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What are the macroeconomic effects of asset purchases?



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ABSTRACT

The impact of announcements of large-scale purchases of government bonds on real GDP and the CPI in the United Kingdom and the United States is explored with a Bayesian VAR, estimated on monthly data from 2009M3 to 2014M5. Four different identification schemes are used, all leaving the reactions of GDP and CPI unrestricted, and the transmission channels of the policy are examined. An asset purchase announcement of 1% of GDP leads to a statistically significant rise of 0.58% (0.25%) and 0.62% (0.32%) rise in real GDP and CPI for the US (UK). The transmission channels differ in the two countries.

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1. Introduction

In response to the 2008–9 financial crisis, both the Bank of England and the Federal Reserve undertook large-scale asset purchases (LSAP), buying government debt as a means of providing monetary stimulus once interest rates were reduced as far as deemed possible. A number of academic studies have examined the effects of this unconventional policy. For example [Chung et al. \(2012\)](#) used the Federal Reserve Board's macroeconomic model to show that real GDP and inflation were respectively 3% and 1% higher as a result of US LSAPs. [Kapetanios et al. \(2012\)](#) used a range of BVAR methods to explore the effects of the Bank of England's purchases, finding that GDP and CPI were raised by 2.5% and 1.5% as a result of the first round of asset purchases in the UK.

This paper takes previous work on asset purchases in four new directions. First, in contrast to most existing studies, three mechanisms are explored through which asset purchases may influence output and prices. Secondly, the passage of time, together with use of monthly data, allows us to estimate our model using only data since March 2009 when the policy was first introduced. This makes our results less susceptible to bias from the introduction of the new policy regime, and hence the Lucas Critique and structural breaks, than any other empirical study of this issue. We also explore whether our results are materially affected if the acute phase of the crisis, in 2009, is omitted from our data. Thirdly, effects found in VAR-based studies (e.g. [Kapetanios et al., 2012](#)) were identified on the assumption that asset purchases led to a rise in real GDP and CPI only through their impact on the long-term interest rate. Here, instead, four different identification schemes are used to identify asset purchase shocks. All of these leave both the transmission mechanism and the responses of real GDP and CPI unrestricted. The possibility implied by [Eggertson and Woodford \(2003\)](#) that, except as a result of signaling the future path of short-term rates, asset purchases have no impact on GDP

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or CPI is therefore not excluded. Finally, all existing VAR studies rely on the imposition of either [Litterman \(1986\)](#) or time-varying parameter priors. Our analysis is carried out using a non-informative normal inverse-Wishart prior, avoiding possible bias from priors that are set too tightly to let the data speak.

Theoretically, asset purchases might affect demand through three different mechanisms. The first is the so-called portfolio balance channel ([Vayanos and Vila, 2009](#)). This relies on the presence of investors with a preferred habitat for a given maturity in the government bond market. If this is the case, purchases of long-term government debt have the effect of reducing yields on debt of the maturities purchased, through their impact on term premia. An alternative mechanism is the signaling channel—the idea that purchases signal that the policy interest rate will remain at its effective lower bound for longer. This was originally suggested by [Eggertson and Woodford \(2003\)](#) and [Bernanke et al. \(2004\)](#). [Gagnon et al. \(2011\)](#) found little evidence to support it but [Bauer and Rudebusch \(2014\)](#) suggested that rather stronger signaling effects were present. A third possible mechanism is that asset purchases help to manage expectations about future economic outcomes and hence reduce economic uncertainty.² All of these channels might lead to wealth effects from higher asset prices and raise consumption and investment.

Our modeling framework allows us to explore which mechanisms may play a role by including relevant variables in the VAR one-by-one. If the portfolio balance (signaling) mechanism is behind the reduction in government bond yields, one should observe a relatively greater reaction of government debt yields (interest rate futures) at longer (shorter) maturities. Inclusion of yields at both maturities makes it possible to establish whether either mechanism is relevant. Further, inclusion of the VIX and a weighted average of implied interest rate futures' volatilities (MOVE) make it possible to examine the impact on uncertainty and risk-taking.

We find that an asset purchase announcement shock worth 1% of nominal GDP, leads to a peak impact of about 0.62% (0.25%) of real GDP and 0.58% (0.32%) in CPI in the US (UK). Conditional forecast exercises, the method of choice for calculating the total impact of QE1 in [Baumeister and Benati \(2013\)](#) and [Kapetanios et al. \(2012\)](#), suggest a total GDP and CPI impact broadly similar to that found by scaling up the peak impacts derived from the impulse response analysis. The overall real GDP and CPI impact of QE1 obtained with our approach is generally only slightly higher than [Baumeister and Benati \(2013\)](#) and [Kapetanios et al. \(2012\)](#) for their US and UK real GDP and CPI responses to spread shocks, despite allowing for more transmission channels. But there is one notable exception: for the UK, our results suggest that the impact on the CPI is almost three times as large as the effect reported in these studies. The implied UK inflation-output trade-off is larger than in the US, meaning that the same change in GDP would have a greater impact on UK inflation. These estimates are, nevertheless, in line with studies of conventional monetary policy for the UK and the US.

In terms of the transmission mechanism, our study suggests that US asset purchases influence yields on medium and long-term government debt, but not interest rate futures, which implies a role for the portfolio rebalancing, rather than the signaling, channel. In contrast, UK purchases do not have clear impacts on either interest rate futures or long rates. In both countries there is evidence that announcements have the effect of reducing measures of financial market and household uncertainty.

The remainder of this paper proceeds as follows. [Section 2](#) explains our model and discusses the details of our identification schemes. [Section 3](#) presents the results and [Section 4](#) concludes.

2. Methodology and data

We use the following VAR model estimated on monthly data:

$$\mathbf{Y}_t = \alpha_c + \sum_{k=1}^L \mathbf{A}_k \mathbf{Y}_{t-k} + \mathbf{e}_t, \quad \mathbf{e}_t \sim N(0, \Sigma) \quad (1)$$

where \mathbf{Y}_t is a vector of the following endogenous variables: the announcement of asset purchases divided by nominal GDP; the log of CPI; the log of real GDP; the yield on the 10-year government bond and the log of real equity prices at time t . \mathbf{A}_k is the array of coefficients associated with the corresponding lagged vector of variables for lag k . \mathbf{e}_t is a vector of residuals at time t . This is assumed to be normally distributed with variance–covariance matrix Σ . When the time-series dimension is small, estimates of \mathbf{A}_k are likely to be imprecise. Previous work has addressed this problem by relying on Bayesian methods of inference and imposing a [Litterman \(1986\)](#), or time-varying parameter, prior. But there is always the risk that tight priors dominate information from the data. Our approach avoids this problem. The model is estimated with a non-informative normal inverse-Wishart prior, as³ in [Uhlig \(2005\)](#) and a lag length, L , of two throughout.⁴

² This is in line with [Woodford \(2003\)](#), who argues that the main transmission mechanism of modern monetary policy is through management of expectations about inflations and real GDP growth.

³ [Jarcinski and Marcet \(2013\)](#) propose imposing priors on the growth rates of variables, as opposed to priors on parameters, as the least controversial way to impose priors in small sample VARs. But it is unclear how to choose suitable priors for variables in our VAR such as real GDP, CPI or asset purchase announcements during this turbulent period of time. That is why it seemed better to use the normal inverse-Wishart prior, with hyperparameters set to small values to ensure that the prior is non-informative ([Uhlig, 2005](#)). See [Appendix D](#) of his paper for more information.

