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Weak discontinuities in electrically conducting and radiating gases

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Abstract

The singular surface theory has been used to determine the law of propagation of weak discontinuities and the problem of growth and decay of waves. The effect of radiative heat transfer has been treated using a differential approximation which is valid over entire optical depth range. The effects of wave geometry and magnetic field with finite electrical conductivity on the global behaviour of the wave amplitude have also been studied. The two cases of diverging and converging waves have been discussed separately.

1 Introduction

With the advancement of space technology, the propagation of waves in gaseous media at very high temperatures becomes and interesting problem. In such studies, the radiation stresses, radiative flux effects and radiation and electromagnetic energy play vital roles in the determination of the flow field. The inclusion of frequency dependence of the radiation field effects transform the governing gasdynamic kinematics into a complex set of nonlinear integrodifferential equations. This is due to the interaction of solar wind of fully ionized plasma with a plasma column of the Earth's atmosphere and compression resulting out of it. As a consequence of interaction, weak wave characteristics emerge and with changing inclinations, they intersect to form a shock wave. A good deal of work have been reported [1,2,3,4] on the problems of radiation gasdynamics with radiative heat flux effects and radiation stresses. In these studies the approximation to the radiative transfer equation is too strong and deals with the simple cases when the gas is optically thin or thick. They have also neglected the time dependence in the radiative heat transfer equation. The neglect of the time dependence in the radiation field suppresses one of the modes of wave propagation that is excited by the radiation. It may be noted that in our recent work [5], we have included the effect of magnetic field with finite electrical conductivity which was not accounted for in the work of Ram and Mosa [6]. However, in both studies, the gas is assumed to be transparent with constant absorption coefficient. But, it is imperative to note that in high temperature gases, the transparent approximation for the radiative transfer equations with constant absorption coefficient is no more valid. The problem, therefore, must be considered incorporating the entire optical depth range from the transparent limit to the optically thick limit and the absorption coefficient should also be taken to be a function of density and temperature. Considering these ideas, Rai and Vishwakarma [7] studied the nature of weak gasdynamic discontinuities in high temperature gases. Since at high temperatures, a gas is likely to fully or partially ionized, the electromagnetic forces start to play a role in the flow field. But this important aspect was not included in the study of [7]. Thus the study of interaction between the radiative field and the electromagnetic field that may arise in the solar photosphere, rocket re-entry and elsewhere, is of vital importance to space scientists. This paper provides a mathematical base as to how and when weak discontinuity will propagate.

The main objective of the present paper is to study the essential features of the effects of the time dependent radiation field interacting with the magnetogasdynamic field with finite electrical conductivity on the propagation of weak discontinuities. The effect of the wave geometry on the wave amplitude is also discussed. A more general differential approximation of the equations of radiative transfer have been used to study the effects of thermal radiation.

2 Law of propagation

The set of non-linear differential equations governing the three dimensional unsteady flow of an electrically conducting and radiating gases are,

$$\frac{\partial \rho}{\partial t} + u_i \rho_{,i} + \rho u_{i,i} = 0, \tag{2.1}$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j u_{i,j} + p_{,i} + p^R_{,i} + \mu H_j (H_{j,i} - H_{i,j}) = 0, \tag{2.2}$$

$$\frac{\partial H_i}{\partial t} + u_j H_{i,j} - H_j u_{i,j} + H_i u_{j,j} - (\sigma \mu)^{-1} H_{i,jj} = 0, \tag{2.3}$$

$$\begin{gathered}\frac{dp}{dt} + 3(\gamma - 1)\frac{dp^R}{dt} + \{\gamma\, p + 4(\gamma - 1)p^R\} u_{i,i} + \\ (\gamma - 1) q_{i,i} - (\gamma - 1)\frac{J^2}{\sigma} = 0,\end{gathered} \tag{2.4}$$

