WAVES AND OSCILLATIONS IN THE CORONA

(Invited Review)

B. ROBERTS
Mathematical Sciences, University of St. Andrews, St. Andrews, Fife KY16 9SS, Scotland
(e-mail: bernie@mcs.st-and.ac.uk)

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Abstract. It has long been suggested on theoretical grounds that MHD waves must occur in the solar corona, and have important implications for coronal physics. An unequivocal identification of such waves has however proved elusive, though a number of events were consistent with an interpretation in terms of MHD waves. Recent detailed observations of waves in events observed by SOHO and TRACE removes that uncertainty, and raises the importance of MHD waves in the corona to a higher level. Here we review theoretical aspects of how MHD waves and oscillations may occur in a coronal medium. Detailed observations of waves and oscillations in coronal loops, plumes and prominences make feasible the development of *coronal seismology*, whereby parameters of the coronal plasma (notably the Alfvén speed and through this the magnetic field strength) may be determined from properties of the oscillations. MHD fast waves are refracted by regions of low Alfvén speed and slow waves are closely field-guided, making regions of dense coronal plasma (such as coronal loops and plumes) natural wave guides for MHD waves. There are analogies with sound waves in ocean layers and with elastic waves in the Earth's crust. Recent observations also indicate that coronal oscillations are damped. We consider the various ways this may be brought about, and its implications for coronal heating.

1. Introduction

The recent discovery by the Transition Region and Coronal Explorer (TRACE) spacecraft of oscillations in coronal loops (Aschwanden *et al.*, 1999; Nakariakov *et al.*, 1999) raises the twin prospects of a greater insight into the process of coronal heating and the use of such oscillations as a diagnostic tool for coronal seismology. The theoretical basis for coronal seismology has been known for some time, with the demonstration that magnetohydrodynamic (MHD) waves in coronal structures may be refracted by regions of low Alfvén speed (Uchida, 1968, 1970; Habbal, Leer, and Holzer, 1979) and thus wave guided by such regions and so able to form distinctive wave packets (Edwin and Roberts, 1983; Roberts, Edwin, and Benz, 1983, 1984). In fact the evidence for MHD waves in the corona has increased dramatically over the last few decades. Their existence in coronal loops had been inferred from modulated radio emission (see Roberts, Edwin, and Benz (1984) and the review by Aschwanden (1987)). Oscillations have been detected in prominences (e.g., Tsubaki, 1988) and given a theoretical description in terms of MHD waves (Joarder and Roberts, 1992, 1993; Oliver *et al.*, 1993; Roberts and

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Joarder, 1994; Joarder, Nakariakov, and Roberts, 1997). Sunspots support a variety of wave phenomena that may be interpreted in terms of MHD waves (see Thomas and Weiss, 1992, and articles therein). Recently, coronal plumes have been shown to harbour waves, and these may be interpreted as slow MHD waves (DeForest *et al.*, 1997; DeForest and Gurman, 1998; Ofman, Nakariakov, and DeForest, 1999; Ofman, Nakariakov, and Sehgal, 1999). Finally, there are global disturbances in the form of EIT waves (Thompson *et al.*, 1999, 2000) and quakes (Kosovichev and Zharkova, 1999).

The fact that the corona is highly inhomogeneous complicates the theoretical description of MHD waves, giving rise to a variety of phenomena including phase mixing, resonant absorption and dispersive ducting. But the ability to measure oscillations in closed structures (prominences, coronal loops) and waves in open structures (plumes) establishes the basis for coronal seismology, suggested earlier on theoretical grounds (Uchida, 1970; Roberts, Edwin, and Benz, 1984; Roberts, 1986). Of course, waves are interesting in their own right, simply because they are there, but their use in coronal seismology, in accelerating the solar wind, and in coronal heating adds greatly to that interest.

Any description of MHD waves in coronal structures modelled as magnetic flux tubes leads to the occurrence of two particular speeds, in addition to the familiar sound speed c_s and Alfvén speed c_{A}. The slow magnetoacoustic speed c_t and the kink speed c_k are defined by

$$\frac{1}{c_t} = \left(\frac{1}{c_s^2} + \frac{1}{c_{\mathrm{A}}^2}\right)^{1/2}, \qquad c_k = \left(\frac{\rho_0 c_{\mathrm{A}}^2 + \rho_e c_{\mathrm{A}e}^2}{\rho_0 + \rho_e}\right)^{1/2}, \tag{1}$$