⁴ *Ex ante* lag length tests such as the Hannan–Quinn or BIC criterion suggest a lag length of 2. When our model was estimated with six lags, it was, as a result of the short time-series, necessary to use a [Litterman \(1986\)](#) prior, with the hyper-parameters estimated from the data following the approach in [Giannone et al. \(2015\)](#). This suggests that a 1% US (UK) asset purchase announcement leads to a peak impact of .53 (.23) and .61(.37) on real GDP and CPI, respectively. These values are almost identical to those found with two lags and described in [Section 3.1](#).

Table 1
Identification schemes.

	p Log CPI	y Log real GDP	AP Asset Purchases	i_t Long Interest Rate	sp_t Log Real Equity Price
Identification Scheme I					
Log CPI	1	0	0	0	0
Log real GDP		1	0	0	0
Asset Purchases			1	0	0
Long Interest Rate				1	0
Log Real Equity Price					1
Identification Scheme II					
Supply Shock	–	+		+	+
Demand Shock	+	+		+	+
Asset Purchase Shock			+	–	+
Identification Scheme III					
Supply Shock	–	+	0		
Demand Shock	+	+	0		
Asset Purchase Shock			+		+
Uncertainty Shock			+		–
Identification Scheme IV					
Variance Decomposition Restrictions					
Supply Shock	–	+		$\frac{Var(Shock)}{Var(Asset\ Purchases)} < MAX(\frac{Var(Shock)}{Var(Asset\ Purchases)})$	
Demand Shock	+	+		$\frac{Var(Shock)}{Var(Asset\ Purchases)} < MAX(\frac{Var(Shock)}{Var(Asset\ Purchases)})$	
Asset Purchase Shock			+	$\frac{Var(Shock)}{Var(Asset\ Purchases)} = MAX(\frac{Var(Shock)}{Var(Asset\ Purchases)})$	

This table shows the restrictions imposed by each of the four identification schemes. Gray shading indicates that the response of the variable (in the column) to the shock (in the row) is unrestricted, + indicates that it is restricted to be non-negative, 1 to be 1, 0 to be zero and – to be non-positive.

2.1. Identification

The challenge for structural VAR models is to disentangle orthogonal, structural economic shocks, $\epsilon_{c,t}$, from the correlated reduced form shocks $e_{c,t}$. This is typically done using a matrix C_0 , such that $C_0 e_{c,t} = \epsilon_{c,t}$. We use four ways of inferring C_0 , zero restrictions, sign restrictions, a combination of zero and sign restrictions, and finally sign variance decomposition restrictions. All of these identification schemes are described in Table 1.

Identification scheme I uses a lower-triangular scheme, with asset purchases ordered after real GDP and prices, but before all of the other variables. The identifying assumptions are therefore that output and prices react with a lag and that aside from responding to these two, asset purchases do not react to any other variable upon impact.

VAR identification schemes that employ timing exclusion restrictions have been criticized in recent years, on the grounds that such restrictions do not naturally emerge from DSGE models. Canova and de Nicoló (2002), Faust and Rogers (2003) and Uhlig (2005) have therefore proposed identifying shocks by means of the implied signs of the impulse responses that they produce. Clearly, for identification restrictions of this type to be valid, they need to be strongly supported by economic theory. In the presence of financial frictions, such as imperfect substitutability between long and short bonds (Harrison, 2012) or preferred habitat investors (Vayanos and Vila, 2009), economic theory does suggest that a rise in asset purchases will lead to a fall in the interest rate on long-term bonds, by reducing term premia. But even in the absence of frictions,

announcements of asset purchases can signal that the short-term interest rate is going to stay lower for longer (Eggertson and Woodford, 2003), depressing the long rate. Secondly, lower yields on longer maturity bonds are likely to lead to some reallocation towards other assets, such as equities, generating a rise in real equity prices. Thus our definition of an asset purchase shock is that it leads to lower long-term rates and higher equity prices.

We also identify aggregate demand and supply shocks. A demand shock would typically lead to a rise in prices and output. The rise in prices, together with the fact that firms may require greater finance for production, is likely to lead to a non-negative response of the long interest rate. The rise in demand would also lead in expected profits and thus to a rise in real equity prices. The sign restrictions used to identify an aggregate supply shock are identical, other than assuming that prices fall rather than rise. This identification scheme, referred to as scheme II throughout the paper, is summarized in Table 1 and implemented with the QR approach presented in Rubio-Ramírez et al. (2010). Unless otherwise noted, all sign restrictions are imposed upon impact and one month thereafter with the exception of asset purchase announcements, where the sign restriction is imposed upon impact and for 5 months thereafter here and also in identification schemes III and IV.

In identification scheme II, the assumption is that asset purchases affect the real economy via portfolio rebalancing from long-term government bonds into equities, to distinguish them from aggregate supply and aggregate demand shocks. But *a priori* it is not clear to what extent the mechanisms that are required for asset purchases to affect the yield on long-term government debt operate in reality. More importantly, to distinguish asset purchase from aggregate supply shocks, it is necessary to assume that long-term interest rates rise in response to an aggregate supply shock. Theoretically, a positive aggregate supply shock may lead to a rise in investment, competition for funds and higher bond yields, but also a decline in bond yields as a result of the monetary policy reaction to lower consumer prices. Empirically, Dedola and Neri (2007) and Peersman and Straub (2009) examine the reaction of the short-term interest rate in response to technology shocks in SVARs for the US and Euro Area, respectively. Peersman and Straub (2009) show a positive medium-term reaction of the short rate to technology shocks, while Dedola and Neri (2007) find no significant effect. While the long rate restrictions are thus consistent with their results, we nevertheless drop them in identification scheme III below.

This is possible, as long as one is willing to make the assumption that asset purchases do not react contemporaneously to aggregate demand and aggregate supply shocks. In that case, the restriction on real equity prices is sufficient to distinguish these shocks from asset purchases. Given that monetary policy makers do not observe aggregate demand or supply shocks within a month, the assumption of a zero contemporaneous reaction of asset purchases to aggregate demand and supply shocks is realistic. An additional advantage is that this allows us to identify a fourth shock, namely a rise in uncertainty/risk premia. This shock is identified as a decline in real equity prices, to which the monetary policy authority reacts with a rise in asset purchases, perhaps as a result of a coincident financial crisis. Unlike demand and supply, these types of shocks can be observed in real time. This identification scheme is referred to as identification scheme III throughout. It is implemented using the procedure in Arias et al. (2014), who generalize the standard QR restrictions algorithm to include zero restrictions as well. Ours is, of course, not the only paper to use a combination of zero and sign restrictions to identify unconventional monetary policy shocks. Gambarcorta et al. (2014) adopt a similar approach.

Identification schemes I–III rely on the idea that shocks can be distinguished based on restrictions on impulse responses. But it is also possible to use variance decomposition restrictions to separate different economic shocks (Faust and Rogers, 2003; Uhlig, 2005). The idea here is that a shock that is variable-specific shocks should explain the largest fraction of the variance in that variable.⁵ In identification scheme IV, asset purchase announcement shocks are assumed to explain the largest fraction of variation in asset purchases upon impact and with a three period delay. This makes it possible to drop the zero restrictions and also the sign restrictions on real equity prices. This scheme is implemented in a fashion similar to identification scheme II, with the QR approach by Rubio-Ramírez et al. (2010), but rather than keeping impulse responses which are consistent with a particular sign, only those consistent with the variance decomposition restrictions in Table 1 are retained.