$$H_{i,i} = 0, \tag{2.5}$$

$$J_i = \varepsilon_{ijk} H_{k,j}, \qquad J^2 = J_k J_k, \qquad p = \rho R T, \tag{2.6a,b,c}$$

where $d/dt = \partial/\partial t + u_i \partial/\partial x_i$, denotes the material time derivative, u_i, q_i, H_i, J_i are respectively the components of velocity, radiative flux, magnetic field and current density, $p-$ denotes pressure, p^R- the radiation pressure, $\rho-$ the density, $\mu-$ magnetic permeability, $\sigma-$ the electrical conductivity, $\gamma-$ the specific heat ratio and $\varepsilon_{ijk}-$ the well known permutation tenser and R is the gas constant. A comma followed by an index (say i) denotes a partial derivative with respect to space variable x_i.

The equation of radiative heat transfer within the differential approximation [8] may be written as the pair of equations

$$\frac{\partial E^R}{\partial t} + q_{i,i} = -\alpha \left(c E^R - 4 a_R T^4 \right) \tag{2.7}$$

$$\frac{1}{c}\frac{\partial q_i}{\partial t} + \frac{c}{3}E^R_{,i} = -\alpha q_i \tag{2.8}$$

where $E^R = 3p^R$ is the radiative energy density per unit volume, α is the absorption coefficient depending on the density and temperature, a_R is the Stefan's constant and c is the velocity of light.

Let us consider a surface of discontinuity in the flow field across which the flow parameters p, ρ, u_i, H_i, q_i and p^R etc. are essentially continuous, but finite discontinuities in their derivatives are permitted. Such a jump discontinuity is defined as weak wave. The first order geometric kinematic compatibility conditions due to [9] are,

$$\left.\begin{aligned} [Z_{,i}] &= Bn_i \\ \left[\frac{\partial Z}{\partial t}\right] &= -BG \end{aligned}\right\} \tag{2.9}$$

where Z may represent any of the flow variables and the scalar function $B = [Z_{,i}]n_i$ is defined over the surface of discontinuity. G is the normal speed of propagation of the moving wave front and n_i is the unit normal component of the wave front.

Evaluating the equations(2.1-2.4), (2.7)and (2.8) on the wave front (forming jumps) and using the first of (2.9) and second order compatibility conditions from [9], we get

$$G\zeta = \rho_0 \lambda_i n_i, \tag{2.10}$$

$$\rho_0 G\lambda_i = \xi n_i + \theta n_i, \tag{2.11}$$

$$\bar{\eta}_i = (\sigma\mu)(H_{0i}\lambda_j n_j - H_{0n}\lambda_i), \tag{2.12}$$

$$3G\theta = \varepsilon_i n_i, \tag{2.13}$$

$$G\varepsilon_i = c^2\theta n_i, \tag{2.14}$$

$$G\xi - \rho_0 a_0^2 \lambda_i n_i + (\gamma - 1)(3G\theta - \varepsilon_i n_i) = 0, \tag{2.15}$$

where

$$\xi = [p_{,i}]n_i, \ \zeta = [\rho_{,i}]n_i, \ \lambda_i = [u_{i,j}]n_j,$$

$$\theta = [p^R_{,i}]n_i, \varepsilon_i = [q_{i,j}]n_j, \bar{\eta}_i = [H_{i,jk}]n_j n_k$$

with $H_{on} = H_{oi}n_i$ and the suffix $'o'$ denotes the evaluation just ahead of the propagation wave surface $\Sigma\,(t)$.

Equations (2.10), (2.11), (2.13), (2.14) and (2.15) from a set of nine homogeneous equations in nine unknowns $\lambda_i, \varepsilon_i, \theta, \xi, \zeta$. This system has a non-trivial solution if the determinant of the coefficient matrix vanishes. Thus we have, $G = +C/\sqrt{3}$ and $G = \pm a_o$.

