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Weighted Conflict Evidence Combination Method Based on Hellinger Distance and the Belief Entropy

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ABSTRACT In the Dempster-Shafer evidence theory, how to effectively measure the degree of conflict between two bodies of evidence is still an open question. To solve this problem, we propose a weighted conflict evidence combination method based on Hellinger distance and the belief entropy. This method uses the probability transformation function to deal with the multi-subset focal elements firstly. Next, the Hellinger distance is introduced to measure the degree of conflict among the evidence. Moreover, improved belief entropy is also employed to quantify the uncertainty of the basic belief assignments. Further, Hellinger distance and the improved belief entropy are combined to construct the weight coefficient concerning evidence, and finally, the Dempster combination rule is used for fusion. The final fusion results of proposed method on fault diagnosis experiment and target recognition experiment are 0.9018 and 0.9895 respectively, apparently higher than that of other methods, revealing the advantages of the proposed method.

INDEX TERMS Dempster-Shafer evidence theory, conflict evidence, Hellinger distance, belief entropy, evidence fusion.

I. INTRODUCTION

MULTI-SENSOR information fusion technology can effectively avoid the limitation of single sensor decision-making by processing and fusing the information obtained by multiple sensors. However, due to external reasons or the sensor itself, the data gathered from multi-sensors could be unreliable or even incorrect, leading to making wrong decision [1]. To solve this problem, experts and scholars have successively proposed some classic theories, such as fuzzy set theory proposed by Zadeh in 1965 [2], evidence theory proposed by Dempster in 1967 [3], developed evidence theory proposed by Shafer in 1976 [4], the rough set theory proposed by Pawlak in 1982 [5], and so on. For several decade years, these theories have been successfully applied in many scientific and engineering fields [6]–[9]. It is noteworthy that these theories are non-isolated, moreover, highly complementary in some aspects. Some scholars have combined these theories, for example, combining fuzzy sets with evidence theory [10], [11] and combining evidence theory with rough sets [12], [13], to analyse and deal with problems, and have achieved good results.

Dempster-Shafer evidence theory, as a powerful tool for multi-sensor information fusion technology, was first proposed by Dempster [3] and popularised by his student Shafer [4]. Dempster-Shafer evidence theory has the advantage of expressing "uncertain" and "unknown", so it can deal with uncertain and imprecise information flexibly. At present, it has been widely used in many fields, such as fault diagnosis [14]–[17], target tracking [18]–[20], multiple attribute decision making [21]–[24], image processing [25], [26], medical diagnosis [27]–[29], risk analysis [30]–[34], and so on.

A. RELATED WORK

With the development of Dempster-Shafer evidence theory, some scholars have found that using classical Dempster-Shafer evidence theory to fuse highly conflicting evidence may lead to counter-intuitive combination results. Many experts and scholars have analysed the reason caused by counter-intuitive results of conflict evidence fusion and proposed some improved methods [35]–[52]. These improvement methods are mainly divided into two categories.

The first is to modify the classical Dempster combination

rule [35]–[37]. The scholars who hold this view think that the main reason for the counter-intuitive result is that some innate defects of the Dempster combination rule. The traditional improvement methods are firstly to reallocate correspondence between the conflict information and the subset, and then, the proportion of the conflict information in the subset is reallocated after subset with conflict receiving to be determined. However, most of the modified combination rules cannot keep the excellent characteristics of the conventional Dempster combination rule such as commutative, associative, and so on.

The second is to pre-process the bodies of evidence without changing the Dempster combination rule. Currently, the most commonly used preprocessing method is to weight the bodies of evidence before fusion. Murphy averages the n bodies of evidence and then used the Dempster combination rule to fuse it $n-1$ times [38]. Still, it does not take into account the relationship between the bodies of evidence and the importance of the evidence. Similar as Murphy average fusion method, Deng et al. introduced Jousselme evidence distance to solve the weight coefficient of fusion bodies of evidence [39], this method considered not only the mutual exclusion of focus elements but also some conflict information between inclusive focus elements. Yuan et al. introduced Jousselme evidence distance and Deng entropy to construct the weight coefficient bodies of evidence [40]. Chen et al. designed a novel weighted method by combining evidence distance and uncertainty of evidence [41]. Liu et al. defined a new probability distance and conflict coefficient related to bodies of evidence, and then used it to design the weight of bodies of evidence [42].

The key of weighted averaging the bodies of evidence to be fused is how to determine the weight coefficient of the bodies of evidence. Research has found that the weight coefficient can be obtained from the conflict of the evidence and the uncertainty of the evidence. From the perspective of effectively measuring the conflict between two bodies of evidence, many experts and scholars have proposed some methods. For example, Jousselme et al. defined a new distance to measure the conflict between two bodies of evidence [43]. Liu defined pignistic probability distance to measure the degree of conflict between two bodies of evidence [44]. Zhang et al. used the improved cosine similarity to measure the degree of conflict between two bodies of evidence [45]. Ma and An used fuzzy nearness and correlation coefficient to measure the degree of conflict between two bodies of evidence [46]. Xiao used the Belief Jensen-Shannon (BJS) divergence to measure the degree of conflict between two bodies of evidence [1], and so on. From the perspective of how to effectively quantify the uncertainty of the evidence, experts and scholars have also proposed some corresponding measurement methods. For example, Deng defined a new uncertainty measure Deng entropy based on Shannon entropy in the framework of evidence theory [47]. Zhou et al. proposed an improved belief entropy based on the scale of frame of discernment [48]. Tang et al. improved on Deng entropy

and defined weighted belief entropy [49]. Pan and Deng defined a new uncertainty measure based on the plausibility function and the belief function [50]. Qin et al. defined a new uncertainty measure based on the number of elements in the frame of discernment and the conflict of basic belief assignments (BBAs) [51], and so on.

Also, some scholars preprocess the bodies of evidence from different perspectives. For example, Jing and Tang proposed the methods based on base basic probability assignment (bBPA) by averaging bBPA and basic probability assignment (BPA), to achieve the purpose of pre-processing the bodies of evidence before fusion [52].

B. CONTRIBUTIONS

From the two aspects of measuring the conflict the bodies of evidence and quantifying the uncertainty of the evidence, we discussed the weighted conflict evidence combination method based on Hellinger distance and the belief entropy. Our main contributions are summarised as follows:

- The Hellinger distance is introduced into the framework of Dempster-Shafer evidence theory, and the Hellinger distance is used to measure the degree of conflict between two bodies of evidence.
- A new belief entropy is defined in the framework of Dempster-Shafer evidence theory, which is used to quantify the uncertainty of the evidence.
- We propose a multi-sensor data fusion algorithm based on Hellinger distance and the belief entropy.

Numerical examples show that the proposed method can effectively measure the degree of conflict between two bodies of evidence and quantify the uncertainty of the evidence. Experimental results related to fault diagnosis and target recognition show that the proposed method has the same supporting element as other methods, moreover, by using the Dempster combination rule fusion, the final basic belief assignment related to the supporting element of proposed methods is higher than that of other methods, thus it is conducive to decision-making.

The remaining chapters of the paper are arranged as follows. Section II introduces some relevant basic theoretical knowledge. Section III introduces the Hellinger distance and analyses some properties of the Hellinger distance. Section IV introduces some methods of quantifying the uncertainty of the evidence and gives the improved method of this paper and compares the improved methods of this paper with previous methods. Section V introduces the algorithm flow proposed in this paper in detail. Section VI based on two numerical examples, the effectiveness of the method proposed in this paper is verified. Finally, Section VII is the conclusion of this paper.

II. PRELIMINARIES

A. DEMPSTER-SHAFER EVIDENCE THEORY

In Dempster-Shafer evidence theory [3], [4], the frame of discernment (FOD) $\Theta = \{\theta_1, \theta_2, \dots, \theta_N\}$ is a collection

of N mutually exclusive elements. The set consisting of all subsets of Θ is denoted as 2^Θ .

Definition 1: Let Θ be the frame of discernment. If the function $m : 2^\Theta \rightarrow [0, 1]$ satisfies $m(\{\phi\}) = 0$ and $\sum m(\{A\}) = 1$ for $\forall \{A\} \subseteq \Theta$, then m is called the basic belief assignment (BBA) on Θ . If $m(\{A\})$ isn't zero, then $\{A\}$ is called the focal element which contains one or more hypotheses, $m(\{A\})$ assigns the basic belief assignment of proposition $\{A\}$, and $\{\phi\}$ is denotes the empty set of Θ .

Definition 2: The belief function $Bel: 2^\Theta \rightarrow [0, 1]$ and the plausibility function $Pl: 2^\Theta \rightarrow [0, 1]$, for $\forall \{A\}, \{B\} \subseteq \Theta$ are

$$Bel(\{A\}) = \sum_{B \subseteq A} m(\{B\}) \quad (1)$$

$$Pl(\{A\}) = \sum_{B \cap A \neq \phi} m(\{B\}) \quad (2)$$

where $Bel(\{A\})$ represents the degree of support for proposition $\{A\}$ to be true, $Pl(\{A\})$ represents the degree of support for proposition $\{A\}$ not to be refused.