At present the theory underlying asset purchases is not sufficiently well understood to devise an identification scheme which would allow us to identify asset purchase announcement shocks perfectly. It is for this reason that we sequentially relax the strongest identification restrictions from the first scheme to the last one. Despite this pecking order, it is nevertheless not possible to claim that one scheme is necessarily better identified or preferable to another. As a result we study the effects of asset purchases in all four cases paying particular attention to results which are significant with at least three of the four schemes adopted in this paper.

2.2. Data

All of the VAR models in this paper are estimated on monthly data for the period when asset purchases were an active policy tool in both the UK and the US, from 2009m3 to 2014m5. Monthly real GDP data for the UK are provided by the National Institute of Economic and Social Research (Mitchell et al., 2005), while monthly real GDP data for the US are taken from Macroeconomic Advisers.

The monthly indices of consumer prices are the official measures. UK Value Added Tax, an important fiscal contributor to CPI movements, was reduced from 17.5 per cent to 15 per cent in January 2009, raised to 17.5 per cent in January 2010 and raised

⁵ Our approach is similar in spirit, but not technique, to the penalty function approach first proposed in Uhlig (2005).

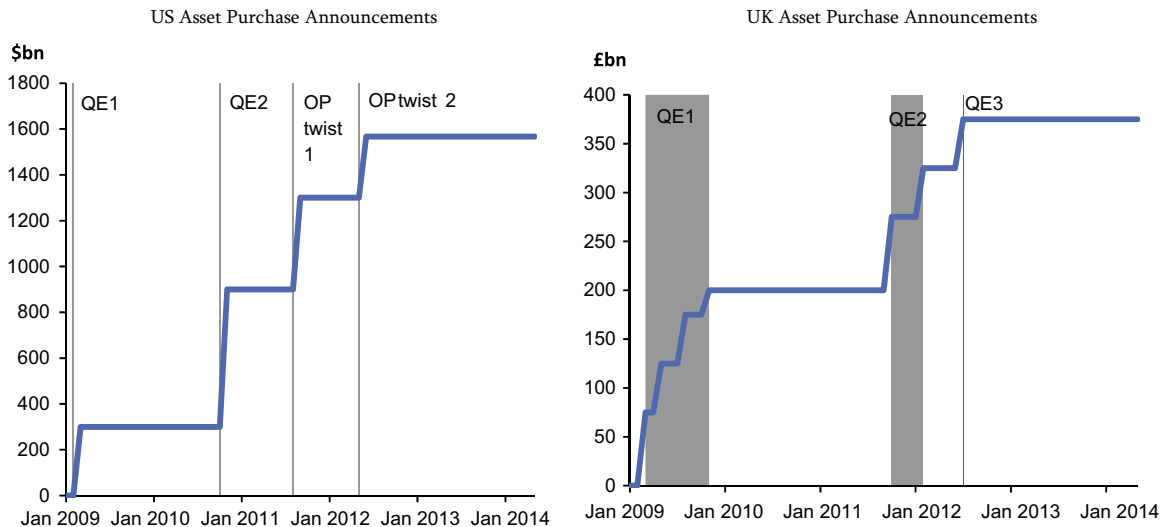


Fig. 1. Asset purchase announcements in the US and the UK. This figure shows total asset purchase announcements in the US and the UK over time, collected from public Federal Reserve and Bank of England statements. The US data do not include announcements about open-ended purchases. These are discussed in Section 3.4.

further to 20 per cent in January 2011. Use of the official CPIY monthly index makes it possible to avoid the distortions this introduces. CPIY excludes the immediate effects of changes to indirect taxes. For the United States the variable used is the consumer price index published by the Bureau of Labour Statistics. Real equity prices are calculated by obtaining monthly averages of daily data for the FTSE100 and S&P500 obtained from Thomson DataStream and deflating by CPIY and CPI, respectively.

The asset purchase announcement series are constructed in the following manner: For the UK, asset purchase announcements are simply cumulated over time. For the US, we treat asset purchases associated with the maturity extension program (Operation Twist) as additional asset purchases, attaching the same weight to them as asset purchase announcements of government bonds financed with the issue of central bank reserves. The effect of giving them a smaller weight is explored in Section 3.4. The asset purchase series are shown in Fig. 1. Unlike the UK, the US also announced open-ended asset purchases. The effects of these are also explored further in Section 3.4. The series for the UK asset purchase announcements are computed from the published Minutes of the Monetary Policy Committee. For the US, these data are taken from Federal Reserve Board announcements.⁶

In order to explore the three possible transmission mechanisms it is necessary to study the impact, if any, of asset purchases on other variables. If the portfolio balance channel is the main transmission channel, one would expect a relatively large impact on the yields of 20 and 30-year government bonds. The signaling mechanism, on the other hand, implies a relatively stronger reaction of the Overnight Index Swap (OIS) futures of the 3-month interest rate, 6 months, 1 year and 2 years ahead. We also examine the impact of our identified asset purchase shocks on two financial market indicators of uncertainty: the VIX (implied stock market volatility) and the MOVE (weighted average of implied interest rate volatilities at different horizons). To tease out whether movements in these variables reflect economic uncertainty or investors' risk appetite, we also look at a survey measure of household uncertainty and the BBB-AAA corporate bond spread. Details of the data are provided in online Appendix E.

3. Results

This section describes our main results. It compares them with earlier work which has studied asset purchases through the impact on the yield curve. The transmission mechanism and the robustness of our findings are also explored.

3.1. Main results

Fig. 2A and B show the results for both countries for each of our four identification schemes, with the row labels indicating the scheme in question.

An inspection of Fig. 2A and B clearly suggests that regardless of identification scheme, real GDP and the CPI always rise in response to an asset purchase shock. This effect is statistically significant throughout, except for identification scheme I for CPI in the UK. Table A1 in online Appendix A shows the maximum impacts of the median and indicates their significance.

⁶ The series are used as percentages of annualised 2009Q1 nominal GDP in order to avoid the endogeneity issues which might arise from scaling by current nominal GDP.

