Wavelength Requirements in Arbitrarily Connected Wavelength-Routed Optical Networks

Stefano Baroni, *Student Member, IEEE,* and Polina Bayvel, *Member, IEEE*

Wavelength division multiplexed optical networks using wavelength routing (WRON's) represent the most promising solution for future high-capacity wide-area network applications. One of the crucial factors which will determine their feasibility is the number of wavelengths required to satisfy the network traffic demand. In this paper, we consider arbitrarily connected networks as physical topologies for WRON's. By analysing a large number of randomly generated networks, bounds on the network wavelength requirements are first evaluated as a function of the physical connectivity. The advantages achievable by multifiber connections and the consequence of single link failure restoration are then assessed for several existing or planned network topologies. The results can be used in the analysis and optimization of the WRON design.

I. Introduction

WAVELENGTH-ROUTED optical networks (WRON's) [1] offer an enormous potential for future high-capacity wide-area network applications [2], [3]. The greatest operational advantage of WRON's is achieved where no wavelength translation or switching is implemented in any of the intermediate wavelength-routing nodes (WRN's), simplifying the management overheads and determining a single-hop logical topology [4]. The network node-pairs are assigned high-capacity all-optical channels, known as lightpaths [5], which transparently connect sources and destinations, providing for the bandwidth requirement of the next generation gigabit applications [6].

The all-optical channel paths are determined by the location of the transmitting and receiving nodes, the transmitted wavelength and the WRN's configuration. The WRN's perform simple optical wavelength routing on the channels, simplifying network management and processing compared to the routing in digital cross-connect systems, with the electronic-equipment savings demonstrated to be significant [7].

The practicability of WRON's depends on the number of wavelengths (N_λ) required to satisfy a given logical connectivity and traffic demand. Several near-optimal lightpath allocation algorithms have recently been reported aimed at minimizing the network wavelength requirements, the wavelengths being a limited resource [8]–[11].

Manuscript received July 8, 1996; revised November 8, 1996. The work of S. Baroni is supported by a Nortel Europe Ltd. through a Ph.D. studentship. The work of P. Bayvel is supported by a Royal Society University Research Fellowship.

The authors are with the Department of Electronic and Electrical Engineering, University College London, London WC1E 7JE, England.

Publisher Item Identifier S 0733-8724(97)01357-1.

In the last few years theoretical lower and upper bounds on the required number of wavelengths have been derived for the *permutation routing* problem [12]–[14]. In this approach each node is equipped with one (tunable) transmitter and receiver and is therefore the origin and destination of one session at any time. The results, obtained considering only the logical connectivity, represent important information-bounds on the logical network layer. However, additional constraints imposed by the physical network layer must be investigated in order to obtain tighter bounds on N_λ, necessary for practical network design, since the number of wavelengths required directly determines the channel spacing and the corresponding device complexity.

The most critical parameter in these networks is the physical topology onto which the logical demand has to be mapped, since it directly determines the lightpath allocation, and hence the complexity of the WRN's and the wavelength requirement. Previous analysis of physical topologies has been limited to topologically regular networks, in which wavelength requirements were analyzed for positive blocking probability [15], [16] and zero-blocking [17], [18]. Whilst these are useful theoretical limits, these results are very difficult to apply to real, national transport networks whose physical topologies, determined by cost and operational constraints, are neither fully nor regularly connected. Therefore it is key to analyze the relationship between the required number of wavelengths and physical connectivity and other topological parameters to enable the design of practical wavelength-routed optical networks. Such networks are likely to have physical topologies with variable connectivity mainly deriving from existing networks, and in this paper, for the first time to the best of our knowledge, we analyze the wavelength requirements in arbitrarily connected WRON's for wide-area backbone applications. Applying a novel lightpath allocation algorithm to analyze a large number of randomly generated, arbitrarily connected networks and several real network topologies, we evaluate the required number of wavelengths as a function of the physical connectivity. We discuss the influence of network topological parameters and compare the results with those of regular topologies. The analysis of several existing or planned network topologies is then used to assess the advantages achievable by selected multifiber connections. The effect on the additional wavelength requirements to guarantee single link failure restoration is also considered. The results can be used to analyze and optimize the WRON design.

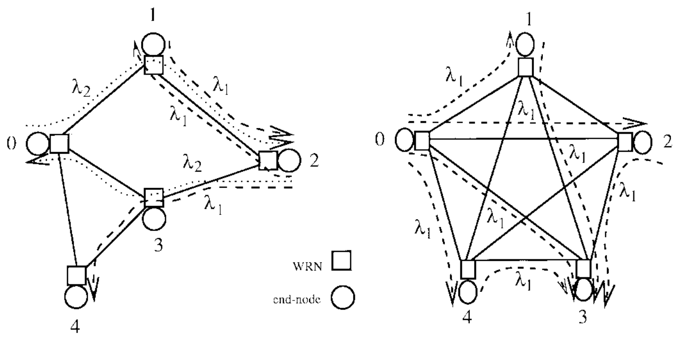


Fig. 1. (a) Example of five-node six-link arbitrarily connected network. (b) Physically fully connected network with $N = 5$.

II. Network Model

The network physical topology consists of N nodes arbitrarily connected by L bidirectional fibers. Each node consists of two parts: the end-node and the WRN. The end-nodes emit and terminate the lightpaths, whilst the WRN's route the lightpaths from sources to destinations. In contrast to regular topologies where the routing is performed according to an appropriate algorithm [17], in arbitrarily connected networks the WRN's are provided with routing tables which are used for the routing functions, as defined by the routing algorithm.

It is assumed that no wavelength translation or opto-electronic processing is performed in the intermediate WRN's, determining a logical single-hop nature of the network. An example of a five-node six-link arbitrarily connected network is shown in Fig. 1(a).

We consider only one bidirectional fiber between each pair of connected nodes. This represents the worst case scenario for wavelength requirement, since in the case of more than one fiber per connection, the number of wavelengths is reduced by the availability of a larger number of alternative physical links. The consequence of removing this constraint is analyzed in Section VI.

We postulate that $(C1)$ any two subparts of the network must be connected by at least two links. This is a fundamental requirement for network reliability, guaranteeing that in the case of a single link failure, the network remains connected and restoration lightpaths can be established along alternative physical paths. As a direct consequence, $(C2)$ the minimum degree of all the nodes is 2 $(\delta_{\min} = 2)$. The analysis of single link failure restoration is presented in Section VII.

The *physical connectivity* α is defined as the normalized number of bidirectional links with respect to a physically fully connected network of the same size [19]

$$\alpha = \frac{L}{L_{FC}} = \frac{2 \cdot L}{N \cdot (N-1)}. \tag{1}$$

A *uniform traffic demand* is assumed, where all the node-pairs are assigned a lightpath consisting of a physical path and a unique wavelength. This is performed with zero wavelength-blocking, i.e., two lightpaths sharing a common physical link are assigned different wavelengths. However, the analysis in this paper can be extended to include a nonuniform traffic demand without loss of generality, but is outside the scope of this paper.

We consider a direct connection between the end-node and the WRN, such that any transmitted wavelength can directly access any of the output fibers. As an example, consider a physically fully connected network with N nodes, as shown in Fig. 1(b) for $N = 5$. Each end-node is connected by a direct physical link to the other $N-1$ end-nodes, and each end-node pair is therefore characterized by a distinct physical path which makes the corresponding lightpath unique. As a consequence the same wavelength can be used from any end-node to all the others [see, for example, the transmitting end-node 0 in Fig. 1(b)]. The same wavelength (λ_1) can be used by all the transmitting end-nodes, such that in a fully connected networks $(\alpha = 1)$ only one wavelength is necessary $(N_\lambda = 1)$. In the arbitrarily connected networks, the reduced number of fibers $(\alpha < 1)$ leads to a higher number of wavelengths required $(N_\lambda > 1)$ and it is the aim of this paper to evaluate this relationship.