This implies that the flow field admits two types of waves present in the gas, one of which propagates with a speed called as radiation induced wave. The other with the speed a_0 is called a modified magnetogasdynamic wave.

The equation (2.12) can be rewritten as,

$$\bar{\eta}_k = \sigma\mu H_0\{I_k - I_n n_k\}\Psi, \tag{2.16}$$

where $\lambda_k = \psi \ n_k$ may be defined as the amplitude of a weak wave. Here I_k represents the components of the unit vector in the direction of magnetic field.

3 Behaviour of radiation induced wave

For a radiation induced wave, we have $G = C/\sqrt{3}$. Substituting for G in equations (2.10), (2.11), (2.13), (2.14), (2.15), we obtain,

$$\lambda^R = \frac{\varepsilon^R}{\rho_o c^2\{1 - 3a_0^2/c^2\}}, \tag{3.1}$$

$$\xi^R = \frac{\varepsilon^R}{c^3\{1 - 3a_0^2/c^2\}}, \tag{3.2}$$

$$\zeta^R = \frac{\sqrt{3}\varepsilon^R}{c^3\{1 - 3a_0^2/c^2\}}, \tag{3.3}$$

$$\theta^R = \frac{\varepsilon^R}{\sqrt{3}c}, \tag{3.4}$$

where $\lambda^R = \lambda_i^R n_i$, $\varepsilon^R = \varepsilon_i^R n_i$ and the superscript R denotes a jump discontinuity associated with a radiation induced wave. To determine these jump discontinuities, we require to determine ε^R first. Differentiating equation (2.6c) with respect to x_i and forming jumps across $\Sigma(t)$ and making use of relations (2.9), (3.3), and (3.2) we get

$$\chi = \frac{\sqrt{3}\varepsilon^R(a_o^2 - a_{oT}^2)}{R\rho_o c^3\{1 - 3a_0^2/c^2\}}, \tag{3.5}$$

where $\chi = [T_{,i}]n_i$ and $a_{oT}^2 = \gamma p_o/\rho_o$ is the isothermal speed of sound.

Differentiating equations (2.7) and (2.8) partially with respect to t and x_i and forming jumps across $\Sigma(t)$, we find on using the second order compatibility conditions [9] and the relations (2.9), (3.1), (3.3), (3.5) and $G = C/\sqrt{3}$, a differential equation of the form

$$\frac{\delta\varepsilon^R}{\delta t} + \varepsilon^R(\Gamma_1 - \Gamma_2) = 0, \tag{3.6}$$

where $\Gamma_1 = (\alpha - \Omega/\sqrt{3})c$,

$$\Gamma_2 = \frac{1}{2R\rho_o c^2\,(1 - 3a_o^2/c^2)}\,\{16\ \alpha\ a_R T_0^3\ (a_0^2 - a_{0T}^2) -$$

$$(\sqrt{3}q_n + cE_0^R - 4a_R T_0^3)\left[R\rho_0\left(\frac{\partial\alpha}{\partial\rho}\right)_0 + \left(\frac{\partial\alpha}{\partial T}\right)_0 (a_0^2 - a_{0T}^2)\right]\right\}$$

This equation governs the growth and decay behaviour of the amplitude of radiation induced weak wave. The mean curvature $\Omega(t)$ at any point of the wave surface has a representation of the form [9]

$$\Omega = \frac{\Omega_0 - K_0 G t}{1 - 2\Omega_0 G t + K_0 G^2 t^2}, \tag{3.7}$$

where $\Omega_0 = (K_1 + K_2)/2$ and $K_0 = K_1 K_2$ are the mean and Gaussian curvature of the wave surface respectively at $t = 0$ with K_1 and K_2 being the principal curvatures and G is the constant speed of wave propagation. Since a radiation-induced wave is divergent, both K_1 and K_2 are negative.