where ρ_0 and c_{A} are the plasma density and Alfvén speed inside the tube, and ρ_e and $c_{\mathrm{A}e}$ are their values in the tube's environment. The speed c_t is sub-sonic and sub-Alfvénic; the speed c_k is the mean Alfvén speed of the medium, intermediate between the Alfvén speed inside the tube and the Alfvén speed in the environment. In the low β coronal plasma, where sound speeds are much smaller than Alfvén speeds, the slow speed c_t is close to the sound speed inside the flux tube. If the magnetic field strength B_0 inside a coronal flux tube (loop) is comparable to the field strength B_e in the environment, so $B_0 \approx B_e$, but the plasma densities are very different, so that the inside of the tube is much denser than its surroundings (i.e., $\rho_0 \gg \rho_e$), then $c_k \approx \sqrt{2} c_{\mathrm{A}}$. Thus the kink speed may be some 41% larger than the Alfvén speed inside the tube. The speeds c_t and c_k are important also in studies of isolated photospheric flux tubes (e.g., Roberts, 1985, 1990a, b, 1992; Hollweg, 1990a; Ryutova, 1990).

2. Wave Equations

The observed complexity of the coronal plasma places severe demands on any theory to model its MHD behaviour. In wave studies it is common to consider a magnetically structured atmosphere in which the effects of gravity and flow are ignored (e.g., Roberts, 1981). A flux tube geometry is of particular interest. Ignoring the effect of twisted magnetic fields (both twisted fields and steady flows can in principle also be included, with interesting and important results; see Bennett, Roberts and Narain (1999) for a recent discussion of twisted tubes), we consider the equations of linear MHD describing a small amplitude perturbative flow $\mathbf{v}$ about an equilibrium magnetic field $\mathbf{B}_0$. The equilibrium field is taken to be unidirectional and aligned with the z-axis of a cylindrical coordinate system (r, θ, z), though it may vary in strength across the field; specifically $\mathbf{B_0} = B_0(r)\hat{\mathbf{z}}$ (for unit vector $\hat{\mathbf{z}}$ aligned with the z-axis).

This model aims to describe waves in a magnetically structured coronal atmosphere but it should be noticed that it ignores the effects of field-line curvature – coronal flux tubes are curved loops – and gravitational stratification. Curvature is likely to be of some importance in describing waves in loops; similarly, stratification may be of some significance, especially in any description of the coupling of a coronal loop to the dense photosphere−chromosphere. Ultimately, both these effects need to be incorporated into a description of the MHD modes, though they are not likely to be first-order effects.

The equilibrium plasma pressure $p_0(r)$ and density $\rho_0(r)$ of a radially structured magnetic atmosphere are such as to maintain total pressure balance: the sum of the plasma pressure p_0 and the magnetic pressure $B_0^2/2\mu$ is a constant,

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(p_0(r) + \frac{B_0^2(r)}{2\mu}\right) = 0 \,. \tag{2}$$

Where the field is strong, the plasma pressure is correspondingly reduced.

Small amplitude motions $\mathbf{v} = (v_r, v_\theta, v_z)$ about the equilibrium (2) satisfy the coupled wave equations (e.g., Roberts, 1981, 1991b)

$$\rho_0(r)\left(\frac{\partial^2}{\partial t^2} - c_\mathrm{A}^2(r)\frac{\partial^2}{\partial z^2}\right)v_r + \frac{\partial^2 p_T}{\partial r\,\partial t} = 0 \,, \tag{3}$$

$$\rho_0(r)\left(\frac{\partial^2}{\partial t^2} - c_\mathrm{A}^2(r)\frac{\partial^2}{\partial z^2}\right)v_\theta + \frac{1}{r}\frac{\partial^2 p_T}{\partial \theta\,\partial t} = 0 \,, \tag{4}$$

$$\left(\frac{\partial^2}{\partial t^2} - c_t^2(r)\frac{\partial^2}{\partial z^2}\right)v_z = -\left(\frac{c_s^2(r)}{c_\mathrm{A}^2(r) + c_s^2(r)}\right)\frac{1}{\rho_0(r)}\frac{\partial^2 p_T}{\partial z\,\partial t} \,. \tag{5}$$

The evolution of the total perturbed pressure $p_T(r, \theta, z)(= p + B_0 B_z/\mu$, for perturbed plasma pressure p and magnetic field component B_z in the direction of the applied field) is described by

$$\frac{\partial p_T}{\partial t} = \rho_0(r)c_\mathrm{A}^2(r)\frac{\partial v_z}{\partial z} - \rho_0(r)(c_s^2(r) + c_\mathrm{A}^2(r))\,\mathrm{div}\,\mathbf{v} \,. \tag{6}$$

Equation (6) results from a combination of the isentropic equation and the induction equation of ideal MHD. Equations (3)–(5) come from the components of the momentum equation in which the magnetic force has been expressed in terms of p_T.

