What are modelling competencies?

Katja Maaß, University of Education, Freiburg

Abstract: *Modelling and application are seen as a highly important topic for maths lessons. But so far the concept "modelling competencies" has not been described in a comprehensive manner. The aim of this paper is to supplement former descriptions of modelling competencies based on empirical data. An empirical study was carried out which aimed at showing the effects of the integration of modelling tasks into day-to-day math classes. Central questions of this study were – among others: How far do math lessons with focus on modelling enable students to carry out modelling processes on their own? What are modelling competencies? Within the theoretical approach, definitions of modelling processes as a basis for definitions of modelling competencies and important views of modelling competencies are discussed. Based on this theoretical approach the transfer into practice is described. Finally we will look at the results of the study. An analysis of the students' abilities and their mistakes lead to more insight concerning the concept of modelling competencies.*

ZDM-Classification: M10, D40, D30

1. Introduction

Modelling and applications have been regarded as important within the didactical discussion (Kaiser-Messmer 1986, I, p.82). The aim is to integrate modelling and applications into daily school routine. But what do we want the students to learn? What do we mean by modelling competencies?

International interest has been shown in the terms competencies and modelling competencies. E.g. the Danish KOM project deals with problems within the Danish school system and looks for possibilities to improve it. The project is based on the question "What does it mean to master mathematics?" (Niss 2004, p.119) For Niss, a definition of the mathematical competencies is necessary to solve problems regarding the school system and the classroom culture. The new German educational standards and curricula are based on competencies too. These include in particular modelling competencies (Kultusministerkonferenz, 2003).

But up to now the concept "modelling competencies" could not be described in a comprehensive manner. This is evident in some questions posed in the Discussion Document for the ICMI-Study in Dortmund (Blum et al. 2002, p. 271):

"Are modelling ability and modelling competency different concepts? Can specific sub-skills and sub-competencies of "modelling competency" be identified? [...] What are characteristic features of the activity of students who have little experience of modelling? What is the role of pure mathematics in developing modelling ability?" (Blum et al. 2002, p.271)

While there is a broad consensus that modelling competencies include certain sub-competencies like setting up a real model or mathematizing such a model, it is often said that those sub-competencies relating to the process of modelling are not sufficient to characterize modelling competencies. Which further competencies are needed?

This paper deals with the results of a study which tries to elucidate – among others – these questions. The central questions of the study were:

1. How do students' mathematical beliefs change during courses of math classes which include modelling exercises?
2. How far do such lessons enable students to carry out modelling processes on their own?
3. What are modelling competencies?
4. Which kind of connections exist between mathematical beliefs and modelling competencies?

This paper will focus on the part of the study which refers to questions 2 and 3. Results to questions 1 and 4 can be found elsewhere (Maaß 2004, p.153, 2005, p.131).

At first we will have a look at the theoretical frame of the study. Then we will see how this theoretical frame can be transferred into practice, e.g. by looking at a teaching unit. Afterwards the methodological approach will be described. Finally the results of the study and there consequences will be explained.

2. Theoretical framework

In order to clarify the state of the art concerning the understanding of modelling competencies we will have to look at different aspects. As there is strong connection between the conception of the modelling process and modelling competencies we will, at first, look at various concepts about the modelling process. After this we will discuss different explanations of modelling competencies. Because metacognition has been identified as an important variable for problem solving processes, we will look at this concept as well. At the end of the chapter empirical results concerning modelling competencies which have already been found will be regarded.

The discussion of all these concepts and empirical results will give a general survey of the discussion about modelling competencies and will by this clarify the theoretical position of this study. Further more, it will show the challenges from a research point of view.

2.1 Concepts of the modelling process

Modelling problems are only one kind of reality-related tasks. Since there is a large variety of reality related tasks we will have a look at a classification of these problems at first (for further classifications see Förster 1997, p.137, Galbraith & Stillman 2001, p.301, Burkhardt 1989, p.5, Galbraith 1995, p.23).

Kaiser (1995, p.67) differentiates between simple word problems, embedding mathematical tasks into everyday language, illustration of mathematical concepts (e.g. the use of temperatures to introduce negative figures), applying mathematical standard routines (application of well-known algorithms to solve reality related problems) and modelling, i.e. complex problem-solving processes.

This study focuses on modelling because its integration into everyday life at school seems necessary regarding the aims of dealing with reality related problems in math classes (Maaß 2004, p.26, Blum and Niss 1991, p.42) and because modelling has not been integrated into day-to-day teaching practice so far.

Within the pedagogical discussion, there are many different views on modelling processes. We can basically differentiate those views according to Kaiser-Messmer (1986/I, p.83 ff.) by showing how much attention each of them pays to mathematics and how much to problems outside mathematics.

De Lange (1989, p.101) considers mathematizing contexts outside mathematics as well as inside mathematics as the most important issue because this leads to discovering mathematical terms. There is a relation back to the real world but it is subordinate. Mathematics is the center of thoughts in other perspectives as well (Lamon 1997, p.35; Matos 1998, p.21; Klaoudatos and Papastravridis 2001, p.327).

In contrast to de Lange, Galbraith does not centre the mathematizing but considers each step to solving a real problem including interpretation and validation as important.

A major point of view in the didactic discussion is Blum's (1985, p.200; 1996, p.18) who also considers the entire modelling process as essential.

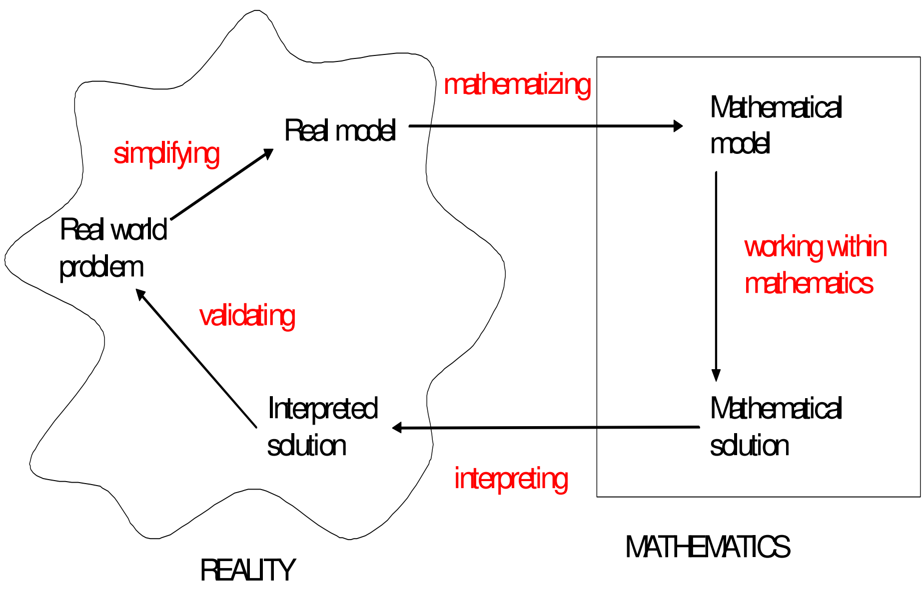


Fig. 1: Modelling process (own representation, following Blum 1996, p.18)

While modelling a real world problem, we move between reality and mathematics. The modelling process begins with the real world problem. By simplifying, structuring and idealizing this problem you get a real model. The mathematizing of the real model leads to a mathematical model. By working within mathematics a mathematical solution can be found. This solution has to be interpreted first and then validated (Blum 1996, p.18). If the solution or the chosen process does not prove to be appropriate to reality, particular steps or maybe even the entire modelling process need to be worked through again (see fig.1).

This illustration of the modelling process is to be seen as a simplified scheme and not as an algorithm which we need to go through in a linear manner. E.g. the construction of the real model often is influenced by one's own mathematical knowledge.

The author agrees with Blum's perspective. The distance to de Lange's results from the aim of this study to examine how the far-spread view of mathematics being abstract and far from reality can be changed by integrating modelling into school. Therefore, it is not sufficient to mathematize the real world problem but also to validate the mathematical results according to the real world problem.

According to the perspectives on the modelling process, opinions on the relevance of the content differ. Whereas de Lange, Matos and Klaudatos and Papastravridis accept inner mathematical contexts as well, others emphasize the importance of realistic, authentic problems (see Galbraith 1995, p.39; Alsina, 1998, p.4). Especially Kaiser-Messmer (1993, p.216) demands a larger number of context-related mathematical problems to be authentic. According to her, the importance of mathematics can only be shown to students by authentic problems. She considers an authentic situation to be an outside mathematical situation embedded in a certain field dealing with phenomena and questions which are relevant for this field and are also regarded as important by experts in this field.

Resulting from the above explanations is a perspective on modelling which is the basis for selection and construction of teaching units within this study (see chapter 3): Modelling problems are authentic, complex and open problems which relate to reality. Problem-solving and divergent thinking is required in solving them. The content needs to be chosen appropriately according to the addressee.

Based on this insight into the different concepts of modelling processes and the definition of the understanding of the modelling process in this study we will now discuss the concept "modelling competency".

2.2 Modelling competencies

In order to specify the term modelling competency we will first take a look at competencies in general: there are various definitions for competencies and they can differ strongly (cp. Böhm 2000, p.309, Jäger 2001, p.160, Jank & Meyer 1994, p.44, Baumert et al. 2001, p.141). These varieties are caused by different origins of the term from various branches of science and the distinction of certain types of competencies. For this study, the following definition from the domain of pedagogy appears to be significant.

Frey defines competency in general as follows: "Competence is the ability of a person … to check and to judge the factual correctness respectively the adequacy of statements and tasks personally and to transfer them into action." (Frey 1999, p.109, cited in Jäger 2001, p.162, personal translation)". Niss (2004, p.120) specifies the term "mathematical competency": "Mathematical competency then means the ability to understand, judge, do and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role…" After this, competencies not only include abilities and skills but also their reflected use in life and the willingness to put these skills and abilities into action. Within the discussion on maths education, Tanner and Jones (1995, p.63) point out that motivation is an essential part for modelling competencies: "Research has shown that knowledge alone is not sufficient for successful modelling: the student must also choose to use that knowledge, and to monitor the process being made."

The exact understanding of modelling competencies and skills is closely related to the definition of the modelling process. The perspectives presented in chapter 2.1 thus imply different views on modelling competencies and skills.

