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## Wavelet Variance Analysis of Output in G-7 Countries

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# Wavelet Variance Analysis of Output in G-7 Countries\*

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## Abstract

The large decline in output volatility experienced by most industrialized countries in the last decades has been thoroughly analyzed using standard time and frequency domain methods. In this paper we investigate the issue of moderation of volatility in G-7 economies and its sources, applying a multi-scaling approach to the industrial production indices of G-7 countries between 1961:1-2006:10. Using the MODWT estimates of wavelet variance we provide a scale-based analysis of variance that allows us to characterize the decline in volatility and to detect the importance of the various explanations of the moderation. The main scale-by-scale results stemming from multi scale analysis of variance are: i) a reduction in volatility which, although displayed by all the G-7 countries, is not uniform across time scales (as the decline is larger at short-term scales than at business cycle scales for France and Italy, and quite uniform across scale for the UK and the US) nor countries (as the decline is significant for a subset of countries only, i.e. France, Italy, the UK and the US); and ii) the moderation has to be attributable to the decline in the variance of both common (in the 1970s) and country-specific (in the 1960s) exogenous disturbances hitting the economy.

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## 1. Introduction

There is actually a large number of recent papers documenting, with the support of strong evidence, the reduction in the variance of output for the US and other industrialized countries over the past forty years.<sup>1</sup> Although there is little disagreement about whether a decline in volatility occurred, the better way to characterize this reduction in volatility (the “Great Moderation” as defined by Stock and Watson, 2002) is still controversial. In particular, two alternative views have been suggested to explain the nature of this moderation: the first states that the reduction of the volatility may be best characterized by a sharp break in the mid 1980s, and thus implicitly consider the last twenty years mainly as the results of the absence of large adverse shocks (see Mc Connell and Perez Quiros, 2000, and Stock and Watson, 2002). The latter states that the evolution of output volatility is nothing else than the result of a large underlying trend of decline which started in the late 1950s and temporarily interrupted only in the 1970s and early 1980s (see Blanchard and Simon, 2001).

In addition, there is also little consensus about the causes of this moderation. The most commonly proposed explanations for the decline in the volatility of overall economic activity fall into three categories: i) changes in the structure of the economies and improved inventory management (“good-practices”),<sup>2</sup> ii) improvements in the conduct of (monetary) policies (“good-policies”),<sup>3</sup> and iii) a reduction in the magnitude and frequency of macroeconomic shocks (“good-luck”). Two main results characterize the empirical evidence on the nature and the causes of the great moderation for the G-7 countries: i) the magnitude of the decline in output volatility is similar, but the timing is not, and ii) there is no favored single cause for this moderation.

In order to distinguish among these three sources of volatility reduction standard time and frequency domain methods have been used.<sup>4</sup> Transforming a (stationary) time series into the frequency domain may help to detect the contribution of each frequency component to the overall variance through the estimates of the spectral density function (*i.e.* the periodogram), as it decomposes the variance by frequency. Thus, the frequency-by-frequency decomposition may be useful to detect the sources of the changes in volatility if each explanation of the moderation can be associated to the shift pattern of

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<sup>1</sup>See Kim and Nelson (1999), Mc Connell and Perez Quiros (2000), Simon (2000), Blanchard and Simon (2001), Chauvet and Potter (2001), Stock and Watson (2002, 2003), van Dijk *et al.* (2002), Doyle and Faust (2002), Herrera and Pesavento (2005).

<sup>2</sup>See Mc Connell and Perez Quiros (2000) and Kahn *et al.* (2001).

<sup>3</sup>See Taylor (1999) and Cogley and Sargent (2001).

<sup>4</sup>Examples of the first approach are Stock and Watson (2002) and Summers (2005), while examples of the latter are Ahmed *et al.* (2002) and Fritsche and Kouzine (2005).

the spectrum at a particular frequency.<sup>5</sup>

Given the potential time-varying nature of output fluctuations, analyzing them using a transform which may capture events both locally in time and frequency may be appealing. Wavelets are particular types of function  $f(x)$  that are localized both in time and frequency domain and used to decompose a function  $f(x)$ , *i.e.* a signal, a surface, a series, etc..., in more elementary functions which include information about  $f(x)$ . In particular, wavelet variance analysis allows for a scale-based analysis of variance of a time series which is different, but related, to spectral analysis. Indeed, while Fourier coefficients are associated with a single frequency, wavelet coefficients are associated with a specific time scale and thus, since each scale may be related to a certain range of frequencies in the data, with a specific band of frequencies. Several applications of wavelet analysis in economics and finance have been first provided by Ramsey and Lampart (1998a, 1998b), Ramsey (2002), but, despite its growing popularity, no attempt has been made, at least to our knowledge, to apply time-frequency analysis to the issue of volatility moderation. Thus, in this paper we apply the wavelet methodology to the analysis of the industrial production series of G-7 countries between 1961:1 and 2006:10. In particular, through the maximal overlap discrete wavelet transform (MODWT) estimator of the wavelet variance (Percival, 1995) we try to distinguish among the competing explanations on moderation examining whether the reduction in volatility is uniform or not across time scales.

The analysis of the variance at different scales derived using wavelet method calls into question the idea that there may be a sort of reference country. Indeed, wavelet variance analysis indicates that the reduction in volatility, although displayed by all the G-7 countries, is not uniform across time scales nor countries. The only regularity, apart from the decline in output volatility, is that there are no regularities. For some countries the decline in volatility is significant (France, Italy, the UK and the US), for some others not. And among these countries, there are some which display larger reductions at short-term scales than at business cycle scales (France and Italy), and some others (the UK and the US) in which the reduction is quite uniform across scales. These results, after isolating periods of major supply disruptions, are consistent with the view that the moderation has to be attributable to the decline in the variance of both common, in the 1970s, and country-specific, in the 1960s, exogenous disturbances hitting the economies.

The structure of the paper is as follows. The main properties of the wavelets and the analytical differences from other filtering methods are dealt with in Section 2, where the characteristics of our data set are also illustrated. In

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<sup>5</sup>See Ahmed *et al.* (2002).

section 3 we present the results from wavelet variance analysis, while section 4 concludes the paper.

## 2. Methodology and implementation

Wavelets are particular types of function  $\omega(x)$  that are localized both in time and frequency domains and are used to decompose a function  $f(x)$ , *i.e.* a signal, a surface, a series, etc., in more elementary functions which include information about  $f(x)$ . The main advantage of wavelet analysis is its ability to decompose macroeconomic time series, and data in general, into their time scale components. Moreover, because of the translation and scale properties, nonstationarity in the data is not a problem when using wavelets and pre-filtering is not needed. Finally, as wavelets are constructed over finite intervals of time and are not necessarily homogeneous over time, they are localized in both time and scale.<sup>6</sup> Thus, two interesting features of wavelet time scale decomposition for economic variables are that, i) since the base scale does not include any non-stationary components, the data need not be detrended or differenced, and ii) the nonparametric nature of wavelets takes care of potential nonlinear relationships without losing detail (Schleicher, 2002).<sup>7</sup>

In this section we first present the basic concepts of wavelet analysis and wavelet transform, then describe the standard discrete wavelet transform and the maximal overlap discrete wavelet transform, and finally introduce the method for calculating wavelet variance.

