Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2010, Article ID 246287, 15 pages
doi:10.1155/2010/246287

Research Article

Weighted Differentiation Composition Operators from the Mixed-Norm Space to the *n*th Weigthed-Type Space on the Unit Disk

Stevo Stević

Mathematical Institute of the Serbian Academy of Sciences and Arts, Knez Mihailova 36/III, 11000 Belgrade, Serbia

Correspondence should be addressed to Stevo Stević, sstevic@ptt.rs

Received 26 March 2010; Accepted 9 May 2010

Academic Editor: Narcisa C. Apreutesei

The boundedness and compactness of the weighted differentiation composition operator from the mixed-norm space to the *n*th weighted-type space on the unit disk are characterized.

1. Introduction

Throughout this paper $\mathbb{D}$ will denote the open unit disk in the complex plane $\mathbb{C}$, $H(\mathbb{D})$ the class of all holomorphic functions on $\mathbb{D}$, and $H^\infty = H^\infty(\mathbb{D})$ the space of all bounded holomorphic functions on $\mathbb{D}$ with the norm $\|f\|_\infty = \sup_{z\in\mathbb{D}}|f(z)|$.

The mixed norm space $H_{p,q,\gamma} = H_{p,q,\gamma}(\mathbb{D})$, $0 < p,q < \infty$, $-1 < \gamma < \infty$, consists of all $f \in H(\mathbb{D})$ such that

$$\|f\|^q_{H_{p,q,\gamma}} = \int_0^1 M_p^q(f,r)(1-r)^\gamma dr < \infty, \tag{1.1}$$

where

$$M_p(f,r) = \left(\frac{1}{2\pi}\int_0^{2\pi} \left|f\left(re^{i\theta}\right)\right|^p d\theta\right)^{1/p}. \tag{1.2}$$

A positive continuous function on $\mathbb{D}$ is called *weight*. Let $\mu(z)$ be a weight and $n \in \mathbb{N}_0$. The nth *weighted-type space* on $\mathbb{D}$, denoted by $\mathcal{W}_\mu^{(n)}(\mathbb{D})$, consists of all $f \in H(\mathbb{D})$ such that

$$b_{\mathcal{W}_\mu^{(n)}(\mathbb{D})}(f) := \sup_{z\in\mathbb{D}} \mu(z)\left|f^{(n)}(z)\right| < \infty. \tag{1.3}$$

The space was recently introduced by this author in [1] as an extension of several weighted-type spaces which attracted a lot of attention in last few decades. For instance, when $n = 0$, the space becomes the weighted-type space $H_\mu^\infty(\mathbb{D})$ (see, e.g., [2–4]), when $n = 1$, the Bloch-type space $\mathcal{B}_\mu(\mathbb{D})$ (see, e.g., [5–7]), and for $n = 2$, the Zygmund-type space $\mathcal{Z}_\mu(\mathbb{D})$. Some information on Zygmund-type spaces on $\mathbb{D}$ and some operators on them can be found, for example, in [8–10] and on the unit ball, for example, in [11, 12].

The quantity $b_{\mathcal{W}_\mu^{(n)}(\mathbb{D})}(f)$ is a seminorm on the nth weighted-type space $\mathcal{W}_\mu^{(n)}(\mathbb{D})$ and a norm on $\mathcal{W}_\mu^{(n)}(\mathbb{D})/\mathbb{P}_{n-1}$, where $\mathbb{P}_{n-1}$ is the set of all polynomials whose degrees are less than or equal to $n-1$. A natural norm on the nth weighted-type space is introduced as follows:

$$\|f\|_{\mathcal{W}_\mu^{(n)}(\mathbb{D})} = \sum_{j=0}^{n-1}\left|f^{(j)}(0)\right| + b_{\mathcal{W}_\mu^{(n)}(\mathbb{D})}(f). \tag{1.4}$$

With this norm the nth weighted-type space becomes a Banach space.

The little nth weighted-type space, denoted by $\mathcal{W}_{\mu,0}^{(n)}(\mathbb{D})$, is a closed subspace of $\mathcal{W}_\mu^{(n)}(\mathbb{D})$ consisting of those f for which

$$\lim_{|z|\to 1} \mu(z)\left|f^{(n)}(z)\right| = 0. \tag{1.5}$$

An analytic self-map $\varphi : \mathbb{D} \to \mathbb{D}$ induces the composition operator C_φ on $H(\mathbb{D})$, defined by $C_\varphi(f)(z) = f(\varphi(z))$ for $f \in H(\mathbb{D})$ (see, e.g., [8, 13–16]).

Let φ be an analytic self-map of $\mathbb{D}$, $u \in H(\mathbb{D})$, and $m \in \mathbb{N}$. Then the weighted differentiation composition operator, denoted by $D_{\varphi,u}^m$, is defined on $H(\mathbb{D})$ by

$$D_{\varphi,u}^m f(z) = u(z) f^{(m)}(\varphi(z)), \quad f \in H(\mathbb{D}). \tag{1.6}$$

Recently there has been some interest in studying some particular cases of operator $D_{\varphi,u}^m$ (see, e.g., [17–25]). For some other products of linear operators on spaces of holomorphic functions see also recent papers [11, 26–32].

Here we study the boundedness and compactness of the operator $D_{\varphi,u}^m$ from $H_{p,q,\gamma}$ to nth weighted-type spaces, where $n \in \mathbb{N}$.

Throughout this paper, constants are denoted by C; they are positive and may differ from one occurrence to the other. The notation $A \asymp B$ means that there is a positive constant C such that $B/C \le A \le CB$.

2. Auxiliary Results

Here we quote some auxiliary results which will be used in the proofs of the main results. The first lemma can be proved in a standard way (see, e.g., in [13, Proposition 3.11] or in [15, Lemma 3]).

Lemma 2.1. *Assume that $m \in \mathbb{N}_0$, $n \in \mathbb{N}$, $p, q > 0$, $\gamma > -1$, φ is an analytic self-map of $\mathbb{D}$ and $u \in H(\mathbb{D})$. Then the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is compact if and only if $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $H_{p,q,\gamma}$ which converges to zero uniformly on compact subsets of $\mathbb{D}$, $D^m_{\varphi,u} f_k \to 0$ in $\mathcal{W}^{(n)}_\mu$ as $k \to \infty$.*

The next lemma is known, but we give a proof of it for the benefit of the reader.

