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WAVELET DOMAIN IMAGE INTERPOLATION VIA STATISTICAL ESTIMATION

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ABSTRACT

We propose a new wavelet domain image interpolation scheme based on statistical signal estimation. A linear composite MMSE estimator is constructed to synthesize the detailed wavelet coefficients as well as to minimize the mean squared error for high-resolution signal recovery. Based on a discrete time edge model, we use low-resolution information to characterize local intensity changes and perform resolution enhancement accordingly. A linear MMSE estimator follows to minimize the estimation error. Local image statistics are involved in determining the spatially adaptive optimal estimator. With knowledge of edge behavior and local signal statistics, the composite estimation is able to enhance important edges and to maintain the intensity consistency along edges. Strong improvement in both the visual quality and the PSNRs of the interpolated images has been achieved by the proposed estimation scheme.

1. INTRODUCTION

Image interpolation involves the problem of resolution enhancement, i.e. recovering a high-resolution image from its smoothed version. In low-resolution images, the lost high-frequency components associated with strong intensity changes are the main information to recover. Strong intensity discontinuity conveys substantial visual information because it often happens around various edges produced by object boundaries. Therefore, how to process the intensity changes around strong edges is the central issue for the interpolation problem.

Classic bilinear and bicubic interpolation methods impose a continuity constraint over the entire image and tend to produce oversmoothed edges. They also implicitly assume that a low-resolution image consists of samples from its high-resolution version, which is not true when the imaging sensor has a spatial averaging characteristic over the sampling lattice. Edge adaptive interpolation schemes [1,2] perform the interpolation in selective directions to avoid smoothing across edges, with extra efforts made to deal with edge related artifacts due to imprecise edge positions and smoothness assumed for the high-resolution image. Wavelet based methods [3,4] take advantage of local signal smoothness (Hölder regularity) observed through multiple scales [5] and actively enhance the high frequency contents. However, the general mathematical conclusion involved is obtained by analyzing continuous signals. When interpolating discrete time signals, ambiguities about the locations and the signs of the extrapolated highband coefficients would arise.

In this work, we propose a new wavelet domain interpolation scheme that explicitly fills in important detailed coefficients to recover the high-resolution image. In contrast to other wavelet domain interpolation methods, we formulate interpolation as a statistical signal estimation problem, i.e. we want to estimate the high-resolution image using the statistical information of the low-resolution signals. We also characterize edge behavior by a parameterized discrete time signal to accurately locate edges from low-resolution samples. A linear composite minimum-mean-squared-error (MMSE) estimator is proposed to solve the estimation problem. The composite estimation involves a parametric edge model and local image statistics. First, local edge behavior is determined from the low-resolution samples and used to synthesize the detailed coefficients. Then, a linear estimator minimizes the estimation error using local statistical information of the enhanced signals. Both analysis and experiments reveal that the knowledge of edge behavior and local image statistics enable the composite estimator to enhance the cross-edge sharpness and maintain the intensity consistency along edges, which are essential for high image quality.

In the following sections, we start by characterizing edge behavior under lowpass filtering, and then derive the optimal MMSE estimator for image interpolation. Finally we show the experimental results which demonstrate the strength of the statistical approach.

2. WAVELET TRANSFORM AND EDGE BEHAVIOR

Figure 1 illustrates a 1D discrete wavelet transform with analysis filters $(\{h(n)\},\{g(n)\})$ and synthesis filters $(\{\tilde{h}(n)\},\{\tilde{g}(n)\})$. Discrete time signal f is decomposed and down-sampled to produce two subband signals, the

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lowpass scaling coefficients f_s and highpass detailed

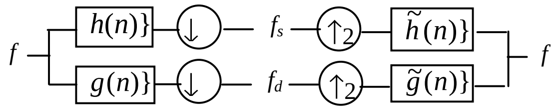

Figure 1. 1D Wavelet transform

coefficients f_d. The synthesis process gives exact reconstruction of f. Wavelet transform on 2D images can be realized by separate 1D transforms on image rows and columns.

To characterize edge behavior under lowpass filtering, we adopt (1) as the underlying continuous edge model, which assumes the ideal step function for real object boundaries and a Gaussian point spread function for the acquisition system [6].

$$e(t;c,A,\sigma) = c + \frac{A}{2}(1 + erf(\frac{t}{\sqrt{2}\sigma})) \qquad (1)$$

Here $erf(t) = (2/\sqrt{\pi})\int_0^t \exp(-x^2/2)dx$. With a normalized sampling rate, the corresponding discrete time edge model is parameterized as

$$e(n;c,A,\sigma,d) = c + \frac{A}{2}(1 + erf(\frac{n-d}{\sqrt{2}\sigma})) \qquad (2)$$

where (A, σ, d) indicate the edge strength, transition speed and sampling bias, and c indicates the base intensity.