### 2.1 Introductory considerations

The wavelet transform maps a function  $f(t)$ <sup>8</sup> from its original representation in the time domain into an alternative representation in the time-scale domain  $w(t, j)$  applying the transformation  $w(t, j) = \psi(\cdot)f(t)$ , where  $t$  is the time index,  $j$  the scale (*i.e.* a specific frequency band) and  $\psi(\cdot)$  the wavelet

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<sup>6</sup>These properties of wavelet analysis may overcome the two main limitations of the Fourier transform, that is i) the loss of time information in the transformation to the frequency domain, and ii) the requirement that the moments of the signals do not appreciably change over time (represented by the assumption of covariance-stationarity).

<sup>7</sup>As many economic and financial time series are non-stationary and, moreover, exhibit changing frequencies over time, much of the usefulness of wavelet analysis has to do with its flexibility in handling a variety of non-stationary signals.

<sup>8</sup>The function may be continuous or discrete. As time series are observed at regular intervals and thus are constituted by a finite-length vector of observations, we restrict our presentation exclusively to the discrete wavelet transformation.

filter.<sup>9</sup> There are two basis wavelet filter functions: the father and the mother wavelets,  $\phi$  and  $\psi$ , respectively. The first integrates to 1 and reconstructs the smooth and low frequency parts of a signal, whereas the latter integrates to zero and describes the detailed and high-frequency parts of a signal.<sup>10</sup> The wavelet filter  $\psi(t_k - t_i, j)$ , given by

$$\psi(t_k - t_i, j) = \frac{1}{\sqrt{j}} \psi_0 \left( \frac{t_k - t_i}{j} \right)$$

is a (normalized) stretched and translated version of a basis wavelet function called mother wavelet  $\psi$  where  $\frac{1}{\sqrt{j}}$  is the normalization factor.

The wavelet function in equation (1) depends on two parameters, scale (or frequency) and time, that provide the time and frequency information simultaneously, hence providing the so-called time-scale or time-frequency representation of the signal.<sup>11</sup> The scale or dilation factor  $j$  controls the length of the wavelet (window), while the translation or location parameter  $k$  refers to the location.<sup>12</sup> The basis wavelet function is stretched (or compressed) according to the scale parameter to extract frequency information (a narrow window yields information on low frequency movements, while a wide window yields information on high frequency movements), and moved on the time line (from the beginning to end) to extract time information from the signal in question.

## 2.2 Discrete wavelet transform

The wavelet transform decomposes a signal into sets of coefficients where each set of coefficients is associated with a spatial scale and each coefficient in a set is associated with a particular location.<sup>13</sup> The wavelet coefficients, the output of the wavelet transform, are obtained through a projection of the signal onto shifted and translated versions of mother and father wavelets and represent, respectively, the underlying smooth behavior of the data at the coarsest scale (the scaling coefficients) and the coarse scale deviations from it (the wavelet coefficients).

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<sup>9</sup>The wavelet transform may be considered as an extension of the Fourier transform in the sense that, with respect to the Fourier transform which concentrates on frequency resolution only, it gives up frequency resolution in order to gain time resolution and replaces the periodic exponential  $\exp(\omega t_i)$  with a localized wavelet  $\psi(t_k - t_i, j)$  which is located around  $t_i$  and stretched according to the scale  $j$ .

<sup>10</sup>The mother wavelet integrates to zero as it reflects the fact that it is used to represent differences in the data that average out to zero (Schleicher, 2002).

<sup>11</sup>Note that the scale factor is inversely related to the frequency of the wavelet.

<sup>12</sup>The location index  $k$  indicates the nonzero portion of each wavelet basis vector.

<sup>13</sup>In wavelet terminology each single coefficient is called an "atom" and the set of coefficients for each scale a "crystal".

Given a stochastic process  $\{X\}$ , if we denote with  $H = (h_0, \dots, h_{L-1})$  and  $G = (g_0, \dots, g_{L-1})$  the impulse response sequence<sup>14</sup> of the wavelet and scaling filters  $h_l$ , and  $g_l$ , respectively, of a Daubechies compactly supported wavelet (with  $L$  the width of the filters), when  $N = L^2$  we may apply the orthonormal discrete wavelet transform (DWT) and obtain the wavelet and scaling coefficients at the  $j$ th level defined as<sup>15</sup>

$$w_{j,t} = \sum_{l=0}^{L-1} h_{j,l} X_{t-l}$$

$$v_{j,t} = \sum_{l=0}^{L-1} g_{j,l} X_{t-l},$$

where  $h_{j,l}$  and  $g_{j,l}$  are the level  $j$  wavelet and scaling filters and, due to downsampling by  $2^j$ , we have  $\frac{N}{2^j}$  scaling and wavelet coefficients.<sup>16</sup>

The DWT is implemented via a filter cascade where the wavelet filter  $h_l$  is used with the associated scaling filter  $g_l$ <sup>17</sup> in a pyramid algorithm (Mallat, 1989) consisting in an iterative scheme in which, at each iteration, the wavelet and scaling coefficients are computed from the scaling coefficients of the previous iteration.<sup>18</sup>

However the orthonormal discrete wavelet transform (DWT), even if widely applied to time series analysis in many disciplines, has two main drawbacks: the dyadic length requirement (*i.e.* a sample size divisible by  $2^j$ ),<sup>19</sup> and the fact that the wavelet and scaling coefficients are not shift invariant due to their sensitivity to circular shifts because of the decimation operation. An alternative to DWT is represented by a non-orthogonal variant of DWT: the

<sup>14</sup>The impulse response sequence is the set of all filter coefficients. The filter coefficients must satisfy three properties: zero mean ( $\sum_{l=0}^{L-1} h_l = 0$ ), unit energy ( $\sum_{l=0}^{L-1} h_l^2 = 1$ ) and orthogonal to its even shifts ( $\sum_{l=0}^{L-1} h_l h_{l+2k} = 0$ ).

<sup>15</sup>The expressions used for DWT (and MODWT) wavelet and scaling coefficients refer to functions defined over the entire real axis, that is  $t \in \mathfrak{R}$  as in this case  $X_t = X_{t \bmod N}$  when  $t < 0$ .

<sup>16</sup>At the  $j$ th level the inputs to the wavelet and scaling filters are the scaling coefficients from the previous level ( $j-1$ ) and the output are the  $j$ th level wavelet and scaling coefficients.

<sup>17</sup>The wavelet and scaling filter coefficients are related to each other through a quadrature mirror filter relationship, that is  $h_l = (-1)^l g_{L-1-l}$  for  $l = 0, \dots, L-1$ .

<sup>18</sup>The only exception is at the unit level ( $j = 1$ ) in which wavelet and scaling filters are applied to original data.

<sup>19</sup>This condition is not strictly required if a partial DWT is performed.

maximal overlap DWT (MODWT).<sup>20</sup>

In the orthonormal Discrete Wavelet Transform (DWT) the wavelet coefficients are related to nonoverlapping differences of weighted averages from the original observations that are concentrated in space. More information on the variability of the signal could be obtained considering all possible differences at each scale, that is considering overlapping differences, and this is precisely what the maximal overlap algorithm does.<sup>21</sup> Thus, the maximal overlap DWT coefficients may be considered the result of a simple modification in the pyramid algorithm used in computing DWT coefficients through not downsampling the output at each scale and inserting zeros between coefficients in the wavelet and scaling filters.<sup>22</sup> In particular, the MODWT wavelet and scaling coefficients  $\tilde{w}_{j,t}$  and  $\tilde{v}_{j,t}$  are given by

$$\tilde{w}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \tilde{h}_{j,l} X_{t-l}$$

$$\tilde{v}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \tilde{g}_{j,l} X_{t-l},$$

where the MODWT wavelet and scaling filters  $\tilde{h}_{j,l}$  and  $\tilde{g}_{j,l}$  are obtained by rescaling the DWT filters as follows:<sup>23</sup>

$$\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}}$$

$$\tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}.$$

The MODWT wavelet coefficients  $\tilde{w}_{j,t}$  are associated with generalized changes of the data on a scale  $\lambda_j = 2^{j-1}$ . With regard to the spectral interpretation of MODWT wavelet coefficients, as the MODWT wavelet filter  $h_{j,l}$  at each scale

<sup>20</sup>The MODWT goes under several names in the wavelet literature, such as the "non-decimated DWT", "stationary DWT" (Nason and Silverman, 1995), "translation-invariant DWT" (Coifman and Donoho, 1995) and "time-invariant DWT".