Equation (3.6) with the help of (3.7) can be integrated to yield

$$\varepsilon^R = \varepsilon_0^R I \exp(-\alpha c t), \tag{3.8}$$

where

$$I = \{(1 - 3^{-1/2} K_1 ct)^{-1/2} (1 - 3^{-1/2} K_2 ct)^{-1/2}\},$$

and ε_0^R is the value of ε^R at $t = 0$.

Since α and c are positive constants, it is obvious from equation (3.8) that $\varepsilon^R \to 0$ at $t \to \infty$ i.e. a radiation induced wave decays rapidly and tends to zero at $t \to \infty$. Since c is very large, it follows from equations (3.1-3.3) that any disturbance caused by a radiation induced weak wave has a negligibly small influence on the non relativistic gasdynamic field.

4 Behaviour of modified gasdynamic weak wave

We know that the flow ahead of this wave will be disturbed by radiation induced waves. But we have seen that for radiation induced waves λ^R, ζ^R, ξ^R and χ are small compared to ε^R and θ^R. Hence, we can study the propagation of modified gas dynamic weak waves into a medium which is in a constant state at rest. The jump discontinuities ζ, ξ, λ and ε, θ related as

$$\xi = \rho_0 a_0 \lambda, \tag{4.1}$$

$$a\zeta = \rho_0 \lambda, \tag{4.2}$$

$$\varepsilon_i n_i = 0, \tag{4.3}$$

$$\theta = 0. \tag{4.4}$$

Differentiating partially the equations (2.2), (2.4).(2.7) and (2.8) with respect to x_k and taking jumps across $\Sigma\,(t)$ and then using (2.6c) (2.9), (2.16),(4.1), (4.2), (4.3), (4.4) and $G = a_o$ and the second order compatibility condition [8], we obtain

$$\rho_0 \frac{\delta \lambda}{\delta t} + (\bar{\xi} - \rho_0 a_0 \bar{\lambda}_i n_i) + \bar{\theta} + \sigma \mu \rho_0 b^2 (1 - I_n^2) \psi = 0, \tag{4.5}$$

$$\begin{aligned} &\rho_0 a_0 \frac{\delta \lambda}{\delta t} - a_0 (\bar{\xi} - \rho_0 a_0 \bar{\lambda}_i n_i) - \\ &2 \rho_0 a_0^2 \Omega \lambda - (\gamma - 1)(3 a_0 \bar{\theta}) - (\gamma - 1) \rho_0 a_0 \lambda^2 = 0, \end{aligned} \tag{4.6}$$

$$\begin{aligned} -3a_0^2 \bar{\theta} - \lambda \Bigg\{ &16 \alpha a_R T_0^3 - \left(c E_0^R - 4 a_R T_0^4 \right) \left(\frac{\partial \alpha}{\partial T} \right)_0 \frac{a_0^2 - a_{0T}^2}{R} - \\ &(c E_0^R - 4 a_R T_0^4) \rho_0 \left(\frac{\partial \alpha}{\partial \rho} \right)_0 \Bigg\} = 0, \end{aligned} \tag{4.7}$$

$$a_0 c^2 \bar{\theta} + c q_n \left[\left(\frac{\partial \alpha}{\partial \rho} \right)_0 \rho_0 + \frac{a_0^2 - a_{0T}^2}{R} \left(\frac{\partial \alpha}{\partial T} \right)_0 \right] \lambda = 0, \tag{4.8}$$

where

$$\mu H_0^2 = \rho_0 b_0^2 \qquad \text{and} \qquad \bar{\lambda}_i = [u_{i,jk}]\, n_j n_k,$$

$$\bar{\xi} = [p_{,jk}]\, n_j n_k, \;\; \bar{\theta} = [p^R_{,jk}]\, n_j n_k, \;\; a_n = q_i n_i.$$