It is assumed that the networks provide $N \cdot (N-1)$ lightpaths, one for each end-node pair. However, the number of actual channels simultaneously active depends on the number of transmitters and receivers at each end-node. If $N-1$ (fixed-tuned) transmitters and receivers are provided to the end-nodes, any of these can simultaneously transmit to all the others, with the $N \cdot (N-1)$ lightpaths active in the network. In this case, no coordination is necessary between end-nodes and WRN's and therefore the management overhead is considerably reduced. As in [17], the *network efficiency* η is defined as the ratio between the maximum number of lightpaths that can be established and the total number of lightpaths provided by the network; in this case, $\eta = 1$. The maximum network throughput (*transport capacity*) is $T_c = N \cdot (N-1) \cdot R_b$ where R_b represents the bit-rate for all channels.

In the case of one (tunable) transmitter and receiver, any end-node can transmit and receive only one channel at a time. Therefore, network coordination is necessary between end-nodes to negotiate the communications to satisfy the traffic demand. The network efficiency is $\eta = 1/(N-1)$ and the maximum throughput $T_c = N \cdot R_b$. In case of only one transmitter and receiver per end-node, more accurate results in terms of N_λ may be achieved considering the permutation routing problem. But this analysis is prohibitive because it implies that all the $N!$ permutations should be considered together with the physical topology to evaluate the maximum network wavelength requirement.

III. Lower Limit

A lower limit on the number of wavelengths can be obtained as follows. Consider a subset C of links, i.e., a *network cut*, whose elimination is a necessary and sufficient condition for originating two disjoint and self-connected subgraphs consisting of K and $N-K$ nodes, respectively. Since each of $K \cdot (N-K)$ node-pairs requires a lightpath through the C links, the minimum number of distinct wavelengths for that particular cut C is given by

$$W_C = \left\lceil \frac{K \cdot (N-K)}{C} \right\rceil \tag{2}$$

where $\lceil x \rceil$ represents the lowest integer greater or equal to x. In general the different cuts C within the network originate different values of W_C and the greatest one determines the theoretical lower limit $W_{\rm LL}$ on the number of wavelengths for the whole network [8], [19]

$$W_{\rm LL} = \max_{{\rm all}\, C} W_C = \max_{{\rm all}\, C} \left\lceil \frac{K \cdot (N-K)}{C} \right\rceil. \quad (3)$$

The cut which produces the lower limit is referred to as the *limiting cut*.

As an example consider a graph consisting of two disjoint and fully-connected subgraphs each of $N/2$ nodes, connected by two links. Then $W_{\rm LL} = \lceil N^2/8 \rceil$ even though the diameter of the graph is three and the minimum nodal degree is $\delta_{\min} = N/2 - 1$.

The lower limit may not always be achieved if particular routing rules are imposed, as it does not determine the routing of lightpaths within the network. However, it is a very useful measure of the efficiency of any lightpath allocation algorithm, and is used to verify the obtained results throughout the paper.

IV. Lightpath Allocation

A heuristic algorithm has been developed for lightpath allocation [19], which includes a number of new modifications compared to those published previously (for example [8]–[11]). First, the physical paths are assigned to all the end-node pairs following the *minimum number of hops* (MNH's) algorithm. This guarantees that each logical path passes through the minimum number of physical links and hence WRN's, minimizing the total and average transit traffic. Any other path-assignment algorithm generates a higher volume of transit traffic and leads to an unnecessary and expensive overdimensioning of the WRN's. This is also particularly important for minimizing the crosstalk penalties associated with the physical limitations of the WRN's.

In a network with N nodes, there exist $N \cdot (N-1)$ node-pairs and therefore $[N \cdot (N-1)]!$ different ways they can be ordered and assigned paths. In our algorithm this is done randomly. Since usually more than one MNH's path exists between each node-pair a certain degree of freedom is available and is used to balance, as evenly as possible, the paths among all the links. This contributes to the reduction of the number of lightpaths to be rerouted in case of a link failure (considering the same probability of failure for all links) and to minimize the network wavelength requirement. The *physical paths allocation* is performed as follows:

1) all the source-destination pairs are randomly ranked and assigned the first found MNH's path;
2) for each node-pair considered: an alternative MNH's path substitutes the one previously assigned if and only if the number of channels (congestion) of the most loaded link in the alternative path is lower than the congestion of the most loaded link in the previously assigned path. This process is repeated for all node-pairs;
3) Step 2) is repeated until no more substitutions are possible.

The wavelengths are then assigned to paths. Again there exist $[N \cdot (N-1)]!$ different ways in which the paths can be ordered and assigned wavelengths. In our algorithm the longest paths are assigned wavelength first. Intuitively these are harder to allocate because a *free* wavelength has to be found on more links. If two or more paths have the same length (in terms of hops) the algorithm randomly selects the one to be assigned wavelength first. The *wavelength allocation* is performed using the following steps:

1) paths with same length (in terms of hops) are grouped in common sets. Sets are ranked in order of decreasing number of hops;
2) randomly select from the first set the first path to be assigned a wavelength;
3) assign to the selected path the lowest wavelength-number previously unused on any edge;
4) if, at least, another path is present in the same set, randomly select the next path and go to Step 3); otherwise if at least another set is present, go to the next set, randomly select the path and go to Step 3).

The highest assigned wavelength-number determines the network wavelength requirement (N_λ).

V. Results

A. Real Networks

The above lightpath allocation algorithm was first applied to several existing or planned network topologies to verify its efficiency of the lightpath allocation algorithm and evaluate network topological parameters such as the physical connectivity α, average internodal distance $\bar{H}$ (in terms of number of hops) network diameter D and the minimum and maximum nodal degree $(\delta_{\min}, \delta_{\max})$. The networks considered are the ARPANet [10], NSFNet [20], the European Optical Network (EON)[1] proposed in [21] and a hypothetical UK topology reflecting the current BT-network [22]. The main topological features of these networks and obtained results are presented in Table I. The networks are ranked in increasing value of α and the dotted lines identify the limiting cuts.[2]

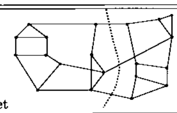
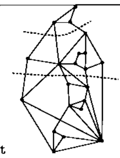
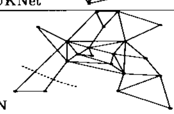
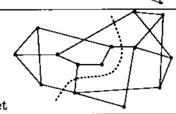
It is important to stress several of the following points:

- α varies from 0.16 to 0.23. This represents the range of α of interest for real networks, and thus the focus of current analysis;
- an increase of α determines a decrease of $\bar{H}$, D and N_λ, as expected;
- the lightpath allocation algorithm works efficiently yielding the number of wavelengths equal or very close to the lower limit $W_{\rm LL}$. In all the networks, N_λ was equal to the number of channels in the most loaded link(s), implying that wavelength translation does not lead to a reduction in the network wavelength requirement.

[1] In the EON [21] a link between Luxemburg and Amsterdam was added in order to satisfy the constraints $(C1)$ and $(C2)$.

[2] In the UKNet the central cut determines the lower limit $W_{\rm LL}$, whilst the upper cut will determine the limit $W''_{\rm LL}$ in the link failure restoration mode, as shown in Section VII.

