

Weak-values Technique for Velocity Measurements

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G. I. Viza et al. Opt Let 38 16 2949 (2013)



Why are velocity measurements important?

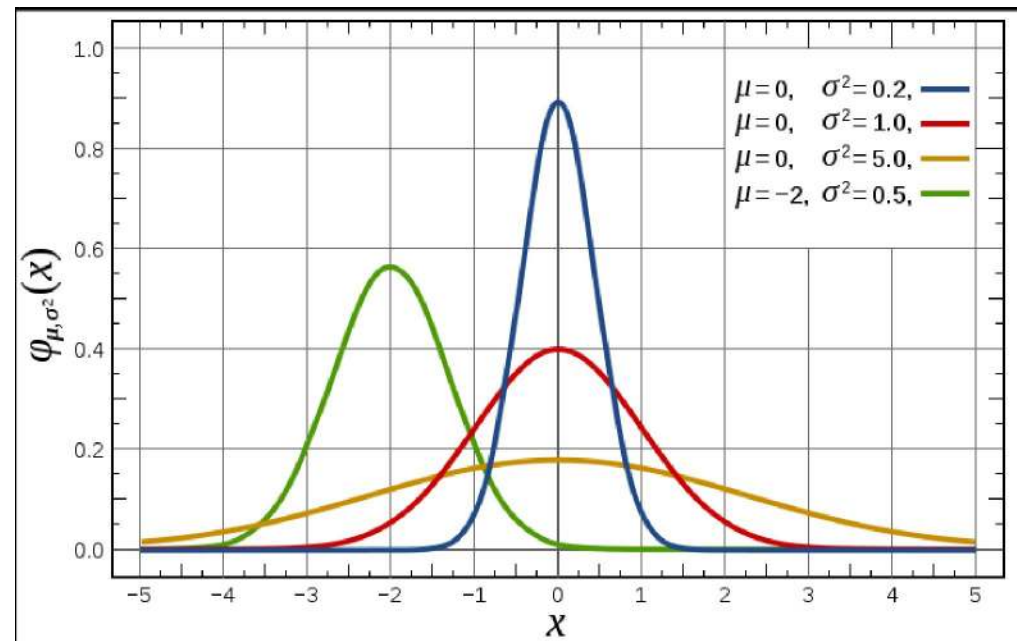
- Y. Yeh and H. Z. Cummins (1964)
 - 40 $\mu\text{m/s}$
- Laser Doppler Anemometry
 - In-vivo medical imaging
 - Quality control vibrometry
- N. Brunner and C. Simmon PRL 105, 010405 (2010)
 - Measuring longitudinal phase with frequency-domain analysis