Lemma 2.2. *Assume that $n \in \mathbb{N}_0$, $0 < p, q < \infty$, $-1 < \gamma < \infty$ and $f \in H_{p,q,\gamma}$. Then there is a positive constant C independent of f such that*

$$\left|f^{(n)}(z)\right| \le C \frac{\|f\|_{H_{p,q,\gamma}}}{\left(1-|z|^2\right)^{(\gamma+1)/q+1/p+n}}. \tag{2.1}$$

Proof. By the monotonicity of the integral means, using the well-known asymptotic formula

$$\int_0^1 M_p^q(f,r)(1-r)^\gamma dr \asymp |f(0)|^q + \int_0^1 M_p^q\left(f^{(n)},r\right)(1-r)^{\gamma+nq} dr, \tag{2.2}$$

and Theorem 7.2.5 in [33], we have that

$$\begin{aligned} \|f\|^q_{H_{p,q,\gamma}} &\ge \int_{(1+|z|)/2}^1 M_p^q\left(f^{(n)},r\right)(1-r)^{\gamma+nq} dr \\ &\ge C M_p^q\left(f^{(n)}, \frac{1+|z|}{2}\right)\left(1-|z|^2\right)^{\gamma+1+nq} \\ &\ge C\left(1-|z|^2\right)^{\gamma+1+nq+q/p}\left|f^{(n)}(z)\right|^q, \end{aligned} \tag{2.3}$$

from which the result follows. □

The following lemma can be found in [34].

Lemma 2.3. *For $\beta > -1$ and $m > 1 + \beta$ one has*

$$\int_0^1 \frac{(1-r)^\beta}{(1-\rho r)^m} dr \le C(1-\rho)^{1+\beta-m}, \quad 0 < \rho < 1. \tag{2.4}$$

A proof of the next lemma can be found in [35, Lemma 2.3].

Lemma 2.4. *Assume $a > 0$ and*

$$D_n(a) = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ a & a+1 & \cdots & a+n-1 \\ a(a+1) & (a+1)(a+2) & \cdots & (a+n-1)(a+n) \\ & & \cdots & \\ \prod_{j=0}^{n-2}(a+j) & \prod_{j=0}^{n-2}(a+j+1) & \cdots & \prod_{j=0}^{n-2}(a+j+n-1) \end{vmatrix}. \tag{2.5}$$

Then $D_n(a) = \prod_{j=1}^{n-1} j!$.

The following formula

$$(f \circ \varphi)^{(n)}(z) = \sum_{k=1}^{n} f^{(k)}(\varphi(z)) \sum_{k_1,\ldots,k_n} \frac{n!}{k_1! \cdots k_n!} \prod_{j=1}^{n} \left(\frac{\varphi^{(j)}(z)}{j!} \right)^{k_j}, \tag{2.6}$$

where the second sum is over all nonnegative integers $k_1, k_2, \ldots, k_n$ satisfying $k = k_1 + k_2 + \cdots + k_n$ and $k_1 + 2k_2 + \cdots + nk_n = n$, is attributed to Faà di Bruno [36]. By using Bell polynomials $B_{n,k}(x_1, \ldots, x_{n-k+1})$ it can be written as follows:

$$(f \circ \varphi)^{(n)}(z) = \sum_{k=0}^{n} f^{(k)}(\varphi(z)) B_{n,k}\left(\varphi'(z), \varphi''(z), \ldots, \varphi^{(n-k+1)}(z)\right). \tag{2.7}$$

For $n \in \mathbb{N}$ the last sum can go from $k = 1$ since $B_{n,0}(\varphi'(z), \varphi''(z), \ldots, \varphi^{(n+1)}(z)) = 0$; however we will keep the summation since for $n = 0$ the only existing term $B_{0,0}$ is equal to 1 and we will use it.

The Leibnitz formula along with (2.6) yields

$$\left(u(z) g(\varphi(z))\right)^{(n)} = \sum_{l=0}^{n} C_l^n u^{(n-l)}(z) \sum_{k=0}^{l} g^{(k)}(\varphi(z)) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right). \tag{2.8}$$

Hence we have the next result.

Lemma 2.5. *Assume that $g, u \in H(\mathbb{D})$ and φ is an analytic self-map of $\mathbb{D}$. Then*

$$\left(u(z) g(\varphi(z))\right)^{(n)} = \sum_{k=0}^{n} g^{(k)}(\varphi(z)) \sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right). \tag{2.9}$$

3. The Boundedness and Compactness of $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$

This section characterizes the boundedness and compactness of the operator $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$.

Theorem 3.1. *Suppose that $m,n \in \mathbb{N}$, $0 < p,q < \infty$, $-1 < \gamma < \infty$, φ is an analytic self-map of the unit disk, $u \in H(\mathbb{D})$, and μ is a weight. Then the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded if and only if for each $k \in \{0,1,\dots,n\}$*

$$I_k := \sup_{z\in\mathbb{D}} \frac{\mu(z)\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+m+k}} < \infty. \tag{3.1}$$

Moreover if $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded, then the following asymptotic relation holds

$$\left\|D^m_{\varphi,u}\right\|_{H_{p,q,\gamma}\to\mathcal{W}^{(n)}_\mu/\mathbb{P}_{n-1}} \asymp \sum_{k=0}^n I_k. \tag{3.2}$$

Proof. First assume that $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded; then there exists a constant C such that

$$\left\|D^m_{\varphi,u} f\right\|_{\mathcal{W}^{(n)}_\mu} \le C\|f\|_{H_{p,q,\gamma}} \tag{3.3}$$

for all $f \in H_{p,q,\gamma}$.