Definition 3: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , and use $m_{1 \oplus 2}$ to represent the new evidence after m_1 and m_2 combination. The Dempster combination rule is defined as

$$m_{1 \oplus 2}(\{A\}) = \begin{cases} 0 & , \{A\} = \{\phi\} \\ \frac{\sum_{B \cap C = A} m_1(\{B\})m_2(\{C\})}{1-k} & , \{A\} \neq \{\phi\} \end{cases} \quad (3)$$

where $k = \sum_{B \cap C = \phi} m_1(\{B\})m_2(\{C\})$ is called the conflict coefficient, which is used to measure the degree of conflict between two BBAs.

B. SOME METHODS OF CONFLICT MEASUREMENT

In Dempster-Shafer evidence theory, the conflict coefficient is used to measure the degree of conflict between BBAs, while the effectiveness of the conflict coefficient is not always guaranteed in some cases. We use Example 1 to demonstrate it.

Example 1: Assuming that the frame of discernment is $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the results collected by two independent sensors are converted into BBAs as shown below.

$$\begin{aligned} m_1 : m_1(\{\theta_1\}) &= m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/3 \\ m_2 : m_2(\{\theta_1\}) &= m_2(\{\theta_2\}) = m_2(\{\theta_3\}) = 1/3 \end{aligned}$$

Using the conflict coefficient to measure the conflict between the evidence m_1 and evidence m_2 in Example 1, we get $k = 2/3$. However, the conflict between two identical belief functions may not equal to 0, leading to somewhat counter-intuitive results [53]. Therefore, experts and scholars have proposed many effective methods to measure the conflict between BBAs. The following is a brief introduction to some classical methods.

1) Jousselme Evidence Distance

Jousselme et al. regarded each group of BBAs as a set of vectors and proposed a conflict expression method that measures the distance between evidence in vector space [43].

Definition 4: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the Jousselme evidence distance between m_1 and m_2 is expressed as

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_2)^T \underline{D}(\vec{m}_1 - \vec{m}_2)} \quad (4)$$

where \vec{m}_1, \vec{m}_2 is the two BBAs in the vector space; \underline{D} is an $2^{|\Theta|} \times 2^{|\Theta|}$ matrix whose elements are $D(A, B) = \frac{|A \cap B|}{|A \cup B|}$, $A, B \in 2^\Theta$; $|A|$ is the cardinality of subset $\{A\}$.

2) Liu Conflict Measurement Method

Liu defined the pignistic probability distance and combined the conflict coefficient and the pignistic probability distance into a two-tuple to judge the degree of conflict in BBAs [44].

Definition 5: Let m be a BBA on the frame of discernment Θ , and its related pignistic probability function $BetP_m : \Theta \rightarrow [0, 1]$ is defined as

$$BetP_m(\{\theta_i\}) = \sum_{A \subseteq \Theta, \theta_i \in A} \frac{1}{|A|} \frac{m(\{A\})}{1 - m(\{\phi\})}, m(\{\phi\}) \neq 1 \quad (5)$$

Definition 6: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the pignistic probability distance between m_1 and m_2 is defined as

$$difBetP_{m_1}^{m_2} = \max_{A \subseteq \Theta} (|BetP_{m_1}(\{A\}) - BetP_{m_2}(\{A\})|) \quad (6)$$

Definition 7: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , and set a binary metric as $cf(m_1, m_2) = \langle k, difBetP \rangle$, if $k > \varepsilon, difBetP > \varepsilon$, then m_1 and m_2 are defined as conflict. Where $\varepsilon \in [0, 1]$ is the threshold of conflict tolerance.

Example 2: Assuming that the frame of discernment is $\Theta = \{\theta_1, \theta_2, \dots, \theta_{10}\}$, the results collected by two independent sensors are transformed into BBAs as shown below.

$$\begin{aligned} \text{case 1 : } & \begin{cases} m_1(\{\theta_1\}) = 1 \\ m_2(\{\theta_2\}) = 1 \end{cases} \\ \text{case 2 : } & \begin{cases} m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = 1/2 \\ m_2(\{\theta_3\}) = m_2(\{\theta_4\}) = 1/2 \end{cases} \\ \text{case 3 : } & \begin{cases} m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/3 \\ m_2(\{\theta_4\}) = m_2(\{\theta_5\}) = m_2(\{\theta_6\}) = 1/3 \end{cases} \\ \text{case 4 : } & \begin{cases} m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/4, \\ m_1(\{\theta_4\}) = 1/4 \\ m_2(\{\theta_5\}) = m_2(\{\theta_6\}) = m_2(\{\theta_7\}) = 1/4, \\ m_2(\{\theta_8\}) = 1/4 \end{cases} \\ \text{case 5 : } & \begin{cases} m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/5, \\ m_1(\{\theta_4\}) = m_1(\{\theta_5\}) = 1/5 \\ m_2(\{\theta_6\}) = m_2(\{\theta_7\}) = m_2(\{\theta_8\}) = 1/5, \\ m_2(\{\theta_9\}) = m_2(\{\theta_{10}\}) = 1/5 \end{cases} \end{aligned}$$

TABLE 1. Conflicting measurement results in case 1-5.

Distance	case 1	case 2	case 3	case 4	case 5
$d_J(m_1, m_2)$	1	0.707	0.577	0.500	0.447
$difBetP_{m_1}^{m_2}$	1	0.500	0.333	0.250	0.200

Table 1 shows the results of solving the above five cases using Jousselme evidence distance and pignistic probability distance. We find that the BBAs in cases 1-5 in Example 2 are all evidence of complete conflict. In this extreme case, the degree of conflict between m_1 and m_2 should reach the maximum value 1. However, the values of $d_J(m_1, m_2)$ and $difBetP_{m_1}^{m_2}$ used to measure conflict between BBAs are getting lower and lower. This means that Jousselme evidence distance and pignistic probability distance are not effective in measuring the degree of conflict between completely conflicting evidence.

Example 3: Assuming that the frame of discernment is $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the results collected by three independent sensors are transformed into BBAs as shown below.

$$\begin{aligned} m_1 : m_1(\{\theta_1\}) &= 0.5, m_1(\{\theta_2\}) = 0.3, m_1(\{\theta_3\}) = 0.1 \\ & m_1(\{\Theta\}) = 0.1 \\ m_2 : m_2(\{\theta_1\}) &= 0.8, m_2(\{\theta_2\}) = 0.1, m_2(\{\theta_3\}) = 0.1 \\ m_3 : m_3(\{\theta_1\}) &= 0.3, m_3(\{\theta_2\}) = 0.5, m_3(\{\Theta\}) = 0.2 \end{aligned}$$

Next, we use Jousselme evidence distance and pignistic probability distance to calculate the conflict between m_1 and m_2 and the conflict between m_1 and m_3 in Example 3.

$$\begin{aligned} d_J(m_1, m_2) &= 0.2582, d_J(m_1, m_3) = 0.2160 \\ difBetP_{m_1}^{m_2} &= 0.2667, difBetP_{m_1}^{m_3} = 0.2333 \end{aligned}$$

We find that evidence m_1 and evidence m_2 support proposition $\{\theta_1\}$, and evidence m_3 supports proposition $\{\theta_2\}$. Therefore, the conflict between evidence m_1 and evidence m_2 is lower than the conflict between evidence m_1 and evidence m_3 . However, the measurements based on Jousselme evidence distance and pignistic probability distance believe that the degree of conflict between evidence m_1 and evidence m_2 is greater than that of between evidence m_1 and evidence m_3 , which is unreasonable.

3) Zhang et al. Similarity Measurement Method

Zhang et al. defined a new conflict measurement method by combining the pignistic probability function and the cosine (cos) correlation coefficient and used $1 - \cos$ to indicate the degree of conflict between evidence [45].

Definition 8: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the cosine of the angle between m_1 and m_2 is expressed as

$$\cos(m_1, m_2) = \frac{\langle \text{PignisticVector } m_1, \text{PignisticVector } m_2 \rangle}{\|\text{PignisticVector } m_1\| \cdot \|\text{PignisticVector } m_2\|} \quad (7)$$

where *PignisticVector* m_1 is the pignistic probability transformation of the evidence vector m_1 .

Example 4: Assuming that the frame of discernment is $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the results collected by two independent sensors are transformed into BBAs as shown below.

$$\begin{aligned} m_1 : m_1(\{\theta_1\}) &= 0.6, m_1(\{\theta_2\}) = 0.1, m_1(\{\Theta\}) = 0.3 \\ m_2 : m_2(\{\theta_1\}) &= 0.7, m_2(\{\theta_2\}) = 0.2, m_2(\{\theta_3\}) = 0.1 \end{aligned}$$

We use the improved cosine correlation coefficient proposed by Zhang et al. to calculate the degree of conflict between evidence m_1 and evidence m_2 in Example 4, and we get $\cos(m_1, m_2) = 1$. However, evidence m_1 and evidence m_2 are not the same two bodies of evidence, so the similarity between them is not 1, so the improved cosine correlation coefficient has certain limitations.