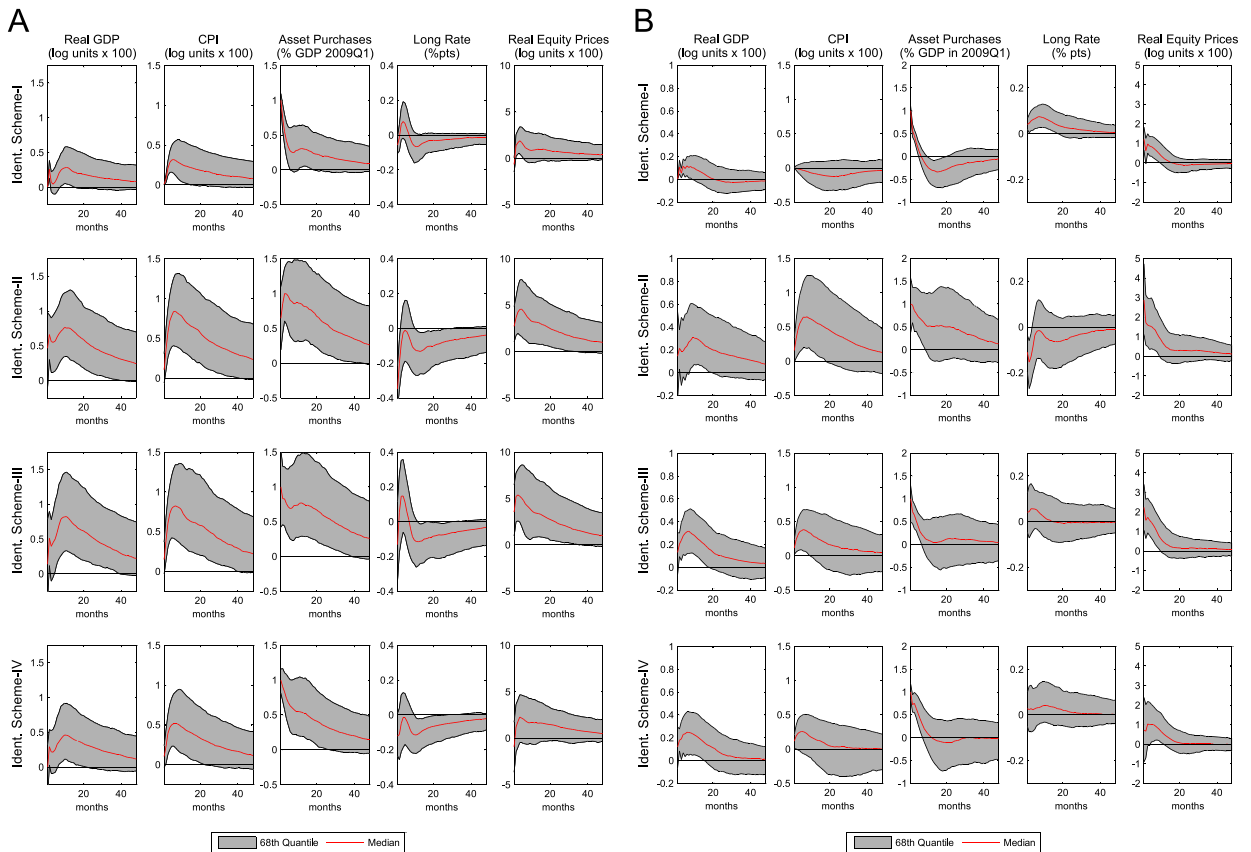


Fig. 2. (A) Results for the standard specification – United States. This figure shows, for each of the variables in our model, the median impulse responses in response to an unexpected 1% asset purchase announcement as a fraction of 2009Q1 GDP, together with 68% Bayesian credible sets. We show results for all four identification schemes for the US. 10,500 simulations, with the first 10,000 as burn-in, were used to generate the responses. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement. (B) Results for the standard specification – United Kingdom. This figure shows, for each of the variables in our model, the median impulse responses in response to an unexpected 1% asset purchase announcement as a fraction of 2009Q1 GDP, together with 68% Bayesian credible sets. We show results for all four identification schemes for the UK. 10,500 simulations, with the first 10,000 as burn-in, were used to generate the responses. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement.

For both countries the maximum values for the impact on both GDP and CPI are higher with identification schemes II, III and IV than they are with scheme I. This probably reflects the role that economic theory plays in identifying the effects with these schemes. Averaging across all four schemes, the maximum impact on GDP is 0.58 in the United States and 0.25 in the United Kingdom (Table A1, online Appendix A). The figures for the CPI are 0.62 and 0.32, respectively.

Baumeister and Benati (2013) and Kapetanios et al. (2012) use a conditional forecasting approach to quantify the impact of QE on real GDP and CPI in the US and the UK respectively. Online Appendix B contains the results of a similar exercise (Waggoner and Zha, 1999) which suggest that QE1 raised GDP in the US (UK) by about 2 (4) percentage points at its peak impact; QE2 (QE2/3) added about 6 (4) percentage points. The CPI in the US was increased by an amount similar to the increase in GDP in each case while in the UK both QE1 and QE2/3 raised the CPI by just under 6 percentage points. As discussed in detail below, scaling up the peak impacts derived from the impulse response analysis yields broadly similar numbers.

To relate our multipliers to those presented in previous work, we compare the effects of the US and the UK QE1 implied by the impulse responses in those studies, to the peak impact implied by the impulse responses in this paper. Baumeister and Benati (2013) for the US and Kapetanios et al. (2012) for the UK argue that the first round of asset purchases in the US and the UK led to fall of about 100 basis points in the spread between the long-term and short-term interest rate. It is then easy to see that the estimates in those papers imply a rise of 1.08 (2.5)% and 0.9 (1.5)% in GDP and CPI in the US (UK), respectively. During QE1, the Federal Reserve and the Bank of England engaged in government bond purchases worth 2% and 14% of annualized 2009Q1 GDP, respectively. Based on the estimates in this paper, this would lead to a rise of 1.12 (3.08)% and 1.2 (4.2)% in US (UK) real GDP and CPI, respectively. When the effect of MBS purchases is included, the estimates for US real GDP and CPI become 1.4 and 1.5. For the UK, the impact on real GDP is slightly higher than previous work, but the