<sup>21</sup>Indeed, the term maximal overlap refers to the fact that all possible shifted time intervals are computed. As a consequence, the orthogonality of the transform is lost, but the number of wavelet and scaling coefficients at every scale is the same as the number of observations.

<sup>22</sup>The DWT coefficients may be considered a subset of the MODWT coefficients. Indeed, for a sample size power of two the MODWT may be rescaled and subsampled to obtain an orthonormal DWT.

<sup>23</sup>Whereas DWT filters have unit energy, MODWT filters have half energy, that is

$$\sum_{l=0}^{L-1} \tilde{h}_{j,l}^2 = \sum_{l=0}^{L-1} \tilde{g}_{j,l}^2 = \frac{1}{2^j}.$$



$j$  approximates an ideal high-pass with passband  $f \in [1/2^{j+1}, 1/2^j]$ ,<sup>24</sup> the  $\lambda_j$  scale wavelet coefficients are associated to periods  $[2^j, 2^{j+1}]$ .

MODWT provides the usual functions of the DWT, such as multiresolution decomposition analysis and variance analysis based on wavelet transform coefficients, but unlike the classical DWT it

- can handle any sample size;
- is translation invariant, as a shift in the signal does not change the pattern of wavelet transform coefficients;
- provides increased resolution at coarser scales.<sup>25</sup>

In addition, MODWT provides a larger sample size in the wavelet variance and correlation analyses and produces a more asymptotically efficient wavelet variance estimator than the DWT.<sup>26</sup>

### 2.3 Wavelet variance analysis

In addition to the features stated above wavelet transform is able to analyze the variance of a stochastic process and decompose it into components that are associated to different time scales. In particular, given a stationary stochastic process  $\{X\}$  with variance  $\sigma_X^2$  and defined the level  $j$  wavelet variance  $\sigma_X^2(\lambda_j)$ , the following relationship holds

$$\sum_{j=1}^{\infty} \sigma_X^2(\lambda_j) = \sigma_X^2$$

where  $\sigma_X^2(\lambda_j)$  represent the contribution to the total variability of the process due to changes at scale  $\lambda_j$ . This relationship says that wavelet variance decomposes the variance of a series into variances associated to different time scales.<sup>27</sup> By definition, the (time independent) wavelet variance for scale  $\lambda_j$ ,  $\sigma_X^2(\lambda_j)$ , is defined to be the variance of the  $j$ -level wavelet coefficients

<sup>24</sup>On the other hand at scale  $\lambda_J$  the scaling filter  $g_{J,l}$  approximates an ideal low-pass filter with passband  $f \in [0, 1/2^{j+1}]$ .

<sup>25</sup>Unlike the classical DWT which has fewer coefficients at coarser scales, it has a number of coefficients equal to the sample size at each scale, and thus is over-sampled at coarse scales.

<sup>26</sup>Wavelet variance is defined in subsection 2.3.

<sup>27</sup>The wavelet variance decomposes the variance of certain stochastic processes with respect to the scale  $\lambda_j = 2^{j-1}$  just as the spectral density decompose the variance of the original series with respect to frequency  $f$ , that is

$$\sigma_X^2(\lambda_j) = \text{var}\{\tilde{w}_{j,t}^2\}.$$

As shown in Percival (1995), provided that  $N - L_j \geq 0$ , an unbiased estimator of the wavelet variance based on the MODWT may be obtained, after removing all coefficients affected by the periodic boundary conditions,<sup>28</sup> using

$$\tilde{\sigma}_X^2(\lambda_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j}^N \tilde{w}_{j,t}^2$$

where  $\tilde{N}_j = N - L_j + 1$  is the number of maximal overlap coefficients at scale  $j$  and  $L_j = (2_j - 1)(L - 1) + 1$  is the length of the wavelet filter for level  $j$ .<sup>29</sup> Thus, the  $j$ th scale level  $j$  wavelet variance is simply the variance of the nonboundary or interior wavelet coefficients at that level (Percival, 1995, and Serrouck *et al.*, 2000). Both DWT and MODWT can decompose the sample variance of a time series on a scale-by-scale basis via its squared wavelet coefficients, but the MODWT-based estimator has been shown to be superior to the DWT-based estimator (Percival, 1995).

Starting from the spectrum  $S_{w_{X,j}}$  of the scale  $j$  wavelet coefficients it is possible to determine the asymptotic variance  $V_j$  of the MODWT-based estimator of the wavelet variance (covariance) and construct a random interval which forms a  $100(1 - 2p)\%$  confidence interval.<sup>30</sup>

$$\sum_{j=1}^{\infty} \sigma_X^2(\lambda_j) = \text{var} X = \int_{-1/2}^{1/2} S_X(f) df$$

where  $\sigma_X^2(\lambda_j)$  is wavelet variance at scale  $\lambda_j$  and  $S_{(\cdot)}$  is the spectral density function.

<sup>28</sup>As MODWT employs circular convolution, the coefficients generated by both beginning and ending data could be spurious. Thus, if the length of the filter is  $L$ , there are  $(2^j - 1)(L - 1)$  coefficients affected for  $2^{j-1}$ -scale wavelet and scaling coefficients, while  $(2^j - 1)(L - 1) - 1$  beginning and  $(2^j - 1)(L - 1)$  ending components in  $2^{j-1}$ -scale details and smooths would be affected (Percival and Walden, 2000).

<sup>29</sup>The quantity estimated in equation (2) is time-independent even in case of nonstationary processes but with stationary  $d$ th-order differences, provided that the length  $L$  of the wavelet filter is large enough to make the wavelet coefficients  $\tilde{w}_{j,t}$  a sample of stationary wavelet coefficients (Serrouck *et al.*, 2000). This is because Daubechies wavelet filters may be interpreted as generalized differences of adjacent averages and are related with difference operator (Whitcher *et al.*, 2000).

<sup>30</sup>For a detailed explanation of how to construct the confidence intervals of wavelet variance, see Gençay *et al.* (2002, p.254-6).

The formulas for an approximate  $100(1-2p)\%$  confidence intervals MODWT estimator robust to non-Gaussianity for  $\tilde{\sigma}_{X,j}^2$  are provided in Gençay *et al.* (2002).<sup>31</sup>

### 3. Empirical results

In order to perform a wavelet analysis of a time series, a number of decisions must be made: which family of wavelet filters to use, what type of wavelet transform to apply, and how boundary conditions at the end of the series are to be handled.

There are several families of wavelet filters available, such as Haar (discrete), symmlets and coiflets (symmetric), daubechies (asymmetric), etc, differing by the characteristics of the transfer function of the filter and by filter lengths.<sup>32</sup> Different wavelet families make different trade-offs between the degree of localization and the degree of approximation of high-pass filters (Lindsay *et al.*, 1996). Daubechies (1992) has developed a family of compactly supported<sup>33</sup> wavelet filters of various lengths, the least asymmetric family of wavelet filters (LA), which is particularly useful in wavelet analysis of time series because it allows the most accurate alignment in time between wavelet coefficients at various scales and the original time series.