For a fixed $w \in \mathbb{D}$, $t \ge (\gamma+1)/q$, and constants $c_1,\dots,c_{n+1}$, set

$$g_w(z) = \sum_{j=1}^{n+1} \frac{c_j}{\prod_{l=0}^{m-1}(j+t+1/p+l)} \widehat{g}_{w,j}(z), \tag{3.4}$$

where

$$\widehat{g}_{w,j}(z) = \frac{\left(1-|w|^2\right)^{j+t-(\gamma+1)/q}}{(1-\overline{w}z)^{1/p+j+t}}, \quad j=1,\dots,n+1. \tag{3.5}$$

By [33, Theorem 1.4.10], we get

$$M_p\left(\widehat{g}_{w,j}, r\right) \le C\frac{\left(1-|w|^2\right)^{j+t-(\gamma+1)/q}}{(1-r|w|)^{j+t}}, \quad j=1,\dots,n+1. \tag{3.6}$$

Applying Lemma 2.3, we have that

$$\begin{aligned}\|\widehat{g}_{w,j}\|^q_{H_{p,q,\gamma}} &= \int_0^1 M_p^q(\widehat{g}_{w,j},r)(1-r)^\gamma dr\\ &\leq C\int_0^1 \frac{\left(1-|w|^2\right)^{q(j+t)-(\gamma+1)}}{(1-r|w|)^{q(j+t)}}(1-r)^\gamma dr\\ &\leq C.\end{aligned}\tag{3.7}$$

Therefore $g_w \in H_{p,q,\gamma}$, and moreover $\sup_{w\in\mathbb{D}}\|g_w\|_{H_{p,q,\gamma}} < \infty$.

Now we show that for each $s \in \{m, m+1, \ldots, m+n\}$, there are constants $c_1, c_2, \ldots, c_{n+1}$, such that

$$g_w^{(s)}(w) = \frac{\overline{w}^s}{\left(1-|w|^2\right)^{s+(\gamma+1)/q+1/p}}, \quad g_w^{(t)}(w) = 0, \qquad t \in \{m, \ldots, m+n\} \setminus \{s\}.\tag{3.8}$$

By differentiating function g_w, for each $s \in \{m, \ldots, m+n\}$, (3.8) becomes

$$\begin{gathered}c_1 + c_2 + \cdots + c_{n+1} = 0,\\ \left(t+p^{-1}+m+1\right)c_1 + \left(t+p^{-1}+m+2\right)c_2 + \cdots + \left(t+p^{-1}+m+n+1\right)c_{n+1} = 0,\\ \vdots\\ \prod_{j=1}^{s-m}\left(t+p^{-1}+m+j\right)c_1 + \cdots + \prod_{j=1}^{s-m}\left(t+p^{-1}+m+n+j\right)c_{n+1} = 1,\\ \vdots\\ \prod_{j=1}^{n}\left(t+p^{-1}+m+j\right)c_1 + \cdots + \prod_{j=1}^{n}\left(t+p^{-1}+m+n+j\right)c_{n+1} = 0.\end{gathered}\tag{3.9}$$

Applying Lemma 2.4 with $a = t + 1/p + m + 1 > 0$ and where $n \to n+1$, we see that the determinant of system (3.9) is different from zero, as claimed.

By $g_{w,k}$, $k \in \{0, 1, \ldots, n\}$, denote the corresponding family of functions which satisfy (3.8) with $s = m + k$. Then, for each fixed $k \in \{0, 1, \ldots, n\}$, inequality (3.3) along with (2.9) and (3.8) implies that for each $\varphi(w) \neq 0$

$$\begin{aligned}&\frac{\mu(w)\left|\varphi(w)\right|^{k+m}\left|\sum_{l=k}^n C_l^n u^{(n-l)}(w) B_{l,k}\left(\varphi'(w), \ldots, \varphi^{(l-k+1)}(w)\right)\right|}{\left(1-\left|\varphi(w)\right|^2\right)^{(\gamma+1)/q+1/p+k+m}}\\ &\quad\leq C\sup_{w\in\mathbb{D}}\left\|D_{\varphi,u}^m\left(g_{\varphi(w),k}\right)\right\|_{\mathcal{W}_\mu^{(n)}} \leq C\left\|D_{\varphi,u}^m\right\|_{H_{p,q,\gamma}\to\mathcal{W}_\mu^{(n)}}.\end{aligned}\tag{3.10}$$

From (3.10) it follows that for each $k \in \{0, 1, \ldots, n\}$,

$$\sup_{|\varphi(z)|>1/2} \frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} \leq C\|D_{\varphi,u}^m\|_{H_{p,q,\gamma}\to\mathcal{W}_\mu^{(n)}}. \tag{3.11}$$

Let

$$h_k(z) = z^k, \quad k = m, \ldots, n+m. \tag{3.12}$$

Then clearly

$$\|h_k\|_{H_{p,q,\gamma}} \leq 1, \quad \text{for each } k \in \mathbb{N}. \tag{3.13}$$

By formula (2.9) applied to the function $f(z) = h_m(z)$ we get

$$\begin{aligned}\left(D_{\varphi,u}^m h_m\right)^{(n)}(z) &= h_m^{(m)}(\varphi(z)) \sum_{l=0}^{n} C_l^n u^{(n-l)}(z) B_{l,0}\left(\varphi'(z), \ldots, \varphi^{(l+1)}(z)\right) \\ &= m! \sum_{l=0}^{n} C_l^n u^{(n-l)}(z) B_{l,0}\left(\varphi'(z), \ldots, \varphi^{(l+1)}(z)\right),\end{aligned} \tag{3.14}$$

which along with the boundedness of the operator $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ and (3.13) implies that

$$m!\sup_{z\in\mathbb{D}} \mu(z)\left|\sum_{l=0}^{n} C_l^n u^{(n-l)}(z) B_{l,0}\left(\varphi'(z), \ldots, \varphi^{(l+1)}(z)\right)\right| \leq \left\|D_{\varphi,u}^m(z^m)\right\|_{\mathcal{W}_\mu^{(n)}} \leq \left\|D_{\varphi,u}^m\right\|_{H_{p,q,\gamma}\to\mathcal{W}_\mu^{(n)}}. \tag{3.15}$$

Now assume that we have proved that for $j \in \{0, 1, \ldots, k-1\}$ and a $k \leq n$

$$\sup_{z\in\mathbb{D}} \mu(z)\left|\sum_{l=j}^{n} C_l^n u^{(n-l)}(z) B_{l,j}\left(\varphi'(z), \ldots, \varphi^{(l-j+1)}(z)\right)\right| \leq C\left\|D_{\varphi,u}^m\right\|_{H_{p,q,\gamma}\to\mathcal{W}_\mu^{(n)}}. \tag{3.16}$$