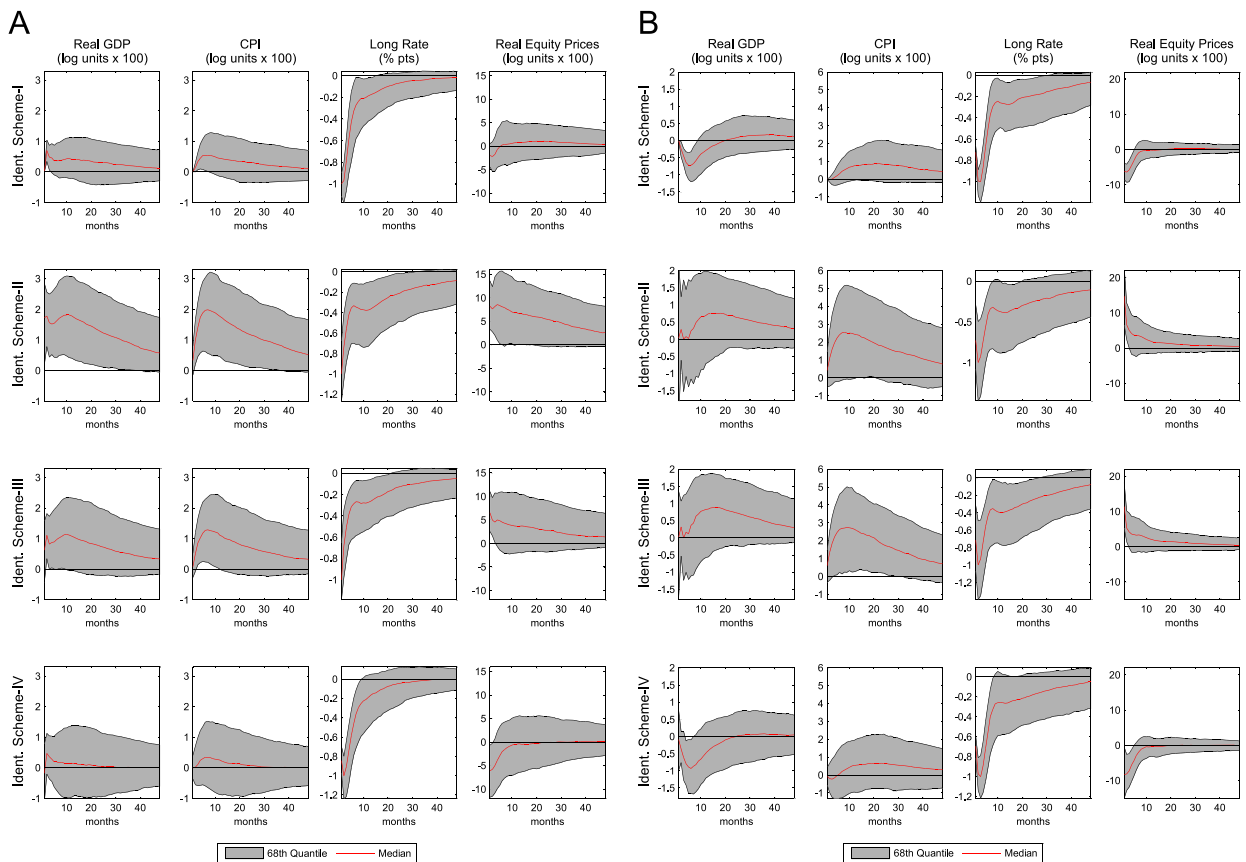


Fig. 3. (A) Results for a shock to the long rate – United States. This figure shows, for each of the variables list above, the median impulse responses in response to a 1% decline in the long rate, together with 68% Bayesian credible sets. We show results for all four identification schemes for the US. 10,500 simulations, with the first 10,000 as burn-in, were used to generate the responses. (B) Results for a shock to the long rate – United Kingdom. This figure shows, for each of the variables list above, the median impulse responses in response to a 1% decline in the long rate, together with 68% Bayesian credible sets. We show results for all four identification schemes for the UK. 10,500 simulations, with the first 10,000 as burn-in, were used to generate the responses. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement.

CPI response is almost three times as large. This difference is not statistically significant but produces estimates for the inflation-output trade-off which are more in line with previous studies of conventional monetary policy in VARs for the UK.⁷

3.2. Comparison with the term spread shock approach

These quantitative differences may arise as a result of the identification scheme or the data on which the model was estimated. Both can be explored. First, our model structure can be modified to explore the role of shocks to the long rate. Since policy rates were constant in both countries, we simply apply the same identification scheme, but with the long rate substituted for asset purchases.⁸ This allows us to explore the role of long rates as a possible transmission route of asset purchases. Fig. 3A and B show the impulse responses from this exercise and Table A2 of online Appendix A their maximum impact. An unexpected shock leading to a 100 basis points decline in the long rate has the effect of raising both GDP and CPI by about 1.06% only in the United States.

These results, and quantitative estimates of QE1 obtained with our approach, which are very similar to previous findings, are consistent with the idea that the US asset purchase announcements affect real activity through their impact on long-term government bond yields. In other words, the spread identification scheme used by previous work might be correct for

⁷ A comparison to previous work can be found in online Appendix D.

⁸ For identification scheme I, we order the long rate after output and prices, but before real equity prices. For identification scheme II, it is assumed that the long rate falls and real equity prices rise in response to unconventional monetary policy. For the identification scheme III, it is assumed that aggregate supply and demand shocks cannot affect the long rate contemporaneously. The risk/uncertainty shock is then identified as a shock that leads to a decline in real equity prices and the long rate, while the unconventional monetary policy shock is identified as a shock that leads to a decline in the long rate and rise in real equity prices. Finally, in identification scheme IV, it is assumed that a long rate shock should explain the greatest fraction in the long rate forecast error variance decomposition. All other restrictions remain the same.

the US. For the UK, however, they suggest that the influence of asset purchases most likely affected GDP and CPI through channels other than the long rate.

In online [Appendix C](#) we show that our results do not depend on the inclusion of the first round of asset purchases in the data set. [Fig. C1](#) shows results estimated over the period 2010m3–2014m5. This omits the period when the financial crisis was at its most extreme. Our results are not greatly affected, suggesting that the impact of the second and third rounds of purchases in the UK and the US was not very different from the impact of the first round. This suggests that asset purchases did not become less effective over time. [Fig. C2](#) looks at results estimated from 2007m1–2014m5. We now find that the effects on GDP are larger than in [Fig. 2A](#) and B. There is no significant effect on CPI in the UK with any of the identification schemes although the median impulse remains positive in all four cases. The inclusion of the UK data before asset purchases were introduced might therefore explain why previous work found a smaller effect on CPI inflation; indeed our estimates ([Table C2](#)) for this period imply an inflation/output trade-off of 0.37 rather than the value of 1.3 implied by our main results. This confirms our view that analysis over this extended period may be subject to the Lucas critique: in this case it seems to bias the UK inflation response to unconventional monetary policy to be substantially lower than we find it to be. For the US, [Tables C1 and C2](#) show that the quantitative magnitudes are larger, but the relative impact on output and inflation remains the same as in the base line case.

In summary, this suggests that the observed differences from previous work arise from both the inclusion of pre-asset purchase data and the difference in identification schemes in the latter. The resulting biases are substantially larger for the UK than the US.