Based on theoretical considerations Blum and Kaiser specify the term modelling competencies by a detailed listing of sub-competencies that are related to their understanding of the modelling process:

Competencies to understand the real problem and to set up a model based on reality: Competency

- to make assumptions for the problem and simplify the situation;
- to recognize quantities that influence the situation, to name them and to identify key variables;
- to construct relations between the variables;
- to look for available information and to differentiate between relevant and irrelevant information;

Competencies to set up a mathematical model from the real model; Competency

- to mathematize relevant quantities and their relations
- to simplify relevant quantities and their relations if necessary and to reduce their number and complexity;
- to choose appropriate mathematical notations and to represent situations graphically;

Competencies to solve mathematical questions within this mathematical model. Competency

- to use heuristic strategies such as division of the problem into part problems, establishing relations to similar or analog problems, rephrasing the problem, viewing the problem in a different form, varying the quantities or the available data etc.;
- to use mathematical knowledge to solve the problem;

Competencies to interpret mathematical results in a real situation: Competency

- to interpret mathematical results in extra-mathematical contexts;
- to generalize solutions that were developed for a special situation;
- to view solutions to a problem by using appropriate mathematical language and/or to communicate about the solutions;

Competencies to validate the solution. Competency

- to critically check and reflect on found solutions;

- to review some parts of the model or again go through the modelling process if solutions do not fit the situation;
- to reflect on other ways of solving the problem or if solutions can be developed differently;
- to generally question the model. (Blum & Kaiser 1997, p.9)

Profke (2000, p.34) gives another detailed listing of modelling sub-competencies. In his listing, Profke does not mention skills to interpret and validate but emphasizes general skills like being curios.

Within the didactical discussion, the concept of what is meant with modelling competencies is often showed by giving analysis schemes to assess students' achievements instead of explicitly stating sub-competencies. Examples for such schemas are given by Money and Stephens (1993, p. 328), Haines and Izard (1995, p.138) and Ikeda and Stephens (1998, p.227):

„(G1) Did the student identify the key mathematical focus of the problem? (G2) Were relevant variables correctly identified? (G3) Did the student "idealize" or simplify the conditions and assumptions? (G4) Did the student identify a principal variable to be analyzed? (G5) Did the student successfully analyze the principal variable and arrive at appropriate mathematical conclusions? (G6) Did the student interpret mathematical conclusions in terms of the situation being modelled?"

These criteria show that Ikeda and Stephens have a similar view of modelling competencies as Blum and Kaiser.

While showing a similar view of modelling as Blum and Kaiser, Niss (2004) explicitly differentiates between active modelling and dealing with finished models: „Analyzing foundations and properties of existing models, including assessing their range and validity, decoding existing models, i.e. translating and interpreting model elements in terms of the reality modelled, performing active modelling in a given context – structuring the field, mathematizing, working with(in) the model, including solving the problems it gives rise to, validating the model, internally and externally, analyzing and criticizing the model, in itself and vis-à-vis possible alternatives, communicating about the model and its results, monitoring and controlling the entire modelling process." In addition to that, Niss (2004, p.124) also differentiates between three dimensions of modelling competencies: "The degree of coverage is the extent to which the person masters the characteristic aspects of the competency at issue …The radius of action indicates the spectrum of contexts and situations in which the person can activate that competency. The technical level indicates how conceptually and technically advanced the entities and tools are with which the person can activate the competency."

Based on the shown perspectives, modelling competencies for this study have been defined as follows: *Modelling competencies include skills and abilities to perform modelling processes appropriately and goal-oriented as well as the willingness to put these into action.*

Because a modelling process similar to the one described by Blum and Kaiser is used it seems reasonable to use their list of sub-competencies (1997, p.9). With regard to the openness the study aimed at (cp. 4.1), only the five main categories were used in the analysis of the data. The subcategories basically remained unconsidered.

The three dimensions described by Niss could not be used here as the study was carried out from 2001 - 2003. However, a similar division was carried out implicitly with the choice of tasks. Tasks were used which differed in context, degree of complexity and the necessary tools.

After the conception of modelling relevant for this study has been made clear we will now consider the concept of metacognition as it seems to be an important issue for the development of modelling competencies as well.

2.3 Metacognition

In connection to the development of competencies, the necessity of developing metacognition is increasingly discussed in the pedagogical discussion. (Sjuts 2003, Baumert et al., 2001, Schoenfeld 1992). The term "metacognition" does not have a standardized definition. Definitions reach from knowledge about one's own thinking up to self regulation in problem solving (Schoenfeld 1992, p. 334).

The term "self regulated learning" is used in the context of the PISA (Baumert et al., 2001, p. 271). It describes the ability of a learner to set his own goals, use appropriate methods and techniques regarding the content and the goal and to review as well as judge his/her own process. Within PISA Boekarts (1999) divides the object of investigation into three components.

Within mathematical education, Sjuts' position is vitally important (2003, p.18). According to his view metacognition is the thinking about one's own thinking and management of one's own thinking. There are three parts of metacognition:

- Declarative metacognition contains the diagnostic knowledge about one's own thinking, the judging thinking about tasks and the strategic knowledge about ways to solve a problem.
- Procedural metacognition contains planning, surveying and judging, which means the monitoring of one's own actions.
- Motivational metacognition: Necessary conditions for the use of metacognition are motivation and the willpower to do so.

Empirical studies refer to the significance of metacognition when solving problems and complex tasks (Sjuts 2003, p.26, Schoenfeld 1992, p.355). The difference between an expert's procedure and that of a beginner is that experts use metacognition. Beginners often experiment without structure and cancel after some time without success. On the other hand, experts review their strategies and come up with a solution with the same or even less effort. (Schoenfeld 1992, p.355).

The development to metacognition or self-regulated learning is regarded as one main task of educational institutions (Baumert et al. 2001, p.271). Metacognition is seen as a basic competency which is relevant for a variety of important competencies such as independent handling with problems or self-regulated learning (Sjuts 2003, p.20).

In order to develop metacognition in school, class needs to be designed appropriately: metacognition cannot be developed without relation to subject knowledge but rather simultaneously. The pursuit of an understanding has to be the centre of class. Comprehension and penetration, specification and systematization, questioning and inquiry as well as cogitating and reflection are things that should be insisted on. The classroom should be stamped by discourse, the exchange of individual perceptions, the discussion about different arguments and the cognitive clarification. This can be achieved in a classroom discussion or with tasks that analyze mistakes (Sjuts 2003, p.20). The students' self-monitoring over the problem-solving processes can be supported by extern monitoring[1] by the teacher (Schoenfeld 1992, p.356) as well as by pointing out previous successes in self-monitoring. It is important to give students the time to get used to those kinds of activities so they can internalize them.

Metacognition might be an important influencing factor on the development of modelling competencies, as problem-solving-strategies are necessary for carrying out modelling processes.

Following Sjuts (2003) *metacognition in this study describes thinking about one's own thinking and controlling one's own thought processes. In addition, still following Sjuts, there are three components of metacognition.* To impart metacognition connected with modelling competencies, the following methods seemed reasonable for this study:

- Imparting metaknowledge about modelling processes meaning declarative metacognition.
- Discussions of different perceptions of students on modelling processes in class
- Productive dealing with students' mistakes and analyzing them
- Demanding planning, monitoring and validating of their own actions – with the scheme of the modelling process helping them
- Comparing and discussing different solutions and reflecting on reasons for that
- Pointing out positive examples of self-monitoring in the course of modelling
- Extern monitoring by the teaching person

[1] The teacher supports the students' self-monitoring by asking why and for what goal they are doing what they do.

Having set the theoretical basis for the study we will now look at previous studies about modelling competencies.

2.4 Empirical studies about modelling competencies

There are only a few detailed studies on modelling competencies compared to the long and intensive discussion on connecting tasks to real world problems. The following part gives an overview over previous studies and their results.

- Mathematical skills are required to acquire modelling competencies (Ikeda 1997, p.52, Galbraith & Clathworthy 1990, p.159, Dunne 1998, p.30).
- Knowledge about the modelling process influences the acquirement of modelling competencies positively (Galbraith & Clathworthy 1990, p.158, Tanner and Jones 1993, p.234).
- The content of the modelling example has a great influence. The context can not only motivate but also distract from solving the problem, e.g. by a strong emotional connection to the students or by too much information (Busse 2001, p.141, Galbraith & Stillman 2001, p.309).
- Treilibs (1979, p.97) recognizes a ”sense of direction” as important for modelling: „A major finding in the study […] is the strong sense of direction exhibited by good modellers and consequently, a great difference in performance between good modellers and other students. This sense of direction is attributed to good modellers being able to work at an operational level as much as possible whilst poor modellers tend to work within the problem context in a step-by-step fashion.” (Treilibs 1979, p.142)
- Different studies show that work in small groups, discussions in groups and working independently support the development modelling competencies (Galbraith & Clathworthy 1990, p.158, de Lange 1993, p.5, p.65, Ikeda & Stephens 2001, p.381, Tanner & Jones 1995, p.65).
- Blomhoej and Jensen (2003, p.128) differentiate between two methods to impart modelling competencies to learners: according to the “holistic approach”, the students need to go through the entire modelling process for them to gain modelling competencies. The “atomistic approach” offers a different perspective. It states that math classes should be limited to the process of mathematizing, working with the mathematical model and interpreting because these activities are closely related to mathematics. Their study shows that both approaches are necessary to acquire modelling competencies.
- Numerous studies show mistakes that appear when learners model problems. Among other things, learners’ difficulties to create a connection between reality and mathematics, as well as simplifying and structuring the reality (Kaiser-Messmer 1986/II, p. 144, Hodgson 1997, p.215, Christiansen 2001, p.317 ff, Haines & Crouch & Davis 2001, p.366) and problems dealing with the mathematical solution (Hodgson 1997, p.215, Haines & Crouch & Davis 2001, p.366) have been discovered.

These previous studies provide different single hypotheses on components of modelling competencies and special difficulties learners have. However, there are few comprehensive studies about modelling competencies and their related weaknesses. Especially metacognitive modelling competencies have hardly been examined. Adding to that, few empirical studies have been conducted which examine the relation between learners’ beliefs on mathematics and modelling competencies.

2.5 Consequences

The previous explanations show that it can be regarded as a challenge to gain empirical evidence which sub-competencies are needed to carry out a modelling process. Theoretical reflections point at many possible sub-competencies of modelling competencies. Metacognition may be an important factor also. In addition to that, various empirical studies hint at single factors which seem to have an influence on modelling competencies. Among these mathematical skills, knowledge about the modelling process, a sense of direction and working in groups seem to have a positive impact on the development of modelling competencies whereas the context can motivate but also distract. Aspects which have not yet taken into account may also have an important influence.

The challenge is to design a research concept which takes care of as many influencing factors as possible. This study cannot give a final definition of modelling competencies. Based on the theoretical frame set above we will however try to formulate empirically based hypotheses leading to a more comprehensive understanding of modelling competencies.

3. Transfer of the theoretical approach into practice

To elucidate the transfer of the theoretical approach into practice we will at first look at the framework given at school. Afterwards some remarks about the basis for the design of the teaching units will be made. Finally an example of a modelling unit will be given to illustrate the realisation in class.

3.1 Framework

Two groups of learners who were parallel classes were chosen for the study. They were at the start of 7th grade (age 13). The lessons of these classes were as usually divided up into 45 minutes lessons. Temporary tests to assess performance had to be conducted within this frame. Moreover, the modelling examples had to relate to the current curriculum for classes 7 and 8 in Baden-Württemberg for official organisation reason.