One solution of the above system of equations is $p_T = 0$ with $v_r = v_z = 0$ and $\partial/\partial\theta = 0$; this describes a torsional Alfvén wave $\mathbf{v} = (0, v_\theta, 0)$ which satisfies the wave equation with Alfvén speed $c_{\mathrm{A}}(r)$. The torsional Alfvén wave exhibits phase mixing (Heyvaerts and Priest, 1983). We return to this topic later.

More generally, when $p_T \neq 0$, it is usual to describe pressure variations (and the associated motions) in terms of a Fourier representation, writing

$$p_T(r, \theta, z, t) = p_T(r) \exp i(\omega t + n\theta - k_z z) \tag{7}$$

for frequency ω, azimuthal number $n = 0, 1, 2, \ldots$, and longitudinal wavenumber k_z. The resulting equations may then be manipulated to yield the ordinary differential equation (e.g., Edwin and Roberts, 1983)

$$\rho_0(r)(k_z^2 c_{\mathrm{A}}^2(r) - \omega^2)\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left\{\frac{1}{\rho_0(r)(k_z^2 c_{\mathrm{A}}^2(r) - \omega^2)} r \frac{\mathrm{d}p_T}{\mathrm{d}r}\right\} = \left(m^2(r) + \frac{n^2}{r^2}\right) p_T \tag{8}$$

with m^2 given by

$$m^2(r) = \frac{(k_z^2 c_s^2 - \omega^2)(k_z^2 c_{\mathrm{A}}^2 - \omega^2)}{(c_s^2 + c_{\mathrm{A}}^2)(k_z^2 c_t^2 - \omega^2)} .$$

Equation (8) possesses singularities at $\omega^2 = k_z^2 c_{\mathrm{A}}^2$ and $\omega^2 = k_z^2 c_t^2$; these singularities generate the Alfvén and slow continua, respectively. The presence of these singularities is an indication of a number of interesting effects connected with the phenomenon of resonant absorption (Goedbloed, 1971, 1983; Goossens, 1991; Goossens and Ruderman, 1995; Tirry and Goossens, 1996), of particular interest in the question of coronal heating.

We consider the specific case of a flux tube of radius a, field strength B_0 and plasma density ρ_0 embedded in a magnetic environment with field strength B_e and plasma density ρ_e:

$$B_0(r) = \begin{cases} B_0, & r < a, \\ B_e, & r > a, \end{cases} \qquad \rho_0(r) = \begin{cases} \rho_0, & r < a, \\ \rho_e, & r > a. \end{cases} \tag{9}$$

The Alfvén, sound and tube speeds within the tube are c_{A}, c_s and c_t, and their values in the external medium are $c_{\mathrm{A}e}$, c_{se} and c_{te}.

For the equilibrium (9) the differential equation (8) may be solved for p_T in terms of Bessel functions $J_n(n_0 r)$ in $r < a$ and $K_n(m_e r)$ in $r > a$, with the result (see McKenzie, 1970; Wentzel, 1979; Wilson, 1980; Spruit, 1982; Edwin and Roberts, 1983; Cally, 1986; Evans and Roberts, 1990)

$$\frac{n_0}{\rho_0(k_z^2 c_{\mathrm{A}}^2 - \omega^2)}\frac{J_n'(n_0 a)}{J_n(n_0 a)} = \frac{m_e}{\rho_e(k_z^2 c_{\mathrm{A}e}^2 - \omega^2)}\frac{K_n'(m_e a)}{K_n(m_e a)} , \tag{10}$$

where

$$n_0^2 = \frac{(\omega^2 - k_z^2 c_A^2)(\omega^2 - k_z^2 c_s^2)}{(c_s^2 + c_A^2)(\omega^2 - k_z^2 c_t^2)}, \qquad m_e^2 = \frac{(k_z^2 c_{se}^2 - \omega^2)(k_z^2 c_{Ae}^2 - \omega^2)}{(c_{se}^2 + c_{Ae}^2)(k_z^2 c_{te}^2 - \omega^2)} .$$

Equation (10) is the dispersion relation describing waves in a magnetic flux tube that is embedded in a magnetic environment; it applies when $m_e > 0$, corresponding to waves that are confined to the tube. The integer n that arises in the description of tube waves defines the geometry of the vibrating tube. The case $n = 0$ corresponds to the sausage wave (a symmetric pulsation of the tube, with the central axis of the tube remaining undisturbed). The case $n = 1$ describes the kink mode (involving lateral displacements of the tube, maintaining a circular cross-section, with the axis of the tube resembling a wriggling snake). Finally, there are fluting modes ($n \geq 2$), which ripple the boundary of the tube. Only the kink mode displaces the central axis of the vibrating tube.

