number of segments in the path \(p\) such that \(N_{s}^{p}=N_{c}^{p}+1\).
    \(\Gamma_{s}\) : the number of segments allowed (to limit the use of converters).
    \(C(\cdot)\) : the cost function.
    \(p^{*}(R)\) : the optimal connection path for connection request \(R\).
```


## BEGIN

```
\(\Gamma_{s} \Leftarrow 1\)
while (all \(k\) edge-disjoint paths in \(Z\) are examined in the ascending order in hop numbers) do
\(Z^{*}=\left\{p^{*}(R) \in Z: N_{s}^{p^{*}(R)}=\Gamma_{s}, C\left(p^{*}(R)\right)=\max _{p(R) \in Z} C(p(R))>0\right\}\)
if \(Z^{*}=\emptyset\) then
\(\Gamma_{s} \Leftarrow \Gamma_{s}+1\)
        else
            The request is accepted, \(p^{*}(R) \in Z^{*}\) (select the first element of \(Z^{*}\) ), and exit.
        end if
    end while
    the request is denied.
    END
```


## 2.C. Algorithms

Consider $k$ shortest paths for connection request $R$. Let $Z$ be the set of $k$ shortest paths, then $Z=\{p(R)\}$. The LLR routing algorithm is given in Algorithm 1.

Note that this LLR algorithm has two variants, LLR-MMM or LLR-MSM, depending on the path cost function $C(\cdot)$ used. These two LLR-based path cost functions take into account the impact of wavelength converters in the routing decision. The number of wavelength converters is limited and converters help to relax the wavelength continuity constraint; therefore, it is easier to find an available channel on a path with wavelength converters than on a path without wavelength converters. Because of the deployment of wavelength converters, at the beginning, the algorithm attempts to select a path without wavelength converters (i.e. $\Gamma_{s}=1$ ) so that wavelength converters can be reserved for future requests. If such paths cannot be found, paths with one wavelength converter (i.e. $\Gamma_{s}=2$ ), and so on, are sought until all the $k$ edge-disjoint shortest paths are examined.

## 2.D. An Illustrative Example

To illustrate the proposed routing algorithm with the two path cost functions, an example of a 6 -node network is shown in Fig. 2. The node $W C$ is a node equipped with a wavelength converter. Other nodes do not have this conversion capability.

A connection request arrives at node $S$ and the destination is node $D$. The cost of the segments and paths are shown in Table 1. In this example, four different segments and four possible paths from node $S$ to node $D$ exist. The wavelengths within brackets, shown in the second and third columns of Table 1, represent the available wavelengths that can be used in their corresponding segments. For both path cost functions, path $p_{1}$ has the largest


Fig. 2. A 6-node network with one wavelength converter.
path cost and, thus, this path should be used to establish a connection from node $S$ to node $D$ in the LLR-MMM or the LLR-MSM with the available wavelength $\lambda_{2}$.

Table 1. Cost of segments and paths for the connection from node $S$ to node $D$.

| Segment/Path | LLR-MMM | LLR-MSM |
| :--- | :--- | :--- |
| $s_{1}=\{S, a, W C\}$ | $C\left(s_{1}\right)=3\left(\lambda_{2}\right)$ | $C\left(s_{1}\right)=5\left(2\left(\lambda_{1}\right)+3\left(\lambda_{2}\right)\right)$ |
| $s_{2}=\{W C, D\}$ | $C\left(s_{2}\right)=4\left(\lambda_{1}\right)$ | $C\left(s_{2}\right)=6\left(4\left(\lambda_{1}\right)+2\left(\lambda_{2}\right)\right)$ |
| $s_{3}=\{S, b, W C\}$ | $C\left(s_{3}\right)=1\left(\lambda_{1}\right.$ or $\left.\lambda_{2}\right)$ | $C\left(s_{3}\right)=2\left(1\left(\lambda_{1}\right)+1\left(\lambda_{2}\right)\right)$ |
| $s_{4}=\{W C, c, D\}$ | $C\left(s_{4}\right)=2\left(\lambda_{2}\right)$ | $C\left(s_{4}\right)=2\left(2 \lambda_{2}\right)$ |
| $p_{1}=\left\{s_{1}, s_{2}\right\}$ | $C\left(p_{1}\right)=3$ | $C\left(p_{1}\right)=5$ |
| $p_{2}=\left\{s_{3}, s_{2}\right\}$ | $C\left(p_{2}\right)=1$ | $C\left(p_{2}\right)=2$ |
| $p_{3}=\left\{s_{1}, s_{4}\right\}$ | $C\left(p_{3}\right)=2$ | $C\left(p_{3}\right)=2$ |
| $p_{4}=\left\{s_{3}, s_{4}\right\}$ | $C\left(p_{4}\right)=1$ | $C\left(p_{4}\right)=2$ |

## 3. An Optimization Model based on Genetic Algorithms

In this section a GA-based optimization model is proposed for the wavelength converter placement problem. Given the network topology, the LLR-MMM or the LLR-MSM routing algorithm, the wavelength assignment algorithm (random or first-fit algorithm whereever applicable), and the number of wavelength converters to be placed, the objective is to minimize the average call blocking probability $B_{p}$ of an optical network with sparse wavelength conversion.

