Wavelet Denoising of the Electrocardiogram Signal Based on the Corrupted Noise Estimation

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Abstract

We present in this paper an algorithm of filtering the noisy real ECG signal. The classical wavelet denoising process, based on the Donoho et al. algorithm, at the 4th level, appears clearly the P and T waves whereas the R waves undergo considerable distortion. This is due to the interference of the WGN and the free noise ECG detail sequences at level 4. To overcome this drawback, our key idea is to estimate the corrupted WGN and consequently remove the noise interfering R waves at the 4th level detail sequence. Our denoising algorithm was applied to a set of the MIT-BIH Arrhythmia Database ECG records corrupted with a 0 dB WGN which provided an output SNR of around 6 dB and an MSE value of around 0.0011. A comparative analysis using the low pass Butterworth filter and the 4th level classical wavelet denoising provides the output SNR values of around 3 dB and MSE value of around 0.0018; which demonstrates the superior performance of our proposed denoising algorithm.

1. Introduction

The electrocardiogram (ECG) signal is the electrical interpretation of the heart activity; it consists of a set of, well defined, successive waves denoted: P, Q, R, S, and T waves. A great intention has been paid to the adequate and accurate analysis of the ECG signal that would lead to cardiac anomalies diagnosis [1-3]. However, as the major part of real signals; the real picked-up ECG signal is corrupted by several sources of noise: EMG (electromyogram) signal (a high frequency signal related to muscle activity), the BLW (the baseline wandering: a low frequency signal caused mainly by the breathing action), the electrode motion (usually represented by a sharp variation of the baseline) …This corrupted noise prevents considerably the accurate analysis of the ECG signal and useful information extraction. Different works have been established to design filtering algorithms aimed to improve the SNR (signal to noise) values and recovering the ECG waves in different noisy environment [4-7]. Recently, the wavelet theory denoising [8] have been widely exploited in noisy ECG filtering. Several wavelet denoising ECG signal algorithms were developed, exploring each a particular parameter: the wavelet function, threshold calculus, and level decomposition … [9-13]. In this context, we develop in this present work a denoising wavelet algorithm based on the corrupted WGN (white Gaussian noise) estimation.

2. Wavelet denoising

2.1. Continuous wavelet transform (CWT)

The CWT -$Wf(s,\tau)$- is the inner product of a time-varying signal f(t) and the set of wavelets $\psi_{s,\tau}(t)$ given by [14-15]:

$$Wf(s,\tau) = <f, \psi_{s,\tau}> = \frac{1}{\sqrt{s}} \int f(t) \psi^* (\frac{t-\tau}{s}) dt \tag{1}$$

The scaling and shifting the mother wavelet (ψ) with factors of s and τ (with s>0), respectively, generate a family of functions called wavelets given by:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi(\frac{t-\tau}{s}) \tag{2}$$

2.2. Discrete wavelet transform (DWT)

A very common discretization of the CWT, which is a very redundant representation, consists of setting the scale and shift value as: $s= s_0^i$ and $\tau=k\tau_0 s_0^i$ with i and k are integers and s_0 is a real value >1. A practical choice of τ_0 and s_0 consists on setting s_0 to 2 and τ_0 to1 that is $s=2^i$ and $\tau=k.\ 2^i$. This is called dyadic wavelet transform. In this case, the wavelet functions become [16]:

$$\psi_{i,k}(t) = 2^{-1/2} \psi(2^{-i} t - k) \tag{3}$$

Y. Meyer has demonstrated that this setting form of scale and shift parameters constitutes an orthonormal basis for L²(R), that is [16]:

$$d_{i,k}(t) \equiv \langle f(t), \psi_{i,k}(t) \rangle \equiv \int f(t)\psi_{i,k}(t)dt \qquad (4)$$

and

$$f(t) = \sum_i \sum_k d_{i,k}(t).\psi_{i,k}(t) \qquad (5)$$

The DWT consists of applying the discrete signal to a bank of octave band filters based on low and high pass filters l(n) and h(n) respectively; more precisely, the function f(t) would be expressed as follows [17]:

$$f(t) = \sum_{k \in Z} a_L(k)\phi_{L,k}(t) + \sum_{j=1}^{L}\sum_{k \in Z} d_j(k)\psi_{j,k}(t) \qquad (6)$$

with:

$d_j(n) = \langle f, \psi_{j,n} \rangle = \Sigma k\ g'(2n-k)\ a_{j-1}(n) \qquad (7)$

$a_L(n) = \langle f, \phi_{j,n} \rangle = \Sigma k\ h'(2n-k)\ a_{L-1}(n)$

where $\phi(t)$ is called the scaling function associated to the wavelet function $\psi(t)$ governed by the following condition:

$$\int \phi(t).dt = 1 \qquad (8)$$

2.3. Denoising

The objective of wavelet based denoising process is to estimate the signal of interest s(t) (equation 9) from the composite one f(i) by discarding the corrupted noise e(i) [18-19]:

$$f(i) = s(i) + e(i) \qquad (9)$$

The underlying model for the noisy signal is the superposition of the signal –s(i)- and a Gaussian zero mean white noise with a variance of σ^2. The threshold value is computed according to the model of the signal of interest to be estimated -s(i)- and the corrupted noise –e(i)-. Donoho and Jonhstone proposed the universal 'VisuShrink' threshold given by [8]:

$$Thr = \sigma\sqrt{2.\log(N)} \qquad (10)$$

In the case of white noise, its standard deviation can be estimated from the median of its detail coefficients (d_j), with j=1..L, and is computed as follows:

$$\sigma = \frac{MAD(|d_j|)}{0.6745} \qquad (11)$$

where MAD is the median absolute deviation of the corresponding sequence.