The restriction $m_e > 0$ imposed on the flux tube dispersion relation means that the amplitude of a wave declines exponentially with radius $r(> a)$, so that far from the tube there is no appreciable disturbance. Inside the tube (for $r < a$) the disturbance is oscillatory if $n_0^2 > 0$ or non-oscillatory (evanescent) if $n_0^2 < 0$. Modes that inside the tube are oscillatory in r are called body waves. In the strongly magnetized coronal plasma, the modes are body waves. The waves are dispersive, the phase speed $c(= \omega/k_z)$ of a tube wave depending upon its wavelength, $2\pi/k_z$, through the combination $k_z a$.

The dispersion relation (10) possesses two sets of modes, namely fast and slow body waves. (There are no surface ($n_0^2 < 0$) waves.) The fast waves are strongly dispersive, and arise only if $c_{Ae} > c_A$; if $c_{Ae} < c_A$, then the fast waves are leaky and propagate energy away from the region of high Alfvén speed. Fast body waves, then, are trapped in regions of *low* Alfvén speed, typically corresponding to regions of *high* plasma density. Regions of low Alfvén speed in a strongly magnetized plasma provide wave guides for fast magneto-acoustic waves (Uchida, 1974; Habbal, Leer, and Holzer, 1979; Edwin and Roberts, 1983; Roberts, Edwin, and Benz, 1984).

There are close analogies between the behaviour of fast magnetoacoustic body modes in a strongly magnetized plasma and Love waves in the Earth's crust and Pekeris sound waves in an internal ocean layer; the fast sausage waves are sometimes referred to as magnetic Pekeris waves and the fast kink waves as magnetic Love waves (Edwin and Roberts, 1983; Roberts, Edwin, and Benz, 1984).

The slow waves (with phase speed between the slow speed c_t and the tube's sound speed) are only weakly dispersive. In a strongly magnetized plasma, their speed is close to the sound speed in the tube. The coronal loop provides an almost rigid tube for the one dimensional ducting of sound waves.

Following the treatment in Roberts, Edwin, and Benz (1984), we may discuss both standing and propagating modes. Standing waves occur in closed structures such as loops, provided the wave has had time to travel the entire length of the loop and back again; for shorter times, the wave has not reached the ends of the loop, where line-tying in the dense lower atmosphere causes reflection, and so the wave propagates freely as if the structure were open. Setting $k_z = N\pi/L$ in a tube (coronal loop) of length L gives the allowed wavelengths $2L/N$ $(= 2\pi/k_z)$ that fit along the closed tube. The integer $(N - 1)$ describes the number of nodes of the vibration along the axis of the tube, with $N = 1$ being the principal mode (and having no nodes within the tube). The period τ $(= 2\pi/\omega)$ of a *slow* wave $(c \approx c_t)$ standing in a loop is

$$\tau = \frac{2L}{Nc_t} \approx \frac{2L}{Nc_s} \,. \tag{11}$$

This generally produces long periods and requires quite some time to set up. For example, in a loop of length $L = 100$ Mm $(= 10^5$ km) with Alfvén speed $c_\mathrm{A} = 10^3$ km s^{-1} and sound speed $c_s = 200$ km s^{-1} it requires some 10^3 s for a slow wave to traverse the extent of the loop and so set up a standing wave with this period (for the principal mode $N = 1$). This is so long that it may be difficult to observe in general, except perhaps in short loops where the period can be reduced to the order of 100 s.

Turning to fast waves, the *kink* mode has speed $c \approx c_k$ and so produces a standing wave of period τ given by

$$\tau = \frac{2L}{Nc_k} \,. \tag{12}$$

We may view this as a global mode of oscillation of a coronal flux tube, moving the whole tube in its vibration. The mode exists for all wavelengths and frequencies as a trapped oscillation of the tube. In the extreme of a coronal tube with internal field strength B_0 comparably with the field strength B_e in the environment but plasma density ρ_0 much greater than the density ρ_e in the environment, so that $B_0 \approx B_e$ and $\rho_0 \gg \rho_e$, the kink speed is $c_k \approx \sqrt{2}c_\mathrm{A}$ and the period (12) of a standing kink mode becomes

$$\tau = \frac{\sqrt{2}L}{Nc_\mathrm{A}} \,. \tag{13}$$

This is much shorter than the period of a slow wave; for the above illustration, we obtain $\tau = 140$ s for the principal mode $N = 1$, and about 100 s must elapse before the wave has had time to become aware of the loop ends.