Applying (2.9) to the function $f(z) = h_{m+k}(z)$, $k \in \{0, 1, \ldots, n\}$, and noticing that $h_{m+k}^{(s)}(z) \equiv 0$ for $s > m+k$, we get

$$\begin{aligned}\left(D_{\varphi,u}^m h_{m+k}\right)^{(n)}(z) &= \sum_{j=0}^{k} h_{m+k}^{(m+j)}(\varphi(z)) \sum_{l=j}^{n} C_l^n u^{(n-l)}(z) B_{l,j}\left(\varphi'(z), \ldots, \varphi^{(l-j+1)}(z)\right) \\ &= \sum_{j=0}^{k} (m+k)\cdots(k-j+1)(\varphi(z))^{k-j} \sum_{l=j}^{n} C_l^n u^{(n-l)}(z) B_{l,j}\left(\varphi'(z), \ldots, \varphi^{(l-j+1)}(z)\right).\end{aligned} \tag{3.17}$$

From (3.17), the boundedness of the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$, the fact that $\|\varphi\|_\infty \le 1$, the triangle inequality, noticing that $(m+k)!$ is the coefficient at $\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}(\varphi'(z),\dots,\varphi^{(l-k+1)}(z))$, and finally using hypothesis (3.16) we get

$$\sup_{z\in\mathbb{B}} \mu(z)\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \le C\left\|D^m_{\varphi,u}\right\|_{H_{p,q,\gamma}\to\mathcal{W}^{(n)}_\mu}. \tag{3.18}$$

Hence by induction, (3.18) holds for each $k \in \{0,1,\dots,n\}$.

From (3.18), for each fixed $k \in \{0,1,\dots,n\}$

$$\begin{aligned}\sup_{|\varphi(z)|\le 1/2} &\frac{\mu(z)\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}(\varphi'(z),\dots,\varphi^{(l-k+1)}(z))\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} \\ &\le C\sup_{z\in\mathbb{B}} \mu(z)\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \le C\left\|D^m_{\varphi,u}\right\|_{H_{p,q,\gamma}\to\mathcal{W}^{(n)}_\mu}.\end{aligned} \tag{3.19}$$

Inequalities (3.11) and (3.19) imply

$$\sum_{k=0}^n I_k \le C\left\|D^m_{\varphi,u}\right\|_{H_{p,q,\gamma}\to\mathcal{W}^{(n)}_\mu}. \tag{3.20}$$

Now assume that (3.1) holds. Then for any $f \in H_{p,q,\gamma}$, by (2.9) and Lemma 2.2 we have

$$\begin{aligned}\mu(z)\left|\left(D^m_{\varphi,u}f\right)^{(n)}(z)\right| &= \mu(z)\left|\sum_{k=0}^n f^{(m+k)}(\varphi(z))\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \\ &\le \mu(z)\sum_{k=0}^n \left|f^{(m+k)}(\varphi(z))\right|\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right|\end{aligned} \tag{3.21}$$

$$\le C\|f\|_{H_{p,q,\gamma}}\sum_{k=0}^n \frac{\mu(z)\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}(\varphi'(z),\dots,\varphi^{(l-k+1)}(z))\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} \tag{3.22}$$

$$\le C\|f\|_{H_{p,q,\gamma}}\sum_{k=0}^n \sup_{z\in\mathbb{D}} \frac{\mu(z)\left|\sum_{l=k}^n C_l^n u^{(n-l)}(z) B_{l,k}(\varphi'(z),\dots,\varphi^{(l-k+1)}(z))\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} \tag{3.23}$$

We also have that for each $s \in \{1, \ldots, n-1\}$

$$\left|\left(D^m_{\varphi,u} f\right)^{(s)}(0)\right| = \left|\sum_{k=0}^{s} f^{(m+k)}(\varphi(0)) \sum_{l=k}^{s} C^s_l u^{(s-l)}(0) B_{l,k}\left(\varphi'(0), \ldots, \varphi^{(l-k+1)}(0)\right)\right|$$
$$\leq C\|f\|_{H_{p,q,\gamma}} \sum_{k=0}^{s} \frac{\left|\sum_{l=k}^{s} C^s_l u^{(s-l)}(0) B_{l,k}(\varphi'(0), \ldots, \varphi^{(l-k+1)}(0))\right|}{\left(1-|\varphi(0)|^2\right)^{(\gamma+1)/q+1/p+m+k}}, \tag{3.24}$$
$$\left|\left(D^m_{\varphi,u} f\right)(0)\right| = |u(0)|\left|f^{(m)}(\varphi(0))\right| \leq C|u(0)| \frac{\|f\|_{H_{p,q,\gamma}}}{\left(1-|\varphi(0)|^2\right)^{(\gamma+1)/q+1/p+m}}.$$

Using (3.23), (3.24), and (3.1) it follows that the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded.

From (3.23) and (3.20) the asymptotic relation (3.2) follows. □

Theorem 3.2. *Suppose that $m, n \in \mathbb{N}$, $0 < p, q < \infty$, $-1 < \gamma < \infty$, φ is an analytic self-map of the unit disk, $u \in H(\mathbb{D})$, and μ is a weight. Then the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu,0}$ is bounded if and only if $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded and for each $k \in \{0, 1, \ldots, n\}$*

$$\lim_{|z|\to 1} \mu(z)\left|\sum_{l=k}^{n} C^n_l u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right| = 0. \tag{3.25}$$

Proof. The boundedness of $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu,0}$ clearly implies that $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_\mu$ is bounded. Applying (2.9) to the function $f(z) = h_m(z)$ and using the assumption $D^m_{\varphi,u}(h_m) \in \mathcal{W}^{(n)}_{\mu,0}$ it follows that

$$\mu(z)\left|\left(D^m_{\varphi,u} h_m\right)^{(n)}(z)\right| = m!\mu(z)\left|\sum_{l=0}^{n} C^n_l u^{(n-l)}(z) B_{l,0}\left(\varphi'(z), \ldots, \varphi^{(l+1)}(z)\right)\right| \longrightarrow 0, \tag{3.26}$$

as $|z| \to 1$, which is (3.25) for $k = 0$.

Assume that we have proved the following inequalities:

$$\lim_{|z|\to 1} \mu(z)\left|\sum_{l=j}^{n} C^n_l u^{(n-l)}(z) B_{l,j}\left(\varphi'(z), \ldots, \varphi^{(l-j+1)}(z)\right)\right| = 0, \tag{3.27}$$

for $j \in \{0, 1, \ldots, k-1\}$ and a $k \leq n$.