TABLE I
MAIN TOPOLOGICAL PARAMETERS AND RESULTS FOR EXISTING OR PLANNED NETWORK TOPOLOGIES. THE DOTTED LINES REPRESENT THE LIMITING CUTS

Network	N	L	α	$\overline{H}$	D	$(\delta_{min},\delta_{max})$	W_{LL}	N_λ	N_λ/W_{LL}
 ARPANet	20	31	0.16	2.81	6	(2,4)	33	33	1
 UKNet	21	39	0.19	2.51	5	(2,7)	19	22	1.16
 EON	20	39	0.2	2.36	5	(2,7)	18	18	1
 NSFNet	14	21	0.23	2.14	3	(2,4)	13	13	1

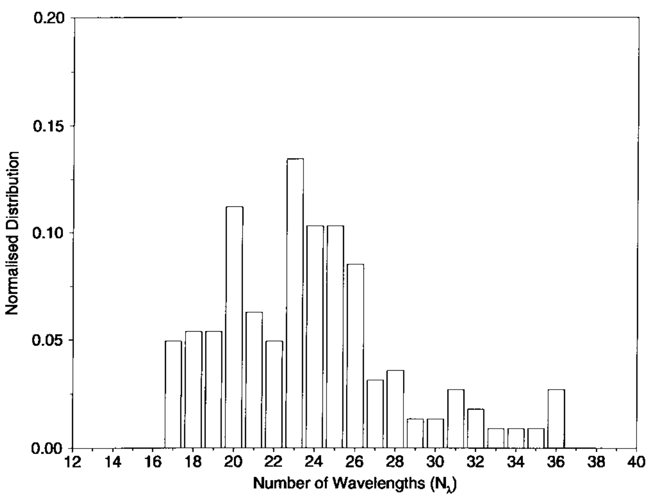


Fig. 2. Normalized distribution of the number of wavelengths, N_λ for RCN's with $N = 14$ and $\alpha = 0.18$.

B. Randomly Connected Networks

To generalize the results for wavelength requirements a large number of random networks satisfying constraints $(C1)$ and $(C2)$ were analyzed for $0.1 \leq \alpha \leq 0.4$. We refer to these networks as *randomly connected networks* (RCN's), and they are generated as follows. Given a number of nodes N and connectivity α, a randomly selected link is added at a time until the value of α is reached. A uniform probability distribution is considered, such that all the $N \cdot (N-1)/2$ links have the same probability to be selected. A new link can be accepted only if it is not already present and the nodal degree of both the interconnecting nodes do not exceed a previously defined maximum degree $\delta_{\max}$, whose value is determined by N and α, as described below. To verify that this random process did not result in an unconnected network, a step was performed to ascertain the constraints $(C1)$ and $(C2)$, with only the connected networks analyzed.

To ensure that the test set of generated networks contained only distinct network topologies, a vector consisting of several topological parameters was assigned to each of them

$$V = (n_2, n_3, n_4, \cdots, n_{\delta_{\max}}, n_{2\cdot D}, n_{3\cdot D-1}, n_{\delta_{\max}\cdot 1}, D, \bar{H})$$

where n_i represented the number of nodes with degree $\delta = i$ and $n_{i\cdot j}$ the number of node-pairs both with degree $\delta = i$, and j hops away from each other. Having different vectors is a sufficient condition for two networks to be topologically different, hence any new generated network was accepted only if its vector was different with respect to the previous ones.

The average nodal degree is given by

$$\overline{\delta} = \frac{2 \cdot L}{N} = \frac{N \cdot (N-1) \cdot \alpha}{N} = (N-1) \cdot \alpha. \qquad (4)$$

Without limiting the nodal degree, a large number of RCN's were generated for different values of N and α and the nodal degree distribution was found to be normally distributed centered around $\overline{\delta}$, with standard deviation σ dependent on N and α. In particular for a given N, σ increased with an increase of α (up to $\alpha = 0.5$). Similarly for a given α, σ increased with an increase of N. Typical values of σ used were between 1.5 and 3. The maximum nodal degree was therefore defined as

$$\delta_{\max} = \lceil \overline{\delta} + 2\sigma \rceil \qquad (5)$$

to retain over 95% of the possible topologies.

Networks with the same N and α can have different physical topologies, hence different wavelength requirements. A statistical analysis showed that a few thousand different topologies were sufficient to generate stable distributions of N_λ.

In Figs. 2–3 the normalized distributions[3] of N_λ for RCN's with $N = 14$ are plotted for different values of α. For $\alpha = 0.18$ (Fig. 2) the distribution assumes a wide range of values, given the different topologies of the generated RCN's. The distribution is bimodal with peaks centered respectively around $N_\lambda = 20$ and 23. The average value is $N_\lambda = 22.6$ and the range which contains 95% of the results is $N_\lambda = 17$–32.

[3] The normalization is performed with respect to the total number of analyzed networks, i.e., the total area of the histogram is 1.

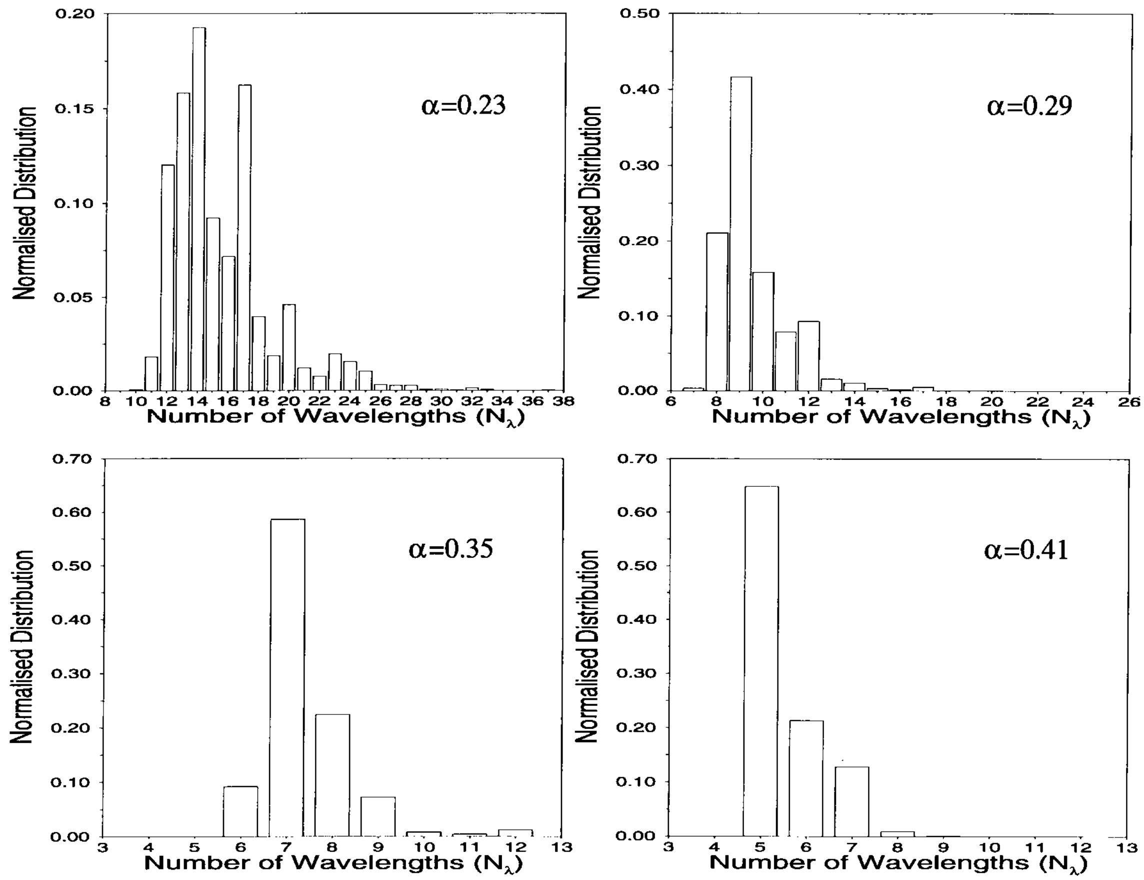


Fig. 3. Normalized distribution of N_λ forRCN's with $N = 14$ for different values of α.