On elimination $\bar{\lambda}_i, \bar{\xi}$ and $\bar{\theta}$ from (4.5-4.8), we have

$$\frac{\delta \lambda}{\delta t} + (Q_0 - a_0 \Omega)\, \lambda - \frac{\gamma + 1}{2} \lambda^2 = 0, \tag{4.9}$$

where,

$$\begin{aligned} Q_0 = \tfrac{1}{2} \sigma \mu b_0^2 \, (1 - I_n^2) + \frac{\gamma - 1}{2 \rho_0 R a_0^2} \Bigg[&16 a_R \alpha T_0^3 (a_0^2 - a_{0T}^2) - \\ &R \rho_0 \left(\frac{\partial \alpha}{\partial \rho} \right)_0 + \left(\frac{\partial \alpha}{\partial T} \right)_0 (a_0^2 - a_{0T}^2)(c E_0^R - 4 a_R T_0^4) \Bigg]. \end{aligned}$$

In the above equation (4.9) the terms containing c_2 in the denominator of Q_0 are neglected. Since a is an arbitrary function of ρ and T hence the sign of Q_0 may be positive or negative depending on the form of a [10]. If β be the angle between the direction of the magnetic field and that of normal to wave front so that $I_i n_i = cos\beta$, then $(1 - I_n^2) = sin^2\beta$ which is the tangential component. Hence using this relation and if $\lambda_i = \psi n_i$ equation (4.9) can be rewritten as

$$\frac{\delta\psi}{\delta t} + \{Q_0 - a_0\Omega\}\psi - \Gamma_0\psi^2 = 0 \tag{4.10}$$

$$Q_0 = \tfrac{1}{2}\sigma\mu b_0^2 \sin^2\beta + \frac{\gamma - 1}{2\rho_0 R a_0^2}\left[16 a_R \alpha T_0^3 (a_0^2 - a_{0T}^2) - \right.$$

$$\left. R\rho_0 \left(\frac{\partial\alpha}{\partial\rho}\right)_0 + \left(\frac{\partial\alpha}{\partial T}\right)_0 (a_0^2 - a_{0T}^2)(cE_0^R - 4a_R T_0^4)\right],$$

$$\Gamma_0 = \frac{1}{2}(\gamma + 1) > 0,$$

which is a differential equation governing the growth and decay behaviour of the weak modified magnetogasdynamic wave.

Let $\sum(t_0)$ represent a weak wave surface at time t_0 and let σ represent the distance measured from $\sum(t_0)$ along the normal trajectories to the family of surfaces $\sum(t)$ in the direction of propagation. Then $\sigma = G\,(t - t_0)$. Hence we can write

$$\frac{\delta\psi}{\delta t} = G\frac{d\psi}{d\sigma}. \tag{4.11}$$

Using eq.(4.11) and $\sigma = Gt$ at time t_0 in eq.(4.10), we get

$$\frac{d\psi}{dt} + [Q_0 - a_0\Omega(t)]\,\psi - \Gamma_0\psi^2 = 0. \tag{4.12}$$

The mean curvature $\Omega(t)$ of the wave surface propagating normal to itself into a uniform medium at rest [10] is

$$2\Omega(t) = \frac{K_1}{1 - K_1 a_0 t} + \frac{K_2}{1 - K_2 a_0 t}. \tag{4.13}$$

Equation (4.12) by using (4.13) can be integrated to yield

$$\psi(t)=\psi_0\exp(-Q_0t)(1-K_1a_0t)^{-1/2}(1-K_2a_0t)^{-1/2}\Big[1- \qquad (4.14)$$

$$\psi_0\Gamma_0\int_0^t \exp(-Q_0t')(1-K_1a_0t')^{-1/2}(1-K_2a_0t')^{-1/2}dt'\Big]^{-1}$$

where ψ_0 is the initial amplitude at $t=0$. Now let us study the physical aspects of the wave amplitude $\psi(t)$. We shall discuss the following two cases of the diverging and converging waves.