To place $K$ converters in the network with $N$ nodes, we first label each node in the network with a unique label from 0 to $N-1$. We define a placement vector $v$ with $K$ elements where the $i$-th element $(i=1, \cdots, K)$ is the label of the node on which the $i$ th converter is located. As an example, a placement vector $(2,5,1,3,9)$ for a 5 -converter placement in the 14 -node NSFNet indicates that the 5 converters are placed at nodes 2,5 , 1,3 and 9 , respectively. Different placements result in different blocking probability $B_{p}$, and finding the best placement (i.e., the one that minimizes the blocking probability) is NPhard. Therefore, we adopt a Genetic Algorithm (GA) to search for the optimal placement in a reasonable time.

A Genetic Algorithm (GA) is an iterative optimization procedure to obtain near-optimal solutions. The basic idea is borrowed from the evolution process in Nature [20], [21]. In our placement problem, the objective function is defined as the average call blocking probability, $B_{p}$. Because the blocking probability for the LLR-MSM and the LLR-MMM algorithms is not available, $B_{p}$ is directly obtained from simulations (described in Section 4).

GA starts its evolution process from a random population of placements. The evolution process is repeated for a predetermined number of times (generations) or until the solution converges to the optimal one (no evolution); thereafter, GA stops and the final solution is the optimal placement with the smallest blocking probability. One generation evolution consists of the selection, to choose placements (chromosomes) that are likely to survive in the next generation; crossover, to combine the good characteristics of placements and generate new placements (combining genes from different chromosomes to form new chromosomes), and finally mutation, to change elements of a placement randomly (i.e., given a placement we move randomly a converter from one node to another). The three steps are executed in this sequence repeatedly. Selection of placements that survive from iteration to iteration obeys a given fitness function $F$, defined here as a negative power transformation of the blocking probability, as shown in Algorithm 2.

In this power transformation, placements with a smaller blocking probability have much higher chance of surviving in the evolution process. $t$ is a very small positive constant ( $t=1 \times 10^{-6}$ in the implementation) to prevent the case where $B_{p}=0$. The parameter $\alpha$ controls the convergence speed of the GA iteration process. When the difference of the objective values of candidate placement vectors is small $\left(\frac{\Delta}{V_{\max }}<0.1\right)$, the absolute value $\alpha$ is assigned with a higher value $(\alpha=-2)$ to increase the difference of the fitness value of each candidate, so that each candidate can be differentiated. When the difference of the objective values of candidate placement vectors is large $\left(\frac{\Delta}{V_{m a x}}>0.9\right)$, the absolute value $\alpha$ is assigned as a smaller value ( $\alpha=-0.5$ ), so that the difference of the fitness value of each candidate is reduced and, hence, it can prevent some candidate with a significant better objective value from dominating the GA process.

```
Algorithm 2 A power transformation from the objective function to the fitness function
    Notations.
    \(B_{p}\) : the objective function, which is the blocking probability in our case.
    \(F\) : the fitness function.
    BEGIN
    \(V_{\max } \leftarrow\) the largest value of the objective function among all vectors in one run.
    \(V_{\min } \leftarrow\) the smallest value of the objective function among all vectors in one run.
    \(\Delta \leftarrow\left|V_{\max }-V_{\min }\right|\)
    if \(\frac{\Delta}{V_{\max }}>0.9\) then
        \(\alpha \leftarrow-0.5\)
    else if \(\frac{\Delta}{V_{\max }}<0.1\) then
\(\alpha \leftarrow-2\)
        else
        \(\alpha \leftarrow-1\)
    end if
    \(t \leftarrow 1 \times 10^{-6}\)
    \(F \leftarrow\left(B_{p}+t\right)^{\alpha}\)
    END
```

In order to reduce the search space, in the selection operation, the number of placements
in each iteration is maintained constant. The universal sampling selection scheme was used in the implementation because the comparison with two other selection schemes, the tournament selection and the roulette wheel selection, indicated that it outperforms them.