Let a discrete time edge signal $f(n;c,A,\sigma,d)$ be smoothed with lowpass filter $\{h(n)\}$ and downsampled by *2*, the low resolution signal f_s still appears in the shape of a slightly modified edge as we expect. However, if we also allow the detailed signal f_d to be produced through highpass filter $\{g(n)\}$ and downsampling, we observe the important fact that for sharp edges with small σ, besides the apparent dependence on A and σ, the local detailed coefficients in f_d also change dramatically and nonlinearly as the sampling bias d changes. Figure 2 (b)-(c) illustrates the effect with σ set to *1* and b set to *0* and *0.5* respectively. Two facts about edge behavior are used in the following analysis. First, a strong intensity change in the high-resolution signal causes a strong intensity change at the corresponding location in the low-resolution signal. Second, the knowledge of edge parameters (A, σ, d) are sufficient for determining the local detailed coefficients.

Now we consider f to be the unknown 1D discrete time signal taken from an entire row or column of a high-resolution image and assume f can be approximated as the composite signal of a number of local edges at different spatial locations

$$f(n) = c + \sum_i \frac{A_i}{2}(1 + erf(\frac{n-d_i}{\sqrt{2}\sigma_i})) \qquad (3)$$

Suppose that only the smoothed signal f_s is available. To recover the significant detailed coefficients of f_d, our basic idea is to obtain an estimate of local edge parameters (A_i,

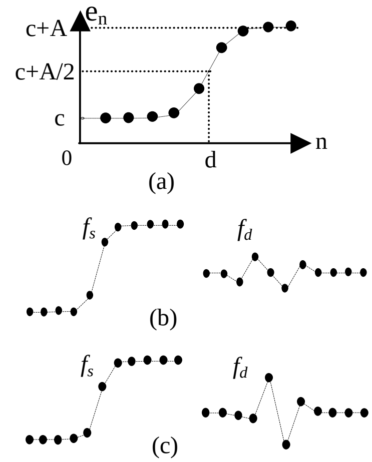

Figure 2. (a) Discrete time edge model; (b)-(c) Scaling coefficients f_s and detailed coefficients f_d. (b) *σ=1, b=0*; (c) *σ=1, b=0.5.*

σ_i, d_i) from the low-resolution signal f_s and use this information to fill in the important coefficients of f_d.

3. MMSE ESTIMATOR FOR IMAGE INTERPOLATION

We formulate image interpolation as an estimation problem, i.e. given the low-resolution image samples $\{f_s(x,y)\}$, we want to estimate the high-resolution samples $\{f(x,y)\}$. To simplify the analysis, here we only consider the problem of resolution enhancement in one direction. To achieve complete resolution enhancement, we apply the interpolation scheme twice. First we interpolate the image horizontally, and then we apply the same scheme vertically to the horizontally interpolated image. Let f_y denote the to-be-interpolated high-resolution image row with vertical index y. $f_{s,y}$ denotes the *y_th* row in the low-resolution image and $f_{d,y}$ denotes the *y_th* row of the unknown detailed coefficients. Figure 3 helps to illustrate the notation. In particular, we consider the estimation of $f_y(x)$, the high-resolution image pixel indexed by (x,y). Under the minimum-mean-squared-error (MMSE) criterion [7], the optimal estimator for $f_y(x)$ is given by the following conditional mean

$$\hat{f}_y(x) = E[f_y(x) \mid f_{s,y+k}; k = 0, \pm 1, \cdots \pm M] \qquad (4)$$

In practice, strong correlation exits only in local image region, so M can be set small. Direct evaluation of (4) requires the knowledge of joint distribution of f_y and $\{f_{s,y+k}\}$. This is difficult to obtain when f_y is unknown. Consequently, we propose the following linear composite MMSE estimator as a replacement,

$$\hat{f}_y(x) = \sum_{k=-M}^{M} a_k \cdot E[f_y(x) \mid f_{s,y+k}] + C \qquad (5)$$

where a_k and C are scalars. Denote $\tilde{f}_y^k = E[f_y \mid f_{s,y+k}]$ with $\tilde{f}_y^k(n) = E[f_y(n) \mid f_{s,y+k}]$, we rewrite (5) as

$$\hat{f}_y(x) = \sum_{k=-M}^{M} a_k \cdot \tilde{f}_y^k(x) + C \tag{6}$$

The estimator is implemented in two steps. First, f_y is estimated as $\tilde{f}_y^k$ using the information from the *k_th* neighboring row $f_{s,y+k}$ in the low-resolution image. We call this step the *estimate by row*. It involves the use of 1D edge model. Then a linear MMSE estimator estimates $f_y(x)$ by minimizing the mean-square-error(MSE) over the linear class $\sum_{k=-M}^{M} a_k \cdot \tilde{f}_y^k(x) + C$. Second order statistics of $\tilde{f}_y^k$ are involved in finding optimal values for $\{a_k\}$ and C.