Two algorithms of thresholding exist: Hard and Soft thresholding algorithms (T_{soft} and T_{hard} respectively) expressed as follows:

$$T_{soft} = sgn(x).(|x|.Thr) \qquad (12)$$

$$T_{hard} = x.1(|x| > Thr)$$

The wavelet based denoising process is summarized as follows: the resulting DWT detail coefficients are thresholded by either shrinkage (soft) or crude (hard) strategy. Reconstructing the original sequence from the thresholded wavelet detail coefficients leads to a de-noised (smoothed) version of the original sequence

3. Algorithm description

Our denoising algorithm is intended to smooth, mainly, the very noisy ECG signal corrupted by a 0 dB WGN. Applying the classical wavelet denoising scheme [8], i.e. the universal threshold and 'soft' strategy, at level 4 appears clearly the P and T waves whereas the R waves undergo a considerable distortion. Our study, carried out on the wavelet coefficients (detail sequences) of both free noise ECG signal and the corrupted WGN signal, show that their amplitudes interfere at the DWT level 4. Our key idea was to discard the WGN signal that has to be estimated in anterior phase. The analysis of the different DWT levels show that the 1st level detail sequence of the noisy ECG signal is highly dominated by the WGN energy (figure 1-a) while the 1st level approximation sequence is, approximately, the superposition of the WGN and a set of peaks which are related to the R waves (figure 1-b). Discarding these peaks will recover the 1st level approximation sequence of the WGN signal which, in conjunction with the 1st level detail sequence, allows the construction of the estimated corrupted WGN by the means of the classical inverse DWT (IDWT).

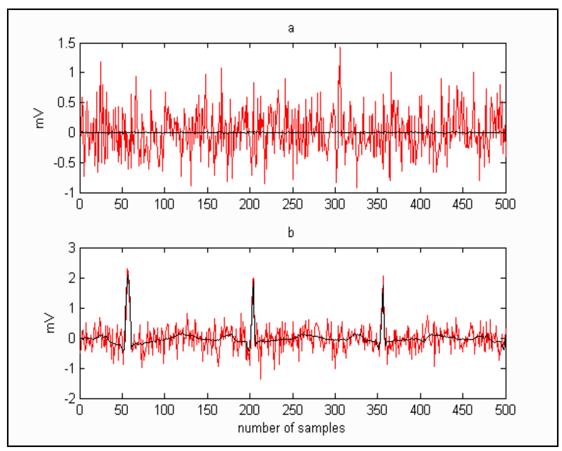

Figure 1. The correlation of the 1st level DWT detail coefficients (the top) and the approximation coefficients (bottom) of the noisy ECG signal (in black) and the corrupted WGN noise (in red).

Our denoising algorithm is composed of two successive phases: the pre-processing and the smoothing process.

The pre-processing phase consists of:

1) High pass filtering of the noisy ECG signal, with a very low cut-off frequency of 0.18 Hz, to suppress the DC offset present at the original free noise ECG signal;

2) Determining the best suitable wavelet function where our experiments have led to use the wavelet function "Symlet 8" based on the reduced value of the MSE (mean square error) value obtained.

The smoothing process consists of:

1) To apply one level DWT to the noisy ECG signal and to estimate the WGN signal;

2) To apply the DWT to the noisy ECG signal at level 4 and identify the resulting 4 detail sequences (cD1, cD2, cD3, and cD4) and the approximation sequence (cA4);

3) To apply the DWT to the estimated WGN signal and identify the 4^{th} level detail sequence (cDN4);

4) To generate the 4^{th} level detail sequence of the ECG free noise (cDF4) given by: cDF4 ≡ cd4 – cDN4;

5) To compute the used denoising threshold (T), given by T ≡ (2*log(N)) 1/2 * median(abs(cD1))/0.6745 where N is the length of the ECG signal;

6) To threshold the set of the detail sequences (cD1, cD2, and cD3), with respect to the computed threshold (T), which results the set of the sequences (cDT1, cDT2, and cDT3);

7) To reconstruct the denoised ECG signal using the IDWT giving the 4 detail sequences (cDT1, cDT2, cDT3, and cDF4) and the approximation sequence (cA4).

