

WEIGHTED HARDY-TYPE INEQUALITIES ON THE CONE OF QUASI-CONCAVE FUNCTIONS

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Abstract. The paper is devoted to the study of weighted Hardy-type inequalities on the cone of quasi-concave functions, which is equivalent to the study of the boundedness of the Hardy operator between the Lorentz Γ -spaces. For described inequalities we obtain necessary and sufficient conditions to hold for parameters $q \geq 1$, $p > 0$ and sufficient conditions for the rest of the range of parameters.

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