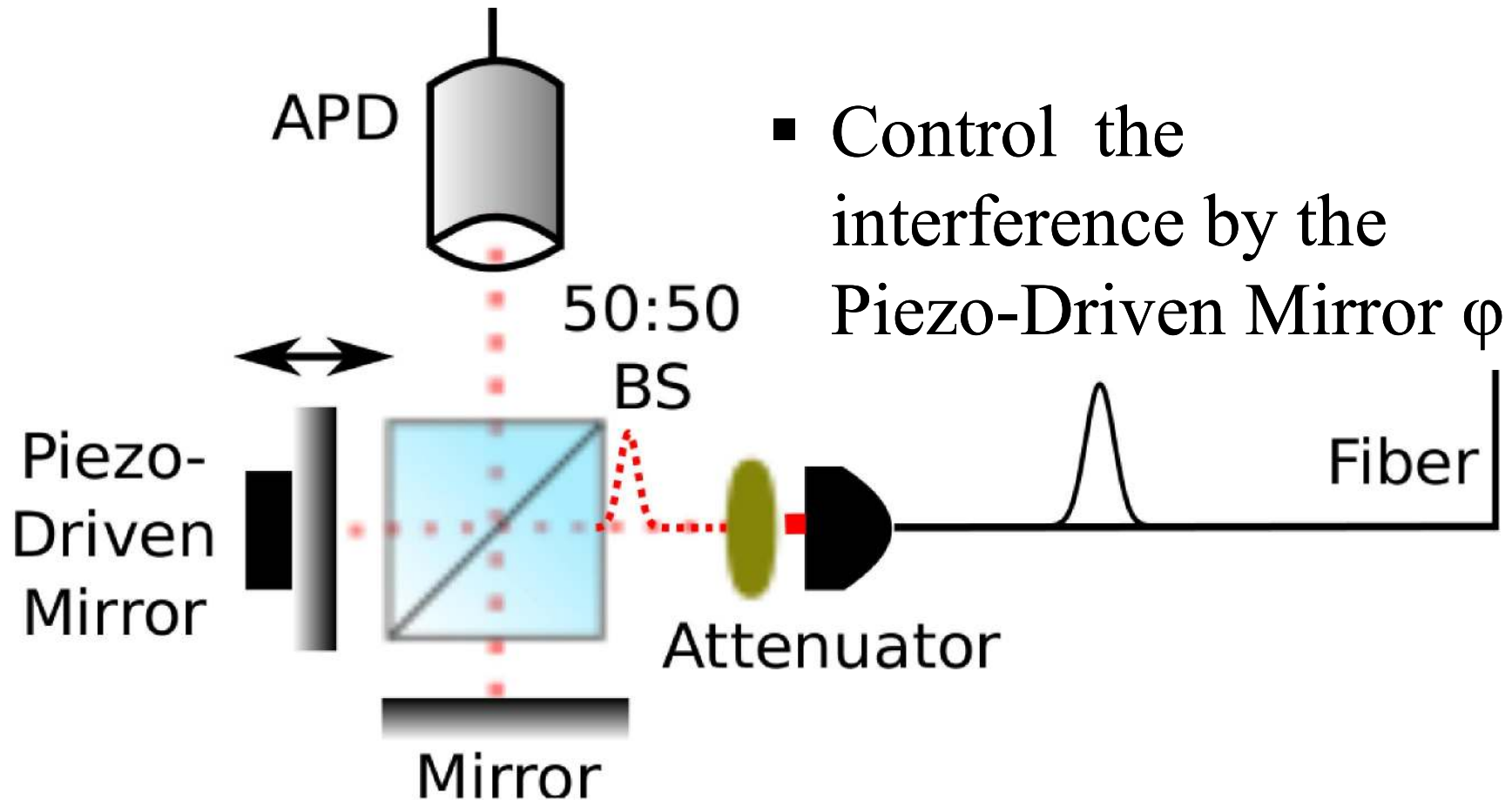
Preparation of State

- Use long temporal Gaussian pulses
- Michelson Interferometer

$$I(t) = I_0(t) \exp\left(\frac{-t^2}{2\tau^2}\right)$$



Small Interaction



Observations

- Time Domain: no change
- Frequency Domain: One pulse is red or blue detuned

$$f_d = \pm 2 \frac{v}{c} f_0$$

- Technologically difficult to measure frequency and much easier to measure arrival time.