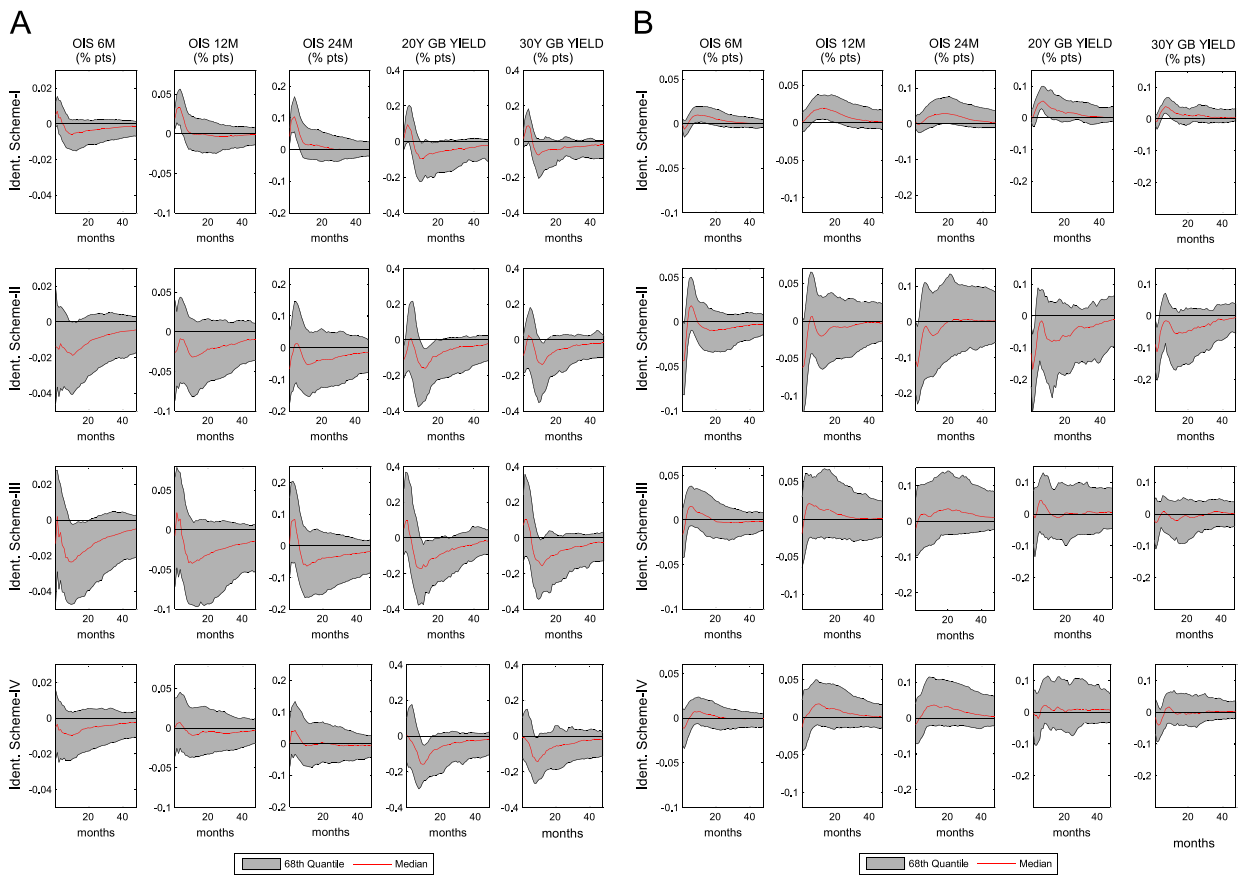


Fig. 4. (A) Results for Portfolio Balance & Signaling Channels – United States. This figure shows, for each of the variables list above, the median impulse responses in response to an unexpected 1% asset purchase announcement as a fraction of 2009Q1 GDP, together with 68% Bayesian credible sets. We show results for all four identification schemes for the US. 10,500 simulations, with the first 10,000 as burn-in, were used to generate the responses. OIS 6M, OIS 12M and OIS 24M are the 3-month rate, 6 months, 12 months and 24 months ahead. 20Y and 30Y GB YIELD are the yields on 20 and 30 year government debt. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement. (B) Results for Portfolio Balance & Signaling Channels – United Kingdom. This figure shows, for each of the variables list above, the median impulse responses in response to an unexpected 1% asset purchase announcement as a fraction of 2009Q1 GDP, together with 68% Bayesian credible sets. We show results for all four identification schemes for the UK. 10,500 simulations, with the first 10,000 as burn-in, were used to generate the responses. OIS 6M, OIS 12M and OIS 24M are the 3-month rate, 6 months, 12 months and 24 months ahead. 20Y and 30Y GB YIELD are the yields on 20- and 30-year government debt. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement.

3.3. Evidence on transmission mechanisms

Economic theory suggests three different ways in which asset purchases can affect demand. First there is portfolio rebalancing. If investors have preferred habitats, then asset purchases will either affect yields with the highest interest rate risk or yields at the maturity purchased through the impact on duration and scarcity, respectively. This should be reflected in a reduction of the term premium rather than a reduction in expected future spot rates. An alternative friction which leads to similar effects comes from the presence of transaction costs leading to imperfect substitutability in the government bond market (Harrison, 2012). These changes, together with associated spillovers into equity and private debt markets, are likely to lead to increases in both consumption and investment.

Secondly, as Bauer and Rudebusch (2014) argue, any announcement of unconventional policy may mean that interest rates will be kept at the zero lower bound for longer. In other words, the expected average short-term interest rate will decline as a result of the announcement. A reduction in either component, the term premium (portfolio rebalancing) or the average expected short-term interest rate (signaling) will lead to a decline in the long-term interest rate with subsequent impacts on demand. But portfolio rebalancing, through the impact on term premia, is likely to have a relatively larger impact on 20 and 30-year maturity government bond yields, while the signaling channel should be reflected in movements of short-term interest rate futures. We include all of these variables into our model to assess which effect is stronger.

Thirdly, asset purchase announcements can help the central bank to manage households' and firms' real GDP growth and inflation expectations. If asset purchases make people more confident that the monetary authorities have a means of supporting the economy despite the fact that short-term interest rates are at the zero lower bound, then the perceived variance of future output and inflation will decline. This is likely to result in a decline in measures of financial market uncertainty. This can reflect either a reduction in household uncertainty about durable consumption or greater risk-taking by investors, as they search for yield. Both of these channels can support demand by raising consumption and reducing premia (spreads) on risky lending.

The reduced-form nature of structural VARs does not allow us to decompose the estimated impacts into contributions from these different transmission channels directly. We can, however, identify variables which we would expect to be affected by asset purchases if each of these mechanisms plays a role in the transmission of this policy.

The extent to which each transmission mechanism should operate in each country clearly depends on the presence of financial frictions in the government bond market. The average maturity of government bonds was 4.2 and 14 years in the US and the UK government bond markets at the end of 2007, which implies greater liquidity premia (transaction costs) in the US government bond market. Similarly, Asian central banks are natural preferred habitat investors in the US government bond market (Turner, 2011). This descriptive evidence would suggest a greater *ex ante* role for the portfolio balance channel in the US, rather than the UK. To examine if this is the case, we include the yields on government bonds of 20 and 30 years maturity, as well as the 3-month rate, 6, 12 and 24 months ahead, as the sixth variable in the VAR. The results are shown in Fig. 4A and B and the maximum impacts in Table A3 of online Appendix A.

Table A3 demonstrates that, for the US, two sets of results show significant effects with at least three of the identification schemes. Twenty-year bond rates are affected by asset purchase shocks in all four schemes, while 30-year bond yields are significant in three out of four schemes. While negative effects are also found more generally, they are not significant. Table A3 also illustrates the problems associated with relying on only one identification scheme. In particular, identification scheme two suggests that most of the included variables are significant, particularly for the UK.

Looking first at the US, Fig. 4A shows that the impact on 20-year debt yields is similar to that found with long (10-year) bond yields shown in Fig. 2A, although of course for scheme II that was an identifying assumption. Interest rate futures tend to move in the right direction, but are not statistically significant. A reasonable conclusion is that these results provide evidence for the portfolio balance channel operating in the US, or that asset purchases affected yields on long-term debt.

In the UK there is greater sensitivity of the results to the identification scheme. Both the long-term government bond yield and interest rate futures react in a statistically significant manner only with scheme II. Thus the responses of OIS and long-term rates do not, overall, provide good evidence that either portfolio balance or signaling play significant roles in the UK.

Christensen and Rudebusch (2012) use several dynamic term structure models to decompose the movements in long rates which they associate with asset purchases. They find that in the US the movements were largely the result of expected future short rates (*signaling*) while in the UK falls in term premia dominated (*portfolio balance channel*). There are several reasons why our results may differ from those of Christensen and Rudebusch (2012). First of all, they estimate their models using daily data stretching from the late 1980s until the end of 2010, since a long sample is needed to mitigate biases in the estimation of their model. As a result they need to make the assumption that asset purchase announcements are just normal shocks to the Treasury bond market. However, if this assumption is violated, their results may be susceptible to the Lucas Critique. Secondly, they look at the immediate impacts of announcements of asset purchases at daily frequency, while we are interested in movements at lower frequencies. Thirdly, their model assumes that the path of interest rates can be represented by a Brownian motion. This assumption may be invalid for the shorter end of the yield curve whose path might be explicitly constrained by the zero lower bound.