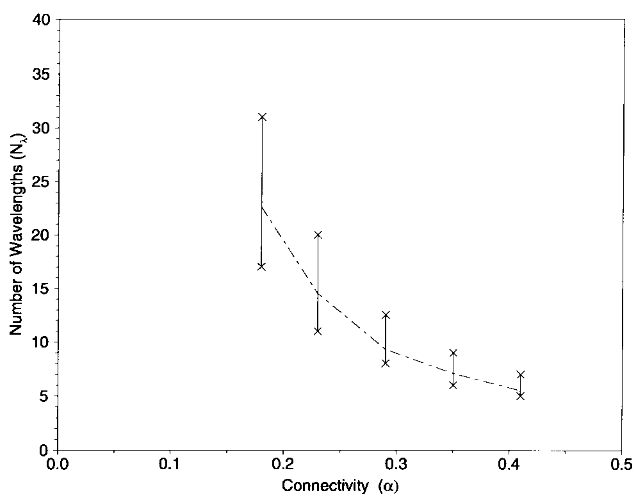


Fig. 4. Wavelength requirements for RCN's with $N = 14$ versus the physical connectivity α. The bars represent the ranges containing the 95%of the results and the dashed lines the mean values fit.

An increase in α leads to a decrease in N_λ, since lightpaths can be mapped onto a greater number of links. Consequently, the distribution shifts toward lower values and becomes narrower (Fig. 3).

The results showed that for low values of the physical connectivity $(\alpha < 0.25)$ small variations of the diameter D generate large variations in N_λ, and networks with higher diameters have a higher wavelength requirement. As α increases $(\alpha > 0.25)$ the influence of the diameter on N_λ becomes smaller, as can be seen by the narrowing of the distribution.

In Fig. 4 the mean values and the ranges containing the 95% of the results are plotted versus the physical connectivity α. As already seen, both the mean values and the ranges decrease with increasing connectivity.

The same analysis was performed for RCN's with a different number of nodes $(N = 20\text{–}50)$. The results showed similar behavior to the case with $N = 14$ and in Fig. 5 the *mean values* of the distributions are plotted versus α. It is interesting to note that the mean values of the wavelength requirements are independent of the network size N. [Similarly, for a given α, the 95% ranges were found comparable and almost independent of N]. A clear trade-off exists between the mean values of N_λ and the connectivity α, relationship *quantified* by the results of Fig. 5. It is shown that on average RCN's achieve a full logical connectivity with a modest number of wavelengths. For example no more than 16 and 8 wavelengths are necessary for $\alpha \geq 0.2$ and 0.3, respectively.

The results of the real networks are also shown. It can be seen that UKNet, EON, NSFNet match well the general behavior, whereas the ARPANet has a slightly higher wavelength requirement, as consequence of its low value of α and high value of the diameter.

A complete analysis of all the possible topologies was performed for networks with $N = 5$ and 6, given their relatively small number. In order to satisfy the constraints $(C1)$ and $(C2)$, at least $L = N$ links are necessary (in the

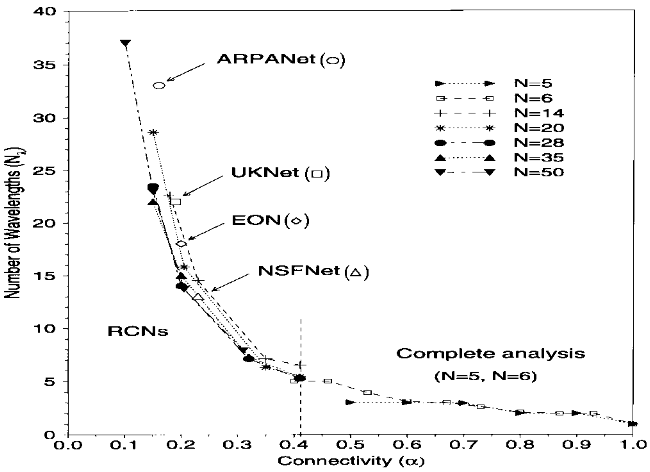


Fig. 5. Mean values of N_λ versus physical connectivity α, as a function of the number of nodes N.

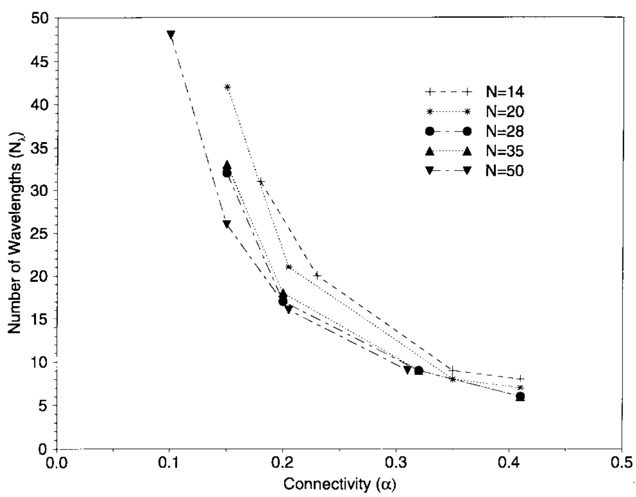


Fig. 6. Number of wavelengths (upper bound) for 95% of the RCN's versus physical connectivity α, as a function of the number of nodes N.

ring configuration), such that the range of α is

$$\frac{2}{N-1} \leq \alpha \leq 1. \tag{6}$$

For $N = 5$ and 6 high values of α were obtained ($\alpha \geq 0.5$ and 0.4, respectively), leading to narrow distributions. The mean values are plotted in Fig. 5. It is shown that as α increases the wavelength requirement decreases reaching $N_\lambda = 1$ for $\alpha = 1$. Moreover it can also be seen that these results correspond well with those obtained for the RCN's, confirming the validity of the RCN's modeling results.

Fig. 6 shows the values of N_λ below which the 95% of the RCN's lie, defining the upper bound on wavelength requirements. It can be noted that, for example, for $\alpha > 0.3$ the 95% of all the generated networks require less than 16 wavelengths.

C. Regular Networks

For comparison with arbitrarily connected networks, two well known and previously studied regular physical topologies were considered, namely the Shufflenet SN(Δ, k) [23] and de Bruijn graph deB(Δ, d) [15] where each directed link was replaced by a bidirectional one. Their topological features and the results are summarized in Table II.