In order to calculate wavelet coefficient values near the end of the series boundary conditions are to be assumed. According to the two main assumptions the series may be extended in a periodic fashion (periodic boundary condition) or in a symmetric fashion (reflecting boundary condition). We apply for wavelet variance analysis the reflecting boundary condition, where the original signal is reflected about its end point to produce a series of length  $2N$  which has the same mean and variance as the original signal. The wavelet and scaling coefficients are then computed by using a periodic boundary condition on the reflected series, resulting in twice as many wavelet and scaling coefficients at each level. Finally, we perform the time scale decomposition analysis using the maximum overlap discrete wavelet transform (MODWT) because of the practical limitations of the DWT, *i.e.* the dyadic length requirement and non shift-invariance.

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<sup>31</sup>The empirical evidence from the wavelet variance suggest that  $N_j = 128$  is a large enough number of wavelet coefficients for the large sample theory to be a good approximation (Whitcher *et al.*, 2000).

<sup>32</sup>In particular, the choice of the filter length depends on a trade-off between leakage and boundary-affected coefficients: a longer length makes the filter closer to an ideal high-pass filter, but reduces the number of boundary-unaffected coefficients.

<sup>33</sup>Wavelet filters with compact support are those in which the mother wavelet filters and scaling filters have finite length.

### 3.1 Time-scale decomposition

Different variables may be used as a proxy for aggregate output. Even if GDP is the most employed measure, its availability on a quarterly basis makes the industrial production index the variable commonly used as a proxy of aggregate output when a large number of observations is needed, as is the case of the wavelet methodology.<sup>34</sup> Thus, we choose to use the industrial production index monthly data between 1961:1-2006:10 for Canada, France, Germany, Italy, Japan, the UK and the US. The data are taken from the OECD database, seasonally adjusted, and expressed in natural logarithm.<sup>35</sup>

We perform a  $J$ -level decomposition<sup>36</sup> applying the *maximal overlap discrete wavelet transform (MODWT)* to the aggregate monthly industrial production series and using the Daubechies least asymmetric (LA) wavelet filter of length  $L = 8$ , denoted as  $LA(8)$ , which is a fourth order filter based on four non-zero coefficients (Daubechies, 1992) as the order of the filter is equal to the number of vanishing moments (half the length of the filter).<sup>37</sup> The application of the MODWT with a number of scales  $J = 6$  produces seven wavelet and scaling filter coefficients:  $v_6, w_6, w_5, w_4, w_3, w_2, w_1$ , where each wavelet scale is associated to a particular time period.

As the MODWT wavelet filter belongs to high-pass filter with passband given by the frequency interval  $[1/2^{j+1}, 1/2^j]$  for scales  $1 \leq j \leq J$ , inverting the frequency range to produce a period of time we obtain, with monthly data, that wavelet coefficients associated to scale  $\lambda_j = 2^{j-1}$  are associated to periods  $[2^j, 2^{j+1}]$ .<sup>38</sup> Thus, scale 1 represents frequencies corresponding to 2-4 month period dynamics, and scales 2, 3, 4, 5 and 6 correspond to 4 – 8, 8 – 16, 16 – 32, 32 – 64, and 64 – 128 month period dynamics, respectively.<sup>39</sup> The first three time scales represent the short-run dynamics of a signal (corresponding to the very high- and high-frequency components), scales 4 through 6 roughly correspond to the standard business cycle time period (Stock and Watson,

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<sup>34</sup>We are aware of the fact that, because of the shift in output from goods to services, the industrial production may not be the most reliable indicator of real economic activity in industrial countries. But in contrast to GDP series, the industrial production index is statistically more reliable than GDP in analyzing the decline in volatility, as the result will not be affected by the change in the structure of the economy.

<sup>35</sup>There are many analysis indicating that the results obtained using the industrial production index are qualitatively similar to those obtained using real GDP (see, for example, Stock and Watson, 2002).

<sup>36</sup> $J$  is the maximum integer such that  $2^j = \log_2 N$ .

<sup>37</sup>Thus, it has the ability to generate stationary series from a series integrated up to level 4 included.

<sup>38</sup>See Serroukh *et al.*, 2000, and Whitcher *et al.*, 2000.

<sup>39</sup>See Whitcher *et al.*, 2000, and Gençay *et al.*, 2003.

2000),<sup>40</sup> while the trend is associated to the low-frequency components of a signal corresponding to the long-run elements. Figure 1 shows the time series of the US industrial production index and the MODWT coefficient sequences  $\tilde{w}_{j,t}$  for levels  $j = 1$  to  $j = J = 6$ .

### 3.2 Wavelet variance analysis

In this section we apply wavelet variance analysis to the industrial production series of G-7 countries. The wavelet variance decomposes the variance of a time series on a scale-by-scale basis through a wavelet multiresolution analysis. Table 1 reports the estimated wavelet variances of industrial production at different wavelet scales,  $\tilde{\sigma}_X^2(\lambda_j)$ , for each G-7 country over the whole sample, as well as the upper and lower bounds for the approximate 95% confidence interval assuming a non-Gaussian process.<sup>41</sup>

In figure 2 we show the plot of  $\tilde{\sigma}_X^2(\lambda_j)$  against  $\lambda_j$  on a log-log scale, as it indicates which scales provide the most important contribution to the variance of the process.<sup>42</sup> The evidence indicates that, at the finest scales, wavelet variance tends to be approximately constant, while, at business cycle scales, it tends to increase as the wavelet scale increases.<sup>43</sup> Two other aspects of the results from wavelet variance are worth mentioning: the first is the wide dispersion of the wavelet variance values among countries at the finest scales, *i.e.* scales 1 to 3, with France and Italy displaying the highest values at all these scales, and the US the lowest values of wavelet variance; the latter is the high contribution of the last two scales to the overall variance.

The wavelet variance estimated values let us investigate some basic properties of the data generation process of a series. In particular, we can determine the relationship between wavelet variance and scale by  $\sigma_X^2(\lambda_j) \propto \lambda_j^{-\alpha-1}$ , where an estimate of  $\alpha$ , the scaling parameter in a pure power law process, may be obtained from the OLS regression of  $\log(\sigma_X^2(\lambda_j))$  on  $\log(\lambda_j^{-\alpha-1})$  (see Gençay *et al.*, 2001). As the estimated scaling parameters  $\hat{\alpha}$  range from  $-1.30$  (France) and  $-2.52$  (the US), according to the relationship between the value of the scaling parameter in a pure power law process and the type of the process (see Percival and Walden, 2000), all industrial production indices of the G-7

<sup>40</sup>According to the modern definition of business cycle, fluctuations in business cycles consist of frequencies between 1.5 to 8 (10) years (18 and 96 (120) months), so that they roughly correspond to scales 4, 5 and 6, as the scales in wavelet filters are dyadic.

<sup>41</sup>The level  $j$  wavelet variance is obtained using the wavelet scaling coefficients  $w_1$  to  $w_6$  not affected by the boundary (Gençay *et al.*, 2002).

<sup>42</sup>Note that lower scales correspond to higher frequencies and vice versa.

<sup>43</sup>In Figure 2 symbols C, G, F, I, J, U and S refer to Canada, Germany, France, Italy, Japan, the UK and the US, respectively.