4) Ma & An Conflict Measurement Method

Ma & An proposed a new conflict measurement method which designs a probability transformation function using the belief function *Bel* and the plausibility function *Pl* to transform the basic belief assignment [46].

Definition 9: Let m be a BBA on the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$, the basic belief assignment function in evidence m is transformed by the following equation

$$P(\{\theta_i\}) = Bel(\{\theta_i\}) + \frac{BEL \cdot Bel(\{\theta_i\}) + (1 - BEL) \cdot Pl(\{\theta_i\})}{\sum_{\theta_j \in \Theta} BEL \cdot Bel(\{\theta_j\}) + (1 - BEL) \cdot Pl(\{\theta_j\})} (1 - BEL) \quad (8)$$

where $BEL = \sum Bel(\{\theta_i\})$.

Definition 10: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the degree of conflict between m_1 and m_2 is expressed as

$$DisSim(m_1, m_2) = 1 - \frac{R(m_1, m_2) + CoC(m_1, m_2)}{1 + R(m_1, m_2) \times CoC(m_1, m_2)} \quad (9)$$

where $R(m_1, m_2) = \frac{\sum_{k=1}^n (P_1(\{\theta_k\}) \wedge P_2(\{\theta_k\}))}{\sum_{k=1}^n (P_1(\{\theta_k\}) \vee P_2(\{\theta_k\}))}$ represents the fuzzy nearness between m_1 and m_2 ; and

$$CoC(m_1, m_2) = \begin{cases} \frac{P_1(\theta_{\max}^{P_1}) + P_2(\theta_{\max}^{P_2})}{2}, & \text{if } \theta_{\max}^{P_1} = \theta_{\max}^{P_2} \\ \frac{P_1(\theta_{\min}^{P_1}) + P_2(\theta_{\min}^{P_2})}{2}, & \text{if } \theta_{\max}^{P_1} \neq \theta_{\max}^{P_2} \end{cases},$$

represents the correlation coefficient between m_1 and m_2 , and $\theta_{\max}^{P_1} = \arg\max P_1(\theta)$, $\theta_{\min}^{P_1} = \arg\min P_1(\theta)$.

5) Xiao Conflict Measurement Method Based on Divergence

Xiao introduced Jensen-Shannon divergence into Dempster-Shafer evidence theory and defined a new conflict measurement method called Belief Jensen-Shannon (BJS) divergence [1].

Definition 11: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the BJS divergence between m_1 and m_2 is expressed as

$$BJS(m_1, m_2) = \frac{1}{2} [S(m_1, \frac{m_1 + m_2}{2}) + S(m_2, \frac{m_1 + m_2}{2})] \quad (10)$$

where $S(m_1, m_2) = \sum_i m_1(\{A_i\}) \log_2(\frac{m_1(\{A_i\})}{m_2(\{A_i\})})$, and $\sum_i m_j(\{A_i\}) = 1 (i = 1, 2, \dots, N; j = 1, 2)$.

Example 5: Assuming the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_7\}$, the results collected by three independent sensors are transformed into BBAs as shown below.

$$\begin{aligned} m_1 : m_1(\{\theta_1, \theta_2\}) &= 0.6, m_1(\{\theta_6, \theta_7\}) = 0.4 \\ m_2 : m_2(\{\theta_4, \theta_5\}) &= 0.6, m_2(\{\theta_6, \theta_7\}) = 0.4 \\ m_3 : m_3(\{\theta_1, \theta_2, \theta_3\}) &= 1 \end{aligned}$$

From Example 5, we find that the degree of conflict between evidence m_1 and m_3 is lower than the degree of conflict between evidence m_2 and m_3 . Using the BJS divergence to measure the conflict between m_1 and m_3 , m_2 and m_3 in Example 5, we can obtain $BJS(m_1, m_3) = BJS(m_2, m_3) = 1$. The results show that BJS divergence can not effectively represent the conflicts between evidence with multi-subset focal elements.

III. HELLINGER DISTANCE MEASURE

A. THE PROPOSED METHOD OF CONFLICT MEASUREMENT

Inspired by the idea of probability transformation function proposed by Ma & An [46], we combine Hellinger distance and Dempster-Shafer evidence theory to characterise the degree of conflict between BBAs. The definition of Hellinger distance is as follows.

Definition 12: Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the Hellinger distance between m_1 and m_2 is expressed as

$$d_H(m_1, m_2) = \frac{\left\| \sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})} \right\|_2}{\sqrt{2}} \quad (11)$$

The Hellinger distance satisfies the following properties.

- (1) Boundedness $0 \leq d_H(m_1, m_2) \leq 1$.
- (2) Symmetry $d_H(m_1, m_2) = d_H(m_2, m_1)$.
- (3) Definiteness $d_H(m_1, m_2) = 0 \Leftrightarrow m_1 = m_2$.
- (4) Triangle inequality $d_H(m_1, m_2) + d_H(m_2, m_3) \geq d_H(m_1, m_3)$.

These properties of Hellinger distance can be proved in the appendix.

B. VERIFY THE PERFORMANCE OF THE PROPOSED METHOD

In Example 1, we combine the probability transformation function and Hellinger distance to get $d_H(m_1, m_2) = 0$.

In Example 2, we combine the probability transformation function and Hellinger distance to measure the degree of conflict between BBAs too, and the results are shown in Table 2. Compared with d_J and $diffBetP$, the proposed method

TABLE 2. Solving Example 2 using Hellinger distance.

Distance	case 1	case 2	case 3	case 4	case 5
$d_H(m_1, m_2)$	1	1	1	1	1

can effectively measure the degree of conflict between completely conflicting BBAs.

In Example 3, we combine the probability transformation function and the Hellinger distance to measure the conflict between evidence m_1 and evidence m_2 and the conflict between evidence m_1 and evidence m_3 . We get $d_H(m_1, m_2) = 0.2132$, $d_H(m_1, m_3) = 0.2443$, and this result satisfies $d_H(m_1, m_2) < d_H(m_1, m_3)$. Therefore, the proposed method is more reasonable than the Jousselme evidence distance and the pignistic probability distance.

In Example 4, we combine the probability transformation function and Hellinger distance to measure the degree of conflict between evidence m_1 and evidence m_2 , we get $d_H(m_1, m_2) = 0.1204$. This result shows that the proposed method is more reasonable than the improved cosine correlation coefficient method.

In the case of multi-subset focal elements, as described in Example 5. We combine the probability transformation function and Hellinger distance to get $d_H(m_1, m_3) = 0.6063$ and $d_H(m_2, m_3) = 1$. This result means $d_H(m_1, m_3) \leq d_H(m_2, m_3)$. So, it may effectively handle the problem that BJS divergence cannot effectively measure the conflict between non-single subset propositions.

Example 6: Assuming the frame of discernment $\Theta = \{\theta_1, \theta_2\}$, the results collected by three independent sensors are converted into BBAs as shown below.

$$\begin{aligned} m_1 : m_1(\{\theta_1\}) &= 0.90, m_1(\{\theta_2\}) = 0.10 \\ m_2 : m_2(\{\theta_1\}) &= 0.80, m_2(\{\theta_2\}) = 0.20 \\ m_3 : m_3(\{\theta_1\}) &= 0.30, m_3(\{\theta_2\}) = 0.70 \end{aligned}$$

Calculating Hellinger distance such as $d_H(m_1, m_2)$, $d_H(m_2, m_3)$, and $d_H(m_1, m_3)$, we can obtain

$$\begin{aligned} d_H(m_1, m_2) &= \frac{\sqrt{(\sqrt{0.90} - \sqrt{0.80})^2 + (\sqrt{0.10} - \sqrt{0.20})^2}}{\sqrt{2}} = 0.1003 \\ d_H(m_2, m_3) &= \frac{\sqrt{(\sqrt{0.80} - \sqrt{0.30})^2 + (\sqrt{0.20} - \sqrt{0.70})^2}}{\sqrt{2}} = 0.3687 \\ d_H(m_1, m_3) &= \frac{\sqrt{(\sqrt{0.90} - \sqrt{0.30})^2 + (\sqrt{0.10} - \sqrt{0.70})^2}}{\sqrt{2}} = 0.4646 \end{aligned}$$

Therefore, we can get the following inequality $d_H(m_1, m_2) + d_H(m_2, m_3) = 0.4690 > d_H(m_1, m_3) = 0.4646$.