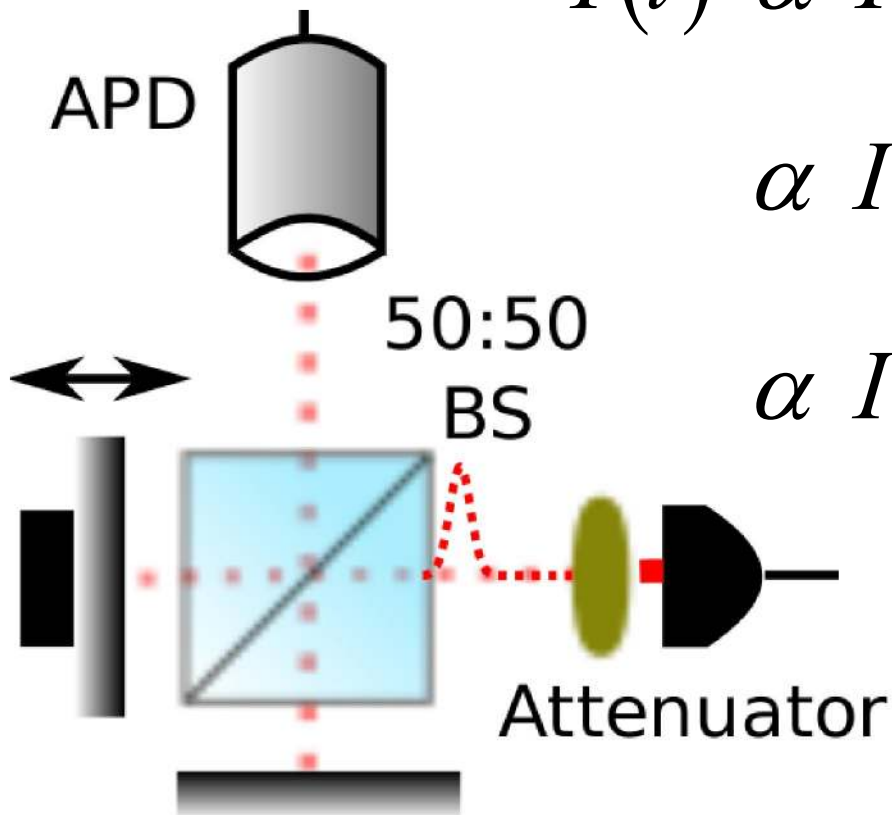
Observations 2

- Phase:
 - Point by point interference
 - Mirror is imparting a phase within the coherence length of the laser
- Pulse:
 - Pulse length \gg Coherence length of the laser
 $\gg \varphi$



Theory

$$\begin{aligned}
 I(t) &\propto I_0(t) \left| 1 - \exp(i2\varphi + i2kx(t)) \right|^2 \\
 &\propto I_0 \sin^2(\varphi) \exp\left(\frac{-t^2}{2\tau^2}\right) \left| \frac{\sin(\varphi + kvt)}{\sin(\varphi)} \right|^2 \\
 &\propto I_0 \sin^2(\varphi) \exp\left[\frac{-1}{2\tau^2} \left(t - \frac{2kv\tau^2}{\varphi}\right)^2\right]
 \end{aligned}$$



Weak-values

- Coupled velocity/frequency to time
- Pre-selection: temporal Gaussian pulse
- Small interaction:
 - Tiny velocity disturbing the frequency domain
- Post-selection: angle close to destructive interference
- Result: A shift in arrival time



Limitations

- **Cramer-Rao Bound**; the lowest bound of the variance over an unbiased estimator. When achieved, the estimator is efficient. $= 1/F$

$$P(t; \delta t) = \frac{1}{\sqrt{2\pi\tau^2}} \exp\left(\frac{-(t-\delta t)^2}{2\tau^2}\right)$$

- **Fisher Information**

$$F = \int dt P(t; \delta t) \left[\frac{d}{d\delta t} \ln(P(t; \delta t)) \right]^2$$



- Fisher Information:

$$N \sin^2(\varphi) \int dt P(t; \delta t) \left[\frac{d}{dt} \ln P(t; \delta t) \right]^2$$
$$\approx \frac{N \varphi^2}{\tau^2}$$

- Uncertainty of δt is bounded by: $\Delta(\delta t) \geq \frac{\tau}{\varphi \sqrt{N}}$
- Error in estimating velocity is bounded by:

$$\Delta v_{CRB} = \frac{\Delta(\delta t) \varphi}{2k\tau^2} = \frac{1}{2k\tau \sqrt{N}}$$



Smallest resolvable velocity

$$P(t; \delta t) = \frac{1}{\sqrt{2\pi\tau^2}} \exp\left(\frac{-(t-\delta t)^2}{2\tau^2}\right)$$