When the mode number N is of low or moderate value (say, $N = 1$–10) and $a \ll L$ – the usual situation in a coronal loop – then the slow modes and the fast kink mode are the only magneto-acoustic waves to be ducted within a loop. The fast sausage mode *leaks* from the loop. This suggests that it is the kink mode that is most likely to be seen as a *standing* wave in a coronal loop.

The conclusion that for standing waves it is the fast kink mode or the slow modes that most likely arise (the sausage wave leaking for low $k_z a$) does not apply when waves are impulsively excited; then all modes are equally possible. The case of impulsively excited waves is in fact of interest. Impulsive excitation of waves may be due to a flare or reconnection or indeed any process that imparts a sudden motion or pressure variation into a loop. Our discussion considers the case of an open magnetic structure, which includes the case of waves in a loop provided the wave has not had time to reach the ends of the loop (and so feel the effect of line-tying). Of particular interest is the fast sausage wave, since this mode is leaky for small $k_z a$ and so unable to exist as a trapped standing wave (unless the mode number N is moderately large, of order L/a). However, when impulsively excited the wave is able to propagate freely along the tube and does so in accordance with its strong dispersive nature. The result is that the sausage wave produces a distinctive wave signature. The distinctive nature of this wave packet makes it potentially useful as a means of coronal diagnostics with high temporal resolution (to be contrasted with the use of standing waves for more moderate temporal resolution). The impulsively generated fast wave produces a signature which consists of three parts: a low-amplitude periodic phase, followed by a larger amplitude quasi-periodic phase, and finally a decay phase. The longest time scales in the motion are those in the periodic phase; the quasi-periodic phase produces high frequency oscillations. An estimate of the time scale τ^{pulse} in the periodic phase is provided by (Roberts, Edwin, and Benz, 1984)

$$\tau^{\text{pulse}} = \frac{2\pi a}{j_0 c_A}\left(1 - \frac{\rho_e}{\rho_0}\right)^{1/2} , \tag{14}$$

where j_0 (≈ 2.40) denotes the first zero of the Bessel function J_0. In this estimate we have taken the Alfvén speed to be much larger than the sound speed. For a tube of radius $a = 10^3$ km and Alfvén speed $c_A = 10^3$ km s^{-1}, with plasma density much higher than that in the environment (so $\rho_0 \gg \rho_e$), this produces a time scale of 2.6 s; such short time scales make for high temporal resolution seismology. In addition to the time scale τ^{pulse}, the decay time of a propagating disturbance carries seismic information too. The diagnostic use of the decay time of a propagating pulse is considered in Roberts, Edwin, and Benz (1984).

We end our discussion with a specific illustration, considering the loop oscillations observed by TRACE. Aschwanden *et al.* (1999) report the detection of oscillations in five loops, with lengths L extending from 90 to 160 Mm and periods τ in the band 258–320 s; oscillation amplitudes are in the range 2.0–5.6 Mm. The oscillations occurred following a flare that began at 14 July 1998, 12.55 UT, their onset apparently being due to a disturbance that propagated from the central flare site with a radial speed of about 700 km s^{-1}. Aschwanden *et al.* conclude that the loop oscillations are most likely standing kink modes. The same event is considered by Nakariakov *et al.* (1999) who report the oscillations of a loop, again interpreted as a standing kink mode, and pay particular attention to the damping of the oscillation. The oscillation has a displacement amplitude of 2 Mm (speed 47 km s^{-1}), with a cyclic frequency $\nu\,(= \omega/2\pi)$ of 3.9 mHz, corresponding to a period of 256 s. The loop is 130 Mm in length. The oscillation lasts for over 20 min; Nakariakov *et al*. estimate the decay time scale to be about 14.5 min (i.e., the oscillation amplitude falls by a factor of e in a time of about 14.5 min).

Consider, then, the application of the period formula (12) to the TRACE oscillations, interpreted as standing kink waves. The shortest loop observed by Aschwanden *et al*. is of length $L = 90$ Mm and has a period of 258 s; for the principal mode, Equation (12) produces a kink speed of almost 700 km s^{-1}. On the other hand, the longest loop observed is of length 160 Mm and has a period of 278 s, producing a kink speed of close to 1000 km s^{-1}. This difference in the kink speed may perhaps be understood as a consequence of differing density contrasts between a loop and its environment. For the loop oscillation observed by Nakariakov *et al*., an interpretation in terms of a standing kink mode in its principal oscillation ($N = 1$) produces a kink speed of $c_k = 1014$ km s^{-1}. Assuming that $\rho_0 = 10\rho_e$ (the actual density ratio is not known) then produces a coronal loop Alfvén speed of $c_{\mathrm{A}} = 752$ km s^{-1}.