Example 7: Assuming the frame of discernment $\Theta = \{\theta_1, \theta_2\}$, the results collected by two independent sensors are converted into BBAs as shown below.

$$\begin{aligned} \text{case 1 : } & \begin{cases} m_1(\{\theta_1\}) = a, m_1(\{\theta_2\}) = 1 - a \\ m_2(\{\theta_1\}) = 0.5, m_2(\{\theta_2\}) = 0.5 \end{cases} \\ \text{case 2 : } & \begin{cases} m_1(\{\theta_1\}) = a, m_1(\{\theta_2\}) = 1 - a \\ m_2(\{\theta_1\}) = 0.9999, m_2(\{\theta_2\}) = 0.0001 \end{cases} \end{aligned}$$

Supposing a varying from 0 to 1, the Hellinger distance between evidence m_1 and evidence m_2 are shown in Fig. 1.

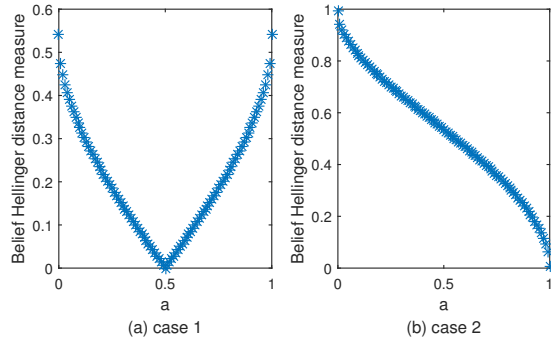


FIGURE 1. The changing trend of Hellinger distance as the parameter changes.

Case 1: As a increases, the Hellinger distance between m_1 and m_2 tends to decrease first and then increase. The conflict between m_1 and m_2 achieves the minimum at $a = 0.5$.

Case 2: As a increases, the Hellinger distance between m_1 and m_2 is always decreasing. The conflict between m_1 and m_2 achieves the minimum at $a = 0.9999$.

Next, we use Example 8 to show the changes of the degree of conflict between BBAs with the number of elements in the proposition changing.

Example 8: Assuming the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_{20}\}$, and the results collected by two independent sensors are converted into BBAs as

$$\begin{aligned} m_1 : m_1(\{A_i\}) &= 0.8, m_1(\{\theta_2, \theta_3, \theta_4\}) = 0.05 \\ m_1(\{\theta_7\}) &= 0.05, m_1(\{\Theta\}) = 0.1 \\ m_2 : m_2(\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}) &= 1 \end{aligned}$$

The elements in proposition $\{A_i\}$ is gradually increasing in order, namely $\{A_i\} = \{\theta_1, \theta_2, \dots, \theta_i\} (i=1, 2, \dots, 20)$.

Given two BBAs, we can conclude that with the number of elements in proposition $\{A_i\}$ increasing, the conflicting between the two BBAs decreases firstly and then increases.

Fig. 2 shows the trend of conflicts between BBAs as the number of elements in proposition $\{A_i\}$ increases. In Fig. 2, we compare the introduced Hellinger distance d_H with some of the evidence conflict measurement methods introduced earlier ($k, d_J, difBetP, 1-cos, DisSim, BJS$).

In Fig. 2, we can find that $d_J, difBetP, 1-cos$, and Hellinger distance d_H have the same trend, they all decrease first and then increase, moreover, when the elements in the proposition $\{A_i\}$ is $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$, the conflict between BBAs is the smallest. $DisSim$ has a strange rise, which is a bit unreasonable; k and BJS cannot measure the degree of conflict between BBAs in Example 8.

IV. UNCERTAINTY MEASUREMENT

A. UNCERTAINTY MEASUREMENT METHODS IN DEMPSTER-SHAFFER EVIDENCE THEORY

In this part, we introduce some typical uncertainty measurement methods in Dempster-Shafer evidence theory. Among them, the specific expression equation related to those meth-

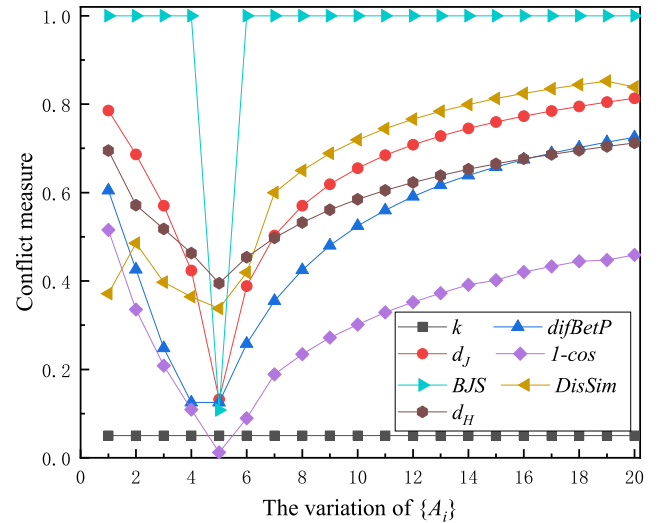


FIGURE 2. Trends of conflict between BBAs.

ods are shown in Table 3, where $|A|$ is the cardinality of the subset $\{A\}$.

B. THE NEW BELIEF ENTROPY

We define a new belief entropy here to tackle shortcomings related to above uncertain measures.

Definition 13: Let m be a BBA on the frame of discernment Θ , the new belief entropy of m is expressed as

$$E_x(m) = - \sum_{A \subseteq \Theta} m(\{A\}) \log_2 \left(\frac{m(\{A\})}{2^{|A|} - 1} \cdot e^{\frac{|A|-1}{2^{|A|}-1}} \right) \quad (12)$$

where $|X| = |A \cup B|$, $B = B_1 \cup B_2 \cup \dots \cup B_i \cup \dots \cup B_s$ ($i = 1, 2, \dots, s; B_i \subseteq \Theta; B_i \neq A; m(\{B_i\}) \neq 0$). $|X|$ is the cardinality of $\{X\}$. Supposing $\{A\}$ composed by single subset proposition, the belief entropy degenerates into Shannon entropy, i.e.

$$\begin{aligned} E_x(m) &= - \sum_{A \subseteq \Theta} m(\{A\}) \log_2 \left(\frac{m(\{A\})}{2^{|A|} - 1} \cdot e^{\frac{|A|-1}{2^{|A|}-1}} \right) \\ &= - \sum_{A \subseteq \Theta} m(\{A\}) \log_2(m(\{A\})) \end{aligned} \quad (13)$$

When the support of some propositions is 0, due to the property of the \log function, calculations cannot be performed at this time. Therefore, we use 1×10^{-12} instead of 0 for $m(\{A\})$ in Eq.(12).

C. NUMERICAL EXAMPLES

In this section, we use several examples to verify the performance of the new belief entropy defined in this paper and compare them with the uncertainty measurement methods proposed by other scholars.

Example 9: Assuming the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_{10}\}$, the results collected by one sensor are converted into BBA as shown below.

TABLE 3. Uncertain measurement methods in Dempster-Shafer evidence theory.

Method	Equation
Deng's method [47]	$E_d(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 \frac{m(\{A\})}{2^{ A -1}}$
Zhou et al.'s method [48]	$E_{Id}(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 \left(\frac{m(\{A\})}{2^{ A -1}} e^{(A -1)/ \Theta } \right)$
Tang et al.'s method [49]	$E_{Wd}(m) = -\sum_{A \subseteq \Theta} \frac{ A }{ \Theta } m(\{A\}) \log_2 \frac{m(\{A\})}{2^{ A -1}}$
Pan & Deng's method [50]	$H_{bel}(m) = -\sum_{A \subseteq \Theta} \frac{Bel(\{A\}) + Pl(\{A\})}{2} \log_2 \frac{Bel(\{A\}) + Pl(\{A\})}{2(2^{ A -1})}$
Qin et al.'s method [51]	$Q(m) = \sum_{A \subseteq \Theta} \frac{ A }{ \Theta } m(\{A\}) \log_2 A + \sum_{A \subseteq \Theta} m(\{A\}) \log_2 \frac{1}{m(\{A\})}$
Höhle's method [54]	$C_H(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 Bel(\{A\})$
Yager's method [55]	$E_Y(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 Pl(\{A\})$
Dubois & Prade's method [56]	$U(m) = \sum_{A \subseteq \Theta} m(\{A\}) \log_2 A $
Klir & Ramer's method [57]	$D(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 \sum_{B \subseteq \Theta} m(\{B\}) \frac{ A \cap B }{ B }$
Klir & Parviz's method [58]	$S(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 \sum_{B \subseteq \Theta} m(\{B\}) \frac{ A \cap B }{ A }$
Chen et al.'s method [59]	$E_i(m) = -\sum_{A \subseteq \Theta} m(\{A\}) \log_2 \frac{m(\{A\})}{2^{ A -1}} \frac{ A }{ \Theta }$

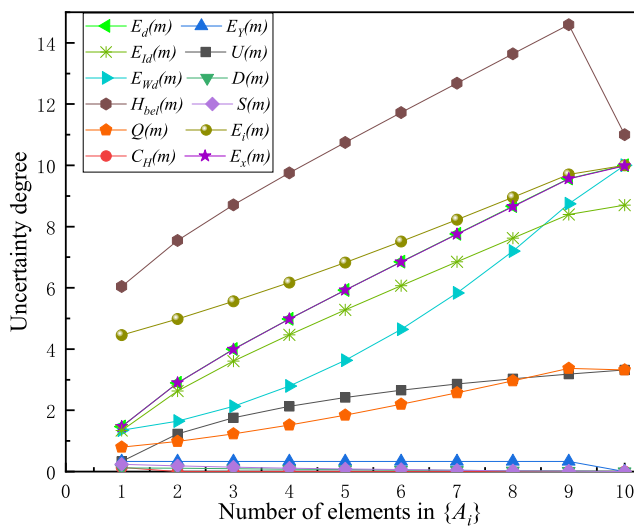


FIGURE 3. Use belief entropy and other uncertainty measures to quantify the uncertainty of the evidence in Example 9.

$$m : m(\{A_i\}) = 0.9, m(\{\Theta\}) = 0.1$$

The elements in proposition $\{A_i\}$ is gradually increasing in order, namely $\{A_i\} = \{\theta_1, \theta_2, \dots, \theta_i\} (i=1, 2, \dots, 10)$.

We use the improved belief entropy defined in this paper and the uncertainty measure methods listed in Table 3 to quantify the uncertainty of the BBA in Example 9, and the results are shown in Fig. 3.

From Fig. 3, we find that the Höhle's method [54], Yager's method [55], Klir & Ramer's method [57], Klir & Parviz's method [58] cannot effectively quantify the uncertainty of the BBA. With the increase of the elements in the proposition $\{A_i\}$, uncertainty measure defined by Deng's method [47], Zhou et al.'s method [48], Tang et al.'s method [49], Dubois & Prade's method [56], Chen et al.'s method [59] and the belief entropy defined in this paper is monotonically increasing. Although the uncertainty measures defined by Pan & Deng's method [50], Qin et al.'s method [51] are also

increasing, when the element in proposition $\{A_i\}$ becomes $\{\theta_1, \theta_2, \dots, \theta_{10}\}$, their results will decrease compared to the previous result. The proposition $\{\theta_1, \theta_2, \dots, \theta_{10}\}$ means that the information is completely unknown. At this time, the uncertainty of BBA should be the largest, so the uncertainty measure defined by Pan & Deng's method [50], Qin et al.'s method [51] is not unreasonable.

Example 10: Let m_1 and m_2 be the two BBAs on the same frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, the distributions of m_1 and m_2 are shown below.

m_1 :

$$\begin{aligned} m_1(\{\theta_1, \theta_2\}) &= 0.4, m_1(\{\theta_3, \theta_4\}) = 0.6, m_1(\{\phi\}) = 0 \\ m_1(\{\theta_1\}) &= m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = m_1(\{\theta_4\}) = 0 \\ m_1(\{\theta_1, \theta_3\}) &= m_1(\{\theta_1, \theta_4\}) = m_1(\{\theta_2, \theta_3\}) = 0 \\ m_1(\{\theta_2, \theta_4\}) &= m_1(\{\theta_1, \theta_2, \theta_3\}) = m_1(\{\theta_1, \theta_2, \theta_4\}) = 0 \\ m_1(\{\theta_1, \theta_3, \theta_4\}) &= m_1(\{\theta_2, \theta_3, \theta_4\}) = m_1(\{\Theta\}) = 0 \end{aligned}$$

m_2 :

$$\begin{aligned} m_2(\{\theta_1, \theta_2\}) &= 0.4, m_2(\{\theta_2, \theta_3\}) = 0.6, m_2(\{\phi\}) = 0 \\ m_2(\{\theta_1\}) &= m_2(\{\theta_2\}) = m_2(\{\theta_3\}) = m_2(\{\theta_4\}) = 0 \\ m_2(\{\theta_1, \theta_3\}) &= m_2(\{\theta_1, \theta_4\}) = m_2(\{\theta_2, \theta_4\}) = 0 \\ m_2(\{\theta_3, \theta_4\}) &= m_2(\{\theta_1, \theta_2, \theta_3\}) = m_2(\{\theta_1, \theta_2, \theta_4\}) = 0 \\ m_2(\{\theta_1, \theta_3, \theta_4\}) &= m_2(\{\theta_2, \theta_3, \theta_4\}) = m_2(\{\Theta\}) = 0 \end{aligned}$$

We use Deng's method [47], Zhou et al.'s method [48], Tang et al.'s method [49], Pan & Deng's method [50], Qin et al.'s method [51], Dubois & Prade's method [56], Chen et al.'s method [59] and the belief entropy defined in this paper to quantify the uncertainty of m_1 and m_2 in Example 10. The results are shown in Table 4 below.

We can find that evidence m_1 assigns basic belief assignments to propositions $\{\theta_1, \theta_2\}$ and $\{\theta_3, \theta_4\}$, moreover, we also find evidence m_1 contains four elements; Similar as evidence m_1 , we can find evidence m_2 contains three elements. Since evidence m_1 contains more elements than that of evidence m_2 , the uncertainty of which should be greater than that of evidence m_2 .

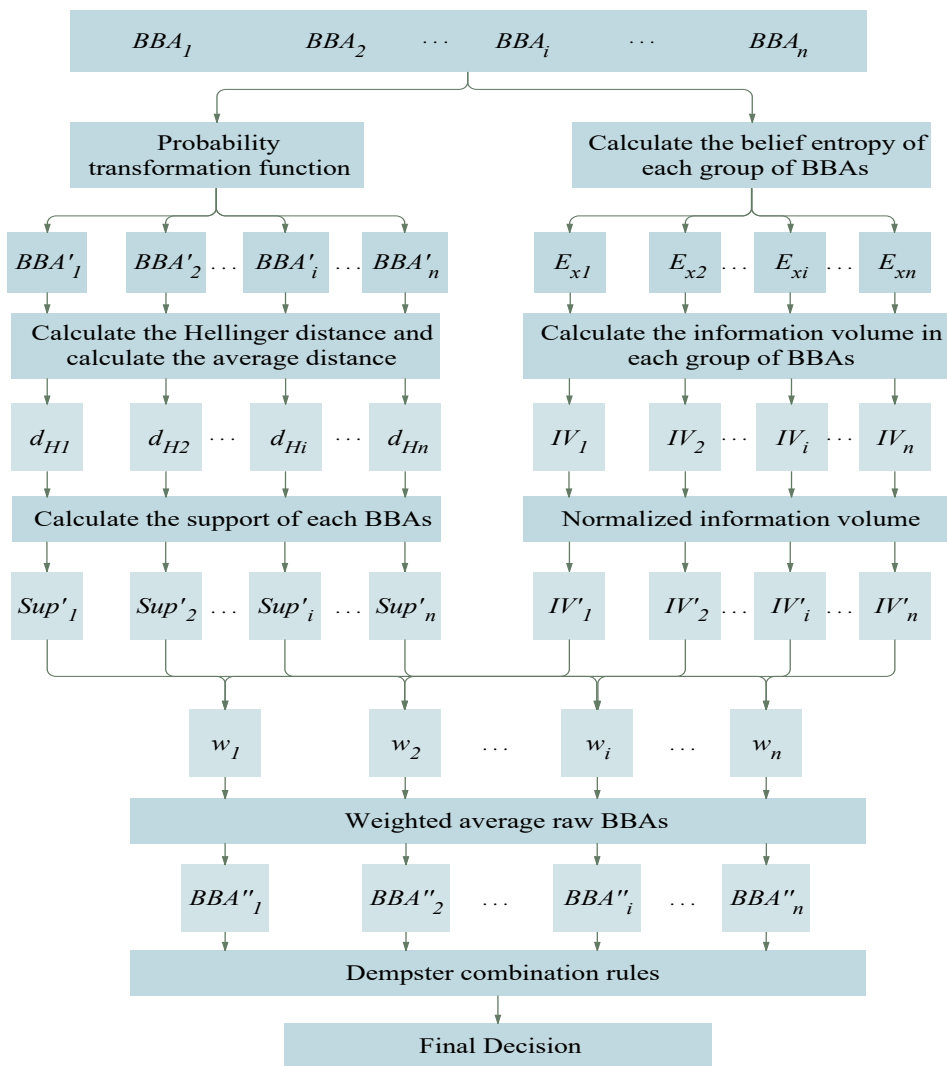


FIGURE 4. The calculation steps of the proposed algorithm.

TABLE 4. Use different methods to quantify the uncertainty of the evidence in Example 10.

Method	m_1	m_2
Deng's method [47]	2.5559	2.5559
Zhou et al.'s method [48]	2.1952	2.1952
Tang et al.'s method [49]	1.2780	1.2780
Pan & Deng's method [50]	3.5559	3.5559
Qin et al.'s method [51]	1.4710	1.4710
Dubois & Prade's method [56]	1	1
Chen et al.'s method [59]	3.5559	3.5559
The proposed method	2.4597	2.3498

From the results in Table 4, we find that Deng's method [47], Zhou et al.'s method [48], Tang et al.'s method [49], Pan & Deng's method [50], Qin et al.'s method [51], Dubois & Prade's method [56], Chen et al.'s method [59] can not

distinguish the uncertainty of evidence m_1 and evidence m_2 , which is inconsistent with our previous analysis. The result of the proposed method is: $E_x(m_1) = 2.4597, E_x(m_2) = 2.3498$, satisfy $E_x(m_1) > E_x(m_2)$.

V. A MULTI-SENSOR INFORMATION FUSION ALGORITHM

Inspired by ideas of Murphy's simple average method [38] and Deng's weighted average method [39], we combine Hellinger distance and belief entropy to propose weighted conflict evidence fusion method described in Fig. 4.

The specific steps of weighted conflict evidence fusion method based on Hellinger distance and belief entropy are as follows.

Step 1: We use Eq.(8) to assign the reliability of non-mono-set propositions in BBAs $m_i (i = 1, 2, \dots, n)$ and $m_j (j = 1, 2, \dots, n)$ into mono-set propositions, and get new BBAs m'_i and m'_j .

TABLE 5. Fusion results of BBAs in Example 11 using seven different methods.

Method	$m_{123A}(\{\theta_1\})$	$m_{123B}(\{\theta_1\})$	$m_{123A}(\{\theta_2\})$	$m_{123B}(\{\theta_2\})$	$m_{123A}(\{\theta_3\})$	$m_{123B}(\{\theta_3\})$
Dempster rule	0	0	0.9574	0.9677	0.0426	0.0323
Yager's method [35]	0.5700	0.5320	0.1478	0.1970	0.0775	0.0775
Murphy's method [38]	0.5235	0.4674	0.4674	0.5235	0.0091	0.0091
Deng et al.'s method [39]	0.7264	0.6823	0.2502	0.2968	0.0234	0.0209
Liu et al.'s method [42]	0.8332	0.7958	0.1454	0.1829	0.0214	0.0213
Ma & An's method [46]	0.8837	0.8513	0.0931	0.1159	0.0232	0.0327
The proposed method	0.9115	0.8941	0.0798	0.0972	0.0087	0.0087

Step 2: Calculate the distance $d_H(m_i, m_j)$ between the transformed evidence m'_i and m'_j , and construct the distance measure matrix $D = [d_H(m_i, m_j)]_{n \times n}$, as shown in Eq.(14).

$$D = \begin{bmatrix} 0 & \cdots & d_H(m_1, m_j) & \cdots & d_H(m_1, m_n) \\ \vdots & & \vdots & & \vdots \\ d_H(m_i, m_1) & \cdots & d_H(m_i, m_j) & \cdots & d_H(m_i, m_n) \\ \vdots & & \vdots & & \vdots \\ d_H(m_n, m_1) & \cdots & d_H(m_n, m_j) & \cdots & 0 \end{bmatrix} \quad (14)$$

Step 3: According to the distance between m'_i and the other BBA m'_j , calculate the average distance d_{Hi} of m'_i .

$$d_{Hi} = \frac{1}{n-1} \left[\sum_{\substack{j=1 \\ j \neq i}}^n d_H(m_i, m_j) \right] \quad (15)$$

Step 4: The average distance d_{Hi} of m'_i is processed and normalised, and the obtained result is regarded as the support degree Sup'_i of m_i described in Eq. (16) and Eq. (17).

$$Sup_i = \left(1 - \sqrt{d_{Hi}} \right) e^{(-\sqrt{d_{Hi}})} \quad (16)$$

$$Sup'_i = Sup_i / \sum_{l=1}^n Sup_l \quad (17)$$

Step 5: Calculate the belief entropy $E_x(m_i)$ of the evidence m_i using Eq. (12). The belief entropy $E_x(m_i)$ is processed and normalised to avoid its zero situations, and the obtained result is defined as the information volume IV'_i of m_i described in Eq. (18) and Eq. (19).

$$IV_i = e^{E_x(m_i)} \quad (18)$$

$$IV'_i = IV_i / \sum_{l=1}^n IV_l \quad (19)$$

Step 6: We use Eq.(17) and Eq.(19) to calculate the trust degree $Cred_i$ of m_i .

$$Cred_i = Sup'_i \times IV'_i \quad (20)$$

Step 7: The weighting factor ω_i of the evidence is obtained by normalisation of trust degree $Cred_i$.

$$\omega_i = Cred_i / \sum_{l=1}^n Cred_l \quad (21)$$

Step 8: We use ω_i to weight the original evidence.

$$m''_i = \sum_{i=1}^n \omega_i \times m_i(A) \quad (22)$$

Step 9: According to the Dempster combination rule, the revised evidence is fused $n-1$ times to obtain the final fusion result.

Next, we introduce examples of robustness testing in Ma & An [46] and compared the proposed method with Ma&An's method and methods proposed by other scholars.

Example 11: Assuming the frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the four independent BBAs are shown below.

$$\begin{aligned} m_1 : m_1(\theta_1) &= 0, m_1(\theta_2) = 0.9, m_1(\theta_3) = 0.1 \\ m_2 : m_2(\theta_1) &= 0.6, m_2(\theta_2) = 0.25, m_2(\theta_3) = 0.15 \\ m_{3A} : m_{3A}(\theta_1) &= 0.75, m_{3A}(\theta_2) = 0.15, m_{3A}(\theta_3) = 0.1 \\ m_{3B} : m_{3B}(\theta_1) &= 0.7, m_{3B}(\theta_2) = 0.2, m_{3B}(\theta_3) = 0.1 \end{aligned}$$

From the four BBAs in Example 11, it can be seen that evidence m_2, m_{3A}, m_{3B} all give the highest degree of trust to the proposition $\{\theta_1\}$, while evidence m_1 gives the highest degree of trust to the proposition $\{\theta_2\}$. We find that the evidence m_1 and several other bodies of evidence have a large conflict. Next, we verify the performance related to dealing with conflicting evidence of the proposed method. Table 5 shows the final fusion results of different methods. The fusion results of evidence m_1, m_2, m_{3A} and m_1, m_2, m_{3B} are represented as m_{123A} and m_{123B} respectively.

The results in Table 5 show that the Dempster combination rule supports proposition $\{\theta_2\}$ due to the zero trust paradox, which is contrary to common sense. Although both Yager's method [35] and Murphy's method [38] support proposition $\{\theta_1\}$, their final fusion result is not good. Deng et al.'s method [39], Liu et al.'s method [42], Ma & An's method [46] and the proposed method all support proposition $\{\theta_1\}$. Compared with the other combination methods, the proposed method has the greatest support for proposition $\{\theta_1\}$, and the result is the most concrete and robust. It can effectively weaken the impact of conflicting evidence and is more conducive to decision-making.

VI. EXAMPLES ANALYSIS

Dempster-Shafer evidence theory is widely used in the fields of fault diagnosis and target recognition. We take the mechanical fault diagnosis in [16] as an example and compare the proposed method with other methods.

Example 12: Assuming the frame of discernment $\Theta = \{F_1, F_2, F_3\}$. The fault data collected by the three sensors S_1, S_2, S_3 at different positions are transformed into three BBAs after modelling, namely m_1, m_2, m_3 . At the same time, two kinds of reliability related to the sensor are considered, one is the static reliability R_i^s which is composed of the evidence sufficiency index μ_i and the importance index ν_i , can be expressed as $R_i^s = \mu_i \times \nu_i$, and the other is the dynamic reliability R_i^d based on $Cred_i$. We use the final comprehensive reliability $R = R^s \times R^d$ as a weighting factor to weight the original evidence. The parameters of the three BBAs and the three sensors are as follows

$$\begin{aligned} m_1 : m_1(\{F_1\}) &= 0.60, m_1(\{F_2\}) = 0.10, \\ & m_1(\{F_2, F_3\}) = 0.10, m_1(\{\Theta\}) = 0.20 \\ m_2 : m_2(\{F_1\}) &= 0.05, m_2(\{F_2\}) = 0.80, \\ & m_2(\{F_2, F_3\}) = 0.05, m_2(\{\Theta\}) = 0.10 \\ m_3 : m_3(\{F_1\}) &= 0.70, m_3(\{F_2\}) = 0.10, \\ & m_3(\{F_2, F_3\}) = 0.10, m_3(\{\Theta\}) = 0.10 \\ S_1 : \mu_1 &= 1, \nu_1 = 1 \\ S_2 : \mu_2 &= 0.6, \nu_2 = 0.34 \\ S_3 : \mu_3 &= 1, \nu_3 = 1 \end{aligned}$$

We give the detailed calculation process of the proposed method below.

Step 1: The reliability of the multi-subset focal elements in the evidence is processed by the probability transformation function.

$$\begin{aligned} m'_1 : m'_1(\{F_1\}) &= 0.8107, m'_1(\{F_2\}) = 0.1606, \\ & m'_1(\{F_3\}) = 0.0287 \\ m'_2 : m'_2(\{F_1\}) &= 0.0607, m'_2(\{F_2\}) = 0.9356, \\ & m'_2(\{F_3\}) = 0.0037 \\ m'_3 : m'_3(\{F_1\}) &= 0.8600, m'_3(\{F_2\}) = 0.1311, \\ & m'_3(\{F_3\}) = 0.0089 \end{aligned}$$

Step 2: Construct the distance measure matrix $D = [d_H(m_i, m_j)]_{n \times n}$.

$$D = \begin{bmatrix} 0 & 0.6166 & 0.0627 \\ 0.6166 & 0 & 0.6446 \\ 0.0627 & 0.6446 & 0 \end{bmatrix}$$

Step 3: Calculate the average distance d_{Hi} of the evidence.

$$\begin{aligned} d_{H1} &= 0.3397 \\ d_{H2} &= 0.6306 \\ d_{H3} &= 0.3536 \end{aligned}$$

Step 4: Calculate the supporting degree Sup'_i of the evidence.

$$\begin{aligned} Sup'_1 &= 0.4237 \\ Sup'_2 &= 0.1694 \\ Sup'_3 &= 0.4069 \end{aligned}$$

Step 5: Calculate the information volume IV'_i of the evidence.

$$\begin{aligned} IV'_1 &= 0.4856 \\ IV'_2 &= 0.2060 \\ IV'_3 &= 0.3085 \end{aligned}$$

Step 6: Calculate the confidence $Cred_i$ of the evidence.

$$\begin{aligned} Cred_1 &= 0.2057 \\ Cred_2 &= 0.0349 \\ Cred_3 &= 0.1255 \end{aligned}$$

Step 7: Calculate the weighting factor ω_i of the evidence.

$$\begin{aligned} \omega_1 &= 0.6080 \\ \omega_2 &= 0.0210 \\ \omega_3 &= 0.3710 \end{aligned}$$

Step 8: The weighted average of the original evidence.

$$\begin{aligned} m(\{F_1\}) &= 0.6255, m(\{F_2\}) = 0.1147, \\ m(\{F_2, F_3\}) &= 0.1608, m(\{\Theta\}) = 0.0990 \end{aligned}$$

Step 9: Fuse two times using Dempster combination rule.

Table 6 shows the fusion results of the seven methods. Dempster combination fusion rule considers the proposition $\{F_2\}$ to be the most likely, while the other six fusion methods support the proposition $\{F_1\}$. The final fusion result of the proposed method supports 0.9018 for the proposition $\{F_1\}$, indicating that this method can effectively deal with the fusion of conflict evidence. Fig. 5 shows histograms comparing the final fusion results based on different methods.

To further verify the effectiveness of the proposed method, we use the following example of target recognition and demonstrate the characteristics of this method through comparative experiments with other methods.

Example 13: There are five different sensors to be used for recognition an air target, the recognition framework can be described as $\Theta = \{A : \text{Civil aircraft}, B : \text{Bomber}, C : \text{Helicopter}\}$, and the information obtained by the five sensors are converted into BBAs as shown below.

$$\begin{aligned} m_1 : m_1(\{A\}) &= 0.41, m_1(\{B\}) = 0.29, m_1(\{C\}) = 0.3 \\ m_2 : m_2(\{A\}) &= 0, m_2(\{B\}) = 0.9, m_2(\{C\}) = 0.1 \\ m_3 : m_3(\{A\}) &= 0.58, m_3(\{B\}) = 0.07, \\ & m_3(\{A, C\}) = 0.35 \\ m_4 : m_4(\{A\}) &= 0.55, m_4(\{B\}) = 0.1, \\ & m_4(\{A, C\}) = 0.35 \\ m_5 : m_5(\{A\}) &= 0.6, m_5(\{B\}) = 0.1, \\ & m_5(\{A, C\}) = 0.3 \end{aligned}$$

We give the detailed calculation process of the proposed method below.

Step 1: The reliability of the multi-subset focal elements in the evidence is processed by the probability transformation

TABLE 6. Fusion results of BBAs in Example 12 using seven different methods.

Method	$m(\{F_1\})$	$m(\{F_2\})$	$m(\{F_2, F_3\})$	$m(\{\Theta\})$	Target
Dempster rule	0.4519	0.5048	0.0336	0.0096	$\{F_2\}$
Murphy's method [38]	0.5376	0.4280	0.0279	0.0065	$\{F_1\}$
Fan and Zou's method [16]	0.8119	0.1096	0.0526	0.0259	$\{F_1\}$
Chen et al.'s method [41]	0.8899	0.0785	0.0243	0.0073	$\{F_1\}$
Yuan et al.'s method [40]	0.8948	0.0739	0.0241	0.0072	$\{F_1\}$
Xiao's method [1]	0.8978	0.0688	0.0254	0.0080	$\{F_1\}$
The proposed method	0.9018	0.0654	0.0250	0.0078	$\{F_1\}$

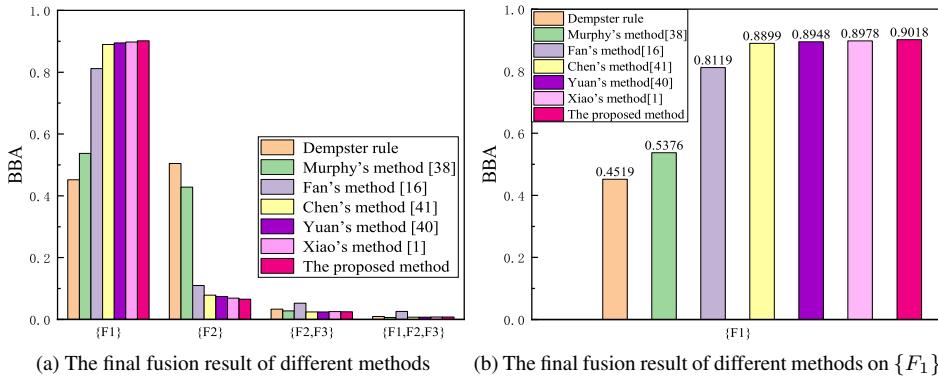


FIGURE 5. Comparison of the final fusion results of different fusion methods in Example 12.

function.

$$\begin{aligned}
 m'_1 : m'_1(\{A\}) &= 0.41, m'_1(\{B\}) = 0.29, m'_1(\{C\}) = 0.3 \\
 m'_2 : m'_2(\{A\}) &= 0, m'_2(\{B\}) = 0.9, m'_2(\{C\}) = 0.1 \\
 m'_3 : m'_3(\{A\}) &= 0.8547, m'_3(\{B\}) = 0.0974, \\
 & m'_3(\{C\}) = 0.0479 \\
 m'_4 : m'_4(\{A\}) &= 0.8130, m'_4(\{B\}) = 0.1391, \\
 & m'_4(\{C\}) = 0.0479 \\
 m'_5 : m'_5(\{A\}) &= 0.8352, m'_5(\{B\}) = 0.1341, \\
 & m'_5(\{C\}) = 0.0307
 \end{aligned}$$

Step 2: Construct the distance measure matrix $D = [d_H(m_i, m_j)]_{n \times n}$.

$$D = \begin{bmatrix} 0 & 0.5621 & 0.3465 & 0.3193 & 0.3488 \\ 0.5621 & 0 & 0.7967 & 0.7596 & 0.7728 \\ 0.3465 & 0.7967 & 0 & 0.0460 & 0.0497 \\ 0.3193 & 0.7596 & 0.0460 & 0 & 0.0324 \\ 0.3488 & 0.7728 & 0.0497 & 0.0324 & 0 \end{bmatrix}$$

Step 3: Calculate the average distance d_{H_i} of the evidence.

$$\begin{aligned}
 d_{H1} &= 0.3942 \\
 d_{H2} &= 0.7228 \\
 d_{H3} &= 0.3097 \\
 d_{H4} &= 0.2893 \\
 d_{H5} &= 0.3009
 \end{aligned}$$

Step 4: Calculate the supporting degree Sup'_i of the evidence.

$$\begin{aligned}
 Sup'_1 &= 0.1896 \\
 Sup'_2 &= 0.0611 \\
 Sup'_3 &= 0.2427 \\
 Sup'_4 &= 0.2577 \\
 Sup'_5 &= 0.2490
 \end{aligned}$$

Step 5: Calculate the information volume IV'_i of the evidence.

$$\begin{aligned}
 IV'_1 &= 0.2016 \\
 IV'_2 &= 0.0673 \\
 IV'_3 &= 0.2391 \\
 IV'_4 &= 0.2596 \\
 IV'_5 &= 0.2325
 \end{aligned}$$

Step 6: Calculate the confidence $Cred_i$ of the evidence.

$$\begin{aligned}
 Cred_1 &= 0.0382 \\
 Cred_2 &= 0.0041 \\
 Cred_3 &= 0.0580 \\
 Cred_4 &= 0.0669 \\
 Cred_5 &= 0.0579
 \end{aligned}$$

Step 7: Calculate the weighting factor ω_i of the evidence.

$$\begin{aligned}
 \omega_1 &= 0.1697 \\
 \omega_2 &= 0.0182 \\
 \omega_3 &= 0.2577 \\
 \omega_4 &= 0.2972 \\
 \omega_5 &= 0.2572
 \end{aligned}$$

Step 8: The weighted average of the original evidence.

TABLE 7. Fusion results of BBAs in Example 13 using seven different methods.

Method	$m(\{A\})$	$m(\{B\})$	$m(\{C\})$	$m(\{A, C\})$	Target
Dempster rule	0	0.1422	0.8578	0	$\{C\}$
Murphy's method [38]	0.9620	0.0210	0.0138	0.0032	$\{A\}$
Deng et al.'s method [39]	0.9820	0.0039	0.0107	0.0034	$\{A\}$
Zhang et al.'s method [45]	0.9820	0.0034	0.0115	0.0032	$\{A\}$
Wang et al.'s method [14]	0.9886	0.0004	0.0091	0.0032	$\{A\}$
Xiao's method [1]	0.9894	0.0002	0.0061	0.0043	$\{A\}$
The proposed method	0.9895	0.0002	0.0061	0.0042	$\{A\}$

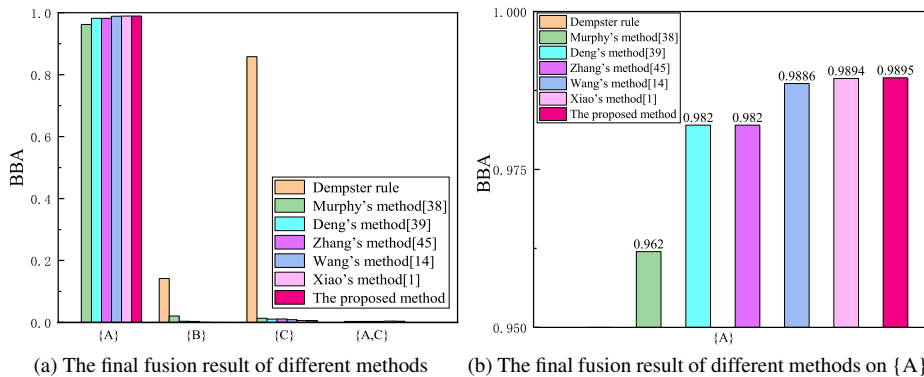


FIGURE 6. Comparison of BBA results obtained by different fusion methods in target recognition.

$$m(\{A\}) = 0.5368, m(\{B\}) = 0.1391, \\ m(\{C\}) = 0.0527, m(\{A, C\}) = 0.2714$$

Step 9: Fuse four times using Dempster combination rule. The final results are shown in Table 7 and Fig. 6.

The data in Example 13 show that the evidence m_2 supports target $\{B\}$, while the other four bodies of evidence all supported target $\{A\}$. It shows that the evidence m_2 is a abnormal evidence, which has an apparent conflict with the other four bodies of evidence. Table 7 shows that the Dempster combination rule will have a zero-trust paradox. The final combined results of several other methods all supported target $\{A\}$. The final fusion result related to the target $\{A\}$ of the proposed method is 0.9895, which is higher than that of other methods.

VII. CONCLUSION

From the perspective of how to effectively measure the conflict between two bodies of evidence and quantify the uncertainty of the evidence itself, we define a weighted conflict evidence combination method based on Hellinger distance and the belief entropy. We use the Hellinger distance and the belief entropy to construct the weight coefficient of the evidence, and then use this coefficient to weight the original evidence, furthermore, use the Dempster combination rule to fuse the weighted evidence.

Finally, the proposed method is verified through fault diagnosis and target recognition cases. The examples show that proposed method reduces the weight of unreliable evidence

to weaken its unfavourable influence on the fusion, which can effectively avoid the counter-intuitive result of the Dempster combination rule in the case of high-conflict evidence fusing.

In the next work, we are going to study how to construct the basic belief assignment in Dempster-Shafer evidence theory and combine it with conflict evidence fusion.

APPENDIX

Proof of the Hellinger distance property:

Let m_1 and m_2 be the two BBAs on the same frame of discernment Θ , the two BBAs m_1 and m_2 are shown as $m_1(\{\theta_1\}), m_1(\{\theta_2\}), \dots, m_1(\{\theta_n\})$ and $m_2(\{\theta_1\}), m_2(\{\theta_2\}), \dots, m_2(\{\theta_n\})$, respectively. It can be found that m_1 and m_2 satisfy $0 \leq m(\{\theta_i\}) \leq 1$ and $\sum_i m(\{\theta_i\}) = 1$.

Proof.(1)

$$d_H^2(m_1, m_2) = \frac{1}{2} \sum_i \left(\left| \sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})} \right| \right)^2 \\ = \frac{1}{2} \sum_i \left| \sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})} \right| \left| \sqrt{m_1(\{\theta_i\})} + \sqrt{m_2(\{\theta_i\})} \right| \\ \leq \frac{1}{2} \sum_i \left| \sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})} \right| \left| \sqrt{m_1(\{\theta_i\})} + \sqrt{m_2(\{\theta_i\})} \right| \\ = \frac{1}{2} \sum_i |m_1(\{\theta_i\}) - m_2(\{\theta_i\})|$$

Since m_i has properties such as $0 \leq m(\{\theta_i\}) \leq 1$ and $\sum_i m(\{\theta_i\}) = 1$, it can be deduced that $0 \leq \sum_i (|m_1(\{\theta_i\}) - m_2(\{\theta_i\})|) \leq 2$, thus leading to $d_H^2(m_1, m_2) \leq 1$. Also, the numerator and denominator in the Eq.(11) are non-negative numbers, so we get that $0 \leq d_H(m_1, m_2) \leq 1$.

Proof.(2)

$$d_H(m_1, m_2) = \frac{\|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}\|_2}{\sqrt{2}}$$

$$= \frac{\|\sqrt{m_2(\{\theta_i\})} - \sqrt{m_1(\{\theta_i\})}\|_2}{\sqrt{2}} = d_H(m_2, m_1)$$

Proof.(3)

$$d_H(m_1, m_2) = d_H(m_1, m_1)$$

$$= \frac{\|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_1(\{\theta_i\})}\|_2}{\sqrt{2}} = 0$$

and,

$$d_H(m_1, m_2) = 0 \Rightarrow \frac{\|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}\|_2}{\sqrt{2}} = 0$$

$$\Rightarrow \|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}\|_2 = 0$$

$$\Rightarrow m_1 = m_2$$

Proof.(4)

$$d_H(m_1, m_2) + d_H(m_2, m_3)$$

$$= [\frac{1}{2} \sum_i |\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}|^2]^{\frac{1}{2}}$$

$$+ [\frac{1}{2} \sum_i |\sqrt{m_2(\{\theta_i\})} - \sqrt{m_3(\{\theta_i\})}|^2]^{\frac{1}{2}}$$

Using Minkowski inequality,

$$[\sum_{i=1}^n (a_i + b_i)^P]^{\frac{1}{P}} \leq [\sum_{i=1}^n (a_i)^P]^{\frac{1}{P}} + [\sum_{i=1}^n (b_i)^P]^{\frac{1}{P}}$$

Where $P \geq 1, a_i > 0, b_i > 0$.

We get,

$$d_H(m_1, m_2) + d_H(m_2, m_3)$$

$$= [\frac{1}{2} \sum_i |\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}|^2]^{\frac{1}{2}}$$

$$+ [\frac{1}{2} \sum_i |\sqrt{m_2(\{\theta_i\})} - \sqrt{m_3(\{\theta_i\})}|^2]^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}} \{[\sum_i |\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}|^2]^{\frac{1}{2}}$$

$$+ [\sum_i |\sqrt{m_2(\{\theta_i\})} - \sqrt{m_3(\{\theta_i\})}|^2]^{\frac{1}{2}}\}$$

$$\geq \frac{1}{\sqrt{2}} [\sum_i (|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}|$$

$$+ |\sqrt{m_2(\{\theta_i\})} - \sqrt{m_3(\{\theta_i\})}|)^2]^{\frac{1}{2}}$$

Moreover, since

$$|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_2(\{\theta_i\})}| + |\sqrt{m_2(\{\theta_i\})} - \sqrt{m_3(\{\theta_i\})}| \geq$$

$$|\sqrt{m_1(\{\theta_i\})} - \sqrt{m_3(\{\theta_i\})}|$$

Then we obtain that

$$d_H(m_1, m_2) + d_H(m_2, m_3) \geq \frac{1}{\sqrt{2}} [\sum_i |\sqrt{m_1(\{\theta_i\})} -$$

$$\sqrt{m_3(\{\theta_i\})}|^2]^{\frac{1}{2}} = d_H(m_1, m_3)$$

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