In order to overcome various difficulties, the “island approach” by Blum and Niss (1991, p.60) was chosen for integrating the modelling examples. Amongst those difficulties was living up to the curriculum’s expectations although integrating modelling examples into “normal lessons” as well as avoiding too great resistance by the learners and their parents who were used to traditional math classes. At the same time the view was met that links to the real world and modelling are only one component in the complex field of teaching and learning mathematics.

3.2 Basis for the design of teaching units

According to the results of several empirical studies (cp. 2.4) teaching methods were chosen which allowed the students to work independently. We regard group work as an appropriate teaching method within this context and under the given school framework. Group work has many advantages which previous studies and literature describe (e.g. Zech 1998, p.358). Therefore, phases of group work alternated with phases of classroom discussion and phases of individual work. Regarding the independent work which we aimed at, classroom discussions were characterized by students discussing while the teacher took a reserved role. During the lessons, much attention was paid to support the development of metacognitive modelling competencies.

Furthermore, the modelling examples were supposed to facilitate imparting various aspects of knowledge about modelling processes (cp. 2.3). The basis for this imparting of knowledge about the modelling process was the scheme of the modelling process (cp. 2.1). Since some of the terms in the diagram might be hard to understand for students, the diagram was simplified: “Mathematizing” was replaced by “describing mathematically”, “validating” was replaced by “evaluating” and the steps “interpreting” and “evaluating” were in contrast to fig.1 not explicitly separated.

Six modelling units were developed regarding the theoretical approach and the given framework. So, open modelling problems were selected for the lessons. The majority of context-related problems were authentic in order to make the students realize the relevance of mathematics (cp 2.1). They address different contexts as well as different mathematical content and they require different amounts of time (1-12 lessons).

The teaching units address questions like: How big is the surface area of a Porsche 911? How many people are in a traffic jam which is 25 km long? (Jahnke 1997, p.70 , Maaß 2005, p.8). How can you display the monthly cell phone bill clearly depending on the usage of the phone? To what extent can the service water in Stuttgart-Waldhausen[2] get warmed by solar panels on the

[2] The name of the town was changed to grant for students’ anonymity.

roofs? How high can a bouncy rubber ball jump when you drop it from a height of 10 meters (Henn 1988, p.143)?

In the following section the modelling unit on the surface of a Porsche 911 will be described as an example.

3.3 Modelling unit "Porsche"

Information on the context

When a new type of car is being developed and before it is produced, the expected costs are exactly calculated. Among other things needed is the calculation for the costs of the paint.

The paint of a Porsche consists of 4 layers with different layer thickness:

- grounding (18-32 µm)
- infill (25-40 µm)
- basecoat (20-25 µm)
- clear coat (30-45µm)

You cannot directly calculate how much paint is needed with the layer thicknesses and the surface area which is to be painted because parts of the water and the dissolver in the paint evaporate. A part of the paint is also sprayed in the air. E.g. twice as much basecoat is used than is actually on the Porsche afterwards.

Thus you can calculate the amount of paint with the layer thicknesses, the factor for additional use and the surface area which is to be painted. But how do you get the surface area?

This real situation and problem does not connect to the actual environment of students but it is realistic and appears to be reasonable for modelling by younger learners for the following reasons:

- The context is easily understood.
- There is not much information on the context needed.
- Even within the mathematical opportunities of a student in 7th grade, different approaches are possible.
- A lot of younger students are interested in fast cars. This especially regards boys but not only boys.

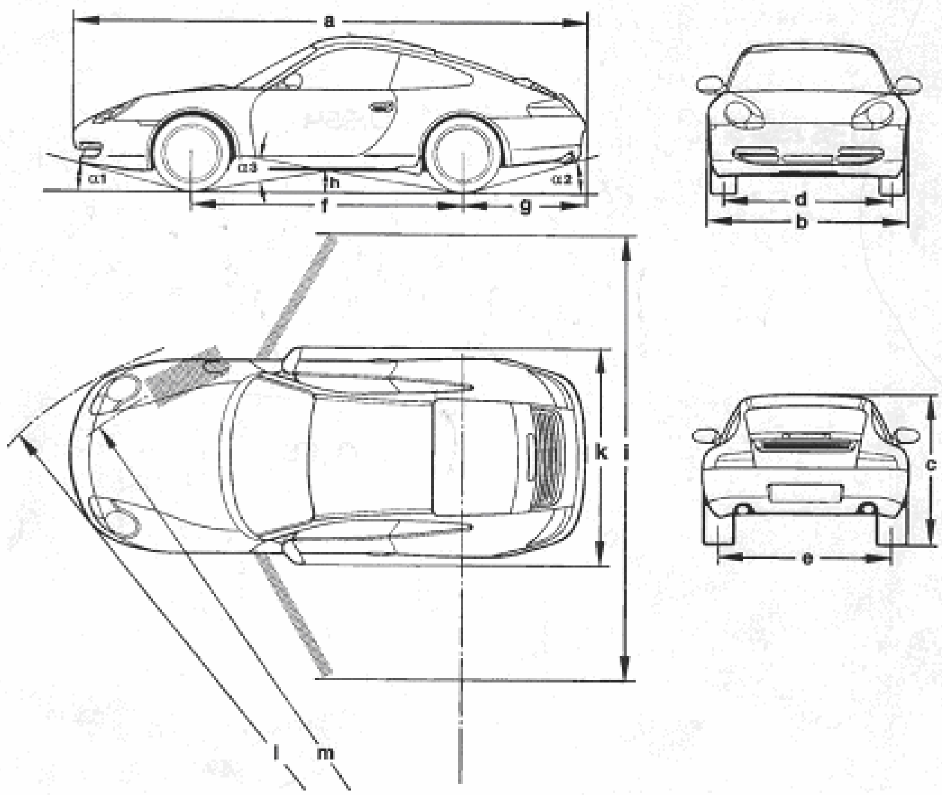
Realization in class

In the following, a course of a unit of lessons is described which was conducted in May 2001 and consisted of three lessons.

At first, the students get information on the "Cost Centre" and the paint of a Porsche at the beginning. After discussing the problem, some students demand information on the dimensions of the new Porsche which are needed to solve the problem. For reasons of secrecy there are no such numbers present of course. Therefore, the present dimensions of the outside car body of a Porsche 911 are looked at (fig. 2).

PORSCHE

Service	4.1 Fahrzeug-Außenmaße



Technische Daten (Maße DIN Leergewicht)

	911 Carrera (996)		Sportfahrwerk	
a	4430		4430	
b	1765		1765	
c	1305		1305	
d	1455	1465[2]	1455	1465[2]
e	1500	1480[3]	1500	1480[3]
f	2350		2350	
g	1055		1055	
h	108[1]		100[1]	
i	3817		3817	
k	1930		1930	
l	5440		5440	
m	5100		5100	
α1	9,2°		8,5°	
α2	14,2°		13,7°	
α3	10,7°		9,8°	

1) Zul.-Gesamtgewicht
2) Mit Scheibenrad 7,5 J x 18
3) Mit Scheibenrad 10 J x 18

Bemerkung: Maße sind für Werkstattbauten, Garagen und Auffahrten.

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Fig. 2: Porsche 911

The information given in the drawing is criticized at first by the students because there are no units given. The hint to think about the units on one's own leads to the assumption that the measures are given in cm. This assumption can be contradicted by the students transferring the centimetres to meters resulting in the students realizing that a Porsche cannot be 44 meters long.

Concrete ideas on how to calculate the surface area are not developed right away. Instead the students hesitate. Some expect a formula to calculate, others think that the area cannot be figured out at all. After some time, they reach a point where they agree that the area can be calculated approximately with details like rear view mirrors and warps being neglected. Now, the students show numerous ideas:

- Approximation of the Porsche by a cuboid which is big enough for the entire Porsche to fit into it – and then calculation of the surface area of the cuboid.
- Approximation of the Porsche by a cuboid which has half the height of the Porsche because the Porsche has windows in the upper half. Regarding the tires also can result in even lower heights.
- Segmentation of the surface of the Porsche into many small triangles and quadrangles (fig. 3). Afterwards, calculating their area, summing them up and converting them according to the scale.

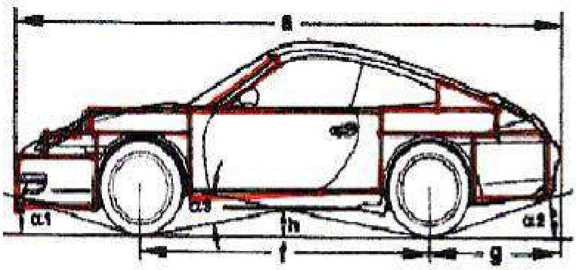


Fig. 3: Segmentation of the surface area

- Covering of a Porsche with paper, cloths or something similar and afterwards measuring the cloths.
- Calling the Porsche AG.

After that, the various ideas, except no. 4 which fails because of the lack of a Porsche, and no. 5 which one student will do at home, are carried out in groups which distribute the work to its members. Problems show up in the following areas:

- Some learners have troubles visualizing the drawing spatially. They fail to assign the sides of the Porsche to the sides of a cuboid.
- Others have troubles with the scale. They do not realize that they just have to measure a length in the drawing whose real length is given in the drawing in order to calculate the scale.

After the calculation, the groups present their findings and their way of getting their solution. Different results lead to a discussion on how exact the results are and to the realization that all the approaches provide unequally exact values which are all just approximations because the problem is simplified. In the course of the discussion, it becomes clear that none of the results can be called true or false but that they have to consider if the modelling was appropriate for this problem. All of the methods seem basically reasonable to get a value. Many learners agree on the opinion that modelling the area with a cuboid is too inaccurate whereas the segmentation into quadrangles and triangles is much more exact but the work that needs to be put into this approach does not seem suitable. Method no. 2 is seen as fitting because in contrast to method no. 1 it regards less area which is not to be painted. It also does not use too much detail. Moreover, the result of this method is pretty much the same as the result of method 3 (depending on the segmentation 14-17 m^2) which even supports method 2.

Talking about how exact the results are supposed to be follows that discussion. It is clearly articulated that a result like 12.1234345 m^2 is not fitting within an estimated calculation like this. It needs to be rounded.

At last, the students regard the Porsche AG again. They want to know how the Porsche AG calculates the surface area because they think that the Porsche AG must use different methods. They get the information that the area is calculated by a CAD-program which is used for developing cars. This calculation by the computers is based on the same principles as the students used which is approximation and segmentation.

4. Methodology

At first some general explanations about the methodological approach will be made. After this the methods of data collection and finally the methods of data evaluation will be explained.

4.1 Basics

The goal of this study was to generate hypotheses on the consequences of using modelling in day-to-day teaching. Since there have only been few empirical studies so far, this study is situated in qualitative research. An elementary goal of qualitative research is to explain complex relations within a day-to-day context instead of explaining singular relations by isolation (Flick & Kardorff & Steinke 2002, p.23). It is rather to discover new things than to proof things that have already been discovered.