Applying formula (2.9) to the function $f(z) = h_{m+k}(z), k \in \{0, 1, \ldots, n\}$, we get (3.17). From (3.17), by using the boundedness of function φ, the triangle inequality, noticing that the coefficient at $\sum_{l=k}^{n} C^n_l u^{(n-l)}(z) B_{l,k}(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z))$ is independent of z, and finally using hypothesis (3.27), we easily obtain

$$\lim_{|z|\to 1}\mu(z)\left|\sum_{l=k}^{n}C_l^n u^{(n-l)}(z)B_{l,k}\left(\varphi'(z),\ldots,\varphi^{(l-k+1)}(z)\right)\right| = 0. \tag{3.28}$$

Hence by induction we get that (3.25) holds for each $k \in \{0,1,\ldots,n\}$.

Now assume that $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is bounded and (3.25) holds for each $k \in \{0,1,\ldots,n\}$. For each polynomial p we have

$$\begin{aligned}\mu(z)\left|\left(D_{\varphi,u}^m p\right)^{(n)}(z)\right| &= \mu(z)\left|\sum_{k=0}^{n}p^{(k)}(\varphi(z))\sum_{l=k}^{n}C_l^n u^{(n-l)}(z)B_{l,k}\left(\varphi'(z),\ldots,\varphi^{(l-k+1)}(z)\right)\right| \\ &\le \sum_{k=0}^{n}\left\|p^{(k)}\right\|_\infty \mu(z)\left|\sum_{l=k}^{n}C_l^n u^{(n-l)}(z)B_{l,k}\left(\varphi'(z),\ldots,\varphi^{(l-k+1)}(z)\right)\right| \longrightarrow 0,\end{aligned} \tag{3.29}$$

as $|z| \to 1$.

From (3.29) we have that, for each polynomial p, $D_{\varphi,u}^m p \in \mathcal{W}_{\mu,0}^{(n)}$. The set of all polynomials is dense in $H_{p,q,\gamma}$, so we have that for each $f \in H_{p,q,\gamma}$, there is a sequence of polynomials $(p_k)_{k\in\mathbb{N}}$ such that $\|f-p_k\|_{H_{p,q,\gamma}} \to 0$ as $k \to \infty$. Thus the boundedness of $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ implies

$$\left\|D_{\varphi,u}^m f - D_{\varphi,u}^m p_k\right\|_{\mathcal{W}_\mu^{(n)}} \le \left\|D_{\varphi,u}^m\right\|_{H_{p,q,\gamma}\to\mathcal{W}_\mu^{(n)}}\|f-p_k\|_{H_{p,q,\gamma}} \longrightarrow 0, \quad \text{as } k \longrightarrow \infty. \tag{3.30}$$

Hence $D_{\varphi,u}^m(H_{p,q,\gamma}) \subseteq \mathcal{W}_{\mu,0}^{(n)}$, from which the boundedness of $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_{\mu,0}^{(n)}$ follows, completing the proof of the theorem. □

Theorem 3.3. *Suppose that $m,n \in \mathbb{N}$, $0 < p,q < \infty$, $-1 < \gamma < \infty$, φ is an analytic self-map of the unit disk, $u \in H(\mathbb{D})$, and μ is a weight. Then the operator $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is compact if and only if $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is bounded and for each $k \in \{0,1,\ldots,n\}$*

$$\lim_{|\varphi(z)|\to 1}\frac{\mu(z)\left|\sum_{l=k}^{n}C_l^n u^{(n-l)}(z)B_{l,k}\left(\varphi'(z),\ldots,\varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} = 0. \tag{3.31}$$

Proof. First assume that $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is bounded and (3.31) holds. By Theorem 3.1 we have that for each $k \in \{0,1,\ldots,n\}$, (3.1) holds.

Let $(f_i)_{i\in\mathbb{N}}$ be a sequence in $H_{p,q,\gamma}$ such that $\sup_{i\in\mathbb{N}}\|f_i\|_{H_{p,q,\gamma}} \le L$ and f_i converges to 0 uniformly on compact subsets of $\mathbb{D}$ as $i \to \infty$. By the assumption, for any $\varepsilon > 0$, there is a $\delta \in (0,1)$, such that for each $k \in \{0,1,\dots,n\}$ and $\delta < |\varphi(z)| < 1$

$$\frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} < \varepsilon. \tag{3.32}$$

We have

$$\begin{aligned}
&\left\|D_{\varphi,u}^m f_i\right\|_{\mathcal{W}_\mu^{(n)}} \\
&\quad= \sup_{z\in\mathbb{D}}\mu(z)\left|\left(D_{\varphi,u}^m f_i\right)^{(n)}(z)\right| + \sum_{j=0}^{n-1}\left|\left(D_{\varphi,u}^m f_i\right)^{(j)}(0)\right| \\
&\quad= \sup_{z\in\mathbb{D}}\mu(z)\left|\sum_{k=0}^{n} f_i^{(m+k)}(\varphi(z))\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \\
&\qquad+ \sum_{j=0}^{n-1}\left|\sum_{k=0}^{j} f_i^{(m+k)}(\varphi(0))\sum_{l=k}^{j} C_l^j u^{(j-l)}(0) B_{l,k}\left(\varphi'(0),\dots,\varphi^{(l-k+1)}(0)\right)\right| \\
&\quad\le \left(\sup_{|\varphi(z)|\le\delta} + \sup_{|\varphi(z)|>\delta}\right)\mu(z)\sum_{k=0}^{n}\left|f_i^{(m+k)}(\varphi(z))\right|\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \\
&\qquad+ \sum_{j=0}^{n-1}\left|\sum_{k=0}^{j} f_i^{(m+k)}(\varphi(0))\sum_{l=k}^{j} C_l^j u^{(j-l)}(0) B_{l,k}\left(\varphi'(0),\dots,\varphi^{(l-k+1)}(0)\right)\right| = J_1 + J_2 + J_3.
\end{aligned} \tag{3.33}$$

Now we estimate J_1, J_2, and J_3:

$$\begin{aligned}
J_1 &= \sup_{|\varphi(z)|\le\delta}\mu(z)\sum_{k=0}^{n}\left|f_i^{(m+k)}(\varphi(z))\right|\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \\
&\le \sum_{k=0}^{n}\sup_{|w|\le\delta}\left|f_i^{(m+k)}(w)\right|\sup_{|\varphi(z)|\le\delta}\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right| \\
&\le \sum_{k=0}^{n}\sup_{|w|\le\delta}\left|f_i^{(m+k)}(w)\right|\sup_{z\in\mathbb{D}}\frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z),\dots,\varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+m+k}} \\
&= \sum_{k=0}^{n}\sup_{|w|\le\delta}\left|f_i^{(m+k)}(w)\right| I_k \longrightarrow 0, \quad \text{as } i \longrightarrow \infty,
\end{aligned} \tag{3.34}$$

where in (3.34) we have used the fact that from $f_i \to 0$ uniformly on compact subsets of $\mathbb{D}$ as $i \to \infty$ it follows that for each $s \in \mathbb{N}$, $f_i^{(s)} \to 0$ uniformly on compact subsets of $\mathbb{D}$ as $i \to \infty$.