TABLE II
MAIN TOPOLOGICAL PARAMETERS AND RESULTS FOR REGULAR NETWORKS

Network	N	L	α	$\overline{H}$	D	N_λ
SN(2,2)	8	12	0.43	1.71	3	5
SN(2,3)	24	48	0.17	2.39	4	19
SN(2,4)	64	128	0.063	3.42	6	68
SN(3,2)	18	45	0.29	1.94	3	8
SN(3,3)	81	243	0.075	2.80	4	45
SN(4,2)	32	112	0.23	2.06	3	11
SN(5,2)	50	225	0.18	2.14	3	15
SN(6,2)	72	396	0.15	2.20	3	18
deB(2,3)	8	13	0.46	1.64	3	6
deB(2,4)	16	29	0.24	2.14	4	12
deB(2,5)	32	61	0.12	2.75	5	30
deB(3,2)	9	21	0.58	1.42	2	3
deB(3,3)	27	75	0.21	2.08	3	12
deB(3,4)	81	237	0.07	2.83	4	47
deB(4,2)	16	54	0.45	1.55	2	4
deB(4,3)	64	246	0.12	2.32	3	23
deB(5,2)	25	110	0.37	1.63	2	6
deB(6,2)	36	195	0.31	1.69	2	8
deB(7,2)	49	315	0.27	1.73	2	9
deB(8,2)	64	476	0.24	1.76	2	10
deB(5,3)	125	610	0.079	2.47	3	38

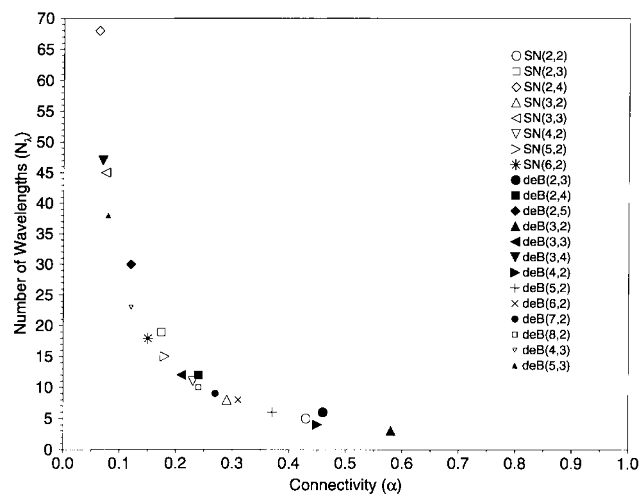


Fig. 7. Number of wavelengths N_λ versus physical connectivity α for regular networksShufflenet and de Bruijn.

In Fig. 7 the required N_λ for the regular networks are plotted versus α. These results lie on the curve describing the mean values for RCN's (Fig. 5), confirming that randomly connected networks have similar topological features of regular topologies [24] and hence similar wavelength requirements.

Conventionally, the analysis of regular topologies has attracted interest because of inherent simplicity of routing rules [17]. However, the presented results lead to an important conclusion that whilst arbitrarily connected networks have similar wavelength requirements, they also have the added advantages of scalability and flexibility, required for network evolution.

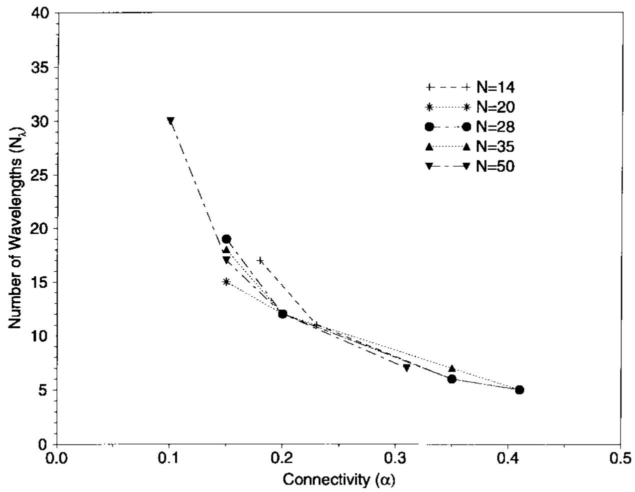


Fig. 8. Minimum values (lower bound) of N_λ for RCN's versus physical connectivity α, as a function of the number of nodes N.

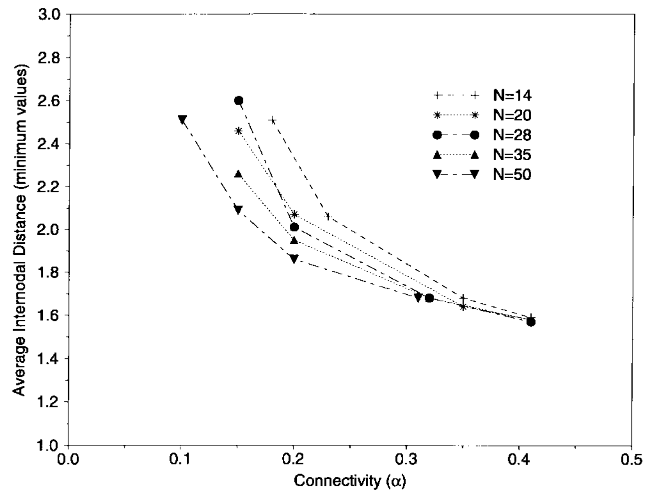


Fig. 9. Minimum values of the mean internodal distance, $\bar{H}$ versus physical connectivity α, as a function of the number of nodes N.

D. Discussion

In Fig. 8 the *minimum values* of the distributions of N_λ for RCN's are plotted versus α. Similarly to the mean values, they are dependent only on α, and independent of N. The following equation

$$N_{\lambda_{\min}} = \frac{k}{\alpha} - (k-1) \tag{7}$$

provides a good fit of the curve with $k = 3$ for $0.1 \le \alpha \le 0.4$.

An analytical relationship which gives a lower limit on N_λ as a function of α can be found as follows. In a network with N nodes and L links, replacing each link with a bundle containing $1/\alpha$ fibers leads to the total number of links of $L/\alpha = L_{FC}$. The derived network has the same number of links as the physically fully-connected one, representing a necessary (but not sufficient) condition for having $N_\lambda = 1$. Suppose the required number of wavelengths for the derived network is $N_\lambda^* = 1$. Assuming equivalence between fibers and wavelengths, it is possible to exchange the $1/\alpha$ fibers per bundle with $1/\alpha$ wavelengths per link in the original network. Hence the required number of wavelengths of the original network is $N_\lambda = 1/\alpha$. However the L_{FC} links in the derived network do not directly connect all the node-pairs, i.e., they are in the *wrong position*, hence $N_\lambda^* = \gamma > 1$, such that

$$N_{\lambda_{\min}} = \frac{\gamma}{\alpha} \tag{8}$$

Only for $\alpha = 1$, $\gamma = 1$ and the minimum value is achieved ($N_\lambda = N_{\lambda_{\min}} = 1$).

The value of γ can be evaluated in the following way. The total number of links utilized by all the network lightpaths is $N \cdot (N-1) \cdot \bar{H}$ where $\bar{H}$ represents the average number of hops between each node-pair, as defined before. Since the total number of unidirectional links in the network is $2 \cdot L$, the minimum N_λ is obtained when all these lightpaths are evenly distributed among all the links

$$N_{\lambda_{\min}} = \frac{N \cdot (N-1) \cdot \bar{H}}{2 \cdot L} = \frac{L_{FC} \cdot \bar{H}}{L} = \frac{\bar{H}}{\alpha}. \tag{9}$$

Again it can be seen that for $\alpha = 1$, $\bar{H} = 1$ and the minimum value is achieved ($N_\lambda = N_{\lambda_{\min}} = 1$).