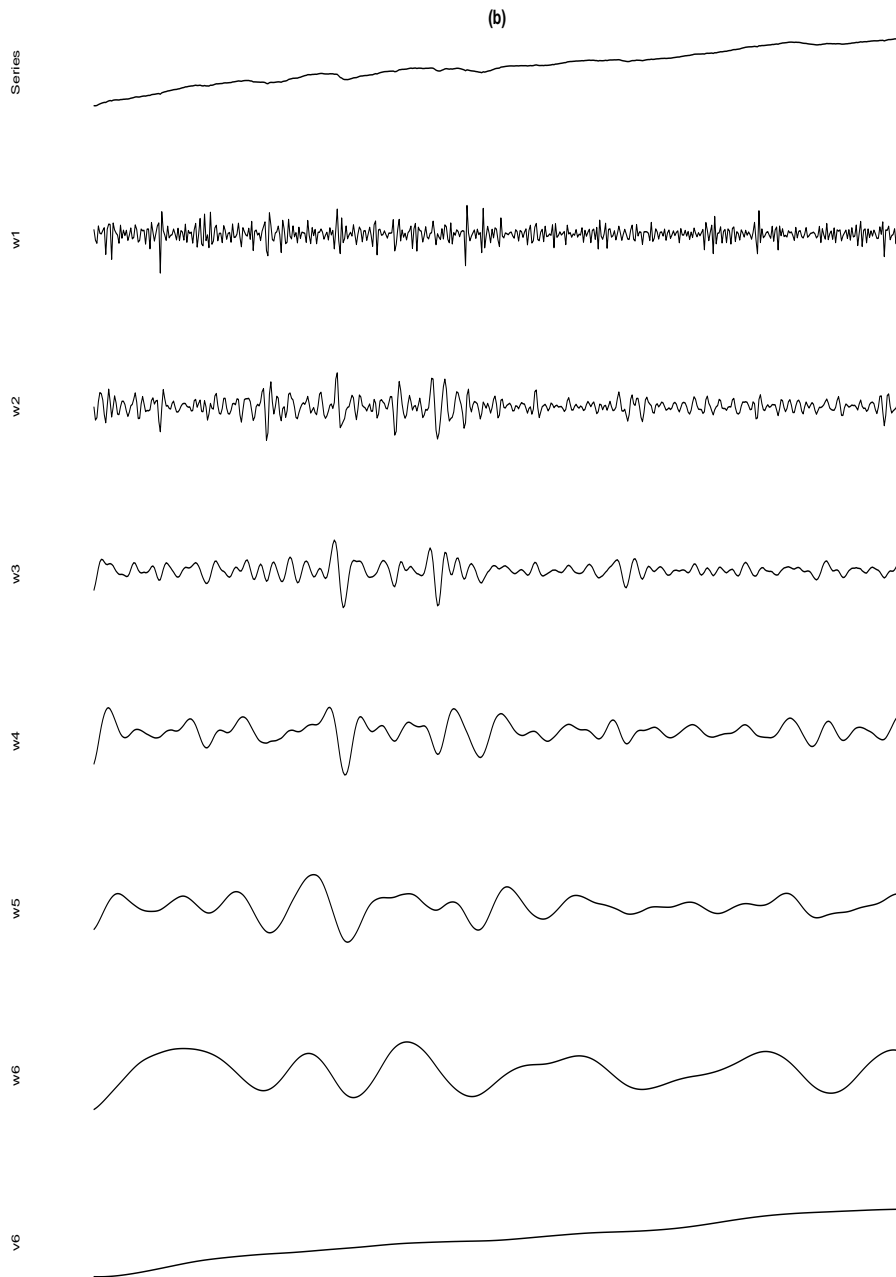


Figure 1: The time series of the US industrial production index and the MODWT coefficient sequences  $\tilde{w}_{j,t}$  for levels  $j = 1$  to  $j = J = 6$ .

Table 1: Multi scale wavelet variance coefficients of G-7 countries with upper and lower bound for the approximate 95% confidence interval assuming a non-Gaussian process.

scale	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$
<b>Canada</b>	<b>2.4e-05</b>	<b>1.6e-05</b>	<b>2.8e-05</b>	<b>1.5e-04</b>	<b>5.3e-04</b>	<b>9.9e-04</b>
lower	1.9e-05	1.4e-05	2.2e-05	9.3e-05	2.8e-04	4.9e-04
upper	2.7e-05	1.8e-05	3.4e-05	2.0e-04	7.8e-04	1.5e-03
<b>France</b>	<b>1.4e-04</b>	<b>1.4e-04</b>	<b>9.9e-05</b>	<b>1.4e-04</b>	<b>2.5e-04</b>	<b>3.9e-04</b>
lower	1.3e-04	1.2e-04	8.0e-05	9.1e-05	1.5e-04	1.8e-04
upper	1.6e-04	1.6e-04	1.2e-04	1.9e-04	3.4e-04	6.2e-04
<b>Germany</b>	<b>6.5e-05</b>	<b>3.6e-05</b>	<b>4.0e-05</b>	<b>1.1e-04</b>	<b>3.9e-04</b>	<b>7.6e-04</b>
lower	5.6e-05	2.9e-05	3.2e-05	7.1e-05	2.2e-04	3.2e-04
upper	7.5e-05	4.2e-05	4.8e-05	1.4e-04	5.8e-04	1.2e-03
<b>Italy</b>	<b>1.0e-04</b>	<b>8.3e-05</b>	<b>9.0e-05</b>	<b>2.4e-04</b>	<b>4.7e-04</b>	<b>5.7e-04</b>
lower	8.6e-05	6.9e-05	7.0e-05	1.5e-04	2.7e-04	2.8e-04
upper	1.2e-04	9.7e-05	1.1e-04	3.2e-04	6.7e-04	8.7e-04
<b>Japan</b>	<b>3.6e-05</b>	<b>1.8e-05</b>	<b>3.5e-05</b>	<b>2.6e-04</b>	<b>7.4e-04</b>	<b>1.1e-03</b>
lower	2.8e-05	1.5e-05	2.7e-05	1.4e-04	4.1e-04	5.2e-04
upper	4.3e-05	2.0e-05	4.3e-05	3.7e-04	1.1e-03	1.7e-03
<b>UK</b>	<b>3.9e-05</b>	<b>3.9e-05</b>	<b>4.9e-05</b>	<b>9.8e-05</b>	<b>2.7e-04</b>	<b>4.2e-04</b>
lower	3.4e-05	3.3e-05	3.8e-05	6.3e-05	1.3e-04	2.1e-04
upper	4.5e-05	4.5e-05	5.9e-05	1.3e-04	3.9e-04	6.3e-04
<b>US</b>	<b>6.0e-06</b>	<b>9.5e-06</b>	<b>3.1e-05</b>	<b>1.2e-04</b>	<b>4.3e-04</b>	<b>8.9e-04</b>
lower	5.3e-06	8.0e-06	2.3e-05	8.1e-05	2.3e-04	4.5e-04
upper	6.8e-06	1.1e-05	3.9e-05	1.5e-04	6.2e-04	1.3e-03