In the crossover operation, two placement vectors exchange partially their elements (genes) and generate new candidate placements. The uniform crossover was used in the implementation. For two parental placements, a mask vector with the same length as a placement vector which consists of 1 s and 0 s is randomly generated. An example is shown in Table 2. If the mask $i$-th bit is 1 , the $i$-th element of two parental vectors are exchanged, otherwise, i.e., if the mask bit is 0 at $i$-th bit, the $i$-th element is not changed. Notice that illegal placements may be created if the uniform crossover is applied directly because duplicated elements may occur in one vector. For the example in Table 2, vector A (5, 4, 6, 1, 8) exchanges the elements for converters 2,3 , and 4 , with vector $\mathrm{B}(7,8,2,9,3)$ and hence two new vectors $\mathrm{A}^{\prime}(5,8,2,9,8)$ and $\mathrm{B}^{\prime}(7,4,6,1,3)$ are generated; however, the placement vector $A^{\prime}$ is not allowed because converters 2 and 5 are placed at the same node 8 . The following technique was used to solve this problem. First, two parental vectors are rearranged (using a permutation of the converter identifiers) so that those converters at the same nodes are placed at the same location in the parental vectors. A modified uniform crossover is shown in Table 3. Vector B $(7,8,2,9,3)$ is rearranged into vector $\mathrm{B}^{*}(7,3,2,9,8)$ so that the child vector A' does not have any duplicated elements.

Table 2. A Uniform Crossover with Illegal Solution Created.

| Vector A | 5 | 4 | 6 | 1 | 8 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Vector B | 7 | 8 | 2 | 9 | 3 |  |
| Mask | 0 | 1 | 1 | 1 | 0 |  |
| Vector A' | 5 | 8 | 2 | 9 | 8 | wrong! |
| Vector B' | 7 | 4 | 6 | 1 | 3 |  |

Table 3. A Uniform Crossover with Legal Solution Created.

| Vector A | 5 | 4 | 6 | 1 | 8 | $\Rightarrow$ | Vector A* | 5 | 4 | 6 | 1 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Vector B | 7 | 8 | 2 | 9 | 3 | $\Rightarrow$ | Vector B* $^{*}$ | 7 | 3 | 2 | 9 | 8 |
|  |  |  |  |  |  |  | Mask | 0 | 1 | 1 | 1 | 0 |
|  |  |  |  |  |  |  | Vector A' | 5 | 3 | 2 | 9 | 8 |
|  |  |  |  |  |  | Vector B' | 7 | 4 | 6 | 1 | 8 |  |

In the mutation procedure, some elements of a placement vector can be changed according to some probability distribution. The single point mutation was used in the implementation. An example is shown in Table 4, in which converter 2 is changed from node 4 to node 2 .

Table 4. A Single Point Mutation.

| Vector A | 5 | 4 | 6 | 1 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Vector A' | 5 | 2 | 6 | 1 | 8 |

The whole wavelength routing and wavelength converter placement problem is the interaction process of the RWA algorithm and the GA, which is depicted in Algorithm 3.

```
Algorithm 3 A GA Optimization Algorithm
    \(v\) : a placement vector.
    \(\mathcal{P}\) : a population of placement vectors.
    \(\mathcal{R} \mathcal{W} \mathcal{A}\) : a routing and wavelength assignment algorithm, i.e., LLR-MSM.
    Sim: the simulation process.
    \(N_{\max }\) : the maximum number of generations.
    \(B_{p}\) : the blocking probability in our case.
    \(F\) : the fitness function.
    \(\mathcal{F}\) : the set of the fitness function for \(v \in \mathcal{P}\)
    PowerTransform: the transform from the blocking probability to the fitness function.
    \(G A\) : the GA process, consisting of the selection, crossover, mutation operations.
    BEGIN
    Generate an initial population \(\mathcal{P}\) randomly.
    for \(\mathrm{i}=1\) to \(N_{\max }\) do
        while \(v \in \mathcal{P}\) do
            \(B_{p} \leftarrow \operatorname{Sim}(v, \mathcal{R} \mathcal{W} \mathcal{A})\).