The goal to examine the consequences of integrating modelling lessons into day-to-day school required integrating modelling units over a longer period of time and still remaining within a class situation as natural as possible. Therefore, the same person took the role of the researcher and the teaching person relating to action research (cp. Kromrey 1998, p.511).

An essential characteristic for the selection of sample survey and evaluation methods was the principle of openness (Strauss & Corbin 1998, p.12): Since hardly anything was known about the consequences of modelling lessons, hypotheses should not already be brought to the study but rather be developed while dealing with the data and be formulated as results. The principle of openness is also supposed to meet the challenges concerning an empirical study heading at a comprehensive definition of modelling competencies (cp. 2.5).

A goal of the study was the creation of typologies. Procedures of creating types, comparing cases and contrasting cases play an important role in qualitative research because thus the complex reality is reduced and made concrete (Kelle & Kluge 1999, p.9). One tool to elucidate the results was the construction of ideal types as described by Weber (1904, see Gerhardt 1990, p. 437). Ideal types are the result of an idealization and therefore have a theoretical character.

4.2 Methods of data collection

The decision on being the teacher and researcher during the study limited the maximum sample number to the total of students in those two 7th grade classes which is 42 teenagers. The field should be covered broadly to create reasonable typologies. Thus, the entire data from 35 students whose development appeared relevant was evaluated over the entire survey period (15 months (04/01 – 07/02).

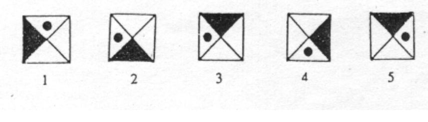
Because of the complexity of the object of investigation, it was decided on a variety of data collection methods. These included

- a test to evaluate the mathematical capacity
- modelling tests
- written class tests and homework
- concept maps to investigate metacognitive modelling competencies
- interviews
- learners diaries and questionnaires (mainly to evaluate the mathematical beliefs of the study, cp. Maaß 2004, p. 120)

Test concerning mathematical capacity

In order to compare the modelling competencies with the mathematical competencies both had to be evaluated. However, it is not easy to define the concept "mathematical competency". Blum, Kaiser, Burghes & Green (1994, p.150) define a so-called mathematical capacity which contains all mathematical knowledge, skills and abilities of a person, independent from their way of development. To evaluate this mathematical capacity they have developed a test. Due to organisational time limits at school the test could not contain time consuming tasks as mathematizing or proofing. However, the test contains as well tasks which are related to abilities developed in math lessons as well as such tasks more related to a person's intelligence. The tasks belong to various areas, e.g. simple calculating, Algebra, dealing with figures etc. As examples two of 28 tasks are shown:

Which square does not fit?



A gardener plants seed in a bowl. He has 20 bowls and needs 10 minutes for each bowl. He starts his work at 7.30 a.m. At what time he will have finished ¾ of his work?

Modelling test

Intending to examine the change of modelling competencies during the evaluation period, the modelling competencies had to be examined before and after the study by the same method. Otherwise a direct comparison is impossible.

A test was developed which consisted of ten tasks. The first test was designed to enable the students who did not have any experience with modelling processes to partially work on the tasks because this was the only way to detect their strengths and weaknesses. The tasks were chosen in a way that all sub-competencies of modelling were tested according to the preliminary definition above. E.g. the test included the following tasks:

- Claudia has a sack of marbles. She gave half of it to Thomas. Then she gave one third of the remaining marbles to Peter. 6 marbles were left in the sack. How many marbles were in the sack at the beginning? (from Baumert et al. 1998, p.76)
- Explain the statement: On average there are 1.2 persons in a car during commuter traffic.
- Class 8a is getting their tests back. They had 45 minutes to work on them. Milena was done after 15 minutes; her work was graded with a 1[3]. It took Rudi 30 minutes and his test was graded with a 2. Can you tell which grade Tanja got who handed in her test after 45 minutes? Give reasons for your answer (from Curkowicz & Zimmermann 2000, p.23)
- How many percent of your time in a year do you spend in school, doing your homework, do you have leisure time?

Some tasks remained as they were in the second test, others were changed slightly to avoid a “dejà-vu” effect for the learners. The tasks were varied in such a way that the same sub-compentencies were evaluated. The following example is a variation of the third task given above:

- Verena is collecting shells on the beach. She puts three of them on a scale. Altogether they weigh 27 g. Afterwards she adds two shells. Which weight the scale is showing now?

Tests, exams and homework

In addition to the tests named above, modelling problems were given in written class tests and exams as well as for homework. These methods of assessment enable a continuous observation of the development of modelling competencies in contrast to the two tests for modelling competencies. Furthermore, in this context it was possible to assign modelling tasks of a greater extent. The following criteria, among others, seemed to be relevant for the design respectively the choice of the tasks:

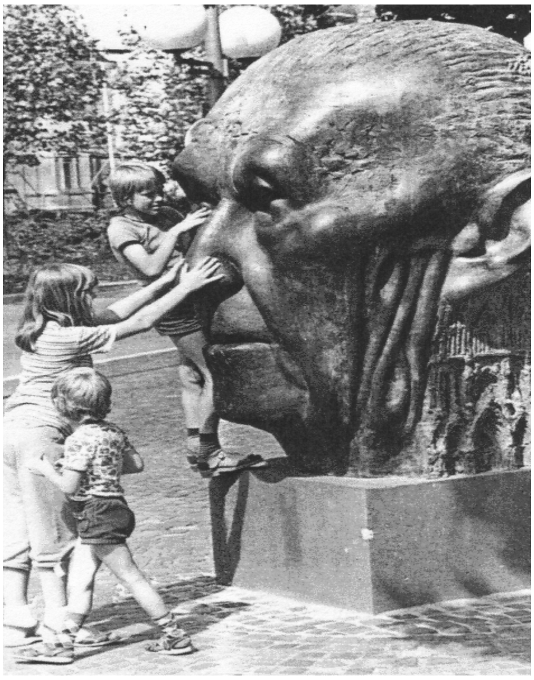
- In contrast to the modelling tests not only sub-competencies but competencies in carrying out a whole modelling process were evaluated.
- The tasks hat to be different concerning the demands the students had to meet. These demands were on the one hand determined by the mathematical content and on the other hand by their connection to the previous lessons.

E.g. the following three tasks were worked on in different tests in the study:

The *Adenauer task* was given in the second test of the study. Here not all the necessary information is given, it has to be estimated. The context was new, but the mathematical challenges were not very high.

Adenauer task (Herget, Jahnke & Kroll 2001, p.20): The following monument is found in the city of Bonn, capital of Western Germany before unification. It shows the head of Konrad Adenauer (1876-1967), first chancellor of Western Germany in 1949 – 1963. What would be the size of the statue if it showed Adenauer from head to toe at the same scale? Explain all your steps.

[3] In Germany there are grades from 1 to 6, 1 is the best, 6 the worst.



The demands in the *account-charge task*, which was given in the third test, were higher. Although the context was very similar to the context "mobile phone charges" which was dealt with in the previous teaching unit, the mathematical demands were very high.

Account charges – task:

Information: unfortunately, a bank account is not free. Different banks charge fees according to different models:

Otto-Leon-Bank: basic fee per month: 4 €, each posting (drawing and depositing money, transfers, debit entry): 0.30€, each account statement: 0.60€, installing a standing order: 2.50€, EC-card: 10€ each year.

Germanic Bank: flat offer: 11€ fee per month, all services included.

Town Bank: basic fee per month: 2 €, each posting and each account statement: 0.50€, installing a standing order: 2€, EC-card: 10€ each year.

Task:

Read the information. We are looking for a clear overview of the monthly fees depending on the usage. Develop various solutions. Describe your ideas for a solution on the basis of the modelling process and compare them shortly. Show one solution in detail.

The *natural gas task* was designed with another focus. It is a very complex task as the students had to develop various models for an unknown question.

Natural gas task (modified after Cukrowicz & Zimmermann 2000b, p.51):

In 1993 the worldwide reserves of natural gas were estimated to be 141.8 Billions cubic meters. Since then 2.5 Billion cubic meters have been used every year on average. Calculate when the reserves of natural gas will be exhausted. Use different assumptions and models. Explain all your steps.

Concept maps

A main focus of the study was the development of metacognitive modelling competencies. One possibility to evaluate them is the use of concept maps. "Roughly speaking, the underlying idea is that something which is inside the mind can be mapped to the outside." (Tergan 1988 cited in Hasemann & Mansfield 1995, p.45)

Within this study the concept maps themselves and not the process of drawing them were evaluated. The students were asked to develop a visual representation containing the terms given (terms relating to the scheme of the modelling process and terms relating to the contexts of the teaching units) and to separate the terms they could not assign. By this a reconstruction of the students' conceptual understanding was possible.

Aiming at a reconstruction of the development of metacognitive modelling competencies the students had to draw one concept map in the middle of the evaluation period and one at the end.

Interviews

Interviews were used to evaluate modelling competencies and metacognition. At the beginning of the interview the students were given a modelling task. The tasks had to be solved within a short time, because there was only limited time for the interviews. To meet a certain nervousness of the students and to have a basis for a communication the students were given the selected task together with a solution which was not appropriate, as is shown in the example:

Ice-cream parlour task

In Leo's small hometown there are four ice-cream parlours. Today Leo is queuing in front of his favourite ice-cream parlour. A ball of ice costs 0.60 €. Leo asks himself how much money the owner will earn on a hot summer day.

Leo is looking for a solution. He asks his friends how many balls of ice they have bought today and calculates the average: (3+4+5):3 = 4 balls per day. He multiplies the result with the number of inhabitants of his hometown (30 000) and divides the result by 4, because there are 4 ice-cream parlours. So the owner must earn about 30 000 · 0.60 € = 18 000 €.

What do you think of Leo's solution? How would you proceed?

The solving of the problem during the interview offered the possibility to observe the students during the process. Afterwards the students were asked to assign their way of solving the problem to the modelling process in order to evaluate their metacognitive modelling competencies.

4.3 Methods of data evaluation

According to the given list of data collection methods the data evaluation will be explained.

Tests concerning mathematical capacity and modelling competencies

Both tests were evaluated in a quantitative way. The maximum of points was 28 (1 point for each task) in the test concerning mathematical capacity and 10 in the modelling test (1 point for each task). To compare the results of both tests the results were represented in a graph.

The modelling test was evaluated also in a qualitative way as explained in the following section.

Tests, exams and interviews

The tests and exams as well as the interviews were analyzed in detail according to the listing by Blum and Kaiser 1997 to reconstruct the modelling competencies.