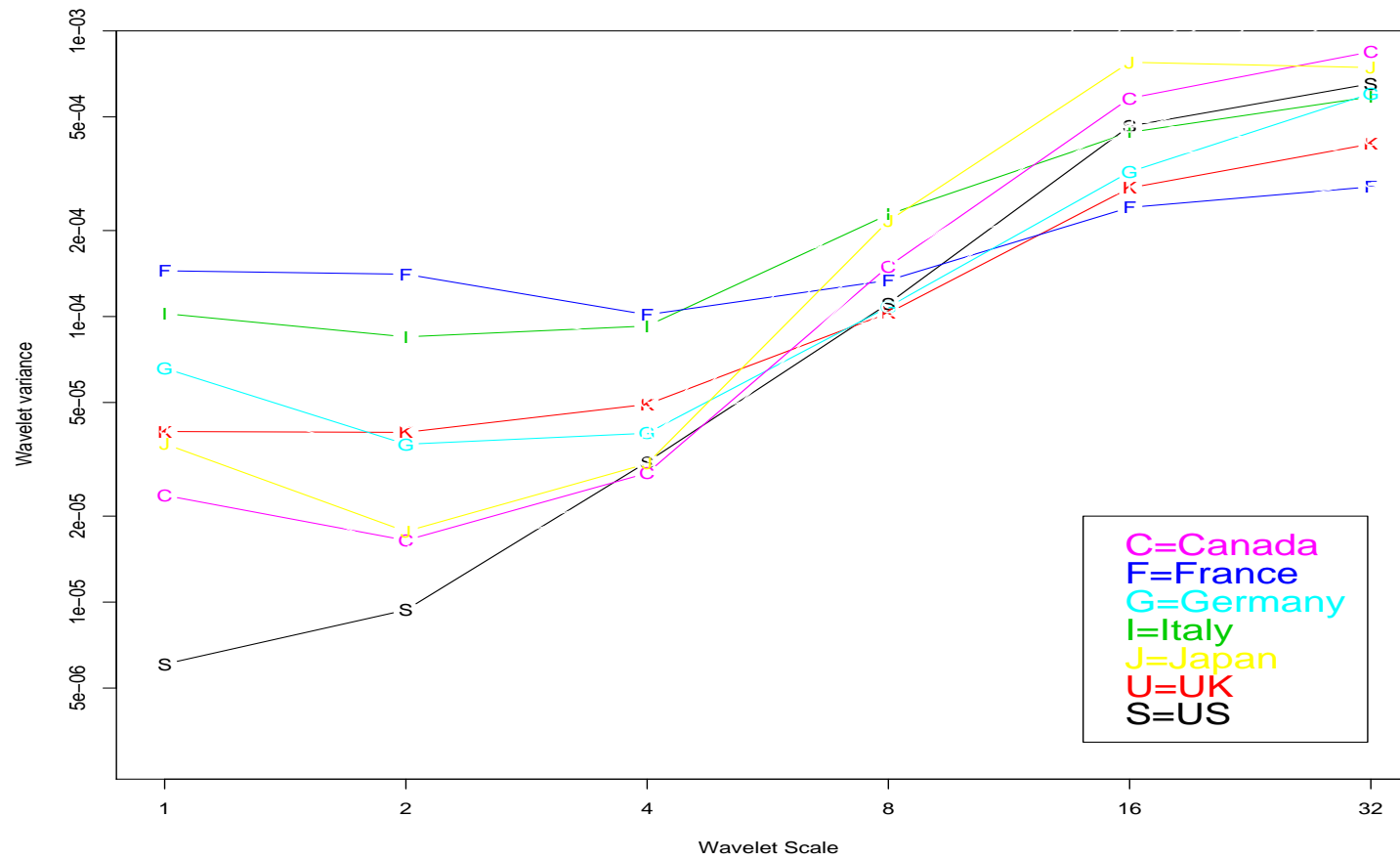


Figure 2: Multiscale wavelet variance coefficients of G-7 countries



countries may be considered as nonstationary long memory processes.<sup>44</sup>

In recent years there has been a renewed interest in the issue of volatility in economic activity following the articles by Mc Connel and Perez Quiros (2000) and Kim and Nelson (1999) who point out a reduction in output volatility in the US in the mid-1980's. Following their independent findings, many empirical studies have analyzed the decline in volatility by comparing changes in volatility in the post-mid-1980's period relative to the pre-mid-1980's period (Stock and Watson, 2003). Thus, in order to further explore the issue of a moderation in output volatility we perform the multiscale wavelet variance analysis splitting the overall sample into two distinct selected sub-periods: period I, the *pre-1984 period*, and period II, the *post-1984 period*.<sup>45</sup>

In Ahmed *et al.* (2002) frequency domain methods are used to investigate the sources of the decline in US growth volatility. In particular, given that the spectrum may decompose the variance by frequency, each competing explanation is associated with a specific shift pattern in the spectrum according to the frequency at which the spectrum itself is expected to shift.

Analogously, wavelet analysis, given its ability to decompose the variance of a series on a scale-by-scale basis, may be a suitable instrument to detect the relative importance of the various explanations for the moderation of volatility. Indeed, once such explanations have been associated to specific time scales on the basis of the inverse relationship between frequency and scale, an informal test about the different explanations for the "moderation" of volatility may be carried out. In particular,

- improvements in inventory management are expected to be associated to smallest scales (highest frequencies), as they help to smooth production within the month or the quarter (Stock and Watson, 2002);
- improvements in (monetary) policy management, in the form of a more aggressive response to inflation, are likely to be associated to business-cycle scales, given the lags in the effects of monetary policy on output;
- exogenous shocks are likely to be associated to different scales depending on their impact, temporary or permanent, and on their nature, demand or supply-side. For example, temporary shocks like strikes and other temporary influences in aggregate production are likely to be associated to short-term scales; on the other hand the effects of permanent shocks,

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<sup>44</sup>Long-memory processes are characterized by autocorrelation values decaying at a very slow rate such that the effects may persist over long time scales (Beran, 1994).

<sup>45</sup>In order to meet the requirement of a sufficient number of wavelet coefficients unaffected by the periodic boundary condition we restrict the estimation of wavelet variance up to scale 5.

like permanent increases in oil prices or productivity shocks, are likely to be associated to all scales. This is because, demand-side effects (such as those on consumers' spending) are likely to be associated to short-term scales, while supply-side effects (such as those on production and capital investment decisions) are likely to be associated to longer scales.<sup>46</sup>

Figure 3 reports the MODWT estimated G-7 multiscale variance over subsamples 1961-1983 (solid lines) and 1984-2006 (dotted lines) where the (.)'s are estimated variances for each scale. Lines with "U" and "L" denote, respectively, the upper and lower bound for the approximate 95% confidence interval assuming a non-Gaussian process. We may examine whether a decline in output volatility over time occurred on a scale-by-scale basis, with the decline represented by a downward shift of the line joining the estimated wavelet variance values. Moreover, as each explanation is associated to a specific scale range, determining if the downward shift is concentrated at particular scales or uniform across scales may provide some insight on the cause of the decline in volatility.

The comparison of the plots, reported in figure 3, confirms the general consensus about the occurrence of this moderation in overall economic activity, as in all G-7 countries (except Japan at the first two scales) the estimated wavelet variance values of period II,  $\tilde{\sigma}_{II}^2(\lambda_j)$ , lie below those of period I,  $\tilde{\sigma}_I^2(\lambda_j)$ . Approximate confidence intervals of estimated wavelet variance values  $\tilde{\sigma}_I^2(\lambda_j)$  and  $\tilde{\sigma}_{II}^2(\lambda_j)$  may be considered as a visual method to statistically testing the hypothesis of equality of wavelet variance across different time periods, that is

$$H_0 : \sigma_I^2(\lambda_j) = \sigma_{II}^2(\lambda_j).$$

In particular, when 95% approximate confidence intervals are non-overlapping, the hypothesis of variance equality may be rejected (Gençay *et al.*, 2002). Thus, the analysis of multiscale variance reported in figure 3 indicates that we can reject the hypothesis of variance equality for France, Italy, the UK and the US.