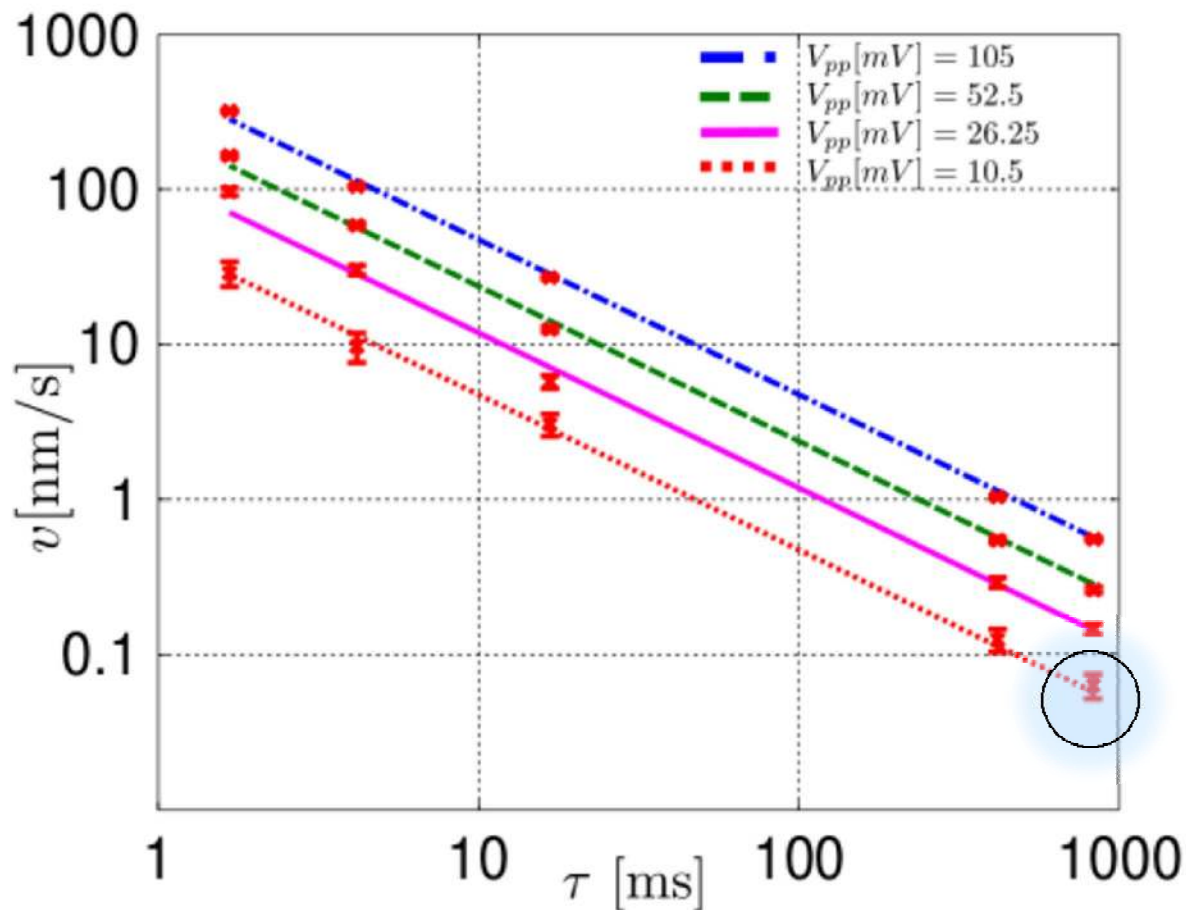
- Our Signal-to-Noise ratio is straight forward for a Gaussian: time shift divided by the uncertainty of the time shift

- $\text{SNR} = \frac{\delta t}{\tau} \varphi \sqrt{N} = \frac{v}{\Delta v}$

Starling, et al. PRA 82, 063822 (2010)