We wrote a description for each task to show in which areas mistakes happened. These descriptions were compared for each student to identify certain weaknesses and generate statements on the development of modelling competencies. The focus was on the defined sub-competencies of modelling competencies, the metacognitive modelling competencies and further noticeable aspects such as how the quality of a solution depends on the complexity of a task or if there is a relation between the quality of the solution and content or between the quality of the solution and the degree of closeness to the previous lessons. Personal profiles were created based on this – including the results from the analysis of the concept maps (see below).

Finally, case-comparing and case-contrasting analyses were conducted according to the "Mehrfeldertafel" by Kelle and Kluge (1999, p.79) on the basis of lists and cards which showed the characteristical weaknesses of the particular cases. Grouping the students by attitude towards the modelling examples, attitude towards mathematics and areas in which mistakes appeared gave an indication which factors influenced modelling competencies. This grouping lead to a construction of idealtypes, too.

Concept maps

The evaluation of the concept maps was interpretative because that way the content of single statements could be viewed and evaluated. To validate hints from concept maps, relevant sections of interviews were looked at.

The following questions, among others, were answered during the analysis of the Concept maps:

- In which way the theoretical terms are arranged?

- In which way the terms relating to the teaching units are assigned to the theoretical terms?
- Which terms could not be assigned at all?
- Which misconceptions can be reconstructed? Which aspects are represented in an appropriate way?

5. Results of the study

The following results are based on the analysis of modelling tests, exams, the concept maps, the interviews and the analysis of learners' diaries and questionnaires (see Maaß 2004, p. 120).

At the beginning we will look at some important basic results about modelling competencies. With the intention of explaining more detailed results about modelling competencies mistakes occurring when carrying out modelling processes will be looked at. After this we will deal with the metacognitive modelling competencies and the misconceptions about the modelling process students have. These explanations, together with further results of the study will finally lead to a list of influencing factors resulting in an empirically based definition of modelling competencies.

5.1 Basic results

One of the most fundamental results of the study is that students at lower secondary level are able to develop modelling competencies. Towards the end of the study, almost all students were qualified to model problems with known as well as with unknown contexts. Not only did they enact sub-competencies but were able to conduct entire modelling processes independently although not always correctly.

Those competencies were visible not only within problems with contents known to the students but also within problems with unknown content.[4]

Differences appeared in relation to the complexity of the tasks. E.g. the modelling of the especially mathematical complex task "account charges" was managed in many cases only approximately or not managed at all. In contrast to that, the modelling of the less complex but unknown "Adenauer-task" succeeded in most of the cases largely or entirely.

At the end of the study, most of the students were able to model simple problems but some failed modelling complex ones. Anyway, an essential part of the students succeeded in modelling complex problems easily.

This result contradicts the opinion – which is especially wide-spread in schools – that real world modelling problems can only be used in higher grades due to their complexity if they can be used at all.

5.2 Mistakes within the modelling process

To gain more information about the competencies needed to carry out modelling processes we will now look at the mistakes which occurred. This however may not be misunderstood in a way that all solutions were incorrect and poor. As said earlier, the opposite was the case. The following list is intended only to show in which parts of the solution process mistakes can occur:

Mistakes in setting up the real model: Sometimes the real model was inadequate, because the assumptions simplified reality too much, some students did not describe the setting up of the real model. Others made wrong assumptions which distorted reality. A mistake of this kind is shown in Fig. 6.

[4] This refers to gradually different levels: at the beginning, tasks were worked on which connected mathematically as well as content-wise to the previous lessons. Later, tasks were chosen which either did not connect mathematically or content-wise as well as tasks which did not connect to the previous lessons at all.

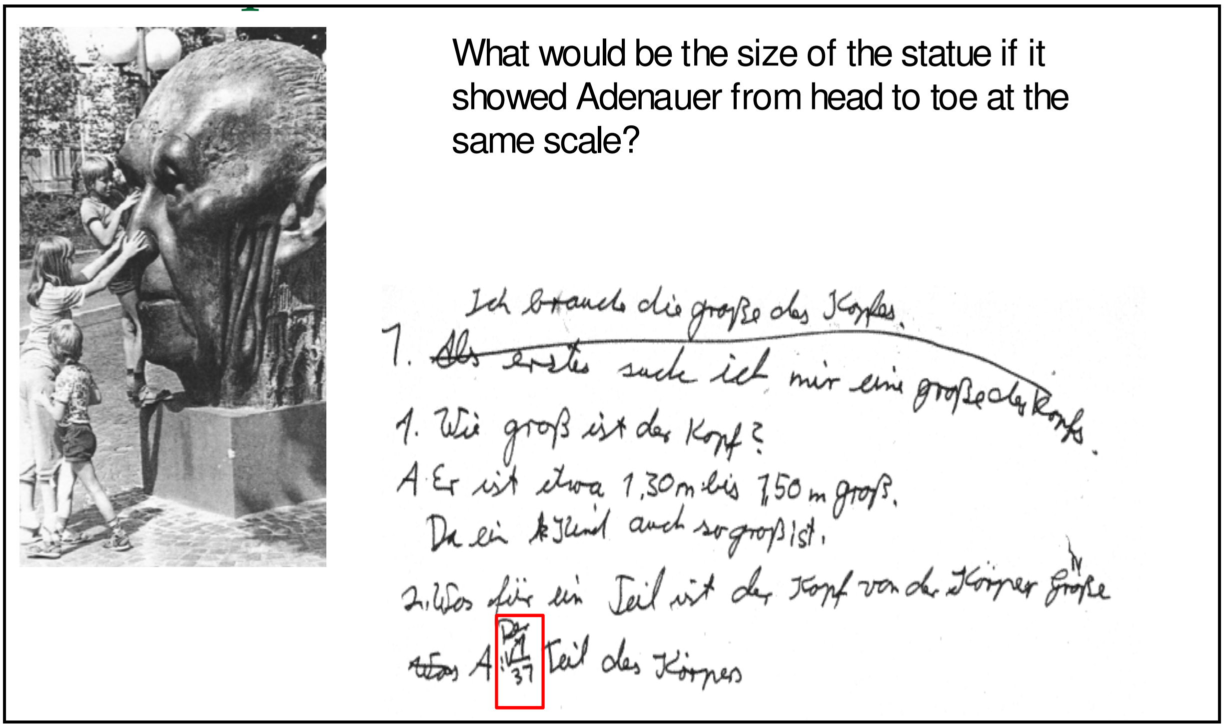


Fig. 6: Assumption which distorts reality

The student deals with the problem in an adequate way: He asks himself how big the head is and he answers that the head is about 1.30m to 1.50m because a child has this size. Then he asks himself what proportion between a body and a head exists. And he writes “The proportion between head and body is 1 to 31!” This shows that he is not able to estimate fittingly the relation between body and head. Additionally, the estimation that the child is about as high as the head simplifies reality too much.

Mistakes in setting up a mathematical model: Some students didn’t use adequate mathematical symbols[5], others used wrong algorithms or formulas. An example of this is shown in Fig. 7. The students were asked to describe three ways to solve the problem and finally calculate a solution by using one way.

[5] E.g. the equal sign was misused repeatedly.

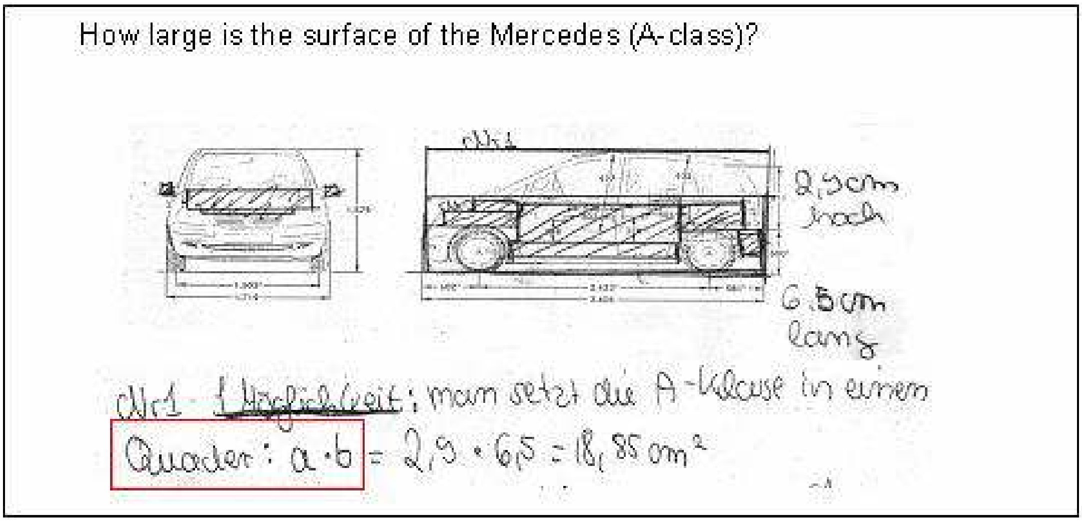


Fig. 7: Use of wrong formulas

This girl describes three correct ways in detail (which is not shown here) and she decides to use a cuboid as a model. However, she has the idea to calculate the surface of the cuboid by calculating a b and so in fact she calculates the area of a rectangle. Further more she forgets to convert the scales.

Mistakes in solving mathematical questions within the mathematical model: As in other mathematical problems, mistakes showed up. Sometimes there were mistakes in the calculation, in other cases the work within the mathematical model was finished without any result. Some students lacked the needed heuristic strategies.

Mistakes in interpreting the solution: Two kinds of mistakes basically appeared: Partly the interpretation of the results was missing; sometimes mathematical solutions were interpreted in a wrong way. An example for this mistake is shown in Fig. 8:

> Explain the statement „There are 1.2 persons in a car on average“
>
> Grown-up – child, Grown-up - dog, Grown-up – shopping bag

Fig.8: Mistake in interpreting the solution

Mistakes in validating the situation: Often the validation of the results was missing; sometimes the inadequacy of a model was realized but not corrected. Very often the validation did not go behind the surface: In Fig. 9 the student just wrote: The proceeding no. 2 is very exact and it is easy to calculate! He does not reflect about the simplification and he does not compare the result with other sizes.

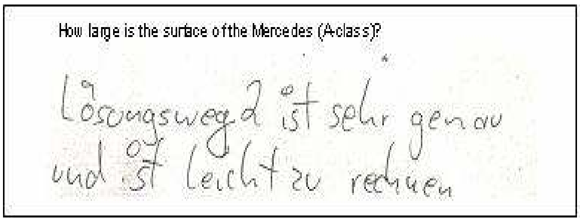


Fig.9: Part of a student’s solution: Superficial validation

Mistakes concerning the whole modelling process: It became clear that often mistakes occurred which did not only concern a step in the modelling process but the entire process: Sometimes many aspects of the real world were described without using them in the modelling.