The fact that

$$J_3 = \sum_{j=0}^{n-1} \left| \sum_{k=0}^{j} f_i^{(m+k)}(\varphi(0)) \sum_{l=k}^{j} C_l^j u^{(j-l)}(0) B_{l,k}\left(\varphi'(0), \ldots, \varphi^{(l-k+1)}(0)\right) \right| \longrightarrow 0, \tag{3.35}$$

as $i \to \infty$, is proved similarly; so we omit it.

By Lemma 2.2 and (3.32) we have that

$$J_2 \leq C \|f_i\|_{H_{p,q,\gamma}} \sum_{k=0}^{n} \sup_{|\varphi(z)|>\delta} \frac{\mu(z) \left| \sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)) \right|}{\left(1 - |\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} < C\varepsilon(n+1)L. \tag{3.36}$$

From (3.34), (3.35), and (3.36) we obtain

$$\lim_{i \to \infty} \left\| D_{\varphi,u}^m f_i \right\|_{\mathcal{W}_\mu^{(n)}} = 0. \tag{3.37}$$

From this and applying Lemma 2.1 the implication follows.

Now assume that $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is compact; then clearly $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is bounded. Let $(z_i)_{i \in \mathbb{N}}$ be a sequence in $\mathbb{D}$ such that $|\varphi(z_i)| \to 1$ as $i \to \infty$. If such a sequence does not exist, then the conditions in (3.31) automatically hold.

Let $g_{w,k}$, $k \in \{0, 1, \ldots, n\}$ be as in Theorem 3.1. Then the sequences $(g_{\varphi(z_i),k})_{i \in \mathbb{N}}$ are bounded and $g_{\varphi(z_i),k} \to 0$ uniformly on compact subsets of $\mathbb{D}$ as $i \to \infty$. Since $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_\mu^{(n)}$ is compact, we have that for each $k \in \{0, 1, \ldots, n\}$

$$\lim_{i \to \infty} \left\| D_{\varphi,u}^m g_{\varphi(z_i),k} \right\|_{\mathcal{W}_\mu^{(n)}} = 0. \tag{3.38}$$

On the other hand, from (3.10) we obtain

$$\left\| D_{\varphi,u}^m g_{\varphi(z_i),k} \right\|_{\mathcal{W}_\mu^{(n)}} \geq \frac{C\mu(z_i) |\varphi(z_i)|^{k+m} \left| \sum_{l=k}^{n} C_l^n u^{(n-l)}(z_i) B_{l,k}(\varphi'(z_i), \ldots, \varphi^{(l-k+1)}(z_i)) \right|}{\left(1 - |\varphi(z_i)|^2\right)^{(\gamma+1)/q+1/p+k+m}}, \tag{3.39}$$

which along with $|\varphi(z_i)| \to 1$ as $i \to \infty$ and (3.38) implies that

$$\lim_{i \to \infty} \frac{\mu(z_i) \left| \sum_{l=k}^{n} C_l^n u^{(n-l)}(z_i) B_{l,k}(\varphi'(z_i), \ldots, \varphi^{(l-k+1)}(z_i)) \right|}{\left(1 - |\varphi(z_i)|^2\right)^{(\gamma+1)/q+1/p+k+m}}, \tag{3.40}$$

for each $k \in \{0, 1, \ldots, n\}$, from which (3.31) holds in this case. □

4. The Compactness of the Operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu,0}$

The compactness of $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu,0}$ is characterized here. The proof of the next lemma is similar to the proof of the corresponding result in [14].

Lemma 4.1. *Suppose that $n \in \mathbb{N}_0$ and μ is a radial weight such that $\lim_{|z|\to 1}\mu(z) = 0$. A closed set K in $\mathcal{W}^{(n)}_{\mu,0}$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(z)\left|f^{(n)}(z)\right| = 0. \tag{4.1}$$

Theorem 4.2. *Suppose that $m, n \in \mathbb{N}$, $0 < p, q < \infty$, $-1 < \gamma < \infty$, φ is an analytic self-map of the unit disk, $u \in H(\mathbb{D})$ and μ is a radial weight such that $\lim_{|z|\to 1}\mu(z) = 0$. Then the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu,0}$ is compact if and only if for each $k \in \{0, 1, \ldots, n\}$*

$$\lim_{|z|\to 1} \frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1 - |\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} = 0. \tag{4.2}$$

Proof. First assume that $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu,0}$ is compact. Then it is bounded and since the test functions in (3.12) belong to $H_{p,q,\gamma}(\mathbb{D})$, we have that (3.25) holds. Beside this the operator $D^m_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{W}^{(n)}_{\mu}$ is compact too, so that (3.31) holds. Hence, if $\|\varphi\|_\infty < 1$, from (3.25) for each $k \in \{0, 1, \ldots, n\}$ we get

$$\begin{aligned} &\frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1 - |\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} \\ &\quad \le \frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1 - \|\varphi\|_\infty^2\right)^{(\gamma+1)/q+1/p+k+m}} \longrightarrow 0, \end{aligned} \tag{4.3}$$

as $|z| \to 1$, hence we obtain (4.2) in this case.