But this decline is not uniform across scales nor countries. In particular, the moderation is generally larger at the finest scales, *i.e.* 1 to 3, than at business cycle scales. As regards the countries displaying a drop in output volatility, the evidence in figure 3 indicates that for the UK and the US the moderation of volatility is quite uniform over all scales. On the contrary, in the case of France and Italy the decrease in volatility is larger at short-term scales than at business cycle scales.

<sup>46</sup>Similarly, in Ahmed *et al.* (2002) the assumption of covariance-stationarity for output growth (required by spectral analysis) implies that the good-luck hypothesis is represented by a parallel downward shift in the spectrum, as it affects all frequencies.

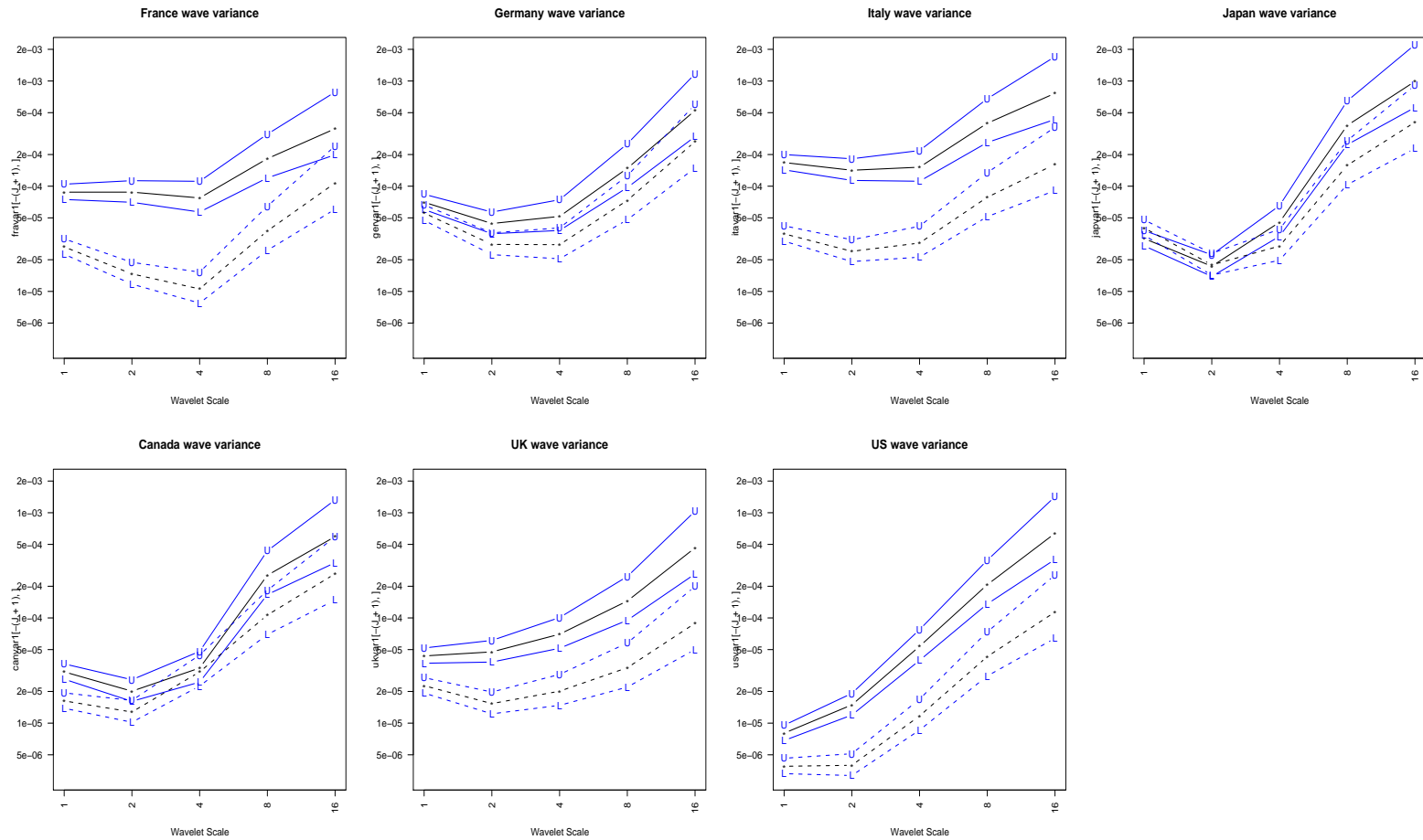


Figure 3: MODWT estimated G-7 multiscale variance over sub-samples 1961-1983 (solid lines) and 1984-2006 (dotted lines).

The choice to break down the overall sample into two sub-samples only, as the result of a one-break case, could be too restrictive for a forty-year sample characterized by the occurrence of both common international shocks, such as the oil price shocks of the 1970s,<sup>47</sup> and domestic shocks, such as the French May of the late 1960s or the German reunification process of the late 1980s. Following this line of reasoning Doyle and Faust (2002), after performing the analysis up to the three-break case for G-7 countries over a similar time span, focused on a three-break case, where the break dates fall in the early 1970s (1972:2), 1980s (1981:1) and 1990s (1992:2). As a consequence, we perform the multiscale wavelet variance analysis splitting the overall sample into four distinct selected sub-periods corresponding roughly to the decades: period 1 *before Bretton Woods collapse* (1961-1971), period 2 the *oil shocks* (1972-1982), period 3 the *slow recovery* (1983-1993) and period 4 the *irrational euphoric years* (1994-2006).<sup>48</sup>

In figure 4 we plot the estimated wavelet variances of G-7 countries at different time scales for the four sub-samples.<sup>49</sup> A simple eyeballing at the estimated wavelet variance values in Figures 4 shows that each G-7 country displays an individual pattern.<sup>50</sup> As a consequence, *they cannot be easily grouped in terms of behavior* and, moreover, may be inappropriate to consider the US as the reference country in the scheme of interpretation for the decline in output volatility. Nonetheless, some regularities seems to emerge both over scales and across countries.

The main regularity refers to the *oil price shocks* period<sup>51</sup> which displays the highest values of the estimated wavelet variance in all G-7 countries at all scales (the only exceptions are Japan, at almost any scale, and Germany at scale 1). For the Euro-zone countries also the *pre-Bretton Woods collapse* period is one characterized by high volatility. Indeed, Germany and Italy present values of the wavelet variance which are similar to those of period

<sup>47</sup>Sharp increases in oil prices occurred in correspondence of the Arab-Israeli war in 1973, the Iranian revolution in 1978 and the Iran-Iraq war in 1980. These oil price shocks represent examples of what Hamilton (2003) identifies as major oil supply disruptions, the only ones that matter for macroeconomic stability.

<sup>48</sup>The choice of the sub-periods is dictated by considering the main shocks, both common and country-specific, affecting G-7 countries over the last forty-five years.

<sup>49</sup>Again, in order to meet the requirement of a sufficient number of wavelet coefficients unaffected by the periodic boundary condition we restrict the estimation of wavelet variance up to scale 4.

<sup>50</sup>In figure 4 the numbered lines 1, 2, 3 and 4 represent the estimated multiscale wavelet variances of the corresponding sub-periods. Confidence intervals here are omitted in order to facilitate the readability of the results.

<sup>51</sup>There is a wide empirical literature regarding the importance of the impact of oil shocks on economic activity (see Hamilton, 1983, 2003; Hamilton and Herrera, 2004).