3.1. Estimate by row

The synthesis of f_y is given by

$$f_y = (f_{s,y} \uparrow 2) * \tilde{h} + (f_{d,y} \uparrow 2) * \tilde{g} \tag{7}$$

where $\uparrow 2$ denotes the linear operation of upsampling by *2*, so we have

$$\tilde{f}_y^k = (f_{s,y} \uparrow 2) * \tilde{h} + (E[f_{d,y} \mid f_{s,y+k}] \uparrow 2) * \tilde{g} \tag{8}$$

We apply the ideas discussed in section 2 and solve the estimate $\tilde{f}_{d,y}^k = E[f_{d,y} \mid f_{s,y+k}]$. Assume the *k_th* neighboring row f_{y+k} can be approximated as the composite signal of a number of edges $c + \sum_i A_i(1 + erf((n - d_i)/\sqrt{2}\sigma_i))/2$. Around d_i there is an edge segment crossing row f_{y+k} that causes the local intensity change on f_{y+k} as $f_{y+k}(n) = e(n; c_i, A_i, \sigma_i, d_i)$. Then the *k_th* neighboring row $f_{s,y+k}(n)$ in the low-resolution image f_s has a local intensity change around $d_i/2$ as

$$\sum_m h(m) \cdot e(2n - m; c_i, A_i, \sigma_i, d_i) \tag{9}$$

So the parameters $(c_i, A_i, \sigma_i, d_i)$ can be estimated from $f_{s,y+k}$ around $d_i/2$ by solving the least square (LS) problem

$$\arg\min_{(c,A,d,\sigma)} \sum_{n \in N(d_i/2)} | f_{s,y+k}(n) - \sum_m h(m) e(2n - m; c_i, A_i, \sigma_i, d_i) |^2 \tag{10}$$

Once the LS estimates $(c_i, A_i, \sigma_i, d_i)$ are obtained, we estimate the direction of the edge segment crossing row f_{y+k}. As illustrated in figure 3, we use the edge segment

$$f_{s,y+k+j}(n) = \sum_m h(m) \cdot e(2n - m; c_i, A_i, \sigma_i, d_i - j/\tan(\theta_i)) \tag{11}$$

to match the local pixels around $(d_i/2, y+k)$ in f_s and determine the edge orientation θ_i. Then the estimates of detailed coefficients around $d_i/2$ in $f_{d,y}$ is given by

$$\tilde{f}_{d,y}^k(n) = \sum_m g(m) \cdot e(2n - m; c_i, A_i, \sigma_i, d_i + k/\tan(\theta_i)) \tag{12}$$

The estimate-by-row step is summarized as follows:

1. Check the *k_th* neighboring row $f_{s,y+k}$ of the low-resolution image. If a strong intensity change is found at some location, perform the LS estimation in (9) to obtain the local edge parameters $(c_i, A_i, \sigma_i, d_i)$. Use (10) to estimate edge orientation θ_i.
2. Use (12) to synthesize the detailed coefficients in $\tilde{f}_{d,y}^k$ around $d_i/2$. Then use (8) to obtain the estimate $\tilde{f}_y^k$.
3. Apply the same procedure to each row of the image. For every row f_y, we obtain *(2M+1)* high-resolution estimates-by- row, $\tilde{f}_y^k = E[f_y \mid f_{s,y+k}]$ $(k=-M,\ldots M)$.

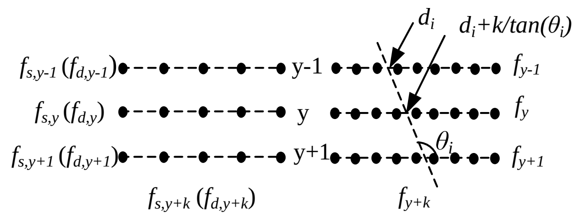

Figure 3. Interpolation lattice

Based on the family of parametric edge models, the estimate-by- row implements a nonlinear estimator for the high-resolution signal using partial information from the low-resolution signal. As we will see, resolution enhancement around strong edges is achieved by the estimates.