Some students lost track of their own proceeding. Fig. 10 shows a solution of the natural gas task (fig. 4). The girl assumes three different developments:

1. The use of natural gas remains the same during the years. 2. It rises. 3. It sinks.

She gives reasons for her assumptions and then she works within her models and interprets the solution. But afterwards she corrects her calculation many times without correcting her interpretations. So she gets the same result in the first and in the second calculation, but in her interpretation she writes: The natural gas will be used up 6 years earlier than in the first case. Possibly she became confused while correcting her solution.

Sometimes the whole modelling process was not described or only in a very short way. In Fig. 11 you can see that there is no explanation at all, only a calculation. In some cases modelling was stopped without any results because the calculation was confusing or the student had chosen a proceeding which he/she couldn't follow. An example of this can be seen in Fig. 12.

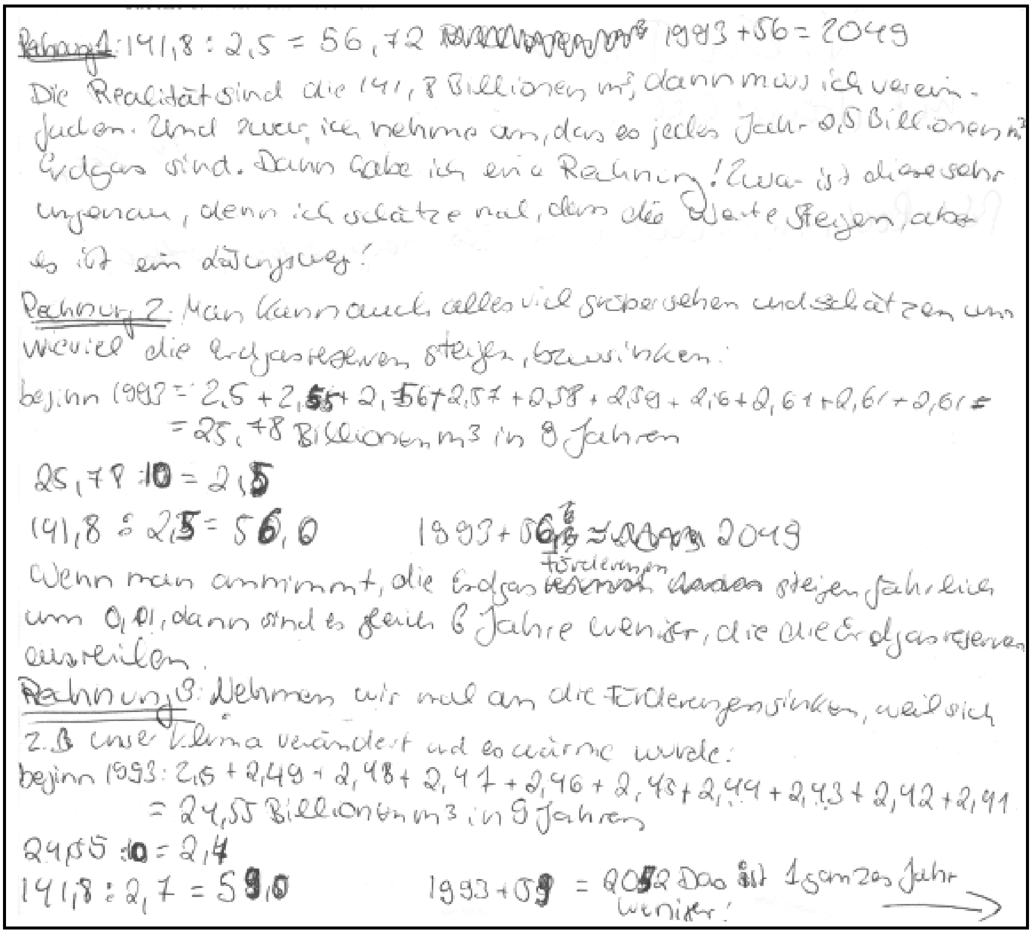

Rechnung 1: 141,8 : 2,5 = 56,72 1993 + 56 = 2049
Die Realität sind die 141,8 Billionen m³, dann muss ich vereinfachen. Und zwar, ich nehme an, dass es jedes Jahr 2,5 Billionen m³ Erdgas sind. Dann habe ich eine Rechnung! Zwar ist diese sehr ungenau, denn ich schätze mal, dass die Werte steigen, aber es ist ein Lösungsweg!
Rechnung 2: Man kann auch alles viel genauer sehen und schätzen um wieviel die Erdgasreserven steigen, bzw. sinken:
beginn 1993: 2,5 + 2,55 + 2,56 + 2,57 + 2,58 + 2,59 + 2,6 + 2,61 + 2,61 + 2,61 = 25,78 Billionen m³ in 9 Jahren
25,78 : 10 = 2,5
141,8 : 2,5 = 56,0 1993 + 56 ≈ 2049
Wenn man annimmt, die Erdgasförderungen steigen jährlich um 0,01, dann sind es gleich 6 Jahre weniger, die die Erdgasreserven ausreichen.
Rechnung 3: Nehmen wir mal an die Förderungen sinken, weil sich z.B. unser Klima verändert und es wärmer würde:
beginn 1993: 2,5 + 2,49 + 2,48 + 2,47 + 2,46 + 2,45 + 2,44 + 2,43 + 2,42 + 2,41 = 24,55 Billionen m³ in 9 Jahren
24,55 : 10 = 2,4
141,8 : 2,4 = 59,0 1993 + 59 = 2052 Das ist 1 ganzes Jahr weniger!

Fig.10: Student's solution with many corrections

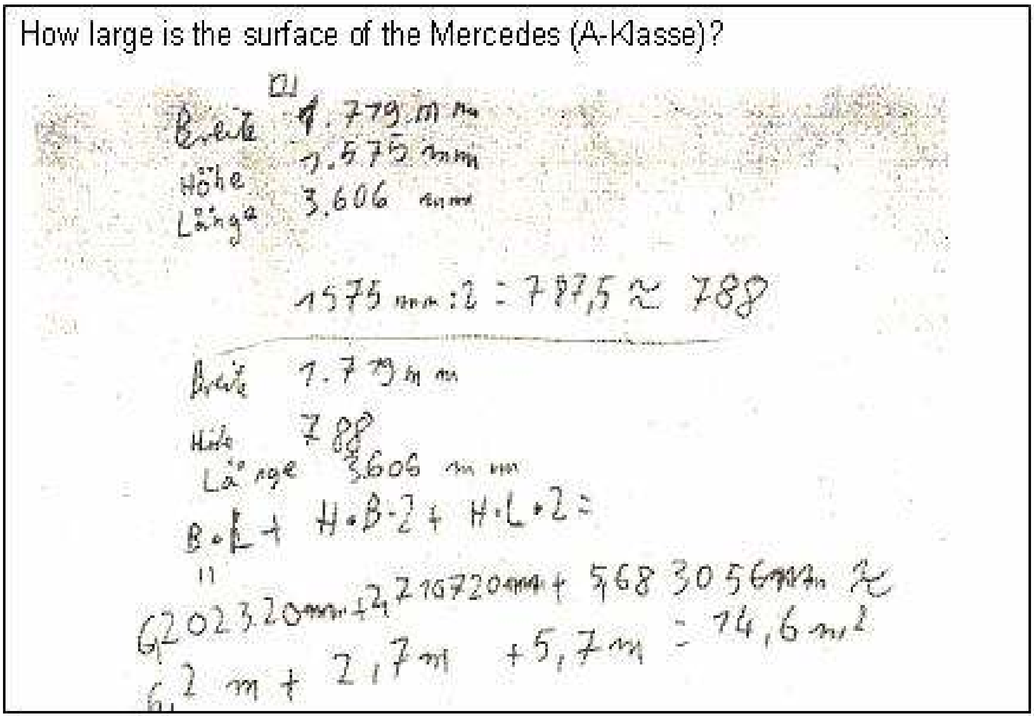
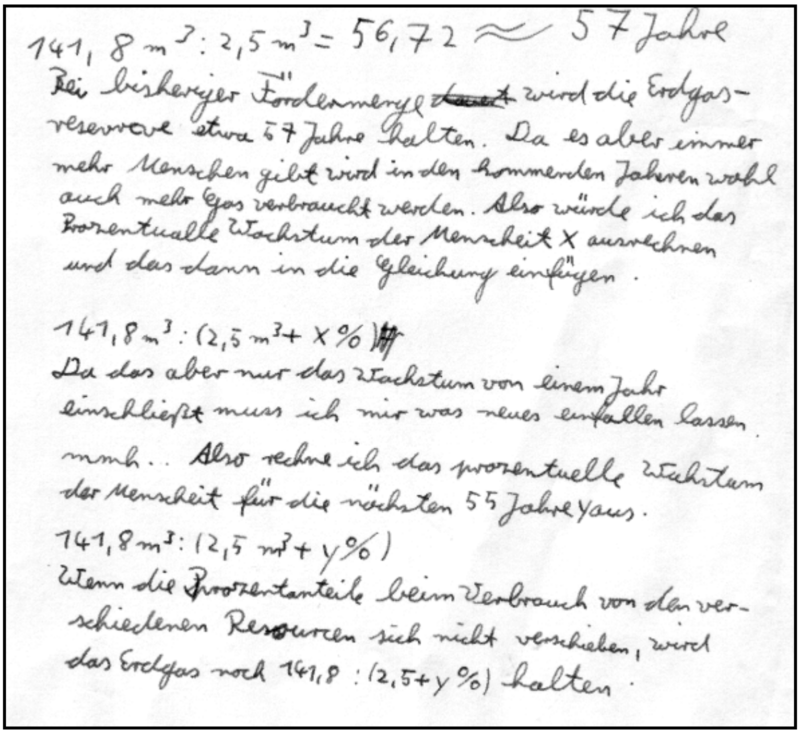


Fig.11: Modelling without written argumentation

Frank's modelling for the natural gas task only succeeds partially because he is not conducting it goal-oriented (fig. 12): Frank only creates two real models with very short descriptions. In his first modelling he assumes that the annual use of gas will remain 2.5 million cubic metres, but he forgets that the data is from 1993 and therefore the gas reserves will not last another 57 years. In the second modelling he assumes that the annual use of gas will grow because the population of the earth grows. A further realization of this idea fails because he does not estimate the rise in population reasonably. At the same time his mathematical work for his demanding idea is incorrect.