3. Damping

The observed extraordinary damping reported in the coronal loop oscillations detected by TRACE raises a major puzzle. Why do the oscillations decay so rapidly, in only a relatively few periods? To answer our question is not easy because as yet there is no theory of coronal oscillations that takes full account of the many complexities of the corona. For example, the theory described above makes no allowance for loop curvature, for stratification or for continuous (as opposed to discrete) field or plasma inhomogeneity, nor is there any realistic modelling of the coupling of the coronal plasma to the denser chromospheric–photospheric plasma. Moreover, the medium has been treated as ideal. So a number of effects crowd in and complicate our picture with some of these effects leading to damping. By damping we mean that the oscillation decays in time; this may indicate a local physical damping with heating resulting or it may indicate a transfer of energy from the visible oscillations to some other agency (with no heating resulting). Both effects are quite possibly occurring. Here we consider briefly some of the effects that require evaluation if we are to explain the observed damping. A fuller discussion of this problem will ultimately require a detailed and probably numerical treatment of a realistic coronal loop. In the meantime, it behoves us to enquire as to what effects are most likely, and where possible to estimate the time scales they produce.

Non-ideal effects, such as viscous and ohmic damping, optically thin radiation, and thermal conduction, act to damp wave motion. Damping due to optically thin (or thick) radiation and thermal conduction have been assessed for an unbounded uniform medium (e.g., Bogdan and Knölker, 1989; Ibañez and Escalona, 1993) or for a slab geometry (Van der Linden and Goossens, 1991; Laing and Edwin, 1994), who generally find that fast waves are not readily damped, requiring at least 20 (and maybe several hundred) damping periods before suffering any appreciable decay. This effect therefore is too slow to explain the observed decay.

An effect that leads to the appearance of damping, but is in fact operating in an ideal medium, is wave leakage. Wave leakage is likely to occur when loop curvature is taken into account in any description of the oscillations or when there is leakage from the ends of the loop (as energy is ducted from the coronal part of the loop into the dense footpoints of the loop). Leakage of fast waves from loop footpoints has been considered by Berghmans and De Bruyne (1995), who conclude that only a very small amount of wave energy is transmitted through to the lower and denser medium from the corona, leading to a decay of order 100 times the transit time of a wave along the loop. This would amount to some 10^4 s, too long to explain the observed decay.

Consider then another contribution to wave leakage: loop curvature. In a coronal loop modelled as a straight magnetic flux tube, we have described a division of modes into sausage and kink oscillations and the global kink mode is trapped within the flux tube for all frequencies. In contrast, the fast sausage modes are trapped at high frequencies but leak at low frequencies. The trapping of the kink mode in a straight flux tube led Nakariakov *et al.* to presume that wave leakage does not occur. But in a *curved* loop, it seems likely that the precise distinction between kink and sausage modes in a straight flux tube is lost, since now a kink-like oscillation will tend to compress the plasma – as the oscillating loop tries to fit into the curved geometry – and so couple with a sausage-like mode. If this is indeed the case (a detailed calculation of the effect is not available), then the fact that the sausage mode leaks at low frequencies suggests that there would be a corresponding leakage in the kink mode, as energy in the kink oscillation is partly transferred into sausage-like oscillations which then leak away by radiating a wave out to infinity. In the absence of a detailed study it is difficult to assess the efficiency of this process, but it would seem that whatever the leakage in the kink mode, due to curvature coupling with the sausage mode, it would not be larger than the leakage experienced by the sausage mode in a straight tube. The leakage of the sausage mode in a straight tube may be estimated as follows (see Spruit, 1982; Cally, 1986). Taking (for simplicity) the plasma β to be zero (corresponding to zero sound speed), leakage of the sausage mode leads to a decay time τ_d which is of order $(L/a)^2$ times the period of the wave. For $a/L = 0.01$, this produces a decay time that is about 10^4 wave periods, and so much longer than the observed decay. This suggests that such leakage is probably not important, but until a proper study of curvature effects becomes available (so far only Cartesian arcade geometries have been examined; see Smith, Roberts, and Oliver, 1997; Terradas, Oliver, and Ballester, 1999) this conclusion must remain tentative.