Results



$$\delta t = \frac{2kv\tau^2}{\varphi}$$

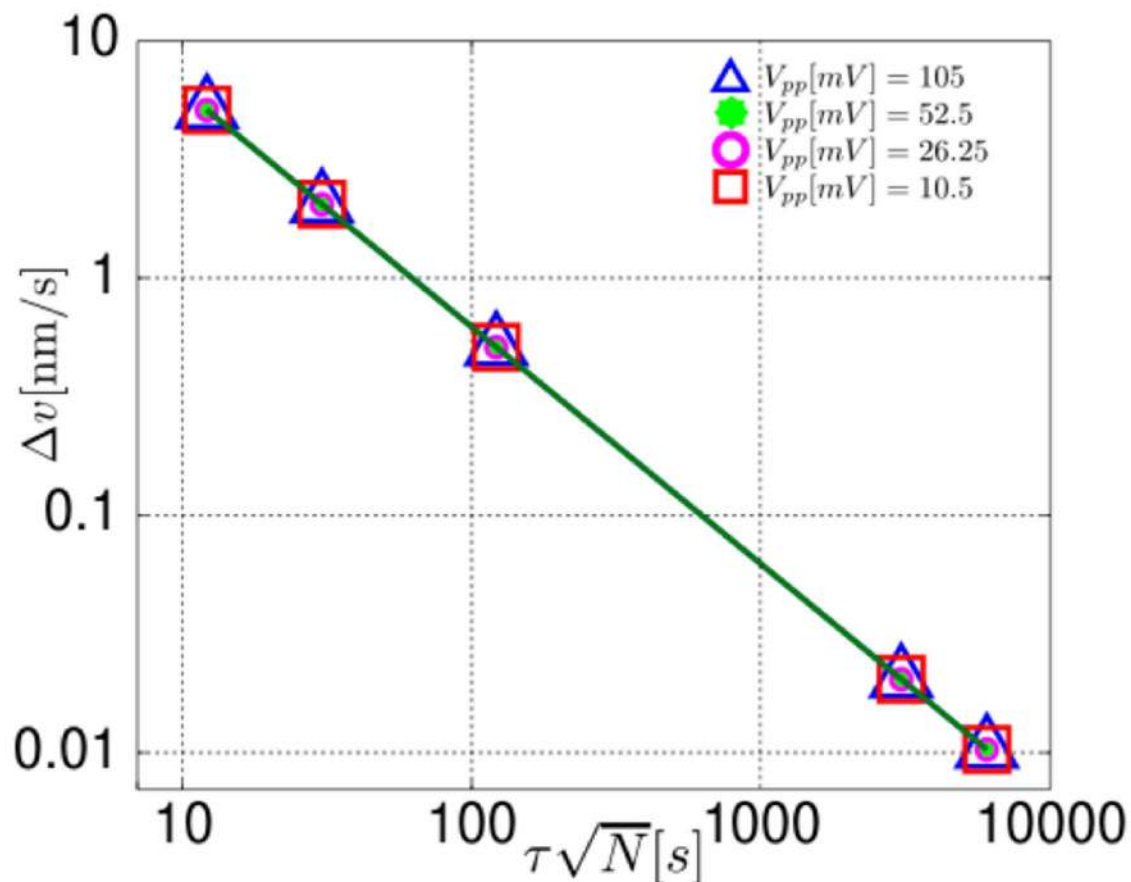
$$\tau = \{1.7 - 833\} \text{ ms}$$

$$\varphi \approx 0.3 \text{ rad}$$

$$v = 60 \pm 11 \text{ pm/s}$$



Uncertainty Results



- Estimator is efficient
- There is not better estimator to produce smaller uncertainties.



Sub picometer/second

$$\delta t = \frac{2kv\tau^2}{\varphi}$$

- What tricks can we do to achieve sub picometer/second?
 - post-selecting angle smaller
 - pulses longer



Results 2

- Our estimator changed from Gaussian to only the tip. Only the top 12% peak to peak intensity variation.

V _{pp} [mV]	Φ [rad]	v [pm/s]
2.0	0.275	1.4±0.5
1.0	0.276	0.6±0.4
0.5	0.279	0.4±0.4



Conclusion and Remarks

- The new estimator is not efficient but it helped us resolve slower velocities.
- Instability of the interferometer limited our integration time. The path length was constant for less than 15mins at a time.
- A weak-values technique allowed us to reach a CRB for velocity measurements.
- Achieved 400fm/s with a $\tau = 50s$.
 - In each run we took (10min) the mirror moved the distance of about 2 hydrogen atoms!



Thank you

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 - Itay Shomroni
 - Barak Dayan
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