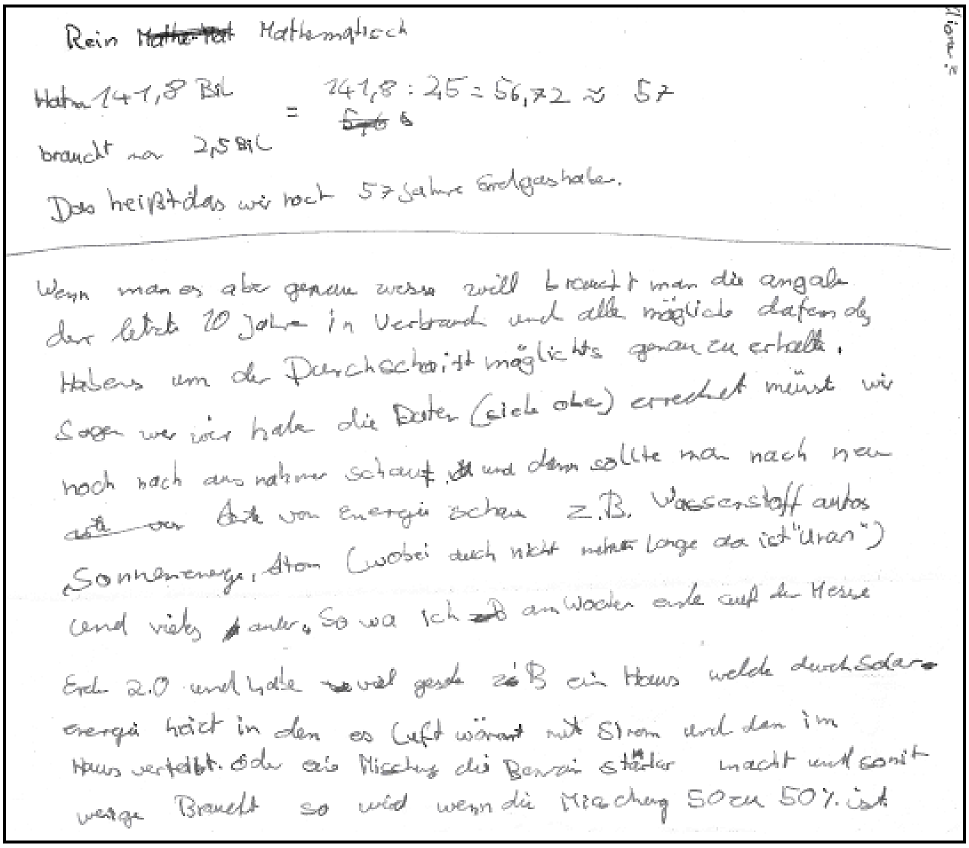

141,8 m³ : 2,5 m³ = 56,72 ≈ 57 Jahre
Bei bisheriger Fördermenge wird die Erdgasreserve etwa 57 Jahre halten. Da es aber immer mehr Menschen gibt wird in den kommenden Jahren wohl auch mehr Gas verbraucht werden. Also würde ich das Prozentuelle Wachstum der Menschheit X ausrechnen und das dann in die Gleichung einfügen.

141,8 m³ : (2,5 m³ + X %)
Da das aber nur das Wachstum von einem Jahr einschließt muss ich mir was neues einfallen lassen.

mmh.. Also rechne ich das prozentuelle Wachstum der Menschheit für die nächsten 55 Jahre y aus.
141,8 m³ : (2,5 m³ + y %)
Wenn die Prozentanteile beim Verbrauch von den verschiedenen Resourcen sich nicht verschieben, wird das Erdgas noch 141,8 : (2,5 + y %) halten.

Fig. 12: Unfinished solution

Some students reported from their own lives' experiences connected to the content without relation to the modelling or their own calculation. The following example shows this:


Rein ~~Mathe~~ Mathematisch
Hatten 141,8 BiL = 141,8 : 2,5 = 56,72 ≈ 57
braucht man 2,5 BiL
Das heißt das wir noch 57 Jahre Erdgas haben.

Wenn man es aber genau wissen will braucht man die angabe der letzten 10 Jahre in Verbrauch und alle möglichen daten dazu um den Durchschnitt möglichst genau zu erhalten. Haben wir um den Durchschnitt möglichst genau errechnet müsst wir sagen wer wir habe die Daten (siehe oben) errechnet müsst wir sagen ... noch nach anderen ... schauen und dann sollte man nach neuen ... Art von Energie schauen z.B. Wasserstoff autos Sonnenenergie, Atom (wobei auch nicht mehr lange da ist "Uran") und vieles andere. So war ich z.B. am Wochen ende auf der Messe Erde 2.0 und habe viel gesehen z.B. ein Haus welche durch Solarenergie heizt in den es Luft wärmt mit Strom und den im Haus verteilt. Oder ein Mischung die Benzin ... macht und somit weniger Braucht so wird wenn die Mischung 50 zu 50 % ist

Fig. 13: Student's own experiences

Although told otherwise, Albert only creates a very simple real model and does not fully recognize time as an influencing variable because he overlooks that the data is from 1993. He also does not describe his creating of a real model. Instead, he describes private experiences from his life in a longer section. The line drawn by him emphasizes that he differentiates between his calculation and his report.

The evaluation also showed that some types of mistakes often occurred simultaneously. It became obvious that mistakes in setting up the real model often went along with mistakes in validating. One reason for that might be that the demanded tasks were new for the students; in contrast to the mathematizing which is practised in regular lessons with the usual simple word problems. Furthermore, it was noticeable that problems with mathematizing, solving the mathematical model and interpreting of complex solutions appeared especially in solutions by students with average or below averages achievements in mathematics.

In summary these mistakes show that it requires far more competencies to solve modelling tasks than solving „standard" mathematical school tasks.

To gain more information about modelling competencies we will now look at metacognitive modelling competencies.

5.3 Metacognitive modelling competencies

A further essential result of the study was that for a great part of the students appropriate metacognitive modelling competencies were reconstructible. While there was only scarce knowledge on the modelling process by the students after the first half of the study, the situation changed immensely until the end of the study. Most of the students showed basic knowledge on the modelling process and beyond that many were able to establish a relation between the tasks and the "metaterms" like "reality", "real model", "mathematical model" and "mathematical solution". A great part of the students appeared to have a connected deeper knowledge on the modelling process after 15 months. This included knowing about the subjectivity of such a process, mistake development and validating a model. In only a few cases, hardly any metacognitive modelling competencies were reconstructible.

Although most of the students developed appropriate metacognitive modelling competencies, misconceptions could be recognized of course. Again, those impart information on metacognitive modelling competencies:

Misconception concerning setting up the real model

- Some students thought that simplifying is the same as guessing. (Concept Map)
- Some students thought they could simplify in such a way that the calculations became as simple as possible.

  "Which steps did you think were most […] important? Well, simplifying is of course important, so that you do not need to calculate too much" (student, 2nd interview, 7/11/02)

  "It's so much more laid-back with the real problems, you leave out this and add that." (student, 1st interview, 1/24/02)

  "Yes, the simplifying was always easy for me, well you could simplify in a way, you find it the most simple…" (student, 2nd interview, 7/11/02)
- Misconceptions concerning the transitions from reality to real model existed. (Concept Map)
- Misconceptions on the real model appeared. (Concept Map)

Misconceptions about setting up the mathematical model

- Some students could not differentiate between the real model and the mathematical model.

  "Well, in written class tests I was not so good, because sometimes I did not understand the difference between the real model and the mathematical model" (student, 1st interview, 1/24/02)

  "Reality is the whole thing here […]. I have to simplify now […] ! And! The mathematical model I can't distinguish right now. (student, 2nd interview, 7/11/02
- The term "mathematical model" could not be explained (Interview)

Which parts of the modelling process did you find easy or difficult? "Well, simplifying reality and finding a mathematical solution was easy. But I have difficulties with the mathematical model." (Albert, 2nd interview, 7/11/02)

"What would be the next step?" Student: "…mathematical model, hhmm, that would be … […] I don't know right now what that would be. (student, 1st interview, 1/24/02)

Misconceptions concerning the mathematical solutions

- Often only a number (and e.g. not a graph or a function) was regarded as a mathematical solution (Concept Map)
- Some students thought that a number always represents an exact and unambiguous result – independent of the way of calculation. (Concept Map)
- Some argued that you can get from the mathematical model to the mathematical solution by rounding. (Concept Map)

Misconceptions concerning the interpretation and validation

- Some students were of the opinion that the validation is always the same.

  "Is it enough if I write for every inquest: The same as usual?" (Frank, Homework on surface area, 1/31/02)

  „Well, validating, it actually is important but I think it is not that important because it was the same in each task so far." (student, 2nd interview, 7/11/02)

- Some students had the impression that the validation represents a debasement of the modelling.

  "Do we have to run down everything now again?" (question of a student in class, 4/22/02)

  "Plus, it is always somehow disappointing writing in the end that everything is simplified and inexact after calculating quite some time." (Frank, learner's diary, handy 12/02/01)

  "And when I come back to reality, I have to first validate and then that it is very inexact." (student, 2nd interview, 7/11/02)

- Some students thought that validating the result or "evaluating" was the same as giving a mark.

  Statement of a student being asked for validation: "We couldn't have done it better." (student, 10/1/01)

  "Evaluation: This will be good!" (student's comment in an exam on the topic handy, 12/17/01)

- Some students could not differ between interpretation and validation. (Concept Map)

General misconception

- Some students thought that it is impossible to make any mistakes because every way of solution is alright. (learners' diaries and interviews)

  "Actually, you cannot make mistakes because nobody can control whether your solution is correct or not." (student, learner's diary, 5/15/01)

  „I only understood the Porsche-task in the last 3-4 lessons fully. When I studied at home then, it was pretty easy since the result is always different. So you can calculate and no one can check if it is correct." (student, learner's diary, 6/26/01)

- Some students regarded the proceeding of experts as exact without knowing much about it whereas they regarded their own proceeding as not exact.

  "I would rather trust the results of the university since our values are estimates and the University Stuttgart has more possibilities to determine the different values." (student, test on the topic "solar energy", 5/7/02)

  "The experts from University modelled the problem in a similar way, but they had far more detailed information. Their real model is more exact than ours." (student, test on the topic "solar energy", 5/7/02)

  "Their values here are all very exact, e.g. demand of warmth, we only had average values" (student, test on the topic "solar energy", 5/7/02)

- Some learners thought mathematics could not help solving real world problems.

"I think it is too mathematical to relate anything to real life." (student, 2nd questionnaire, 7/2/02)

- The entire proceeding within the modelling process is unclear (Concept Map)
- The connection from the knowledge on the modelling process to the modelling examples could not be established, i.e. the attribution of different terms in the modelling process such as real model, simplification or mathematical to one's own proceeding does not succeed (Concept Map).

Summing up, these misconceptions refer to the complexity of metacognitive knowledge of the modelling process.

Altogether it seems to be a highly complex task to develop modelling competencies which should not be underestimated. The huge variety of possible mistakes and misconceptions shows the high performance of the majority of students who developed adequate modelling competencies.

5.4 Factors influencing modelling competencies

Having identified the development of modelling competencies as a very complex challenge we have to ask what factors influence this development.

The following results are based on the analysis of the mistakes and misconceptions (see 5.1), the evaluation of the mathematical capacity and the evaluation of the attitude concerning modelling tasks and mathematics with the help of learners' diaries and questionnaires (Maaß 2004, p.120)

Influencing factors:

A. Sub-competencies in carrying out the single steps of the modelling process.

The explanations above have shown that competencies in carrying out the single steps of the modelling process are necessary. Among these sub-competencies are competencies in setting up a real model and in solving mathematical problems within this model. We will now have a closer look at the mathematical capacity required for these two sub-competencies.