Viscous and ohmic damping are generally considered to be very small in the coronal plasma, but the presence of density inhomogeneity on a small scale is expected to strongly enhance dissipation through the process of resonant absorption (see, for example, reviews by Hollweg, 1990b; Goossens, 1991; see also Mok and Einaudi, 1985; Ruderman, Tirry, and Goossens, 1995). We can most easily *illustrate* the effect by examining the process of *phase mixing* of an Alfvén wave (Heyvaerts and Priest, 1983). This is not to imply that phase mixing is the actual cause of the damping; but resonant absorption processes are operating in the system, and do so in a manner similar to phase mixing and it is phase mixing that is easier to describe mathematically. The Alfvén wave in an inhomogeneous plasma with Alfvén speed $c_A(x)$ structured in x (the Cartesian coordinate perpendicular to the applied magnetic field) propagates transverse motions $v(x, z, t)\hat{\mathbf{y}}$ according to the wave equation (Heyvaerts and Priest, 1983)

$$\frac{\partial^2 v}{\partial t^2} = c_A^2(x)\frac{\partial^2 v}{\partial z^2} + \nu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\frac{\partial v}{\partial t} \ , \tag{15}$$

where ν is the coefficient of kinematic viscosity (considered dominant over ohmic diffusivity). An approximate solution of Equation (15) is (cf., Heyvaerts and Priest, 1983; Roberts, 1988; Hood, Ireland, and Priest, 1997; Ruderman, 1999)

$$v = u(t)\sin(k_z z)\cos(k_z c_A(x)t) \ , \tag{16}$$

where the amplitude $u(t)$ is given by

$$u(t) = u_0 \exp\{-[\tfrac{1}{2}k_z^2 \nu(t + \tfrac{1}{3}c_A'^2 t^3)]\} \ , \tag{17}$$

the prime ($'$) denoting the derivative of the Alfvén speed $c_A(x)$.

In a uniform medium (for which c_A is constant) the decay in amplitude $u(t)$ is exponential in time t, producing a decay time τ_d given by

$$\tau_d = \frac{2}{\nu k_z^2} \ . \tag{19}$$

For a standing wave with $k_z = N\pi/L$ this produces a decay time

$$\tau_d = \frac{2L^2}{\nu N^2 \pi^2} \ . \tag{20}$$

This time is very long; with a coronal viscosity of $\nu = 4 \times 10^9$ m^2 s^{-1} and a loop of length $L = 10^5$ km, the principal ($N = 1$) mode decays by a factor of e in 5×10^5 s (some 139 hr). However, in a magnetically structured plasma the simple exponential decay of a uniform medium persists only for a time of order $\sqrt{3}/c_A'$ after which the t^3-dependence in Equation (17) dominates, and phase mixing is established. In the corona, the transition time is likely to be short, about one second. So phase mixing effects are expected to dominate. Damping then proceeds on a time scale τ_d given by

$$\tau_d = \left[\frac{6}{\nu k_z^2 c_A'^2}\right]^{1/3} \ . \tag{21}$$

Write $c'_{\mathrm{A}} \approx c_{\mathrm{A}}/l$, corresponding to an Alfvén speed that varies spatially on a scale of order l. Then phase mixing produces damping (of the $N = 1$ mode) on a scale

$$\tau_d = \left[\frac{6L^2 l^2}{\nu \pi^2 c_{\mathrm{A}}^2}\right]^{1/3} . \tag{22}$$

Assuming a scale l of variation in the Alfvén speed c_{A} that is one-tenth of the loop length ($l = L/10$), then (22) produces a decay time scale in a loop of length $L = 10^5$ km and Alfvén speed $c_{\mathrm{A}} = 10^3$ km s^{-1} of 530 s (for $\nu = 4 \times 10^9$ m^2 s^{-1}), dramatically shorter than in a uniform medium and comparable with that observed.