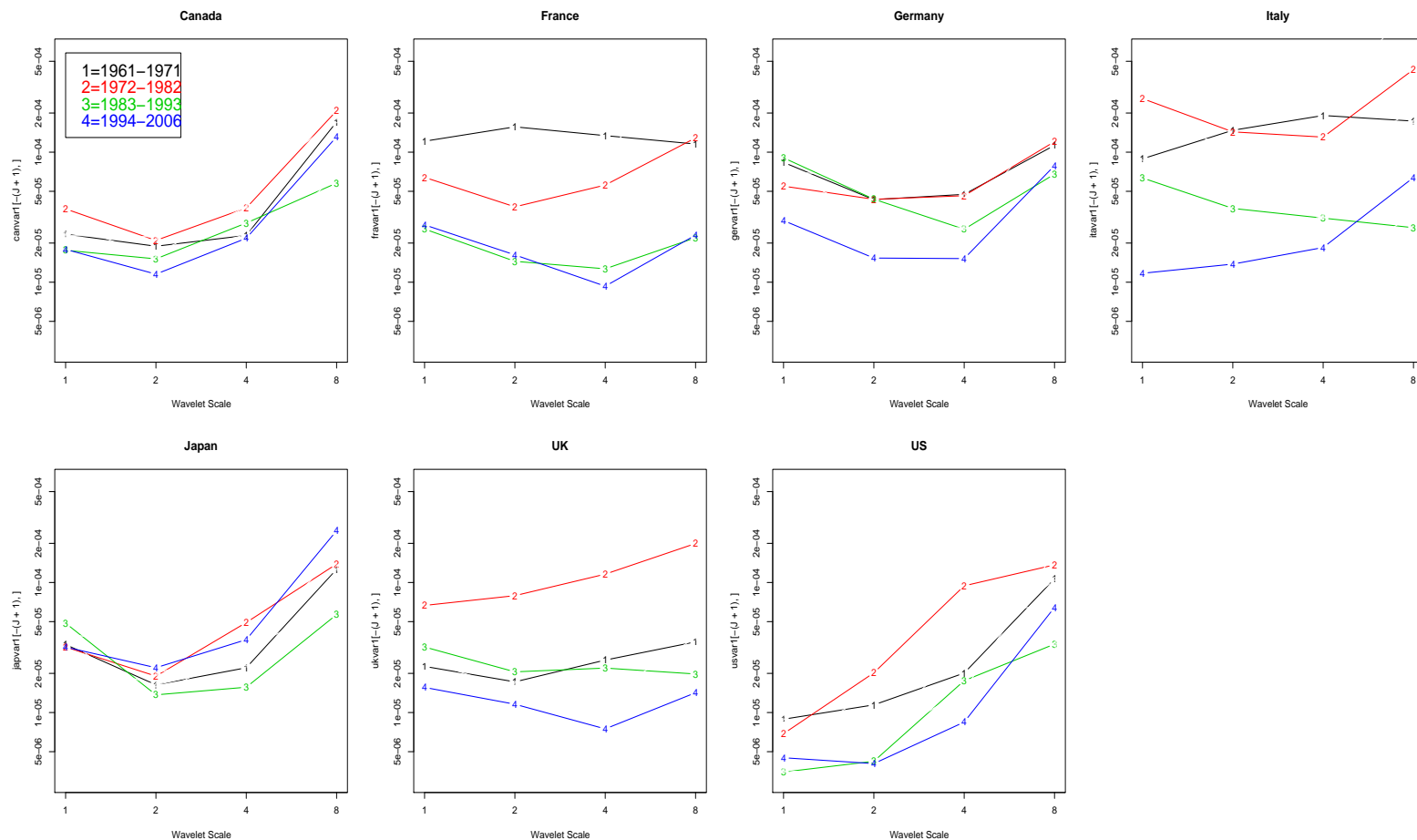


Figure 4: MODWT estimated multiscale wavelet variance over sub-samples 1961-1971 (period 1), 1972-1982 (period 2), 1983-1993 (period 3) and 1994-2006 (period 4) represented by solid lines 1, 2, 3 and 4, respectively.

2, while France displays the highest values of the estimated wavelet variance in absolute terms. At the other extreme, in period 4 the values of the estimated wavelet variances are the lowest ones everywhere, with the exception of Japan whose variance values are similar to that of the oil price shocks period. As regards the *slow recovery*, the values of the estimated wavelet variance are generally intermediate between the highest and lowest volatility periods (exceptions are Germany, whose variance at the shortest scales is also higher than those of the first two decades, and, again, Japan which displays the lowest values in the third decade).

Again, when we use confidence intervals as a visual method to test the hypothesis of equality of wavelet variance across sub-periods within each G-7 country, we are able to reject the hypothesis of equality of wavelet variance in many cases: between periods 1 and 2 versus periods 3 and 4 for France, between period 2 versus periods 3 and 4 for Italy and between period 2 versus all other periods for the UK.<sup>52</sup>

The results from the informal statistical test on wavelet variance suggest that the volatility reduction of the last decades has to be attributed to a fall in the variance of exogenous common structural disturbances hitting international economies from the 1980s.<sup>53</sup> In addition, the large decline in volatility displayed by France and Italy at the finest scales (*i.e.* higher frequencies) may be associated, consistently with the hypothesis proposed in this paper about the sources of the moderation in volatility, to the occurrence of large transitory country-specific exogenous disturbances impacting the labor markets of these countries in the 1960s.

## 4. Conclusions

In this paper we apply a multi-scaling approach to investigate the occurrence and the sources of the decline in output volatility using data on the industrial production index of the G-7 countries between 1961:1-2006:10. The analysis is performed using the MODWT estimator of wavelet variance as it decomposes the variance of a series on a scale-by-scale basis, and thus may be a suitable instrument to detect the relative importance of the various explanations for the

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<sup>52</sup>The rise in standard deviation that characterized France in the late 1960s (*French May*), Germany in the late 1980s (*German reunification*), Italy in the late 1960s (*Hot Autumn*) and Japan in the late 1990s has been well documented in the empirical literature investigating breaks in the variability in the G-7 economies (Doyle and Faust, 2002).

<sup>53</sup>Similar conclusions are also reached in Stock and Watson (2002), Blanchard and Simon (2001) and Ahmed *et al.*, (2001) who state that "our results support the *good-luck* hypothesis as the leading explanations [...] although *good-practices* and *good-policy* are also contributing factors".

moderation of volatility. Indeed, looking at wavelet variance we may examine whether there is a decline in output volatility over time on a scale-by-scale basis, with the decline represented by a downward shift of the line joining the estimated wavelet variance values. Moreover, as each explanation may be associated to a specific scale range, determining if the downward shift is concentrated at particular scales or uniform across scales may provide some insight on the nature of the decline in volatility.

The evidence from wavelet variance analysis indicates that the reduction in variance, although common to all G-7 countries, is not uniform across countries (as the decline is significant for a subset of countries only, *i.e.* France, Italy, the UK and the US) nor scales (as the decline is larger at short-term scales than at business cycle scales for France and Italy, and quite uniform across scale for the UK and the US). Moreover, when we perform the multiscale wavelet variance analysis splitting the overall sample into four distinct selected sub-periods corresponding roughly to the decades, the results from the informal statistical test based on the wavelet variance provide evidence of the importance for the moderation in output volatility of the common (in the 1970s) and country-specific (in the 1960s) exogenous disturbances hitting international economy

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