3.2. Linear MMSE

The linear MMSE (6) finds the optimal linear combination of the estimates-by-row that minimizes the estimation MSE. Only first and second order statistics of $\tilde{f}_y^k$ are involved to solve the optimal $\{a_k\}$ and C [7],

$$\begin{aligned} C &= E[f_y(x)] - \sum_{k=-M}^{M} a_k \cdot E[\tilde{f}_y^k(x)] \\ Cov(f_y(x), \tilde{f}_y^k(x)) &= \sum_{j=-M}^{M} a_j Cov(\tilde{f}_y^j(x), \tilde{f}_y^k(x)) \end{aligned} \tag{13}$$

where $Cov(\cdot,\cdot)$ denotes covariance function. The following facts are easily verified:

$$\begin{aligned} E[\tilde{f}_y^k(x)] &= E[f_y(x)] \\ Cov(f_y(x), \tilde{f}_y^k(x)) &= Cov(\tilde{f}_y^k(x), \tilde{f}_y^k(x)) \end{aligned} \tag{14}$$

Denote $C_{(x,y)}(j,k) = Cov(\tilde{f}_y^j(x), \tilde{f}_y^k(x))$. (14) allows $\{a_k\}$ and C to be solved by *2M+2* linear equations,

$$\begin{aligned} C_{(x,y)}(k,k) &= \sum_{j=-M}^{M} a_j C_{(x,y)}(j,k) \qquad (k = 0, \pm 1, \cdots \pm M) \\ C &= (1 - \sum_{k=-M}^{M} a_k) E[f_y(x)] \end{aligned} \tag{15}$$

and $E[f_y(x)]$ be evaluated as $E[f_y(x)] = \frac{1}{2M+1} \sum_{k=-M}^{M} \tilde{f}_y^k(x)$. $C_{(x,y)}(j,k)$ is adaptively evaluated using the neighboring pixels around (x,y) in $\{\tilde{f}_y^k(x)\}$,

$$C_{(x,y)}(j,k)=\frac{1}{|N(x,y)|}\sum_{(x',y')\in N(x,y)}[(\tilde{f}_{y'}^{j}(x')-E[f_{y'}(x')]) \cdot(\tilde{f}_{y'}^{k}(x')-E[f_{y'}(x')])]\qquad (j,k=0,\pm1,\cdots\pm M)$$

The linear MMSE estimate is summarized as follows:

1. For sample (x,y), evaluate the local statistics $E[f_y(x)]$ and $C_{(x,y)}(k,j)$ using the estimates $\{\tilde{f}_y^k(x)\}$ in the local region. Solve (15) to get optimal $\{a_k\}$ and C. Use (6) to compute the composite estimate of $f(x,y)$.
2. Apply the same procedure to every sample in f.

The linear MMSE estimator combines the estimate results by partial information in an optimal way such that the estimation MSE is minimized. This estimator imposes spatially adaptive filtering on signals interpolated with partial information and subsequently assures the intensity consistency along the locally enhanced edges.

4. EXPERIMENT RESULTS

We applied the linear composite MMSE estimation algorithm to image interpolation. Compared with bilinear and bicubic schemes, the MMSE estimation scheme produces clearer boundaries in the interpolated images. In addition to sharpening strong edges, the linear estimation step assures the intensity consistency along edges and avoids unnecessary artifacts around enhanced edges.

Figure 4 shows the example of a subregion from the interpolated *Lena* image by different methods, where the image is interpolated to four times the original size. Since the wavelet coefficient synthesis deals with sparsely located strong edges, as we expect, the improvement of the visual quality is more obvious in the regions containing strong edges than in the regions with many weak edges such as in textures. In Table 1, we show an example of the PSNR improvement by the proposed estimation approach with M set to 1. The numbers were obtained from interpolating the smoothed Lena image to 2 and 4 times the size. The full size (512x512) image was first smoothed by the lowpass filter and downsampled. Then the smoothed image was interpolated to full size and compared with the original image. A 4~5dB gain is achieved by the proposed estimation algorithm.

5. CONCLUSIONS

In this work, we used signal estimation techniques to solve the interpolation problem. We proposed a linear composite MMSE estimator to recover the fine-resolution image from the low-resolution samples. An edge model based nonlinear estimator synthesizes the important detailed coefficients with partial information obtained from low-resolution samples. A standard linear estimator then minimizes the MSE of the recovered image using knowledge of local statistics of previously estimated signals. Subsequently, strong edges are enhanced by the model based coefficient synthesis while the intensity consistency along edges is maintained by the subsequent linear estimator. The interpolation results demonstrate the strength of the statistical approach in achieving better image quality and better signal estimation. Further study includes extending the statistical analysis to the problem of multiscale image modeling.

(a) (b) (c)

Figure 4. Images produced by different interpolation methods. (a) Bilinear; (b) Bicubic; (c) Linear composite MMSE estimator.

Method	× 2 (dB)	× 4 (dB)
Bilinear	28.84	23.35
Bicubic	29.11	23.26
Proposed	33.78	28.56

Table 1. PSNR comparison of different interpolation methods

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