The comparison of the mathematical capacity of the learners at the beginning of the study and the modelling competencies at the end of the study examined by a final modelling test shows in a cross-section survey a connection between these two aspects.

Fig. 14 shows that for a fixed number of points in the test „mathematical capacity", the range of the number of points in the modelling test is not more than 1/3 of all possible points.[6] This means a good mathematical capacity can have a positive impact on the modelling competencies. However the graph shows that variation is possible.

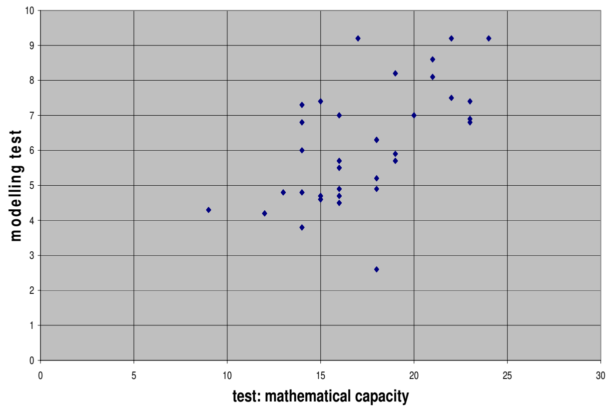


Fig. 14: Connection between mathematical capacity and modelling competencies

B. Metacognitive modelling competencies

Relations between the meta-knowledge about the modelling process and the modelling competencies could be reconstructed for many students. It could be seen that single weaknesses

[6] Two extreme values in this diagram, P(18/2.6) and Q(17/9.2), have been neglected in this statement: A detailed analysis of the learners indicated that selectively worse performances occurred. One of the students achieved clearly more than 2.6 points in the first modelling test. This result seemed to relate more to the form on that day than to the modelling competencies. The other students showed outstanding mathematical capacity in class. The 17 points in the potential test do not reflect her mathematical capacity adequately and also suggest an off-day by the student.

in modelling matched with misconceptions in the metacognitive modelling competencies:

- Misconceptions about the real model could be reconstructed together with deficits in setting up the real model.
- Misconceptions about the validation occurred together with deficits in doing so.
- Parallel developments could be seen. E.g. somebody who became more successful in setting up a real model also corrected misconceptions about the real model.
- There were relations between the quality of meta-knowledge and the competencies in modelling a problem. Normally, very good modellers also had a high meta-knowledge about the modelling process whereas bad modellers had a low meta-knowledge.

Independent from the reconstructible connections between the two areas of competency, the knowledge on the modelling process was seen as helpful by many students. Being asked whether the knowledge helped solving the tasks, they said:

“Well. Yes, because you have an order. And you know better how to do it. Now. With the modelling process.” (Elli, 2nd interview, 7/11/02)

“Yes, I imagined it and then I imagined my different single steps in the model.” (Student, 1st interview, 1/24/02)

“I thought the tasks weren’t that difficult at all, because since they are divided up into four parts you could imagine that easily-I think. […] Yes I actually used it for each task and did them after that pattern, and then solved it like I thought would be the most reasonable way…” (Student, 2nd interview, 7/11/02)

All learners in their comments refer to the knowledge they had on the modelling process and the scheme of it as a help to give orientation. Some of them refer to the visual possibilities of imagination – as shown in the 2nd and 3rd statement.

C. A sense of direction

It became obvious in section 5.2 that some mistakes were related to the entire modelling process: sometimes the learners stopped the modelling without result, mentioned a lot of information on the content without using it for their work or they lost track of their own proceedings. These mistakes point out that the above described sub-competencies are not enough to run through a modelling process. Moreover, the learners should keep an overview over their proceedings and aim at a goal when modelling a problem. An essential part of modelling competencies seems to be competencies for a goal-oriented proceeding. A “sense of direction” must exist.

D. Competencies in arguing in relation to the modelling process

The results of the study show that some students don’t argue in relation to the modelling process but in relation to their private experiences. Others do not describe their proceeding. Important parts of the argumentation are missing. These failures show clearly that the students must learn to argue and to write down their argumentation in relation to the modelling process. Competencies in arguing are necessary.

E. Attitude towards modelling examples and mathematics

The analysis which contrasted and compared cases pointed to a connection between the modelling competencies and the attitudes towards modelling examples and context-free mathematics.

Typical reaction patterns have been observed. Since the data show a close connection between a positive attitude towards modelling examples or mathematics, respectively, and the corresponding performance, four categories can be distinguished (fig. 15).

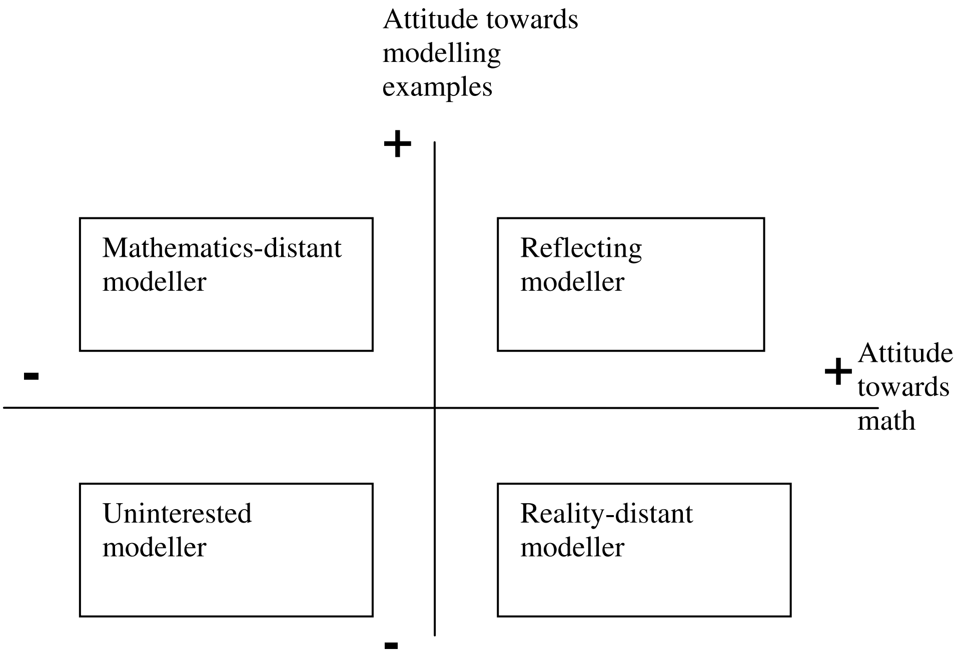


Fig. 15: Types of modellers

In the following, the reaction patterns including failure patterns of the four ideal types will be described in detail.

Ideal type I: "The reality-distant modeller"

Reality-distant modellers have a positive attitude concerning context-free mathematics. They reject modelling examples and are not interested in the contexts of the real-world problem. In conclusion, an affective barrier is set up which mainly results in a lack of competency to solve problems closely connected to context-related mathematics, which means that they have problems with the construction of the real model, the validation and partially also the interpretation.

Ideal type II: "The mathematics-distant modeller"

Non-mathematic modellers clearly give preference to the context of the real-world problem. In contrast, they show negative attitudes towards mathematics and only low performance in maths lessons. These students are very enthusiastic about modelling examples. With the help of their competencies on structuring and analysing of problems they are able to construct the real model and validate the solution quite well. Lack of ability is found in constructing the mathematical model, in finding a mathematical solution and in interpreting complex solutions.

Ideal type III: "The reflecting modeller"

Reflecting modellers have positive attitudes towards mathematics itself as well as towards modelling examples. They show an appropriate performance in mathematics. Deficits on modelling are hardly to be found.

Ideal type IV: "The uninterested modeller"

Uninterested modellers are neither interested in the context of the real-world problem nor in mathematics itself. There are deficits in mathematical competencies. While dealing with modelling tasks, problems occur in every part of the modelling process.

Altogether, these reaction patterns show that the attitudes concerning modelling tasks and mathematics have a high impact on the development of modelling competencies.

Especially, a negative attitude towards the modelling tasks basically appeared to hinder the development of modelling performances. This was especially the case for setting up a real model and for validating the solution which are the steps that have the strongest connection to context-related mathematics. Unreasonable assumptions were made, the validation was superficial and often written explanations were missing for both aspects. The assumption made above is supported by the fact that those students basically managed setting up a real model and validating better whose attitudes towards the modelling examples were positive.

While students who perform well in mathematics were able to largely eliminate existing weaknesses in setting up a real model and validating, students who do not perform that well in mathematics were not able to.

The observed modelling mistakes as well as the influencing factors described above suggest a deeper understanding of modelling competencies. Before this study, the following listing under A (see 2.1) was chosen to preliminary describe the sub-competencies needed to run through the steps of a modelling process.

Based on the results of the study this list is now to be supplemented according to the above list of influencing factors. This new definition is not to be seen as complete since several essential aspects such as linguistic components which are definitely important have not been object to this study. For this reason they could not be included into the description of modelling competencies.

Modelling competencies include abilities and skills to conduct modelling processes adequately and in a goal-oriented way; as well as the willingness to put these abilities and skills into practice.

In Detail modelling competencies contain

A. Sub-competencies to carry out the single steps of the modelling process
    - Competencies to understand the real problem and to set up a model based on reality.
    - Competencies to set up a mathematical model from the real model.
    - Competencies to solve mathematical questions within this mathematical model.
    - Competencies to interpret mathematical results in a real situation.
    - Competencies to validate the solution.

B. Metacognitive modelling competencies

C. Competencies to structure real world problems and to work with a sense of direction for a solution

D. Competencies to argue in relation to the modelling process and to write down this argumentation

E. Competencies to see the possibilities mathematics offers for the solution of real world problems and to regard these possibilities as positive.

6 Consequences and implications

The results of the study show clearly that modelling competencies include more competencies than just running through the steps of a modelling process. This adds to the findings of previous studies. Besides competencies in conducting the steps, important factors are the development of metacognitive modelling competencies, structuring facts, competencies in mathematical arguing and a positive attitude.

These results lead to the following considerations: The various sub-competencies of modelling competencies should be paid attention to in class. The results of the study show that integrating modelling competencies into day-to-day school can be seen as a challenge. Providing the teachers with relevant tasks is not enough. They need to gain independent experiences with modelling first. Furthermore, they need to get to know and test teaching methods that allow them to include modelling examples appropriately and support the relevant competencies.

Therefore, effective teacher training courses need to be developed which are connected to teachers' needs and add to their knowledge as well as support the necessary competencies and understandings.

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Author

Katja Maaß
University of Education
Kunzenweg 21
D-79117 Freiburg
Germany
Email: katja.maass@ph-freiburg.de