Asset purchases can, of course, also have a direct impact on the real economy by reducing uncertainty and managing expectations about future economic outcomes. Deaton (1992) shows how uncertainty depresses the current level of consumption and Dixit and Pindyck (1994) how it reduces investment. Woodford (2003) argues that the one of the most

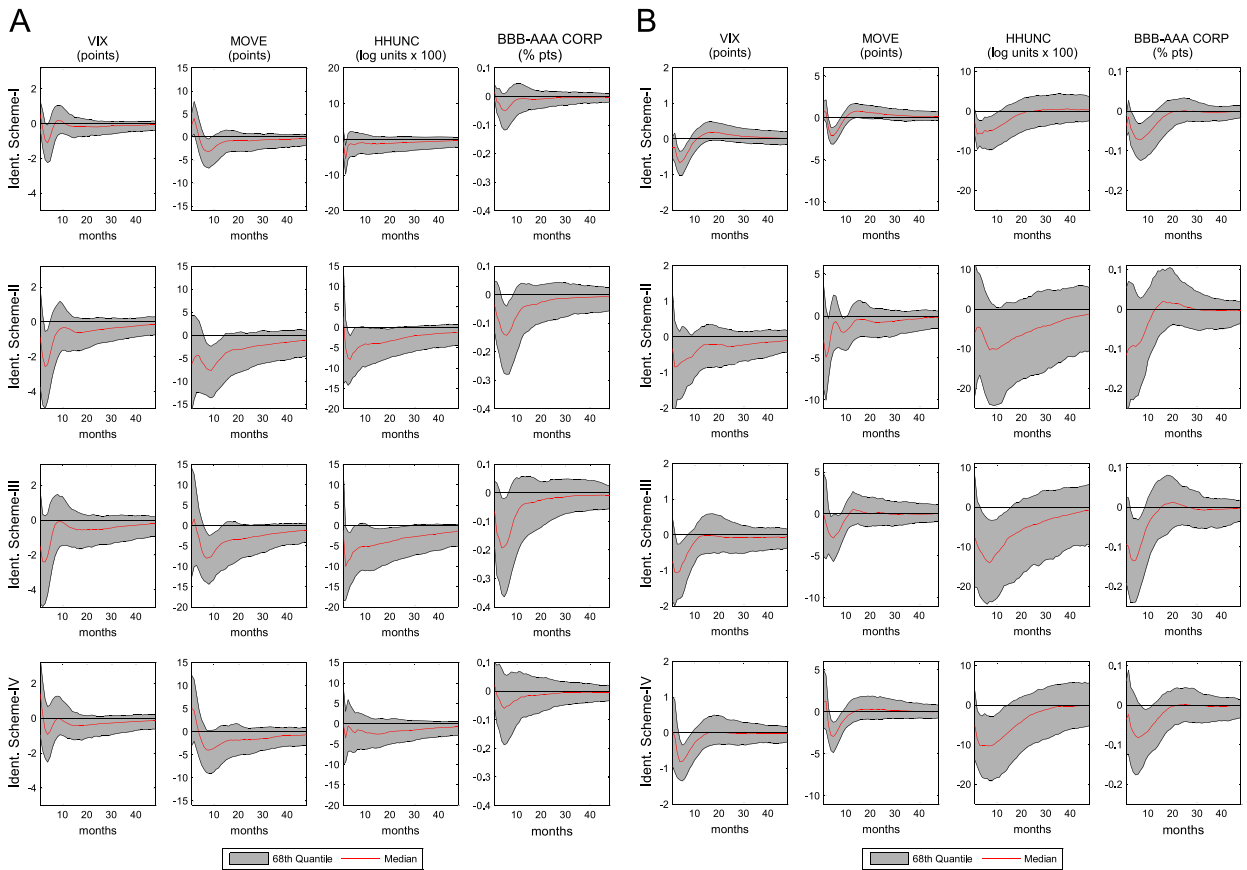


Fig. 5. (A) Results for the uncertainty channel – United States. This figure shows the response functions of the VIX, the MOVE, a measure of household uncertainty (HHUNC) and the spread between BBB and AAA corporate bonds to an asset purchase shock. Results are shown for the US for each of our four identification schemes. Five hundred simulations were used to generate the responses. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement. (B) Results for the uncertainty channel – United Kingdom. This figure shows the response functions of the VIX, the MOVE, a measure of household uncertainty (HHUNC) and the spread between BBB and AAA corporate bonds to an asset purchase shock. Results are shown for the UK for each of our four identification schemes. Five hundred simulations were used to generate the responses. The units of the vertical axes are shown for each column, while the horizontal axis indicates the number of monthly time periods since the announcement.

important transmission channels of monetary policy is the management of expectations about future economic outcomes while [Boivin et al. \(2012\)](#) argue that there is empirical support for this view. Expectations management is likely to reduce uncertainty, having effects on demand which do not need to be transmitted through financial markets, although it may reduce market risk premia.

We explore the impact on two measures of financial market uncertainty for that purpose: the implied volatility of the share price index (VIX) and interest rate futures (swaptions) in each country (MOVE). Two of the interpretations taken by previous work are that these measures reflect real economic uncertainty ([Bloom, 2009](#)) or investors' risk appetite ([Bruno and Shin, forthcoming](#)). To disentangle these two different interpretations, household survey measures of uncertainty and the BBB–AAA corporate bond spread into are included as a sixth variable in our VAR model to establish whether there is a significant response to asset purchase shocks. The results are shown in [Fig. 5A and B](#) and the peak impacts are found in [Table A4](#) of online [Appendix A](#).

[Fig. 5A and B](#) and [Table A4](#) demonstrate that both the VIX and MOVE show significant movements in the UK, while only MOVE does so in the US. [Bloom \(2009\)](#) argues that the VIX is a reflection of uncertainty. On the other hand, [Adrian and Shin \(2010\)](#), [Bruno and Shin \(forthcoming\)](#) and [Miranda-Agrippino and Rey \(2013\)](#) argue that the VIX is a reflection of investor's risk appetite. Interestingly, the reaction of household uncertainty over durable purchases suggests that the first interpretation is relevant for both countries. Similarly, the fact that corporate bond spreads react significantly in three of the four identification schemes for the UK only is stronger evidence that the risk-taking channel plays a role in the UK.

3.4. Robustness

We examine the robustness of our results from two perspectives. First, we investigate whether they may be subject to omitted variable bias and then explore whether they are materially affected by the way in which the announcements of asset purchases are defined.

3.4.1. Omitted variable bias

Due to the short sample size, our baseline model consists of five variables. But it is well known that small VARs may suffer from omitted variable bias. The asset purchase shock may reflect the reaction of the monetary authority to coincident developments, such as domestic fiscal policy, the Euro Area crisis, real oil prices and monetary expansion by the European Central Bank. This can be explored by including the domestic government budget balance to GDP ratio, the public debt to GDP ratio, the spread between Italian and German 10-year government bond yields, the natural logarithm of real oil price in the US dollars/UK sterling, the ratio of the ECB's total assets on its balance sheet to Euro Area GDP, the trade balance to GDP ratio and the real exchange rate one by one in our VAR. The impulse response charts are shown in online [Appendix C, Figs. C3–C9](#) while [Tables C3–C9](#) make clear the maximum effects and statistical significance.

A comparison of the base results for the mean maximum effects in [Table A1](#) with those in [Tables C3–C9](#) shows that these results are robust to the inclusion of additional variables. For the United States all four identification schemes continue to show a significant impact on CPI; for the United Kingdom scheme I does not show a significant effect, just as it did not in our basic model. However, in the majority of cases our main effects of interest are statistically different from zero and they are quantitatively very similar to the estimated effects from the model presented in [Fig. 2A](#) and [B](#).

3.4.2. Definition of the announcement series

Our model assumes that macroeconomic variables tend to respond to announcements, rather than actual asset purchases. But it is worth verifying whether our results are robust to using the actual amount of assets purchased instead. Similarly, in contrast to the UK, the nature of asset purchases in the US has changed over time, with the Federal Reserve engaging in Operation Twist and open-ended purchases, as well as purchases of mortgage-backed securities. This means that a number of assumptions were needed to create the asset purchase announcement series for the US and it is demonstrated that our results are robust to all of them in this section.

There were six possible alternatives for the United States; only the first of these is also relevant to the United Kingdom. Looking at the amount of assets purchased rather than the announcements, the impulse response effects ([Fig. C10](#)) remained significantly above zero in all four cases.

In our asset purchase announcement series, announcements associated with the Federal Reserve's maturity extension program (also known as Operation Twist) receive the same weight as asset purchases of government bonds that were financed through the issue of central bank reserves. While it is clearly difficult to pinpoint the right weight for Operation Twist announcements, we also explore results with a weight of one half, probably a reasonable lower bound, in [Fig. C9](#). These results are similar to our baseline results, but not significant for identification scheme IV for the real GDP response. If the portfolio channel is an important part of the transmission mechanism in the United States, as the earlier results suggest, then perhaps it is not surprising that significance is reduced when Operation Twist is down-rated.

The Federal Reserve Board also announced open-ended purchases of government bonds at a rate of \$US 45 billion per month in 2012. It is unclear how to translate the magnitude of this announcement to one that is comparable to other US asset purchase announcements. At the time of the announcement, guidance was also provided that the federal funds rate would stay low until unemployment had reached the 6.5% threshold. FOMC minutes that accompanied the announcement suggested that this would be met in 2015, implying that purchases would continue for at least 3 years. One way of calculating the economic impact of the open-ended asset purchase announcement is therefore to calculate the present value⁹ of an asset that pays \$US 45 billion each month, for 36 months. This suggests that the economic impact of the open-ended asset purchase announcement was about \$US 1217 billion. Financial markets may of course take a different view and an examination of OIS rate futures data suggest that they expected a rise in the 3-month OIS rate 24, but not 12 months ahead. Assuming that open-ended QE will expire after 18 months yields an economic impact of \$US 702 billion, which is similar to the impact of the second asset purchase announcement (\$US 600 billion). In December 2013, the FOMC announced that the rate at which assets were purchased would slow. This announcement is treated as an unwinding of the open-ended purchases, meaning that \$US 1217 billion and \$US 702 billion are subtracted from the total asset purchase announcement series for the 18 and 36 months cases, respectively. This is shown in [Figs. C12 and C13](#). The GDP response is significant with all four schemes while the CPI response is significant with the first three with all our different assumptions about how long open-ended purchases last.

In addition to government bonds, the Federal Reserve also purchased large quantities of mortgage-backed securities. Most of these purchases were made before March 2009, when government bond purchases began, and from September 2012, when open-ended purchases of mortgage backed securities at a rate of US\$ 40 billion USD per month were announced. Following the same approach as for government bonds the present discounted value of mortgage-backed securities for 18 and 36 months respectively is added and then subtracted. We also add the MBS purchases before 2009m3 to our series. The impact of asset purchases ([Figs. C14 and C15](#)) on GDP is still statistically significant in at least three out of four identification schemes. While the evidence for CPI is weaker, the major part of the 68% Bayesian credible sets is above zero.

A comparison of [Figs. C12–C15](#) with [Fig. 2A](#) and [Tables C12–C15](#) with [Table A1](#) suggests that the peak responses in these four variants are generally lower. This is to be expected since the total sum of purchases is larger, and we should not look for robustness in terms of the coefficients. Our findings about statistical significance are, however, robust to the definition of asset purchases used.

⁹ The yield on the 10-year government bond in the month prior to the announcement was the discount rate used for our calculation.

4. Conclusion

In response to the ‘Great Recession’, central banks deployed a range of novel monetary policy tools, but their impact on the economy is still not well understood. In this paper we study purchases of government bonds by the Bank of England and the Federal Reserve. We find a significant impact of asset purchases on GDP and CPI without making the identifying assumption that this is positive and with models estimated only on the period in which asset purchases were carried out; models estimated over a longer period may be more susceptible to the Lucas critique. The analysis also makes more use of non-informative priors than does any other paper in this literature.

Our results suggest that an asset purchase shock that results in the central bank purchasing government bonds worth 1% of nominal GDP, leads a rise of about 0.62% (0.25%) of real GDP and 0.58% (0.32%) in CPI in the US (UK). These results are robust to including a number of different additional variables in the VAR. Similarly, using the actual amount of assets purchased as the main variable of interest and, for the US, various perturbations to the definition of asset purchases makes little difference to our findings. Our estimates¹⁰ of the impact of asset purchases on real GDP and CPI are similar to studies that identify unconventional monetary policy as a compression in the spread between the long and the short rate (Baumeister and Benati, 2013; Kapetanios et al., 2012), with one notable exception: for the UK, the CPI response is almost three times larger than documented by previous work; the implied inflation-output trade-off is, however, consistent with studies of the conventional monetary policy transmission mechanism and may help explain why the UK inflation was higher than expected after asset purchases began.

For the United States long-term bond yields respond to asset purchases while short-term swap rates do not. This suggests that the portfolio balance may play a role while it is unlikely that signaling is important. Asset purchases reduce measures of financial market and household uncertainty in both countries. In addition, in the UK there is a rise in the appetite for risk. This suggests that managing expectations through reducing uncertainty (Bloom, 2009) may be relevant for both countries, and the risk-taking channel (Bruno and Shin, 2015) for the transmission mechanism in the UK.

Our results have important implications for policy. In both the UK and the US asset purchases were an effective means of supporting GDP in the aftermath of the financial crisis, and they retained their effectiveness beyond the acute phase of the crisis. This should provide considerable reassurance for those who are concerned that, with interest rates still at or close to their lower bound, monetary authorities will find it difficult to respond to renewed global demand weakness.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.jmoneco.2016.03.010>.

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¹⁰ Previous studies used conditional forecasts in their VAR models to compare a ‘QE’ to a ‘No QE’ scenario and assess the total macroeconomic impact of the policy. We repeat this type of exercise in online Appendix B. It turns out that, at least in our case, the total effect obtained from the conditional forecasts is quantitatively broadly similar to multiplying the asset purchase announcement by the corresponding peak impulse response effect, justifying our use of peak impacts in this paper.

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