The distributions of the average internodal distance $\bar{H}$ for all the generated RCN's were analyzed and it was found that they can be fitted well with a normal distribution. In Fig. 9, the *minimum values* of the distributions ($\bar{H}_{\min}$) are plotted versus the physical connectivity α. It is shown that for any network size an increase in the connectivity results in a decrease in the minimum values of $\bar{H}$, as expected. As shown for low values of α, $\bar{H}_{\min}$ decreases with an increase of N. The influence of the network size decreases with an increase of α and for $\alpha \ge 0.3$, $\bar{H}_{\min}$ is almost independent of N. Given the spread of the results, it is not possible to find a quantitative relationship between $\bar{H}_{\min}$ and α independent of N. However the results of Fig. 9 can be used in (9) to calculate the minimum required number of wavelength $N_{\lambda_{\min}}$. For example for $N = 28$ and $\alpha = 0.25$, $\bar{H}_{\min} = 1.9$, hence from (9) $N_{\lambda_{\min}} = 7.6$. From (7) for $\alpha = 0.25$, $N_{\lambda_{\min}} = 10$. This difference is due to the uneven distribution of the lightpaths among the network links, which decreases as the connectivity increases. For example, for $\alpha = 0.4$, $\bar{H}_{\min} = 1.6$ (independent of N), hence $N_{\lambda_{\min}} = 4$. From (7) for $\alpha = 0.4$, $N_{\lambda_{\min}} = 5.5$. This confirms that by increasing the number of links, the lightpaths can be more evenly spread among them, determining a lower wavelength requirement approaching the lower limit and providing a more efficient link utilization.

It is worthwhile to note that exact lower bounds on the number of wavelengths have been analytically derived in [25]. They are in very good agreement with the results reported in this paper.

VI. Multifiber Networks

Given the growth of opto-electronic technology, networks requiring 8–16 wavelengths could be implemented in the next decade [26]. From the results shown in Section V, it follows that if single-fiber connections are used, practical topologies need a physical connectivity $\alpha > 0.2$. This condition may not be achieved, especially for networks with a high number of nodes, since physical links are expensive to install. An obvious alternative approach is the use of multifiber connections between the nodes, an option particularly attractive where the

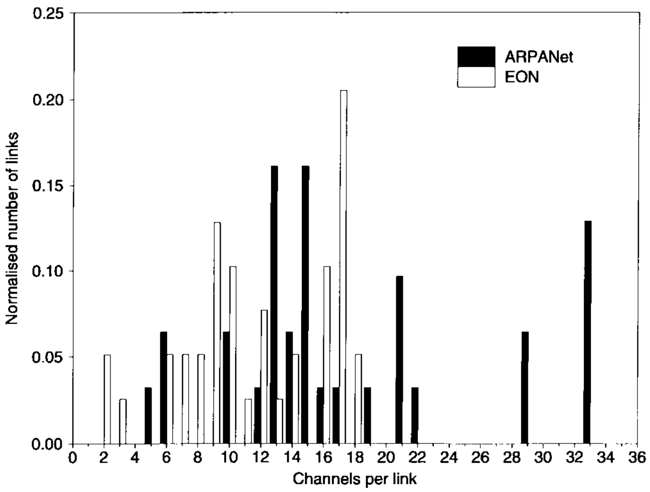


Fig. 10. Links loading in ARPANet and EON.

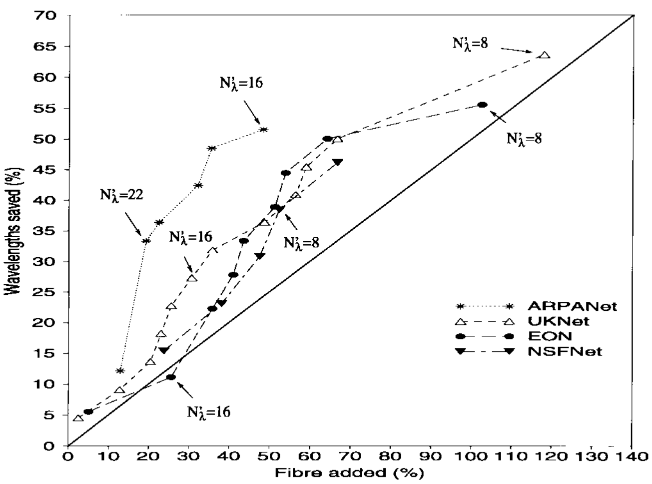


Fig. 11. Wavelength saving versus percentage fiber added. The solid line represents the savings achievable with a nonselective duplication of the network links.

physical topology is already defined and free multiple fibers are available.

By replacing each connection with a bundle containing P bidirectional fibers, a new lower limit $W'_{\rm LL}$ can be obtained from (3) substituting C with $C' = P \cdot C$. For example for $P = 2$ (100% fiber added), $W'_{\rm LL} = W_{\rm LL}/2$ and 50% of wavelength saving can be achieved (see solid line in Fig. 11). However, depending on the physical topology, the selective duplication of only a few links may lead to a significant reduction of N_λ, as shown below.

Consider as an example the normalized distribution of the number of lightpaths passing through the links in the ARPANet and EON (Fig. 10). As shown, the ARPANet has a very different loading between its links. This is due to its physical topology where the links in the limiting cut are much more loaded compared to the others. In this case the duplication of only these links results in a large reduction of N_λ. Conversely, in the EON the links are quite evenly loaded, hence N_λ can be reduced only at the expense of adding more fibers.

This is illustrated in Fig. 11, where the percentage of saving in the required number of wavelengths is plotted as a function of extra links added. For the ARPANet setting $P = 2$ in 6 bidirectional links (corresponding to 19% of the total fiber) allows to achieve $N'_\lambda = 22$ (a reduction of 33% in N_λ). To obtain $N'_\lambda = 16$ (saving 51% of the wavelengths) it is sufficient to set $P = 3$ in the four links carrying 33 lightpaths, and $P = 2$ in the eight links carrying 17-to-29 channels, with a total fiber added of only 48%. As shown, this curve lies well above the solid line, confirming that a selective duplication of links results in a significant reduction of N_λ.

Similarly, for the UKNet, EON and NSFNet, a required number of wavelength $N'_\lambda = 16$ or 8 can be achieved by adding fiber. For example, in the EON, $N'_\lambda = 16$ is obtained by setting $P = 2$ in the 5 most loaded links (25% fiber added), and $N'_\lambda = 8$ with $P = 3$ for them and $P = 2$ for the 20 links carrying 9-to-16 channels (102% fiber added). However, for these three networks, reductions in N_λ are achieved at the cost of more fiber added, as witnessed by the less steep slope of their curves (close but still above the solid line).

In summary, an optimized topology must have links loaded as evenly as possible, and hence the replacement of heavily loaded links with multifiber connections leads to a reduction of N_λ.

VII. Link Failure Restoration

In transport network applications no channel blocking is allowed, therefore link failures must be accompanied by a complete restoration of the lightpaths involved.

Consider a network cut with C links. If a link carrying w lightpaths fails, these channels must now be distributed over $C'' = C - 1$ links. Hence a new lower limit $W''_{\rm LL}$ can be obtained from (3) by replacing C with C''. If the new limiting cut results from the original one, the increment in the lower limit is $100/C''$% (see LL variation curve in Fig. 12)

$$W''_{\rm LL} = W_{\rm LL}\left(1 + \frac{1}{C''}\right). \tag{10}$$

Consider, for example, the EON where the limiting cut consists of $C = 2$ links. The new lower limit originates from the same cut determining an increment of 100% ($W''_{\rm LL} = 36$). Similarly, the new limiting cuts of ARPANet and NSFNet derive from the original ones ($C = 3$ and 4, respectively) therefore an increment of 50 and 33% is respectively obtained ($W''_{\rm LL} = 50$ and $W''_{\rm LL} = 17$). In the UKNet, on the contrary, the new lower limit does not derive from the original limiting cut, since the upper cut consisting of only $C = 3$ fibers sets $W''_{\rm LL} = 27$, with an increment of about 50% (see footnote 2).

This implies that all the original cuts C where $W_C \simeq W_{\rm LL}$ must consist of as many links as possible, to limit the increment in the lower limit and hence in the number of additional wavelengths required for restoration.

As already discussed, the lower limit may not be achieved if routing rules are imposed. In particular we consider the case of only the lightpaths passing via the failed link rerouted, without involving the other network channels. This is a consequence of the fact that in transport network applications live-traffic cannot be interrupted.

The restoration algorithm works as follows. The lightpaths to be rerouted are ranked in order of decreasing original length

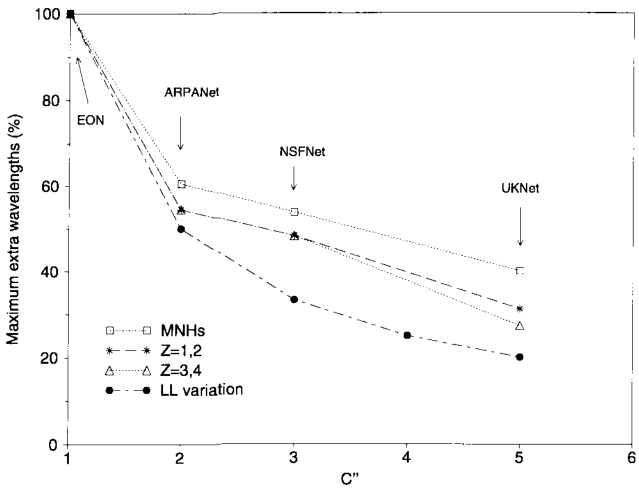


Fig. 12. Maximum extra wavelengths versus number of links in the new limiting cut C''.

TABLE III
ADDITIONAL WAVELENGTH REQUIREMENTS FOR SINGLE LINK FAILURE RESTORATION

Network	C''	W''_{LL}	N''_λ	$(\frac{\Delta N_\lambda}{N_\lambda})_M$ (%)	$(\frac{\Delta N_\lambda}{N_\lambda})_{av}$ (%)
EON	1	36	36	100	29.8
UKNet	2	27	37	68.2	22.5
	5	22	31	40.1	-
ARPANet	2	50	53	60.6	21.1
NSFNet	3	17	20	53.8	33.3

(in terms of hops) and assigned alternative MNH's paths. Since for each node-pair more than one MNH's paths may exist, the one requiring the lowest wavelength number is assigned. This allows the wavelength of the channel to be restored to be changed, if necessary, whilst maintaining end-to-end logical transparency. MNH's paths are selected for the reasons stated in Section II.

The four real topologies were analyzed and the new wavelength requirements N''_λ evaluated for all the possible single link failure restoration scenarios. The results are shown in Table III, where C'' represents the number of links in the new limiting cut, $\Delta N_\lambda = N''_\lambda - N_\lambda$ and $(\Delta N_\lambda/N_\lambda)_M$ and $(\Delta N_\lambda/N_\lambda)_{\rm av}$ give the maximum and average increment in the wavelength requirement (in %). It can be noted that the maximum increments are slightly higher than the theoretical values expected from the lower limit variations. For example in the UKNet and ARPANet the maximum increments are respectively 68.2 and 60.6% with respect to the 50% expected by the new lower limit. In the NSFNet the maximum increment (53.8%) is considerably higher than the theoretical values (33.3%). For the UKNet the maximum increment of N''_λ given by the failure of a link in the original cut ($C'' = 5$) is also shown. Its value (40.1%) is higher than the expected one (25%). This higher values obtained can be explained by the limitations imposed by the rerouting algorithm: only the lightpath involved in the cut are rerouted over only the MNH's paths.

In Fig. 12, the results of $(\Delta N_\lambda/N_\lambda)_M$ for the considered networks are plotted versus C'' (see curve MNH's). For

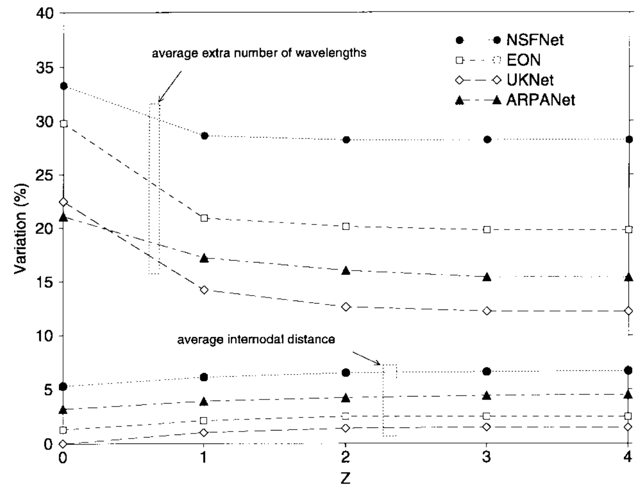


Fig. 13. Average percentage variation in the number of wavelengths $(\Delta N_\lambda/N_\lambda)_{\rm av}$ and mean internodal distance $\bar{H}$ versus the number of additional hops Z.

$C'' = 2$ only the result of the ARPANet is shown, and the one for $C'' = 5$ is obtained by the UKNet original limiting cut. A clear trade-off between the number of links C'' in the new limiting cut and the maximum increment of the wavelength requirement is shown.

In the second step, the reduction of $(\Delta N_\lambda/N_\lambda)_M$ achievable by removing the MNH's constraint was analyzed. In this case for any rerouted lightpath, the restoration path was selected from those whose length was less or equal to the MNH's one plus Z hops, with $Z = 1$–4. The results are shown in Fig. 12. It can be seen that setting $Z = 1$ significant improvements can be achieved and the theoretical values approached. Further increases of Z do not lead to any decrease of $(\Delta N_\lambda/N_\lambda)_M$.

Similar results and tradeoff between $(\Delta N_\lambda/N_\lambda)_M$ and C'' were obtained for several analyzed RCN's. In conclusion, the design of fault-tolerant networks must maximize the number of links in the cuts where the wavelength requirement $W_C \simeq W_{\rm LL}$.

Fig. 13 shows the average increment in the wavelength requirement considering all the possible link failures versus Z. It can be seen that a decrease in $(\Delta N_\lambda/N_\lambda)_{\rm av}$ can be achieved by setting $Z = 1$ and again further increase of Z does not lead to any improvement. Therefore restoration lightpaths slightly longer than the MNH's can be used to reduce the wavelength requirement. It is interesting to note that on average no more than 20–30% extra wavelengths are necessary to fully restore the logical connectivity. Fig. 13 also shows that the increment in the average internodal distance (in terms of hops) was always negligible. Similar results were obtained for several considered RCN's (not shown here).

In all the analyzed situations, the wavelength requirement N''_λ was found to be always equal or very close to the new lower limit W''_{LL}. This implies that wavelength translation does not introduce significant advantages in terms of N''_λ for link failure restoration either.

VIII. CONCLUSION

In this paper the wavelength requirements in arbitrarily connected wavelength-routed optical networks have been studied. A heuristic algorithm for the lightpath allocation was presented and a large number of randomly connected networks were analyzed in order to evaluate bounds on the number of wavelengths necessary to satisfy a uniform logical connectivity. The results showed that the wavelength requirement strongly depends on the physical connectivity, whilst it is almost independent of the network size. Moreover it is shown that WRON's can provide high transport capability with a modest number of wavelengths and that wavelength translation does not lead to a reduction of N_λ.