We note that the damping time τ_d that operates when phase mixing is established may be rewritten in terms of the viscous Reynolds number $R = l c_{\mathrm{A}}/\nu$, based upon the Alfvén speed, c_{A}, and the spatial scale of inhomogeneity, l. Writing also $\tau_{\mathrm{A}} = l/c_{\mathrm{A}}$ for the transit time across the scale l of a wave moving with speed c_{A}, Equation (22) becomes

$$\tau_d = \left(\frac{6L^2}{\pi^2 l^2}\right)^{1/3} R^{1/3}\tau_{\mathrm{A}} \approx 0.85 \left(\frac{L^2}{l^2}\right)^{1/3} R^{1/3}\tau_{\mathrm{A}} . \tag{23}$$

Thus, for $l = L/10$ we obtain $\tau_d \approx 3.9R^{1/3}\tau_{\mathrm{A}}$, and for $l = L/100$ (comparable with the radius of a loop) we obtain $\tau_d \approx 18.2R^{1/3}\tau_{\mathrm{A}}$. These results of phase mixing compare reasonably well with scalings deduced from numerical studies of resonant absorption; for example, in a slab geometry (see Ofman, Davila, and Steinolfson, 1994, 1995; Erdélyi and Goossens, 1995), Nakariakov *et al.* (1999) quote $\tau_d \approx 32.6R^{0.22}\tau_{\mathrm{A}}$.

However, it is in fact more appropriate to use the *shear* viscosity coefficient of a plasma than the compressional viscosity we have used, reflecting the strong anisotropy of the coronal plasma, and this has the effect of reducing the effective viscosity coefficient by a factor $(\omega_i \tau_i)^{-2}$ where ω_i is the ion gyro-frequency and τ_i is the ion collision time (see Braginskii, 1965; Hollweg, 1985, 1987; see also Ofman, Davila, and Steinolfson, 1994; Erdélyi and Goossens, 1995). Under coronal conditions, this results in a severe reduction in the effective value of ν. Consequently, the theoretically estimated damping time by this process is much longer than the observed damping time. If we turn the result around, and set τ_d to be the observed damping time, then an effective Reynolds number R of order 10^5 or 10^6 results (Nakariakov *et al.*, 1999), much smaller than the usual estimate of 10^{14} or so. Similar considerations apply if we consider ohmic damping instead.

Finally, following a suggestion by M. S. Ruderman (1999, private communication), we consider the time scale for which a global oscillation decays as a result of the development of localized oscillations (Alfvén waves) in regions where plasma inhomogeneity is strong. The importance of the effect (resonant absorption) in coronal physics was pointed out by Ionson (1978), following a related development for cold plasma oscillations by Sedlacek (1971). In an *incompressible* medium with plasma inhomogeneity on a spatial scale l, Lee and Roberts (1986; see also Rae and Roberts, 1981) have shown that global disturbances of a magnetic structured medium decay on a time scale τ_d given by

$$\tau_d = \frac{16c_k}{\pi l k_z^2 (c_{\mathrm{A}e}^2 - c_{\mathrm{A}}^2} \, . \tag{24}$$

Applied to a standing wave of period $2L/c_k$ in a loop length $L = 130$ M, the observed decay time is produced by a plasma inhomogeneity of scale $l \approx 15 \times 10^3$ km if we assume $c_{\mathrm{A}e}^2$ or $l \approx 6 \times 10^3$ km for $c_{\mathrm{A}e}^2 = 10c_{\mathrm{A}}^2$. These scales are broadly consistent with the cross-sectional size of a loop. Thus, the process seems capable of producing an oscillation decay that is comparable with that observed by Nakariakov *et al.* The resonance decay time (24) is not an indicaton of plasma heating but rather of a mode conversion process, as global oscillations transfer energy into localized Alfvén waves (Lee and Roberts, 1986). Of course, heating may also be involved, since small-scale oscillations are produced and so readily damped by non-adiabatic processes (e.g., viscosity) operating efficiently in the resonant absorption layers.

4. Concluding Remarks

The discovery of damped oscillations in coronal loops (Aschwanden *et al.*, 1999; Nakariakov *et al.*, 1999), interpreted as the global kink mode, raises the prospect of a proper development of coronal seismology in which both the period of the oscillation and its damping time provide important (and largely independent) sources of information about the corona. This seismic information offers the means of determining local conditions (e.g., magnetic field strength) within coronal structures (loops and plumes), a task that has not hitherto proved possible. Such progress is made possible by the use of high resolution observations combined with reasonable time resolution. The addition of high time resolution makes possible a finer scale of seismology, using rapid pulsations within a loop as well as its global mode of oscillation. Such a programme of development requires detailed observational studies combined with more sophisticated theoretical models of oscillations. Our current theoretical understanding is limited to simple models only but there is now a pressing need to extend our understanding to more realistic configurations with an appropriate incorporation of certain physical effects, including field line curvature, non-adiabaticity, plasma inhomogeneity, the presence of flows, and an appropriate representation of the coupling of a coronal structure to the dense medium at its base. Such a programme has in view the twin goals of coronal seismology and a greater understanding of coronal heating. The prospects for such theoretical developments, spurred by observations, are bright.

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