Case 1. Diverging Waves

For diverging waves both the initial principal curvatures K_1 and K_2 are negative, so that the solution (4.14) can be expressed in the form

$$\psi(t)=F(t)\left[\frac{1}{\psi_0}\Gamma_0\int_0^t F(\tau)d\tau\right]^{-1}, \qquad (4.15)$$

where

$$F(t)=\exp(-Q_0t)\ (1+|K_1|\ a_0t)^{-1/2}\ (1+|K_2|\ a_0t)^{-1/2}.$$

Let $Q_0>0$. Then the function $F(t)$ is non negative and monotonically decreases and tends to zero as $t\to\infty$. If $\psi(t)<0$, then $\psi(t)$ is also negative and $\lim_{t\to\infty}|\psi(t)|=0$.

This situation arises for weak expansion waves which decay in time and damp out ultimately. From (4.15), we get

$$\frac{d|\psi|}{dQ_0}=\frac{-Q_0\ |\psi|}{1+\Gamma_0|\psi_0|w(t)}<0, \qquad (4.16)$$

where $w(t)=\int_0^t F(\tau)d\tau$.

Equation (4.16) shows that the decrease of the wave amplitude $|\psi|$ will be accelerated under the radiation and magnetic field effects.

When $\psi_0 > 0$, then $\psi(t)$ is also positive and there exists a critical value ψ_c of ψ given by

$$\psi_c = [\Gamma_0 w(\infty)]^{-1} > 0 \quad \text{for} \quad Q_0 \neq 0, \tag{4.17}$$

such that for $\psi_0 > \psi_c$, a weak discontinuity will terminate into shock wave at time $t_c > 0$. The critical time t_c can be determined by the following relation

$$w(t_c) = \int_0^{t_c} \exp(-Q_0 t)\ (1 + |K_1| a_0 t)^{-1/2}\ (1 + |K_2| a_0 t)^{-1/2}\ dt = \frac{1}{\psi_0 \Gamma_0}, \tag{4.18}$$

which clearly shows that the solution (4.14) will break down at $t = t_c$. This situation will arise for weak compressive waves. For $\psi_0 < \psi_c$, the solution (4.14) is valid for all times and $\psi(t)$ decreases monotonically and vanishes ultimately i.e. a shock formation is disallowed and weak wave will damp out. If $\psi_0 = \psi_c$, then $\lim_{t\to\infty} |\psi(t)| = Q_0/\Gamma_0$,which shows that the wave will neither terminate into a shock wave nor damp out into a shock wave. It ultimately takes a stable form with constant amplitude. And if $Q_0 = 0$, then ψ_c=0 and hence, all weak compressive waves will terminate into shock waves within a finite time t_c, no matter, however small its initial amplitude ψ_0 may be.

From (4.17) and (4.18), we get

$$\frac{d\psi_c}{dQ_0} = \Gamma_0 \psi_c^2 \int_0^{\infty} t\ F(t)\ dt > 0, \tag{4.19}$$

$$\frac{dt_c}{dQ_0} = \frac{Q_0 w(t_c)}{F(t_c)} > 0. \tag{4.20}$$

The equation (4.19) and (4.20) show that both ψ_c and t_c increase or decrease with the increase or decrease of Q_0, since Q_0 increases under radiation effect and also under resistance of the magnetic field with finite electrical conductivity σ. Thus a shock formation is either delayed or disallowed due to thermal radiation and magnetic field effects.

To study the curvature effects on diverging waves, we differentiate (4.17) and (4.18) with respect to $|K_1|$, getting the following inequalities:

$$\frac{d\psi_c}{d|K_1|} = \frac{1}{2}\Gamma_0\ \psi_c^2\ a_0 \int_0^\infty t\ (1 + |K_1|a_0 t)^{-1} F(t)\ dt > 0, \tag{4.21}$$

$$\frac{dt_c}{d|K_1|} = \frac{a_0}{2F(t_c)} \int_0^{t_c} \frac{tF(t)}{1 + |K_1|a_0 t}\ dt > 0. \tag{4.22}$$

The equations (4.12) and (4.22) show that ψ_c and t_c increase with curvature effect.