            \(F \leftarrow\) PowerTransform \(\left(B_{p}\right)\).
        end while
        \(\mathcal{P} \leftarrow G A(\mathcal{F})\).
    end for
    END
```

One simulation usually takes several minutes to obtain the call blocking probability. The crossover probability and the mutation probability are carefully chosen so that the GA generates near optimal solutions in relative fewer iterations. In our implementation, the crossover probability is set higher to ensure that the algorithm searches more solution space; the mutation probability is set to 0.1 to ensure that the algorithm generate new candidates by mutation in each iteration on average and the population of the candidates is relatively stable.

## 4. Numerical Results

In this section numerical results are provided in order to illustrate the performance of the proposed LLR algorithm with two path cost functions. Performance evaluations were conducted for the Shortest Path Routing, Fixed Alternate Routing, LLR-MMM and LLR-MSM with the random and the first-fit wavelength assignment schemes. In FAR, LLR-MMM and LLR-MSM, 2 edge-disjoint paths ( $K=2$ ) are used. The performance of the GA model for the wavelength converter placement is also discussed, and compared with two popular heuristic algorithms, Total Outgoing Traffic (TOT) [12] and the K Minimum Dominating Set (K-MDS) placement [22].

The investigations were conducted in the 14-node NSFnet network, as shown in Fig. 3 , Each link has a single fiber $\left(M_{l}=1\right)$ and each fiber has 40 wavelengths ( $K=40$ ). The connection requests arrive according to a Poisson process and all call holding times are exponentially distributed with a unit mean. All processing times, including the call setup and release time, are negligible. When a connection request is aborted, it is not retried and is cleared immediately from the network.

For each set of simulation points, there are 10 batches in one simulation run and the length of each batch is $10^{5}$ units of mean interarrival time of the connection request. The initial $10 \%$ is discarded to avoid the effect of the transient states. The size of the simulation points shown in the figures below is large enough to cover $95 \%$ confidence intervals. The
whole simulation model was constructed based on SimLib 2.2 [23].


Fig. 3. The 14-node NSFnet network.

## 4.A. Performance of LLR Algorithms

In this section the performances of the LLR-MMM, LLR-MSM, SP and FAR algorithms under the random and the first-fit wavelength assignment schemes are presented. Fig. 4, 5 and 6, show the blocking probability under different algorithms when the network has no conversion, full conversion, and sparse conversion with the first-fit wavelength assignment and the random wavelength assignment, respectively. In Fig. 6, the network has 5 nodes with wavelength converters and the placements are obtained using the K-MDS algorithm. The LLR-MSM routing significantly outperforms all other algorithms under the three different situations, and the difference increases when the network loading decreases.

Moreover, the performance of the LLR-MMM routing is the same as that of FAR, except in Fig. 7 and 8 because, in the case of a single fiber, the cost of a path is either 0 or 1 in LLR-MMM and, thus, LLR-MMM is equivalent to FAR with first fit. If a link can install multiple fibers, LLR-MMM is expected to outperform FAR. The LLR-MMM routing has a lower blocking probability than that of FAR because random wavelength assignment is used in FAR, see Fig. 7 and 8 , In general, the First Fit wavelength assignment has a lower blocking probability than that of the random wavelength assignment. Since the LLR-MSM and the First-Fit algorithm result in the best system performance, we focus on the converter placement study using the LLR-MSM and First-Fit in the next section.

## 4.B. Performance of GA Model

In this section the performance of the GA model is compared with that of two heuristic placement algorithms: TOT and K-MDS. Since the LLR-MSM routing outperforms others, we concentrate on the GA model which uses the LLR-MSM. In Table 5 the wavelength converter placements under different placement algorithms are shown. It is interesting to observe that in different algorithms the same results are obtained if the number of wavelength converters is small (i.e. 1 or 2). However, when the number of wavelength converters is sufficiently large, different wavelength converter placements are obtained.

A comparison of the performances of different placement algorithms with different number of wavelength converters, is depicted in Table 6. The blocking probability with the converter placement obtained using the proposed GA model converges much faster to
the blocking probability with full conversion than the blocking probability with the converter placement obtained using TOT and K-MDS. In addition, the performance difference increases when the number of wavelength converters increases. When the number of wavelength converters is small, the obvious solution is to place wavelength converters in the congestion region of the whole network, i.e., the nodes with the highest call blocking probability, and all placement algorithms can find these congestion regions using their heuristic approaches. However, when the number of wavelength converters is large, the heuristics of TOT and K-MDS are not good enough to obtain a near-optimal solution; on the other hand, the GA model can still converge to the global optimal solution.