4. Results

To evaluate our denoising algorithm we have utilized the well-known free uploaded MIT-BIH Arrhythmia Database; a set of ECG data records sampled at a rate of 360 Hz with 11 bit resolution over a 10 mV range [18].

As it has been stated, in the previous section, a pre-processing phase precedes the application of our denoising algorithm which consists on high pass filtering of the 0 dB noisy ECG signal. We should note that, along this work, the wavelet function 'Symlet 8' is used in the DWT process.

The figure 2 shows the 0 dB noisy ECG signal (a data segment of the record '100.dat') and its estimated (denoised) version, using our algorithm, correlated to the original free noise ECG signal which provides an output SNR of around 6 dB and an MSE value of around 0.0011.

A comparative study will assess the filtering performance of our designed denoising algorithm. The figure 3 illustrates the different denoised versions of the noisy ECG data (figure 2-a) using the low pass Butterworth filter and the 4^{th} level classical wavelet denoising with provide the output SNR values of 3 dB and 3.65 dB and the MSE values of 0.0018 and 0.0017 respectively.

These obtained results of the comparative analysis, using the low pass Butterworth filter and the 4^{th} level classical wavelet denoising technique, emphasizes the higher performance of our designed algorithm for the very noisy ECG signal smoothing.

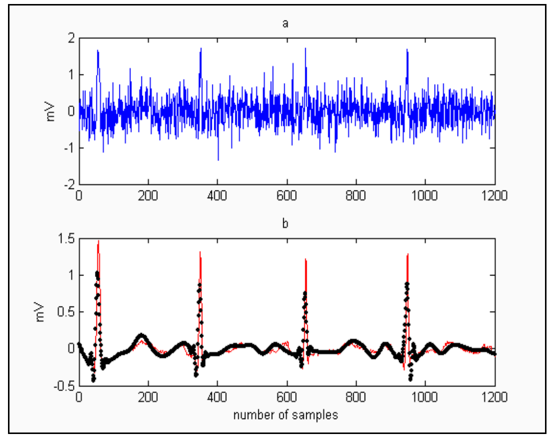

Figure 2. The application of our denoising algorithm; a: the 0 dB noisy ECG data segment of the record'100.dat'; b: the correlation of the original free noise ECG signal (in continuous line) and the denoised one (discontinuous line).

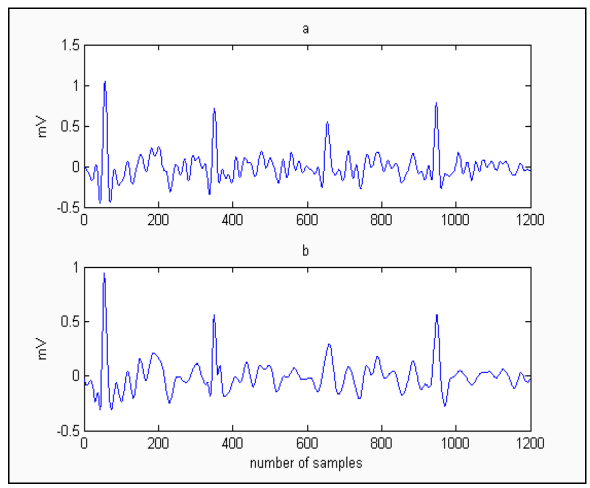

Figure 3. The comparative analysis brought on the same 0 dB noisy ECG signal of figure 2; top: the low pass Butterworth filtered signal; bottom: the 4^{th} classical DWT denoising signal.

5. Discussion and conclusions

We present in this work a smoothing algorithm based on the wavelet denoising approach applied to the electrocardiogram signal. Our key idea was to estimate the white Gaussian noise that is corrupted to the original ECG signal, by an adequate processing and combining the 1^{st} level DWT detail and approximation sequences of the noisy ECG signal, which, in turns, will be discarded from the 4^{th} level DWT detail coefficients. Introducing this (noise-discarded) 4^{th} level DWT detail sequence to the 3^{rd} level classical wavelet denoising generates the estimated (denoised) ECG signal. Our algorithm was carried out on a set of the MIT-BIH Arrhythmia Database ECG records corrupted with a WGN of an SNR of 0 dB where the obtained results showed a high efficiency for suppressing the corrupted noise and highly recovering the ECG signal components. However, even thought this improved output SNR value obtained, with our denoising algorithm, the latter one suffers from some limitations that are neglecting the P and T waves of the 1^{st} level approximation coefficients of the noisy ECG signal and concentrating, for the corrupted noise estimation, on the R waves alone. This is justified, in our designing approach by the fact that these P and T waves are roughly submerged in the noise that requires a robust routine for their extraction. A major drawback that it suffers from our denoising algorithm is its high dependency on the R waves picks in the 1^{st} level approximation coefficients of the noisy ECG signal. Nevertheless, the comparative study based on the low pass Butterworth filter and the 4^{th} level classical wavelet denoising demonstrates the superior filtering performance of our algorithm where it is, obviously, noticed the higher P and T waves extraction and least R waves distortion.

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