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WEAK ASYMPTOTIC STABILITY FOR SEMILINEAR FRACTIONAL DIFFERENTIAL EQUATIONS WITH FINITE DELAYS

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Abstract. This paper investigates a class of semilinear fractional differential equations with finite delays. Based on the α-resolvent theory, the fixed point theory for condensing maps and the local estimates of solutions, we prove the existence of solutions to the suggested system when the nonlinear part is superlinear. In the case, the nonlinear part is sublinear we study the weak asymptotic stability of the zero solution by applying a new Halanay type inequality. An application to a class of partial differential equations will be given.
Key Words and Phrases: Weak asymptotic stability, Halanay inequality, measure of non-compactness, condensing map.
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