Case 2. Converging Waves

In this case the principal curvatures K_1 and K_2 are positive. Let t^* be the least of two roots of the equation

$$(1 - K_1 a_0 t)(1 - K_2 a_0 t) = 0,$$

and let $K_1 > K_2$ so that $t^* = (K_1 a_0)^{-1}$.

The solution (4.14) is not valid for $t > t^*$. If $Q_0 > 0$ or $Q_0 < 0$, $K_1 \neq K_2$ and $\psi_0 < 0$, then $\psi(t)$ is also negative and $\lim_{t\to\infty} |\psi(t)| = \infty$. This means that within a finite time all such converging waves form a cusp an $t = t^*$. If $\psi_0 > 0$, then there exists a critical value ψ^* of ψ_0, given by

$$\psi^* = \left[\int_o^{t*} \xi(t)dt\right]^{-1},$$

where

$$\xi(t) = \exp(Q_0 t)\ (1 + K_1 a_0 t)^{-1/2}\ (1 + K_2 a_0 t)^{-1/2} \quad \text{for} \quad K_1 \neq K_2,$$

such that for $\psi_0 < \psi^*$ a cusp will be formed at $t = t^*$, otherwise a shock wave will be formed at time $t_c < t^*$, where t_c is given by

$$\int_o^{t_c} \xi(t)dt = \frac{1}{|\psi_0|\Gamma_0}, \qquad t_c < t^*. \tag{4.23}$$

If $K_1 = K_2 > 0$, we have $\psi^* = 0$ so that all compressive weak waves will terminate into shock waves before a cusp could be formed, no matter, however small ψ_0 may be. The effects of radiation and magnetic field on converging waves are of the same nature as in the case of diverging waves.

If we differentiate (4.23) with respect to $K_1 > 0$ partially we get

$$\frac{dt_c}{dK_1} = \frac{a_0}{2\xi(t_c)} \int_o^{t_c} t\xi(t)(1 + K_1 a_0 t)^{-1} dt < 0. \qquad (4.24)$$

The inequality (4.24) shows that t_c decreases under curvature effect. Hence, from the equations (4.20) and (4.24) we can conclude that under curvature effects the stability of a diverging wave increases while that of converging wave decreases.

5 Conclusion

It has been observed that the time dependent radiation field gives rise to radiation induced weak wave, which has a negligible influence on the nonrelativistic flow properties of the gas dynamic field. It has also been concluded that these waves are ultimately damped. It is further concluded that if the initial wave amplitude is numerically larger than the critical value ($\psi_0 > \psi_c$), a weak discontinuity will break down and a shock type discontinuity will be formed after a finite critical time t_c and with an initial amplitude less than the critical one ($\psi_0 < \psi_c$) resulting in a decay of the weak discontinuity. The effects of thermal radiation and magnetic field with finite electrical conductivity have stabilizing effect on the propagation of weak discontinuities in the sense that they delay the process of shock formation. It is also concluded that under the curvature affects the stability of diverging wave increases while that of the converging wave decreases.

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Slabi diskontinuiteti u elektroprovodnim i zračečim gasovima

UDK 533.1; 534.13

U radu se koristi teorija singularnih površi za odredjivanje zakona prostiranja slabih diskontinuiteta kao i rasta i

slabljenja ovih talasa. Efekat radijacionog prenosa toplote je rešavan korišćenjem diferencijalne aproksimacije

koja je ispravna preko čitavog opsega optičke dubine. Uticaji geometrije talasa s jedne i magnetnog polja sa

konačnom električnom provodnošću sa druge strane na globalno ponašanje amplitude talasa su takodje proučeni.

Posebno su diskutovana dva specijalna slučaja divergirajučih i konvergirajučih talasa.