A comparison of the performance of different placement algorithms under different network loadings is presented in Fig. 9. With 4 converters obtained using the GA model, the call blocking probability is very close to the blocking probability obtained with full conversion; however, the blocking probability using the placement obtained using TOT and K-MDS is still quite far from the blocking probability obtained with full conversion. The differences in terms of the blocking performance increases when the network loading decreases.

In Fig. 10, the conversion gain with converters obtained using GA is depicted. With only 4 nodes out of 14 nodes in the whole network, equipped with converters, the blocking probability is already very close to the blocking probability obtained with full conversion. Note that although TOT and K-MDS can obtain a solution very quickly compared with GA, the processing time to obtain a solution is not our major concern because the optimization should be processed off-line and only GA gives the optimal solution.

In Fig. 11, the convergence speed of the GA model is depicted when the network load is at 454.5 Erlangs. Within around 12 iterations in the GA process, the placement is almost converging to the optimal solution empirically. When the converter number is really large (8 converters in the 14 -node NSFNet), the GA can not help much in optimizing the placement. It is because over half of nodes in the network are equipped with converters, even the random placement can achieve the near-optimal system performance.

Table 5. Wavelength Converter Placement under Different Placement Algorithms in the LLR-MSM Routing.

| No. of wavelength converters | TOT | K-MDS | GA-Simul |
| :---: | :---: | :---: | :---: |
| 1 | 5 | 5 | 5 |
| 2 | 5,9 | 5,9 | 5,9 |
| 3 | $5,7,9$ | $5,9,12$ | $5,9,10$ |
| 4 | $3,5,7,9$ | $5,9,11,12$ | $3,5,7,9$ |
| 5 | $1,3,5,7,9$ | $5,9,11,12,13$ | $1,3,5,9,11$ |

## 5. Conclusion and Future Work

In this paper we discussed the routing and the wavelength converter placement problem in optical networks with sparse wavelength conversion. We proposed a new dynamic routing algorithm with two new path cost functions: the LLR-MMM and the LLR-MSM. They are based on the concept of the LLR with sparse converter placement. Simulation results showed that the LLR-MSM and LLR-MMM outperform other traditional routing algorithms.

We also discussed the development of a GA model for the wavelength converter placement problem. To minimize the call blocking probability, the optimal location of wavelength converters are found. Simulation results indicated that with a small number of con-

Table 6. Performance Comparison in the Network using the First-Fit Wavelength Assignment under the Uniform Traffic at the Load of 454.5 Erlangs.

| No. of wavelength converters | GA | K-MDS | TOT |
| :---: | :---: | :---: | :---: |
| No Conversion | 0.014329 | 0.014329 | 0.014329 |
| 1 | 0.008699 | 0.008699 | 0.009628 |
| 2 | 0.006113 | 0.006898 | 0.006898 |
| 3 | 0.004849 | 0.006208 | 0.006332 |
| 4 | 0.003460 | 0.005554 | 0.004111 |
| Full Conversion | 0.002970 | 0.002970 | 0.002970 |

verters, placed using the GA model the blocking probability can converge to the blocking probability obtained with full conversion.

Because analytical blocking probability models for the LLR-MSM and the LLR-MMM are not available in a closed form, a simulation model was developed to obtain the blocking probability in the GA framework. This results in a longer search time in the GA process. Though it is an off-line design problem, using simulation for even larger network in the GA framework might not be affordable. In our future work, we will concentrate on developing an analytical model for obtaining the call blocking probability of the proposed dynamic routing algorithm LLR-MSM and LLR-MMM.

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Fig. 4. Performance comparison in the network with no conversion and the First-Fit wavelength assignment.


Fig. 5. Performance comparison in the network with full conversion and the First-Fit wavelength assignment.


Fig. 6. Performance comparison in the network with partial conversion, K-MDS placement and the First-Fit wavelength assignment.


Fig. 7. Performance comparison in the network with no conversion and the Random wavelength assignment.


Fig. 8. Performance comparison in the network with full conversion and the Random wavelength assignment.


Fig. 9. Performance comparison in the network using the LLR-MSM routing and the FirstFit wavelength assignment with 4 wavelength converters under the uniform traffic.


Fig. 10. The effect of the number of wavelength converters on the performance of our GA model using the LLR-MSM routing and the First-Fit wavelength assignment under the uniform traffic.


Fig. 11. The convergence speed of the GA model using the LLR-MSM routing and the First-Fit wavelength assignment under the uniform traffic at the Load of 454.5 Erlangs.