Now assume $\|\varphi\|_\infty = 1$. Let $(\varphi(z_i))_{i\in\mathbb{N}}$ be a sequence such that $|\varphi(z_i)| \to 1$ as $i \to \infty$. Then from (3.31) we have that for every $\varepsilon > 0$, there is an $r \in (0, 1)$ such that for each $k \in \{0, 1, \ldots, n\}$

$$\frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1 - |\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} < \varepsilon \tag{4.4}$$

when $r < |\varphi(z)| < 1$, and from (3.25) there exists a $\sigma \in (0, 1)$ such that for $\sigma < |z| < 1$

$$\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right| < \varepsilon\left(1 - r^2\right)^{(\gamma+1)/q+1/p+k+m}. \tag{4.5}$$

Therefore, when $\sigma < |z| < 1$ and $r < |\varphi(z)| < 1$, we have that

$$\frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} < \varepsilon. \tag{4.6}$$

On the other hand, if $|\varphi(z)| \leq r$ and $\sigma < |z| < 1$, from (4.5) we obtain

$$\begin{aligned}&\frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{\left(1-|\varphi(z)|^2\right)^{(\gamma+1)/q+1/p+k+m}} \\ &\quad < \frac{\mu(z)\left|\sum_{l=k}^{n} C_l^n u^{(n-l)}(z) B_{l,k}\left(\varphi'(z), \ldots, \varphi^{(l-k+1)}(z)\right)\right|}{(1-r^2)^{(\gamma+1)/q+1/p+k+m}} < \varepsilon.\end{aligned} \tag{4.7}$$

Combining the last two inequalities we obtain (4.2), as desired.

Now assume that (4.2) holds. Taking the supremum in (3.22) over f in the unit ball of $H_{p,q,\gamma}$, then letting $|z| \to 1$ is such obtained inequality and using (4.2) we get

$$\lim_{|z|\to 1} \sup_{\|f\|_{H_{p,q,\gamma}} \leq 1} \mu(z)\left|\left(D_{\varphi,u}^m f\right)^{(n)}(z)\right| = 0. \tag{4.8}$$

Hence by Lemma 4.1 the compactness of the operator $D_{\varphi,u}^m : H_{p,q,\gamma} \to \mathcal{W}_{\mu,0}^{(n)}$ follows. □

References

[1] S. Stević, "Composition operators from the Hardy space to the Zygmund-type space on the upper half-plane," *Abstract and Applied Analysis*, vol. 2009, Article ID 161528, 8 pages, 2009.

[2] K. D. Bierstedt and W. H. Summers, "Biduals of weighted Banach spaces of analytic functions," *Australian Mathematical Society Journal Series A*, vol. 54, no. 1, pp. 70–79, 1993.

[3] S. Stević, "Essential norms of weighted composition operators from the Bergman space to weighted-type spaces on the unit ball," *Ars Combinatoria*, vol. 91, pp. 391–400, 2009.

[4] W. Yang, "Weighted composition operators from Bloch-type spaces to weighted-type spaces," *Ars Combinatoria*, vol. 93, pp. 265–274, 2009.

[5] S. Stević, "Norms of some operators from Bergman spaces to weighted and Bloch-type space," *Utilitas Mathematica*, vol. 76, pp. 59–64, 2008.

[6] S. Stević, "Essential norm of an operator from the weighted Hilbert-Bergman space to the Bloch-type space," *Ars Combinatoria*, vol. 91, pp. 123–127, 2009.

[7] S. Stević, "Norm estimates of weighted composition operators between Bloch-type spaces," *Ars Combinatoria*, vol. 93, pp. 161–164, 2009.

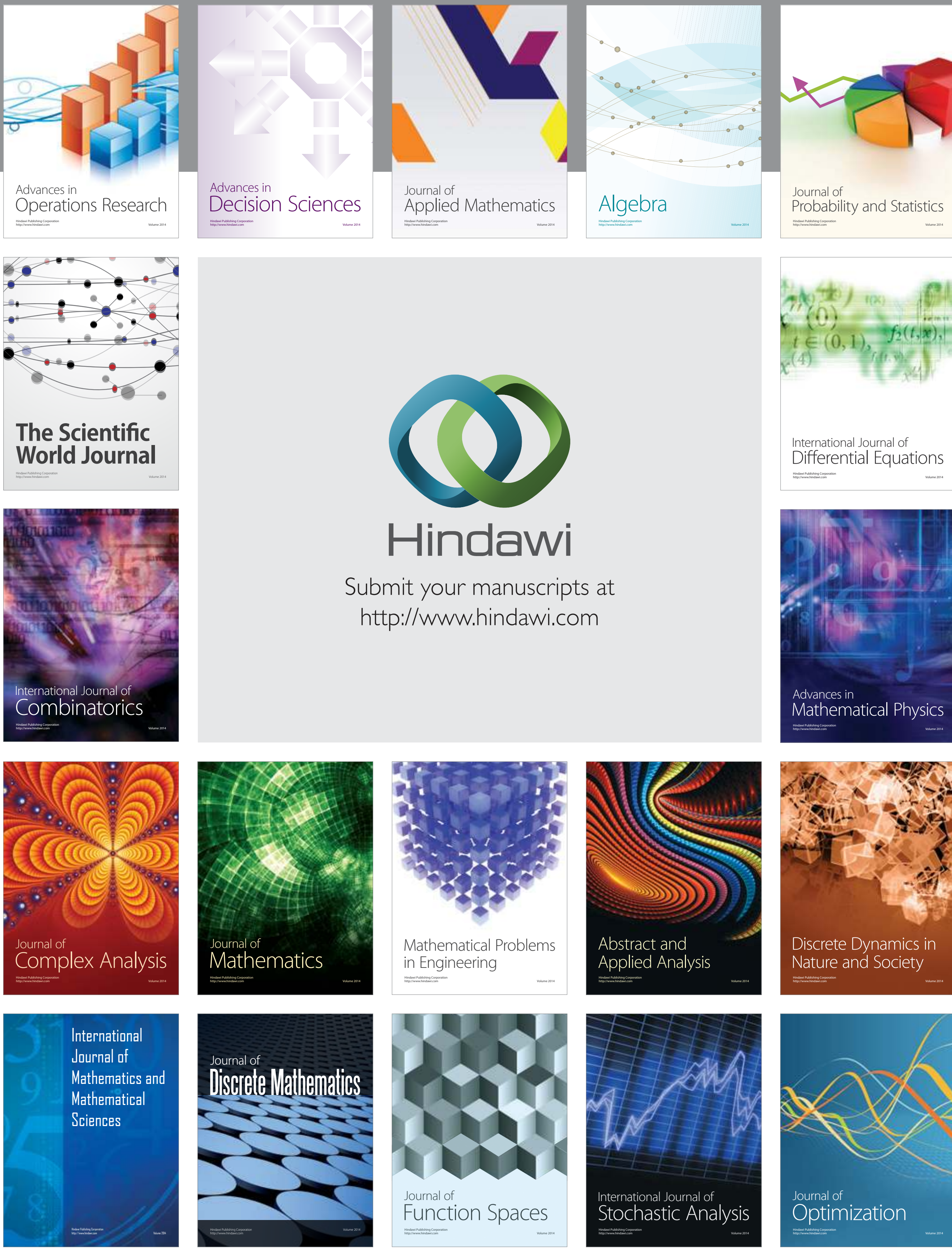
[8] B. R. Choe, H. Koo, and W. Smith, "Composition operators on small spaces," *Integral Equations and Operator Theory*, vol. 56, no. 3, pp. 357–380, 2006.