The advantages achievable by the use of multifiber connections was demonstrated by the analysis of several existing or planned network topologies. It was shown that the selective duplication of heavily loaded links can result in a significant reduction in the wavelength requirement.

Finally, the consequence of single link failure restoration was analyzed. A clear trade-off between the number of links C in the cuts where $W_C \simeq W_{\mathrm{LL}}$ and the maximum increment of the wavelength requirement was identified. Moreover the analysis of several real networks has shown that on average no more than 20–30% extra wavelengths are necessary to fully restore the logical connectivity. Also, in this case, wavelength translation did not result in significant reduction in the wavelength requirement.

These algorithms and results can be applied to design and optimize the architecture and topology of wavelength-routed optical networks.

ACKNOWLEDGMENT

The authors would like to thank J. E. Midwinter, F. P. Kelly, and R. J. Gibbens for invaluable discussions and suggestions followed in Sections III and V. They would also like to thank P. A. Kirkby, R. Barry, Y. Aumann, P. Sasieni, and R. Ramaswami for useful comments.

REFERENCES

[1] G . R. Hill, "A wavelength routing approach to optical communication networks," in *Proc. IEEE INFOCOM'88*, 1988, pp. 354–362.

[2] S. B. Alexander et al., "A precompetitive consortium on wide-band all-optical networks," *J. Lightwave Technol.*, vol. 11, pp. 714-735, May/June 1993.

[3] C. A. Brackett *et al.*, "A scalable multiwavelength multihop optical network: A proposal for research on all-optical networks," *J. Lightwave Technol.*, vol. 11, pp. 736–753, May/June 1993.

[4] B. Mukherjee, "WDM-based local lightwave networks—Part I: Single-hop systems," *IEEE Network*, vol. 6, pp. 12–27, May 1992.

[5] I. Chlamtac, A. Ganz, and G. Karmi, "Purely optical networks for terabit communication," in *Proc. IEEE INFOCOM'89*, 1989, pp. 887–896.

[6] P. E. Green, Jr., *Fiber Optic Networks*. Englewood Cliffs, NJ: Prentice Hall, 1993.

[7] P. Roorda, C.-Y. Lu, and T. Boutilier, "Benefits of all-optical routing in transport networks," in *Proc. OFC'95*, 1995, pp. 164–165.

[8] N. Nagatsu, Y. Hamazumi, and K.-I. Sato, "Optical path accommodation designs applicable to large scale networks," *IEICE Trans. Commun.*, vol. E-78, pp. 597–607, Apr. 1995.

[9] I. Chlamtac, A. Ganz, and G. Karmi, "Lightpath communications: An approach to high bandwidth optical WAN's," *IEEE Trans. Commun.*, vol. 40, pp. 1171–1182, July 1992.

[10] R. Ramaswami and K. N. Sivarajan, "Routing and wavelength assignment in all-optical networks," *IEEE/ACM Trans. Networking*, vol. 3, pp. 489–500, Oct. 1995.

[11] N. Wauters and P. Demeester, "Wavelength routing algorithms for transparent optical networks," in *Proc. ECOC'95*, 1995, pp. 855–858.

[12] A. Aggarwal, A. Bar-Noy, D. Coppersmith, R. Ramaswami, B. Schieber, and M. Sudan, "Efficient routing and scheduling algorithms for optical networks," in *Proc. 5th Annu. ACM-SIAM Symp. Discrete Algorithms*, Jan. 1994, pp. 412–423.

[13] R. A. Barry, "Wavelength routing for all-optical networks," Ph.D. dissertation, Dep. Elec. Eng. Comput. Sci., M.I.T., Cambridge, MA, Aug. 1993.

[14] R. K. Pankaj and R. G. Gallager, "Wavelength requirements of all-optical networks," *IEEE/ACM Trans. Networking*, vol. 3, pp. 269–280, June 1995.

[15] K. N. Sivarajan and R. Ramaswami, "Lightwave networks based on de Bruijn graphs," *IEEE/ACM Trans. Networking*, vol. 2, pp. 70–79, Feb. 1994.

[16] E. D. Lowe and M. J. O'Mahony, "Wavelength contention blocking in circuit switched WDM optical networks," in *Proc. ECOC'94*, 1994, pp. 889–892.

[17] M. Ajmone Marsan, A. Bianco, E. Leonardi, and F. Neri, "Topologies for wavelength-routing all-optical networks," *IEEE/ACM Trans. Networking*, vol. 1, pp. 534–546, Oct. 1993.

[18] H. A. Jäger, "WDM-gridconnect as a transport structure," *IEEE Photon. Technol. Lett.*, vol. 7, pp. 576–578, May 1995.

[19] S. Baroni, P. Bayvel, and J. E. Midwinter, "Influence of physical connectivity on the number of wavelengths in dense wavelength-routed optical networks," in *Proc. OFC'96*, 1996, pp. 25–26.

[20] R. Ramaswami and K. N. Sivarajan, "Design of logical topologies for wavelength-routed all-optical networks," in *Proc. IEEE INFOCOM'95*, 1995, pp. 1316–1325.

[21] M. J. O'Mahony, D. Simeonidou, A. Yu, and J. Zhou, "The design of a european optical network," *J. Lightwave Technol.*, vol. 13, pp. 817–828, May 1995.

[22] S. Appleby and S. Steward, "Mobile software agents for control in telecommunications networks," *BT Technol. J.*, vol. 12, pp. 104–116, Apr. 1994.

[23] M. G. Hluchyj and M. J. Karol, "Shufflenet: An application of generalized perfect shuffles to multihop lightwave networks," *J. Lightwave Technol.*, vol. 9, pp. 1386–1397, Oct. 1991.

[24] C. Rose, "Mean internodal distance in regular and random multihop networks," *IEEE Trans. Commun.*, vol. 40, pp. 1310–1318, Aug. 1992.

[25] D. J. Wischik, "Routing and wavelength assignment in optical networks," *Part III Mathematics Tripos*, University of Cambridge, May 1996.

[26] R. Alferness and P. Kaiser, in *OFC'96, 2nd Int. Workshop Optic. Networking*, Mar. 1, 1996.

Stefano Baroni (S'95) was born in Reggio Emilia, Italy, in 1968. He received the Dr.Ing. degree in electronic engineering from the University of Parma, Italy, in October 1993. Since November 1994, he has been pursuing the Ph.D. degree at the Department of Electronic and Electrical Engineering in University College London.

From November 1993 to October 1994, he was with the Telecommunication Group of the Department of Information Engineering at the University of Parma working on the design and performance testing of a prototype of a 622 Mb/s FSK multichannel coherent optical transmission system. His research interests include the analysis and design of wavelength routing and allocation algorithms for application in WDM optical communication networks.

Polina Bayvel (S'87–M'89) received the B.Sc. (Eng.) degree in electronic enginering in 1986 and the Ph.D. degree specializing in nonlinear fiber optics and its applications, in 1990, both from University College London (London University).

After a Visiting Fellowship at the Fiber Optics Department of the General Physics Institute (U.S.S.R. Academy of Sciences), she worked as a Principal Systems Engineer, Network Technology Division, BNR Europe Ltd., Harlow (1991–1993, and 1990–1991 at STC Submarine Systems, Greenwich) on the design and planning of optical fiber transmission networks. In December 1993, she was awarded a Royal Society University Research Fellowship and joined the Electronic Engineering Department, University College London. Her current research interests include multiwavelength optical communications networks, wavelength routing algorithms, transmission limitations, and the associated WDM devices.

Dr. Bayvel is a member of the IEE and is a Chartered Engineer.