[9] S. Stević, "Composition operators from the Hardy space to Zygmund-type spaces on the upper half-plane and the unit disk," *Journal of Computational Analysis and Applications*, vol. 12, no. 2, pp. 305–312, 2010.

[10] X. Zhu, "Volterra type operators from logarithmic Bloch spaces to Zygmund type spaces," *International Journal of Modern Mathematics*, vol. 3, no. 3, pp. 327–336, 2008.

[11] S. Stević, "On an integral operator from the Zygmund space to the Bloch-type space on the unit ball," *Glasgow Mathematical Journal*, vol. 51, no. 2, pp. 275–287, 2009.

[12] K. Zhu, *Spaces of Holomorphic Functions in the Unit Ball*, vol. 226 of *Graduate Texts in Mathematics*, Springer, New York, NY, USA, 2005.
[13] C. C. Cowen and B. D. MacCluer, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, Fla, USA, 1995.
[14] K. Madigan and A. Matheson, "Compact composition operators on the Bloch space," *Transactions of the American Mathematical Society*, vol. 347, no. 7, pp. 2679–2687, 1995.
[15] S. Stević, "Composition operators between H^∞ and α-Bloch spaces on the polydisc," *Zeitschrift für Analysis und ihre Anwendungen*, vol. 25, no. 4, pp. 457–466, 2006.
[16] S. Stević, "Norm of weighted composition operators from Bloch space to H_μ^∞ on the unit ball," *Ars Combinatoria*, vol. 88, pp. 125–127, 2008.
[17] R. A. Hibschweiler and N. Portnoy, "Composition followed by differentiation between Bergman and Hardy spaces," *The Rocky Mountain Journal of Mathematics*, vol. 35, no. 3, pp. 843–855, 2005.
[18] S. Li and S. Stević, "Composition followed by differentiation between Bloch type spaces," *Journal of Computational Analysis and Applications*, vol. 9, no. 2, pp. 195–205, 2007.
[19] S. Li and S. Stevich, "Differentiation of a composition as an operator from spaces with mixed norm to Bloch α-spaces," *Rossiĭskaya Akademiya Nauk. Matematicheskiĭ Sbornik*, vol. 199, no. 12, pp. 117–128, 2008.
[20] S. Li and S. Stević, "Composition followed by differentiation between H^∞ and α-Bloch spaces," *Houston Journal of Mathematics*, vol. 35, no. 1, pp. 327–340, 2009.
[21] S. Ohno, "Products of composition and differentiation between Hardy spaces," *Bulletin of the Australian Mathematical Society*, vol. 73, no. 2, pp. 235–243, 2006.
[22] S. Stević, "Norm and essential norm of composition followed by differentiation from α-Bloch spaces to H_μ^∞," *Applied Mathematics and Computation*, vol. 207, no. 1, pp. 225–229, 2009.
[23] S. Stević, "Products of composition and differentiation operators on the weighted Bergman space," *Bulletin of the Belgian Mathematical Society. Simon Stevin*, vol. 16, no. 4, pp. 623–635, 2009.
[24] W. Yang, "Products of composition and differentiation operators from $Qk(p,q)$ spaces to Bloch-type spaces," *Abstract and Applied Analysis*, vol. 2009, Article ID 741920, 14 pages, 2009.
[25] X. Zhu, "Generalized weighted composition operators from Bloch type spaces to weighted Bergman spaces," *Indian Journal of Mathematics*, vol. 49, no. 2, pp. 139–150, 2007.
[26] S. Stević, "Generalized composition operators from logarithmic Bloch spaces to mixed-norm spaces," *Utilitas Mathematica*, vol. 77, pp. 167–172, 2008.
[27] S. Stević, "On a new operator from the logarithmic Bloch space to the Bloch-type space on the unit ball," *Applied Mathematics and Computation*, vol. 206, no. 1, pp. 313–320, 2008.
[28] S. Stević, "On a new operator from H^∞ to the Bloch-type space on the unit ball," *Utilitas Mathematica*, vol. 77, pp. 257–263, 2008.
[29] S. Stević, "On a new integral-type operator from the Bloch space to Bloch-type spaces on the unit ball," *Journal of Mathematical Analysis and Applications*, vol. 354, no. 2, pp. 426–434, 2009.
[30] S. Stević, "On an integral-type operator from logarithmic Bloch-type and mixed-norm spaces to Bloch-type spaces," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, no. 12, pp. 6323–6342, 2009.
[31] S. Stević, "On an integral-type operator from logarithmic Bloch-type spaces to mixed-norm spaces on the unit ball," *Applied Mathematics and Computation*, vol. 215, no. 11, pp. 3817–3823, 2010.
[32] W. Yang, "On an integral-type operator between Bloch-type spaces," *Applied Mathematics and Computation*, vol. 215, no. 3, pp. 954–960, 2009.
[33] W. Rudin, *Function Theory in the Unit Ball of* $\mathbb{C}^n$, Classics in Mathematics, Springer, Berlin, Germany, 2008.
[34] A. L. Shields and D. L. Williams, "Bonded projections, duality, and multipliers in spaces of analytic functions," *Transactions of the American Mathematical Society*, vol. 162, pp. 287–302, 1971.
[35] S. Stević, "Composition operators from the weighted Bergman space to the n-th weighted spaces on the unit disc," *Discrete Dynamics in Nature and Society*, vol. 2009, Article ID 742019, 11 pages, 2009.
[36] Wikipedia, http://en.wikipedia.org/wiki/Fa%C3%83_di_Bruno%E2%80%99s_formula.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Differential Equations
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization