Ann Oper Res (2013) 205:141–168
DOI 10.1007/s10479-012-1247-6

What do robust equity portfolio models really do?

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Published online: 17 November 2012

Abstract Most of previous work on robust equity portfolio optimization has focused on its formulation and performance. In contrast, in this paper we analyze the behavior of robust equity portfolios to determine whether reducing the sensitivity to input estimation errors is all robust models do and investigate any side-effects of robust formulations. Therefore, our focus is on the relationship between fundamental factors and robust models in order to determine if robust equity portfolios are consistently investing more in the factors opposed to individual asset movements. To do so, we perform regressions with factor returns to explain how robust portfolios behave compared to portfolios generated from the Markowitz's mean-variance model. We find that robust equity portfolios consistently show higher correlation with the three fundamental factors used in the Fama-French factor model. Furthermore, more robustness among robust portfolios results in a higher correlation with the Fama-French three factors. In fact, we show that as equity portfolios under no constraints on portfolio weights become more robust, they consistently depend more on the market and large factors. These results show that robust models are betting on the fundamental factors instead of individual asset movements.

Keywords Robust portfolio optimization · Robustness of equity portfolios · Fundamental factors · Fama-French three-factor model · Regression analysis

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1 Introduction

Portfolio management is one of the key revenue-generating activities offered by banks. The importance of this non-interest fee financial activity that banks provide to bank customers, both retail and institutional, has become increasing important in recent years as interest income has declined due to the increase in loan defaults. Consequently, banks compete for client assets based on performance and, in turn, performance depends on a bank's ability to develop reliable models for managing assets.

One of the most popular models used in portfolio management is the mean-variance model by Markowitz (1952). The model has been applied in practice both at the asset level and asset class level. In this paper, we focus on the latter and take a closer look at equity portfolios. Although the mean-variance model is extensively employed by practitioners, it possesses several impractical aspects. Equity portfolios constructed using the Markowitz model involve very extreme or non-intuitive weights, which forces practitioners to use a variety of constraints to control the weights. Although the constraints resolve unrealistic weights, multiple constraints easily result in weights becoming heavily dependent on the limits. For example, optimization using no-shorting constraints could form a portfolio with most of its assets having zero weight. In addition to unrealistic portfolio weights, one of the major problems with the mean-variance model is the high sensitivity of the parameter estimations to small changes in the inputs. Best and Grauer (1991) reported how much input parameters affect portfolio performance; they found that portfolio weights show extreme sensitivity to changes in the inputs. With no definite method to estimate the true input values, sampling errors in estimates of asset mean and covariance directly affect the calculation of portfolio weights. By simulation, Broadie (1993) showed that the estimation error can be surprisingly large when comparing the true efficient frontier with the actual efficient frontier.

In order to address the sensitivity issue, robust portfolio models have been proposed. Robust models include methods to improve the accuracy of inputs and to apply robust optimization frameworks to portfolio optimization. For example, portfolio resampling, which is based on resampling from the estimated inputs, is one way to reduce estimation errors. The Black-Litterman model combines investors' views with the market equilibrium to improve the accuracy of the inputs. In addition, worst-case optimization incorporates uncertainty directly into the optimization process; the uncertainty set for the input parameters in the mean-variance framework is assumed to be known based on the probability distributions of the uncertain parameters, and the worst possible scenario from the chosen uncertainty set is optimized. Among the several frameworks briefly introduced above, we focus on the worst-case optimization approach in this paper.

Along with the development of various robust models, much effort has been devoted to test the performance of these robust portfolios. Many researchers have conducted out-of-sample performance tests to contrast the classical mean-variance model and robust models but there has not been a dominating conclusion to its performance. For example, Santos (2010) tested the performance of both robust portfolio optimization and mean-variance optimization with empirical and simulated data. Results for simulated data indicate that the robust portfolio significantly outperforms the mean-variance portfolio. Although statistical improvement was not detected when testing with empirical data, Santos concluded that the robust approach would reduce portfolio maintenance cost because the robust portfolio had more stable weights. In contrast, Scherer (2007) found that based on out-of-sample tests, the robust model underperformed the mean-variance model. In summary, although robust models decrease the sensitivity in parameter estimation errors—the principal motivation for these models—and they can be easily understood at the conceptual level, it is not a trivial task to measure how successfully the proposed models achieve their goals under practical settings.

Although many studies have focused on developing robust formulations and testing their performance, little effort has been put into analyzing the behavior of robust portfolios. On the theoretical front, Kim et al. (2012) focus on worst-case optimization and provide mathematical rationale for the relationship between factor returns and portfolios formed from the robust model with an ellipsoidal uncertainty set. They analytically find that as the robustness of a portfolio is increased, the weights of the portfolio move closer to the portfolio whose variance is maximally explained by factors.

In this paper, we look for empirical support for their theoretical results in order to further analyze the behavior of portfolios obtained from robust optimization. We attempt to find out if simply decreasing the sensitivity to estimation errors is all robust models do; there could be unintended side-effects of robust optimization, which further explain the behavior of robust portfolios. Specifically, we evaluate whether robust equity portfolios consistently invest more in the fundamental factors of companies. If robust equity portfolio returns are reasonably explained by these factors, it would mean that robust portfolio optimization is betting on these factors instead of individual asset movements. Therefore, we compute in-sample returns for robust portfolio optimization and classical mean-variance optimization using empirical data and compare the two approaches through factor analysis using the returns generated from a factor model. There are several commercially available factor models employed by quantitative equity managers by banks in managing client accounts, a popular one is the fundamental risk factor model proposed by Fama and French (1993). In addition, we compare the behavior of robust equity portfolios with various levels of robustness; robustness of a portfolio is increased as the confidence interval around the estimated expected return of the uncertainty set is increased.

The main contribution of this study is our empirical evidence that robust equity portfolios are more dependent on the fundamental factor model such as the Fama-French three-factor model. By performing regression analysis between portfolio returns and factor returns, we show that robust models invest more on the factors compared to Markowitz's mean-variance portfolios. Furthermore, we find that this conjecture also holds among robust portfolios; correlation with factor returns increases as the robustness of a portfolio is increased. Additionally, we notice that robust equity portfolios tend to bet on the market and large factors under no constraints on portfolio weights. In particular, unconstrained robust equity portfolios using a 5-year rebalancing period appears to also bet on the growth factor in addition to the market and large market capitalization factors.

The organization of the paper is as follows. Section 2 briefly reviews robust models that have been widely used, focusing on worst-case optimization because this is the basis for our robust formulations. Section 3 describes the data and Sect. 4 outlines our test model for portfolio optimization and factor analysis. Section 5 reports the results and interpretations of our findings; further analysis of the portfolios is included in Sect. 6. Our conclusions are summarized in Sect. 7.

2 Robust portfolio models

As previously mentioned, research in robust models became popular in order to resolve high sensitivity to estimation errors in the mean-variance framework.

Michaud (1998) proposed a robust framework combining Monte Carlo resampling and bootstrapping. As an approach to stabilize portfolio weights that are sensitive to inputs, the model initially computes weights by solving an optimization problem for each point within the confidence region of the parameters, and then taking the average. This approach, known as the Resampling Efficiency technique, gives more stable and realistic weights compared with the classical mean-variance method (Michaud and Michaud 2008).

The robust framework introduced by Black and Litterman (1991, 1992), based on the capital asset pricing model (CAPM), creates a robust model by making the input variables robust. When an investor has his or her own view on the asset returns, the expected return can be improved using a Bayesian framework. Hence, the expected return in the Black-Litterman model becomes a linear combination of the market equilibrium and the investor's view.

The worst-case approach, the focus of this paper, assumes the distributions of uncertain parameters such as mean and covariance of returns, and creates a robust portfolio by maximizing the return in the worst case for each parameter (Fabozzi et al. 2007a, 2007b). Since the expected return affects portfolio performance more than its covariance (Chopra and Ziemba 1993), we concentrate on robust portfolios using a box or an ellipsoid as the uncertainty set for expected returns. In the classical mean-variance framework, portfolio optimization formulation is,

$$\begin{aligned} &\min_{w} w'\Sigma w - \lambda \mu' w \\ &\text{s.t.} \quad w'\iota = 1 \end{aligned} \tag{1}$$

where μ is the expected returns, Σ is the covariance, w is the portfolio weights, λ is the risk coefficient, and ι is a vector of ones. In the above formulation, λ represents the risk-seeking coefficient where setting it to zero gives the minimum-variance portfolio. For the return μ in the above objective function, if we assume the uncertainty set as a box with estimation error less than a small number δ, the uncertainty set becomes,

$$U_\delta(\mu) = \left\{\mu \;\middle|\; |\mu_i - \hat{\mu}_i| \le \delta_i,\ i = 1, \dots, N\right\}$$

where $\hat{\mu}$ is the estimate of the expected returns. In other words, δ is a constant that sets the size of the confidence region. Formulating a min-max problem using the above uncertainty set gives,

$$\begin{aligned} &\min_{w} \max_{\mu \in U_\delta(\hat{\mu})} \; w'\Sigma w - \lambda \mu' w \\ &\text{s.t.} \quad w'\iota = 1 \end{aligned}$$

This min-max problem is equivalent to solving the following minimization problem, which is the optimization problem for our first robust model,

$$\begin{aligned} &\min_{w} w'\Sigma w - \lambda\big(\hat{\mu}' w - \delta' |w|\big) \\ &\text{s.t.} \quad w'\iota = 1 \end{aligned} \tag{2}$$

Similarly, the uncertainty set for μ when assumed to be an ellipsoid is defined as (see Goldfarb and Iyengar 2003),

$$U_\delta(\mu) = \left\{\mu \middle| (\mu - \hat{\mu})' \Sigma_\mu^{-1} (\mu - \hat{\mu}) \le \delta^2\right\}$$

The optimization problem with this uncertainty for expected asset returns is formulated as,

$$\begin{aligned} &\min_{w} \max_{\mu \in U_\delta(\hat{\mu})} \; w'\Sigma w - \lambda \mu' w \\ &\text{s.t.} \quad w'\iota = 1 \end{aligned}$$

Consequently, by differentiating the above min-max problem and solving the first-order condition forms the following simpler problem, which is our second robust optimization model,

$$\begin{aligned} &\min_{w} w'\Sigma w - \lambda\left(\hat{\mu}'w - \delta\sqrt{w'\Sigma_\mu w}\right) \\ &\text{s.t.} \quad w'\iota = 1 \end{aligned} \tag{3}$$

where Σ_μ is the covariance matrix of estimation errors for the expected returns.

While the theory of robust optimization is well developed, there are still many shortcomings in attempting to implement these models. In order to further understand robust models, this paper tackles the two worst-case optimization models that use a box and an ellipsoid as uncertainty sets. Hereinafter, when referring to robust models we mean the worst-case optimization formulations using these two uncertainty sets.

3 Data

There are two basic ways to estimate asset returns. The first approach involves using the returns of every individual security; the second is using factor-level returns. Forming portfolios using individual security returns become computationally expensive in even the simplest mean-variance framework. As noted by Fabozzi et al. (2007a, 2007b), since accurately measuring return covariance matrices especially requires a large number of observations, a common practice is to estimate the mean and variance at a factor level. Kim and Mulvey (2009) pointed out that an effective security segmentation scheme is to group stocks at an industry level as it is defined in a straightforward way and has small misclassification errors. Therefore, we perform our analysis primarily at an industry level and confirm the results at the individual security level using sampling techniques to resolve computational limitations.

In fact, Fama and French (1997) introduced industry portfolios that were used to validate their factor model. These industry portfolios assign each stock traded on the NYSE, AMEX, and NASDAQ to an industry portfolio using the corresponding Standard Industrial Classification (SIC) code. Among many variations, the 49 industry portfolios are used in this study, which is the version with the largest number of industries available in French's data library.

In addition, return data for fundamental factors are required for regression analysis. Fundamental factors refer to underlying basic sources of randomness that influence individual asset returns. Common factors include size, earnings/price ratio, leverage, and momentum. To investigate the behavior of robust equity portfolios in our study, the three-factor model proposed by Fama and French (1993, 1995) is used. The three-factor model explains a security's return by excess return on the market portfolio and an additional two factors referred to by Fama and French as SMB (small minus big) and HML (high minus low). Excess market return is defined as the difference between the value-weighted return on all NYSE, AMEX, and NASDAQ stocks and the one-month Treasury rate. SMB, which is the size factor, is the return on portfolios of small capitalization stocks minus return on portfolios of big capitalization stocks. Similarly, HML, which is the book-to-market ratio factor, is the return on portfolios of value stocks minus the return on portfolios of growth stocks.

Security-level returns that are used as a check of our industry-level results are retrieved from the Center for Research in Security Prices (CRSP) database. Portfolios are created from randomly sampled 200 securities. In order to eliminate any bias against the three Fama-French fundamental factors, 50 securities are selected from each of the four groups: large-cap value, large-cap growth, small-cap value, and small-cap growth stocks. Securities are split into these four groups using market capitalization and book-to-market ratio. Furthermore, since the listed stocks for the NYSE, AMEX and NASDAQ stock markets change over time, sampling is done at each rebalancing period among the existing stocks during that time period. For example, for a portfolio using a 5-year rebalancing period starting in 1970, the first sampling will be done among stocks that existed from January 1970 to December 1974.

In our empirical analysis, daily returns from 1970 to 2009 are collected for both the 49 industry portfolios and the Fama-French three factors.[1] In addition, daily returns of individual securities in the CRSP database are compiled for the same period as well.

4 Test model

To explain the behavior aspect of robust models, we test whether these models consistently put more weight on the fundamental factors. For the analysis, we focus on three portfolios: the mean-variance portfolio, the robust portfolio with a box uncertainty, and the robust portfolio with an ellipsoid uncertainty formed by solving optimization problems (1), (2), and (3), respectively.[2] For the remainder of the paper, we will refer to portfolios optimized through the mean-variance model as MV, portfolios formed using robust portfolio optimization with box uncertainty set as R1, and robust portfolios with uncertainty set defined as an ellipsoid as R2.

We perform in-sample tests on portfolio returns because our main goal is to find out how the robust portfolios put weights on the fundamental factors during the process of constructing portfolios. We compute returns for the mean-variance and robust portfolios from 1970 to 2009 using 5-year, 3-year, and 1-year rebalancing periods. With these portfolio returns, we attempt to compare the correlation between the three fundamental factors by linear regression on the returns. Our analysis is based on the time-series regression model introduced by Jensen et al. (1972), and also used by Fama and French (1993) in distinguishing the common risk factors. From the three-factor model, portfolio returns can be expressed as a linear equation,

$$R_p = a + \beta_M[R_M - R_f] + \beta_{SMB}[SMB] + \beta_{HML}[HML] + \varepsilon$$

where β_M, β_{SMB}, and β_{HML} are corresponding coefficients for the Fama-French three factors. Therefore, the R^2 values of the linear regression will indicate how much of the return is dependent on the three fundamental factors.

Another reason for comparing the performance between the robust and mean-variance portfolios is to investigate two corresponding portfolios that take the same level of risk. Since the risk coefficient determines how much risk investors are willing to take, portfolios with the same value of λ will correspond to each other with respect to risk. For example, when λ is set to zero, the mean-variance and robust optimization formulations are the same and all models compute the minimum-variance portfolio,

$$\begin{aligned} &\min_w w'\Sigma w \\ &\text{s.t.} \quad w'\iota = 1 \end{aligned}$$

Therefore in this experiment, we compare MV, R1, and R2 that have the same value of λ to analyze their characteristics. We select various values of λ to correctly compare factor

[1] Data obtained from the online data library of Kenneth R. French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

[2] For the estimation error covariance matrix Σ_μ in the robust models with ellipsoidal uncertainty sets, we use the diagonal matrix containing the estimation variances, which is known to work well in practice for robust optimization. For further details, see Stubbs and Vance (2005).

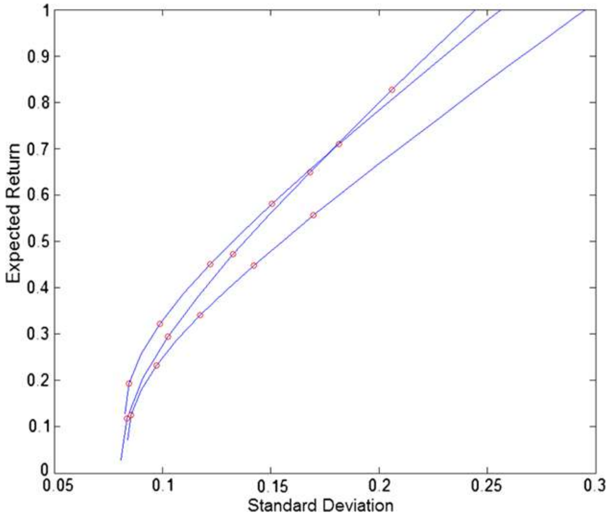


Fig. 1 Mean-variance efficient frontier: no constraints. This figure shows three separate efficient frontiers for mean-variance portfolios with no constraints. *Circles* show portfolios with λ as 0.01, 0.03, 0.05, 0.07, and 0.09 (starting from *left-bottom*)

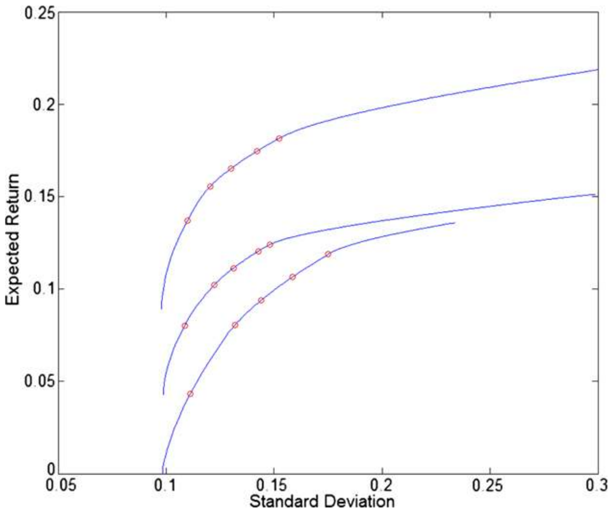


Fig. 2 Mean-variance efficient frontier: no-shorting constraints. This figure shows three separate efficient frontiers for mean-variance portfolios with no-shorting constraints. *Circles* show portfolios with λ as 0.1, 0.2, 0.3, 0.4, and 0.5 (starting from *left-bottom*)

loadings; λ values between 0.01 and 0.5 are used, where portfolios with no constraints focus on 0.01, 0.03, 0.05, 0.07, and 0.09 (as shown in Fig. 1), and portfolios with no-shorting constraints focus on 0.1, 0.2, 0.3, 0.4, and 0.5 (as shown in Fig. 2).[3]

Besides using portfolios with predetermined values of λ, we also compare the mean-variance portfolio with maximum Sharpe ratio with its corresponding robust portfolios. For this evaluation, we first find Markowitz's mean-variance portfolio with maximum Sharpe ratio and use that portfolio's risk level to calculate the returns of robust portfolios R1 and R2.

In addition to comparing the mean-variance and robust models, we investigate a pattern among robust equity portfolios with different robustness. The robustness can be controlled

[3]The values of λ are chosen to represent five portfolios with standard deviation less than 0.3 that are equally spread-out when plotted on the mean-variance efficient frontier.

by changing the value of δ; a higher value of δ enlarges the uncertainty set, which makes it more robust.[4] We observe robust equity portfolios with various confidence levels between 1 % and 99 % to confirm whether more robust portfolios with higher confidence levels are betting more on the Fama-French three factors. For this test, we fix the value of λ and compare the correlation among all confidence levels for each MV, R1, and R2. We also pay close attention to the regression coefficients for the three factors. A pattern in the coefficients along with an increase in robustness will provide further insight on the specific factors and directions the robust equity portfolios bet on.

Finally, since non-negative weights are often imposed in reality, we perform the entire analysis under no constraints and also under no-shorting constraints on portfolio weights. Furthermore, we confirm the outcome using individual security returns; 40 samples of security returns from 1970 to 2009 are retrieved to form 40 portfolio returns for each MV, R1, and R2. For each sample, a new list of 200 securities is randomly selected every rebalancing period. In other words, each sample consists of a total of 200 securities but its composition is updated every time the portfolio is rebalanced. The empirical results for both industry-level and security-level analysis are presented in the next section.

5 Empirical test results

In this section, we compare the results of the factor analysis between the returns of the mean-variance and robust portfolio models. We mainly focus on industry-level results and match the outcomes by analyzing individual securities.

5.1 Industry-level results

We present the results under two conditions: no constraints and no-shorting constraints on portfolio weights. We look for patterns under no constraints as a generic case and confirm using no-shorting constraints. Tables 1, 2, 3, 4, and 5 report the regression outcomes when no constraints are imposed, and Tables 6, 7, and 8 do the same when we optimize using non-negative weights. We primarily focus on results using a 5-year rebalancing period unless observed patterns differ using shorter rebalancing periods.

For the no constraints case, R^2 values from the three-factor regression for MV, R1, and R2 are presented in Table 1. Initially, we confirm that R^2 values are very similar when the value of λ is set to zero since the minimum-variance portfolio is formed in all three cases. In addition, the results of the factor analysis for multiple points on the efficient frontier with various risk levels clearly show significant differences in the R^2 values among MV, R1, and R2. In particular, once the value of λ reaches 0.05, the coefficient of determination reaches 0.6 for R1 and 0.7 for R2. The two robust models, R1 and R2, consistently have higher correlation than MV for all values of λ. Moreover, this observation is persistent throughout all three rebalancing periods. Further significance lies in the fact that even though only confidence levels of 90 % and 95 % are included in Table 1, this pattern continues for all confidence levels providing strong evidence that the robust models bet more on the three fundamental factors than the mean-variance model.

[4] The value of δ does not directly represent confidence levels (90 %, 95 %, etc.). Asset returns are assumed to follow a normal distribution when setting the confidence interval. For example, a 95 % confidence level for the box model uses $\delta_i = 1.96\sigma_i/\sqrt{T}$, where T is the sample size (Fabozzi et al. 2007a, 2007b, 2010). For the ellipsoid model, we assume the square of estimation error δ^2 follows a χ^2 distribution with degrees of freedom as the number of assets in the portfolio (Fabozzi et al. 2007a, 2007b, 2010).

Table 1 R^2 values for MV, R1, and R2 formed under no constraints

λ	Confidence	5-year			3-year			1-year		
		MV	R1	R2	MV	R1	R2	MV	R1	R2
0	–	0.1621	0.1638	0.1621	0.1962	0.1934	0.1962	0.2224	0.2211	0.2224
0.01	90 %	0.3572	0.3872	0.4439	0.3783	0.4370	0.5028	0.3674	0.4593	0.5242
	95 %		0.3965	0.4477		0.4517	0.5065		0.4789	0.5290
0.03	90 %	0.5174	0.5687	0.6495	0.4933	0.6091	0.6963	0.3417	0.5831	0.6999
	95 %		0.5832	0.6540		0.6261	0.7013		0.6112	0.7067
0.05	90 %	0.5427	0.6338	0.7241	0.4854	0.6594	0.7583	0.2875	0.6086	0.7640
	95 %		0.6513	0.7289		0.6762	0.7634		0.6362	0.7709
0.07	90 %	0.5343	0.6659	0.7622	0.4569	0.6789	0.7887	0.2486	0.6063	0.7960
	95 %		0.6843	0.7672		0.6968	0.7939		0.6343	0.8029
0.09	90 %	0.5159	0.6818	0.7859	0.4272	0.6843	0.8073	0.2201	0.5958	0.8147
	95 %		0.7002	0.7911		0.7035	0.8126		0.6265	0.8215
Maximum Sharpe ratio	90 %	0.4251	0.6497	0.8423	0.0021	0.4968	0.8356	0.0015	0.1569	0.6574
	95 %		0.6797	0.8481		0.6303	0.8434		0.2402	0.6909

We also look at the mean-variance portfolio with maximum Sharpe ratio. In order to properly compare portfolios taking the same risk level in both models, the value of λ that maximizes the Sharpe ratio in the mean-variance framework is used when forming the corresponding robust portfolios. As shown in the last row of Table 1, there is a noticeable difference between the R^2 values for regression analysis on the return series using the Fama-French three-factor model. Compared with the R^2 value in the mean-variance framework, the R^2 values in the robust models when assuming confidence intervals of 90 % and 95 % for the parameter distribution are both significantly higher. Particularly, R2 having uncertainty set as an ellipsoid has the highest correlation with R^2 values greater than 0.8. R1 with uncertainty set as a box shows R^2 values not as high as R2 but values over 0.6, which is a meaningful increase from MV. The difference in correlation is larger for shorter rebalancing periods; the coefficient of determination is less than 0.002 for MV but at least 0.1 and up to 0.7 for R1 and R2 when using a 1-year rebalancing period. These findings coincide with the preliminary conclusion from Table 1 that the robust equity portfolios depend more on the Fama-French three factors.

We test further to see how the robust equity portfolios behave depending on their robustness. Portfolio optimization constructs more robust portfolios as the confidence level increases. We run the optimization with confidence levels from 1 % to 99 % with 10 % increments. As reported in Tables 2 and 3, the R^2 value shows a steady rise as the confidence increases for a fixed value of λ. The results show high statistical significance as most of the p-values of regression coefficients are significant at the 5 % level. For example, Panel A of Table 3 has an R^2 value around 0.36 at a confidence level of 0 % but constantly increases to 0.42 at a confidence level of 99 %, and all coefficients for the three factors show significance at the 1 % level. Tables 2 and 3 also include R^2 values from performing simple linear regression between each Fama-French factor and the portfolio returns; we use the R^2 values to investigate how much of the portfolio returns are explained by a single factor. We find that not only does the correlation from the three-factor regression increase with robustness, but robustness also leads to an increase in the correlation from simple regression between the robust portfolio return and each fundamental factor (i.e., market return, SMB, and HML).

Table 2 β and R^2 values for R1 under no constraints using a 5-year rebalancing period

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
Panel A. Value of λ is set to 0.01					Panel B. Value of λ is set to 0.03				
0 %	0.6493**	0.0766**	0.1921**	0.3576	0 %	0.7639**	0.0423**	0.1430**	0.5179
	(0.3227)	(0.0058)	(0.0008)			(0.4995)	(0.0178)	(0.0131)	
1 %	0.6467**	0.0757**	0.1950**	0.3546	1 %	0.7643**	0.0427**	0.1441**	0.5181
	(0.3189)	(0.0059)	(0.0006)			(0.4994)	(0.0177)	(0.0128)	
10 %	0.6506**	0.0752**	0.1903**	0.3594	10 %	0.7648**	0.0408**	0.1432**	0.5196
	(0.3253)	(0.0060)	(0.0009)			(0.5012)	(0.0183)	(0.0131)	
20 %	0.6466**	0.0765**	0.1928**	0.3545	20 %	0.7658**	0.0383**	0.1420**	0.5219
	(0.3194)	(0.0057)	(0.0007)			(0.5039)	(0.0190)	(0.0134)	
30 %	0.6527**	0.0760**	0.1920**	0.3615	30 %	0.7678**	0.0349**	0.1411**	0.5259
	(0.3268)	(0.0060)	(0.0009)			(0.5082)	(0.0201)	(0.0137)	
40 %	0.6530**	0.0741**	0.1905**	0.3623	40 %	0.7677**	0.0322**	0.1421**	0.5263
	(0.3282)	(0.0063)	(0.0010)			(0.5085)	(0.0208)	(0.0135)	
50 %	0.6528**	0.0716**	0.1912**	0.3624	50 %	0.7702**	0.0310**	0.1421**	0.5303
	(0.3283)	(0.0067)	(0.0009)			(0.5125)	(0.0213)	(0.0136)	
60 %	0.6577**	0.0706**	0.1899**	0.3683	60 %	0.7722**	0.0270**	0.1419**	0.5343
	(0.3347)	(0.0070)	(0.0011)			(0.5167)	(0.0226)	(0.0138)	
70 %	0.6620**	0.0679**	0.1881**	0.3739	70 %	0.7762**	0.0239**	0.1419**	0.5410
	(0.3412)	(0.0076)	(0.0013)			(0.5234)	(0.0239)	(0.0141)	
80 %	0.6678**	0.0668**	0.1878**	0.3809	80 %	0.7825**	0.0190**	0.1400**	0.5522
	(0.3484)	(0.0080)	(0.0015)			(0.5351)	(0.0258)	(0.0150)	
90 %	0.6725**	0.0633**	0.1872**	0.3872	90 %	0.7918**	0.0132	0.1369**	0.5687
	(0.3551)	(0.0089)	(0.0016)			(0.5523)	(0.0284)	(0.0165)	
95 %	0.6797**	0.0597**	0.1872**	0.3965	95 %	0.7996**	0.0076	0.1330**	0.5832
	(0.3647)	(0.0099)	(0.0018)			(0.5677)	(0.0308)	(0.0181)	
99 %	0.6945**	0.0524**	0.1832**	0.4166	99 %	0.8159**	−0.0015	0.1250**	0.6137
	(0.3865)	(0.0121)	(0.0026)			(0.5998)	(0.0353)	(0.0218)	
Panel C. Value of λ is set to 0.05					Panel D. Value of λ is set to 0.07				
0 %	0.7739**	0.0266**	0.1180**	0.5431	0 %	0.7634**	0.0189**	0.1048**	0.5341
	(0.5308)	(0.0225)	(0.0201)			(0.5245)	(0.0240)	(0.0229)	
1 %	0.7744**	0.0273**	0.1194**	0.5431	1 %	0.7637**	0.0187**	0.1050**	0.5346
	(0.5305)	(0.0224)	(0.0198)			(0.5249)	(0.0241)	(0.0229)	
10 %	0.7772**	0.0237**	0.1185**	0.5487	10 %	0.7700**	0.0138*	0.1044**	0.5454
	(0.5364)	(0.0237)	(0.0203)			(0.5359)	(0.0261)	(0.0236)	
20 %	0.7813**	0.0186**	0.1170**	0.5568	20 %	0.7781**	0.0083	0.1056**	0.5588
	(0.5448)	(0.0255)	(0.0210)			(0.5490)	(0.0285)	(0.0240)	
30 %	0.7851**	0.0144*	0.1177**	0.5635	30 %	0.7826**	0.0040	0.1046**	0.5673
	(0.5514)	(0.0272)	(0.0211)			(0.5577)	(0.0303)	(0.0247)	
40 %	0.7878**	0.0119	0.1179**	0.5682	40 %	0.7862**	−0.0005	0.1038**	0.5745
	(0.5561)	(0.0282)	(0.0213)			(0.5649)	(0.0321)	(0.0253)	
50 %	0.7913**	0.0082	0.1178**	0.5749	50 %	0.7927**	−0.0030	0.1038**	0.5853
	(0.5627)	(0.0297)	(0.0216)			(0.5756)	(0.0336)	(0.0259)	

Table 2 (*Continued*)

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
60 %	0.7970**	0.0056	0.1177**	0.5844	60 %	0.8018**	−0.0046	0.1027**	0.6001
	(0.5722)	(0.0311)	(0.0222)			(0.5906)	(0.0349)	(0.0272)	
70 %	0.8035**	0.0037	0.1177**	0.5949	70 %	0.8131**	−0.0059	0.1026**	0.6180
	(0.5827)	(0.0323)	(0.0228)			(0.6085)	(0.0363)	(0.0285)	
80 %	0.8135**	−0.0005	0.1157**	0.6124	80 %	0.8253**	−0.0085	0.1006**	0.6389
	(0.6005)	(0.0346)	(0.0244)			(0.6297)	(0.0383)	(0.0306)	
90 %	0.8253**	−0.0056	0.1125**	0.6338	90 %	0.8401**	−0.0130*	0.0969**	0.6659
	(0.6224)	(0.0374)	(0.0266)			(0.6571)	(0.0414)	(0.0336)	
95 %	0.8344**	−0.0100	0.1089**	0.6513	95 %	0.8502**	−0.0160**	0.0951**	0.6843
	(0.6403)	(0.0399)	(0.0287)			(0.6757)	(0.0435)	(0.0355)	
99 %	0.8502**	−0.0143*	0.1060**	0.6795	99 %	0.8646**	−0.0174**	0.0974**	0.7078
	(0.6690)	(0.0430)	(0.0315)			(0.6987)	(0.0455)	(0.0364)	

For each confidence level, β and R^2 values are shown, and significance at the 1 % and 5 % levels are given by ** and *, respectively. R^2 value from simple linear regression representing the relationship between a single factor and portfolio returns is shown in parenthesis

Table 3 β and R^2 values for R2 under no constraints using a 5-year rebalancing period

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
Panel A. Value of λ is set to 0.01					Panel B. Value of λ is set to 0.03				
0 %	0.6493**	0.0766**	0.1921**	0.3576	0 %	0.7639**	0.0423**	0.1430**	0.5179
	(0.3227)	(0.0058)	(0.0008)			(0.4995)	(0.0178)	(0.0131)	
1 %	0.6937**	0.0589**	0.1824**	0.4143	1 %	0.8148**	0.0164*	0.1248**	0.6056
	(0.3840)	(0.0106)	(0.0027)			(0.5920)	(0.0288)	(0.0220)	
10 %	0.7000**	0.0569**	0.1809**	0.4227	10 %	0.8216**	0.0152*	0.1209**	0.6178
	(0.3931)	(0.0113)	(0.0031)			(0.6051)	(0.0297)	(0.0238)	
20 %	0.7029**	0.0561**	0.1801**	0.4266	20 %	0.8246**	0.0146*	0.1192**	0.6232
	(0.3973)	(0.0116)	(0.0033)			(0.6108)	(0.0301)	(0.0247)	
30 %	0.7047**	0.0555**	0.1796**	0.4291	30 %	0.8267**	0.0141*	0.1180**	0.6270
	(0.3999)	(0.0118)	(0.0034)			(0.6149)	(0.0304)	(0.0253)	
40 %	0.7062**	0.0552**	0.1791**	0.4312	40 %	0.8287**	0.0137*	0.1170**	0.6305
	(0.4021)	(0.0120)	(0.0035)			(0.6186)	(0.0307)	(0.0258)	
50 %	0.7078**	0.0549**	0.1787**	0.4333	50 %	0.8304**	0.0136*	0.1161**	0.6337
	(0.4044)	(0.0121)	(0.0037)			(0.6219)	(0.0308)	(0.0263)	
60 %	0.7096**	0.0544**	0.1781**	0.4357	60 %	0.8320**	0.0132*	0.1150**	0.6367
	(0.4070)	(0.0123)	(0.0038)			(0.6251)	(0.0311)	(0.0268)	
70 %	0.7109**	0.0541**	0.1779**	0.4375	70 %	0.8338**	0.0127*	0.1140**	0.6400
	(0.4090)	(0.0124)	(0.0039)			(0.6287)	(0.0314)	(0.0273)	

In fact, plotting the yearly returns and standard deviations from the in-sample test with various confidence levels, we see that the portfolios with higher confidence tend to move

Table 3 (*Continued*)

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
80 %	0.7129**	0.0536**	0.1773**	0.4402	80 %	0.8359**	0.0122*	0.1127**	0.6440
	(0.4119)	(0.0126)	(0.0040)			(0.6329)	(0.0317)	(0.0280)	
90 %	0.7156**	0.0528**	0.1765**	0.4439	90 %	0.8389**	0.0117	0.1108**	0.6495
	(0.4159)	(0.0129)	(0.0042)			(0.6388)	(0.0321)	(0.0290)	
95 %	0.7182**	0.0519**	0.1755**	0.4477	95 %	0.8412**	0.0111	0.1092**	0.6540
	(0.4200)	(0.0133)	(0.0045)			(0.6436)	(0.0325)	(0.0297)	
99 %	0.7224**	0.0507**	0.1743**	0.4536	99 %	0.8456**	0.0102	0.1064**	0.6624
	(0.4262)	(0.0138)	(0.0048)			(0.6525)	(0.0331)	(0.0312)	
Panel C. Value of λ is set to 0.05					Panel D. Value of λ is set to 0.07				
0 %	0.7739**	0.0266**	0.1180**	0.5431	0 %	0.7634**	0.0189**	0.1048**	0.5341
	(0.5308)	(0.0225)	(0.0201)			(0.5245)	(0.0240)	(0.0229)	
1 %	0.8454**	−0.0019	0.0960**	0.6703	1 %	0.8585**	−0.0130*	0.0804**	0.7025
	(0.6621)	(0.0374)	(0.0348)			(0.6963)	(0.0427)	(0.0424)	
10 %	0.8539**	−0.0035	0.0916**	0.6867	10 %	0.8684**	−0.0136*	0.0755**	0.7215
	(0.6791)	(0.0387)	(0.0375)			(0.7160)	(0.0438)	(0.0458)	
20 %	0.8576**	−0.0041	0.0897**	0.6937	20 %	0.8724**	−0.0139**	0.0735**	0.7292
	(0.6865)	(0.0393)	(0.0387)			(0.7240)	(0.0443)	(0.0473)	
30 %	0.8600**	−0.0044	0.0882**	0.6984	30 %	0.8751**	−0.0139**	0.0719**	0.7345
	(0.6914)	(0.0396)	(0.0396)			(0.7295)	(0.0445)	(0.0484)	
40 %	0.8620**	−0.0047	0.0870**	0.7024	40 %	0.8773**	−0.0142**	0.0708**	0.7390
	(0.6955)	(0.0398)	(0.0404)			(0.7341)	(0.0448)	(0.0492)	
50 %	0.8639**	−0.0049	0.0858**	0.7061	50 %	0.8794**	−0.0143**	0.0696**	0.7431
	(0.6994)	(0.0400)	(0.0411)			(0.7384)	(0.0450)	(0.0500)	
60 %	0.8658**	−0.0052	0.0847**	0.7098	60 %	0.8813**	−0.0143**	0.0684**	0.7470
	(0.7033)	(0.0403)	(0.0418)			(0.7424)	(0.0452)	(0.0509)	
70 %	0.8677**	−0.0056	0.0834**	0.7136	70 %	0.8835**	−0.0145**	0.0673**	0.7512
	(0.7073)	(0.0406)	(0.0426)			(0.7468)	(0.0454)	(0.0517)	
80 %	0.8699**	−0.0060	0.0821**	0.7181	80 %	0.8858**	−0.0145**	0.0659**	0.7559
	(0.7120)	(0.0410)	(0.0434)			(0.7516)	(0.0456)	(0.0527)	
90 %	0.8728**	−0.0063	0.0799**	0.7241	90 %	0.8889**	−0.0145**	0.0639**	0.7622
	(0.7183)	(0.0413)	(0.0448)			(0.7581)	(0.0459)	(0.0541)	
95 %	0.8752**	−0.0067	0.0784**	0.7289	95 %	0.8914**	−0.0146**	0.0627**	0.7672
	(0.7233)	(0.0416)	(0.0457)			(0.7633)	(0.0461)	(0.0550)	
99 %	0.8794**	−0.0069	0.0752**	0.7376	99 %	0.8957**	−0.0145**	0.0601**	0.7760
	(0.7325)	(0.0420)	(0.0477)			(0.7724)	(0.0465)	(0.0569)	

For each confidence level, β and R^2 values are shown, and significance at the 1 % and 5 % levels are given by ** and *, respectively. R^2 value from simple linear regression representing the relationship between a single factor and portfolio returns is shown in parenthesis

towards the lower-left direction on the mean-variance plane. In Markowitz's mean-variance plane, portfolios towards the lower-left are risk-averse portfolios and ones towards the upper-right are risk-seeking portfolios. Each curve in Fig. 3 represents R1 portfolios with the same

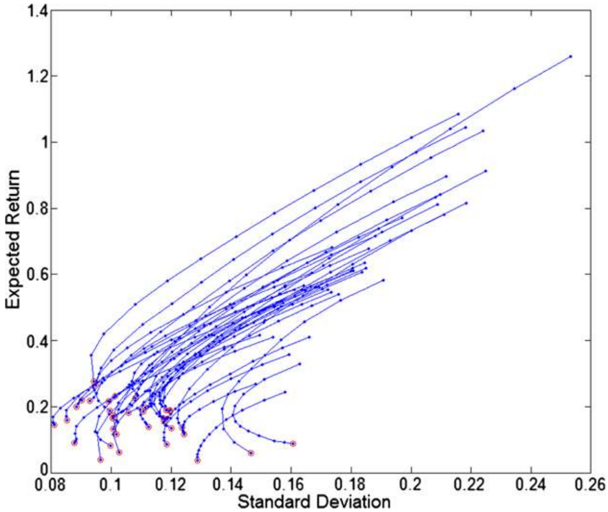


Fig. 3 R1 portfolios with no constraints with various confidence levels (for $\lambda = 0.1$). *Connected dots* are portfolios formed with identical data. *Circles* towards the *left-bottom* are portfolios with confidence 99 % and the confidence level decreases towards the *right-top*

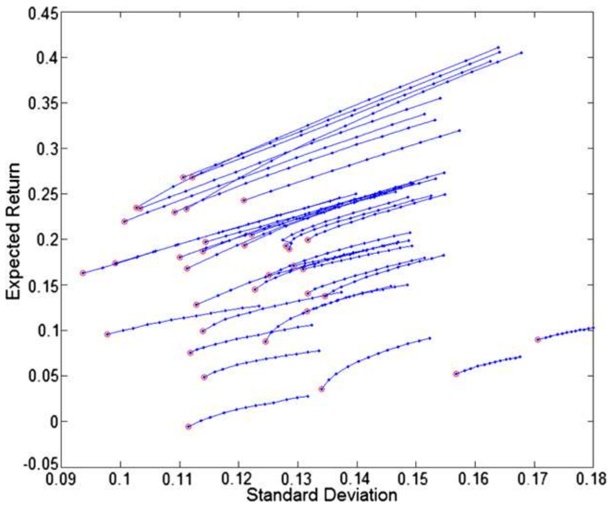


Fig. 4 R2 portfolios with no constraints with various confidence levels (for $\lambda = 0.1$). *Connected dots* are portfolios formed with identical data. *Circles* towards the *left-bottom* are portfolios with confidence 99 % and the confidence level decreases towards the *right-top*

value of λ for a specific 5-year period.[5] The right-most point for each curve is the portfolio with 1 % confidence level and the circle towards the lower-left is the portfolio with 99 % confidence. Therefore, the plot confirms that the robust portfolios which depend more on the three fundamental factors are the more conservative ones. In fact, the plotted curve shows a very similar shape to efficient frontiers of the mean-variance model. Figure 4 represents R2 portfolios in the same manner as Fig. 3, and also indicates that more robust portfolios are plotted towards the left-bottom in the mean-variance plane.

Tables 4 and 5 contain regression coefficients presented in Tables 2 and 3 along with those obtained using 3-year and 1-year rebalancing periods. We find that for each value of λ, the coefficient for the market factor always increases as the confidence level increases for

[5] The first curve uses return data from 1970 to 1974, the second curve uses return data from 1975–1979, and so on.

Table 4 Values of β for Fama-French three factors under no constraints for R1

λ	Confidence	5-year β_M	5-year β_{SMB}	5-year β_{HML}	3-year β_M	3-year β_{SMB}	3-year β_{HML}	1-year β_M	1-year β_{SMB}	1-year β_{HML}
0.01	10 %	0.6506	0.0752	0.1903	0.6989	0.0660	0.2332	0.6546	0.0730	0.1424
	30 %	0.6527	0.0760	0.1920	0.7041	0.0660	0.2344	0.6660	0.0722	0.1454
	50 %	0.6528	0.0716	0.1912	0.7162	0.0676	0.2384	0.6803	0.0704	0.1482
	70 %	0.6620	0.0679	0.1881	0.7284	0.0663	0.2403	0.6981	0.0682	0.1512
	90 %	0.6725	0.0633	0.1872	0.7473	0.0637	0.2458	0.7258	0.0620	0.1505
0.03	10 %	0.7648	0.0408	0.1432	0.7761	0.0456	0.1695	0.6366	0.0600	0.1043
	30 %	0.7678	0.0349	0.1411	0.7919	0.0465	0.1789	0.6721	0.0544	0.1134
	50 %	0.7702	0.0310	0.1421	0.8119	0.0459	0.1903	0.7101	0.0488	0.1226
	70 %	0.7762	0.0239	0.1419	0.8317	0.0432	0.1991	0.7524	0.0426	0.1261
	90 %	0.7918	0.0132	0.1369	0.8564	0.0354	0.1998	0.8025	0.0310	0.1220
0.05	10 %	0.7772	0.0237	0.1185	0.7645	0.0379	0.1385	0.5899	0.0539	0.0890
	30 %	0.7851	0.0144	0.1177	0.7916	0.0357	0.1513	0.6330	0.0442	0.0974
	50 %	0.7913	0.0082	0.1178	0.8200	0.0325	0.1639	0.6861	0.0354	0.1082
	70 %	0.8035	0.0037	0.1177	0.8491	0.0285	0.1728	0.7487	0.0297	0.1143
	90 %	0.8253	−0.0056	0.1125	0.8786	0.0215	0.1725	0.8164	0.0243	0.1146
0.07	10 %	0.7700	0.0138	0.1044	0.7419	0.0332	0.1213	0.5521	0.0515	0.0817
	30 %	0.7826	0.0040	0.1046	0.7755	0.0278	0.1344	0.5951	0.0387	0.0876
	50 %	0.7927	−0.0030	0.1038	0.8110	0.0234	0.1470	0.6558	0.0279	0.0999
	70 %	0.8131	−0.0059	0.1026	0.8480	0.0194	0.1556	0.7338	0.0254	0.1094
	90 %	0.8401	−0.0130	0.0969	0.8842	0.0140	0.1556	0.8144	0.0251	0.1115
0.09	10 %	0.7576	0.0081	0.0965	0.7192	0.0304	0.1107	0.5218	0.0504	0.0778
	30 %	0.7732	−0.0042	0.0955	0.7557	0.0223	0.1223	0.5633	0.0355	0.0818
	50 %	0.7867	−0.0103	0.0928	0.7961	0.0165	0.1340	0.6263	0.0241	0.0952
	70 %	0.8131	−0.0111	0.0905	0.8393	0.0125	0.1418	0.7148	0.0231	0.1068
	90 %	0.8458	−0.0170	0.0839	0.8823	0.0096	0.1426	0.8075	0.0264	0.1097

both R1 and R2. In other words, as portfolios become more robust, they depend more on the market portfolio. Similarly, the coefficient for SMB appears to decrease as the confidence level increases. This implies that the robust equity portfolios depend more on the large factor as robustness leads to betting on big stocks. We also recognize that the coefficient for HML tends to decrease as the confidence level increases when using a 5-year rebalancing period. Although not as clear in shorter rebalancing periods, this pattern indicates that the robust portfolios invest on the growth factor. Therefore, at least when using a 5-year rebalancing period, robustness in portfolios leads to betting on the market, large, and growth factors. In general, unconstrained robust portfolio optimization evidently put more weight on market and large factors.

Results using non-negative portfolio weights presented in Table 6 exhibit a similar increasing pattern in the R^2 value as portfolios become more robust. In agreement with the findings when there are no constraints, R1 and R2 show a higher correlation than MV for all values of λ. Moreover, we see that the value of R^2 reaches 0.9 for R2 for all three rebalancing periods. Consistently, a high R^2 value is shown for both R1 and R2 when comparing the maximum Sharpe ratio mean-variance portfolio to the corresponding robust portfolios. Tables 7 and 8 present results under no-shorting constraints that correspond to values in

Table 5 Values of β for Fama-French three factors under no constraints for R2

λ	Confi-dence	5-year			3-year			1-year		
		β_M	β_{SMB}	β_{HML}	β_M	β_{SMB}	β_{HML}	β_M	β_{SMB}	β_{HML}
0.01	10 %	0.7000	0.0569	0.1809	0.7779	0.0671	0.2409	0.7466	0.0641	0.1338
	30 %	0.7047	0.0555	0.1796	0.7837	0.0669	0.2401	0.7534	0.0638	0.1320
	50 %	0.7078	0.0549	0.1787	0.7878	0.0666	0.2396	0.7580	0.0634	0.1305
	70 %	0.7109	0.0541	0.1779	0.7915	0.0667	0.2390	0.7626	0.0630	0.1291
	90 %	0.7156	0.0528	0.1765	0.7974	0.0655	0.2380	0.7690	0.0624	0.1267
0.03	10 %	0.8216	0.0152	0.1209	0.8854	0.0412	0.1807	0.8382	0.0438	0.0961
	30 %	0.8267	0.0141	0.1180	0.8918	0.0407	0.1784	0.8486	0.0429	0.0922
	50 %	0.8304	0.0136	0.1161	0.8961	0.0404	0.1766	0.8551	0.0423	0.0895
	70 %	0.8338	0.0127	0.1140	0.9001	0.0402	0.1746	0.8612	0.0416	0.0869
	90 %	0.8389	0.0117	0.1108	0.9057	0.0397	0.1717	0.8693	0.0406	0.0831
0.05	10 %	0.8539	−0.0035	0.0916	0.9074	0.0255	0.1483	0.8673	0.0324	0.0835
	30 %	0.8600	−0.0044	0.0882	0.9146	0.0253	0.1453	0.8796	0.0317	0.0796
	50 %	0.8639	−0.0049	0.0858	0.9191	0.0250	0.1432	0.8870	0.0313	0.0769
	70 %	0.8677	−0.0056	0.0834	0.9233	0.0249	0.1411	0.8935	0.0309	0.0743
	90 %	0.8728	−0.0063	0.0799	0.9289	0.0247	0.1380	0.9018	0.0304	0.0709
0.07	10 %	0.8684	−0.0136	0.0755	0.9150	0.0153	0.1301	0.8810	0.0256	0.0804
	30 %	0.8751	−0.0139	0.0719	0.9231	0.0155	0.1272	0.8951	0.0256	0.0767
	50 %	0.8794	−0.0143	0.0696	0.9281	0.0157	0.1252	0.9031	0.0256	0.0742
	70 %	0.8835	−0.0145	0.0673	0.9327	0.0160	0.1233	0.9101	0.0256	0.0719
	90 %	0.8889	−0.0145	0.0639	0.9387	0.0165	0.1205	0.9185	0.0259	0.0691
0.09	10 %	0.8768	−0.0196	0.0663	0.9184	0.0086	0.1189	0.8881	0.0215	0.0808
	30 %	0.8842	−0.0197	0.0630	0.9277	0.0096	0.1164	0.9040	0.0222	0.0772
	50 %	0.8889	−0.0196	0.0608	0.9334	0.0104	0.1148	0.9128	0.0227	0.0749
	70 %	0.8933	−0.0194	0.0587	0.9385	0.0113	0.1132	0.9202	0.0233	0.0729
	90 %	0.8992	−0.0187	0.0560	0.9450	0.0126	0.1110	0.9290	0.0243	0.0705

Tables 2 and 3. The R^2 values display an increasing trend as the confidence level increases for all values of λ for both R1 and R2. In addition, it appears that the robust models under no-shorting constraints also bet more on the market portfolio as robustness increases; the coefficient for the market return mostly increases as the confidence level increases. However, trends in the coefficients for the other two Fama-French factors are not as notable as in the case where there is an absence of restrictions on the weights. In summary, even when restrictions on portfolio weights are imposed, the empirical results under no-shorting constraints agree with our finding that the robust equity portfolios depend more on the Fama-French three factors.

5.2 Security-level results

Consistent with our industry-level analysis, the security-level results from the 40 samples also show that robust portfolios are more dependent on the three Fama-French factors than the mean-variance portfolios. Figure 5 displays the R^2 value for MV, R1, and R2 when the value of λ is set to 0.03 and 0.07 with a confidence level of 95 % under no constraints. We clearly observe the following pattern for the 40 samples: R2 has the highest correlation

Table 6 R^2 values for MV, R1, and R2 formed under no-shorting constraints

λ	Confidence	5-year MV	5-year R1	5-year R2	3-year MV	3-year R1	3-year R2	1-year MV	1-year R1	1-year R2
0	–	0.7705	0.7705	0.7705	0.7617	0.7617	0.7617	0.7586	0.7586	0.7586
0.1	90 %	0.7258	0.7546	0.8787	0.6876	0.7380	0.8851	0.6334	0.6707	0.8923
	95 %		0.7556	0.8800		0.7412	0.8865		0.6748	0.8936
0.2	90 %	0.6644	0.7214	0.8917	0.6124	0.6961	0.8944	0.5424	0.6097	0.9030
	95 %		0.7253	0.8929		0.7066	0.8957		0.6163	0.9043
0.3	90 %	0.6180	0.6783	0.8970	0.5524	0.6460	0.8973	0.4827	0.5700	0.9069
	95 %		0.6862	0.8982		0.6576	0.8987		0.5840	0.9082
0.4	90 %	0.5867	0.6424	0.8997	0.5102	0.6070	0.8987	0.4431	0.5412	0.9089
	95 %		0.6508	0.9009		0.6239	0.9001		0.5614	0.9102
0.5	90 %	0.5679	0.6145	0.9013	0.4814	0.5732	0.8995	0.4146	0.5180	0.9101
	95 %		0.6225	0.9025		0.5960	0.9009		0.5441	0.9114
Maximum Sharpe ratio	90 %	0.6537	0.7113	0.9047	0.5285	0.6462	0.9037	0.5109	0.5902	0.9105
	95 %		0.7136	0.9056		0.6675	0.9049		0.6117	0.9116

Table 7 β and R^2 values for R1 under no-shorting constraints using a 5-year rebalancing period

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
Panel A. Value of λ is set to 0.1					Panel B. Value of λ is set to 0.2				
0 %	0.8643**	0.0084	0.0332**	0.7258	0 %	0.7999**	−0.0147*	−0.0338**	0.6644
	(0.7248)	(0.0340)	(0.0656)			(0.6633)	(0.0362)	(0.0903)	
1 %	0.8643**	0.0093	0.0331**	0.7255	1 %	0.8003**	−0.0146*	−0.0338**	0.6652
	(0.8601)	(0.0415)	(0.0738)			(0.8732)	(0.0424)	(0.0816)	
10 %	0.8670**	0.0087	0.0359**	0.7288	10 %	0.8035**	−0.0153**	−0.0321**	0.6694
	(0.8656)	(0.0415)	(0.0742)			(0.8800)	(0.0428)	(0.0813)	
20 %	0.8699**	0.0086	0.0392**	0.7322	20 %	0.8071**	−0.0157**	−0.0304**	0.6745
	(0.8677)	(0.0415)	(0.0743)			(0.8820)	(0.0427)	(0.0811)	
30 %	0.8726**	0.0088	0.0423**	0.7352	30 %	0.8106**	−0.0158**	−0.0285**	0.6792
	(0.8692)	(0.0415)	(0.0744)			(0.8835)	(0.0427)	(0.0809)	
40 %	0.8757**	0.0050	0.0395**	0.7434	40 %	0.8147**	−0.0158**	−0.0263**	0.6846
	(0.8704)	(0.0415)	(0.0745)			(0.8846)	(0.0426)	(0.0808)	
50 %	0.8782**	0.0048	0.0426**	0.7462	50 %	0.8191**	−0.0161**	−0.0235**	0.6904
	(0.8715)	(0.0415)	(0.0745)			(0.8857)	(0.0426)	(0.0807)	
60 %	0.8800**	0.0061	0.0505**	0.7448	60 %	0.8250**	−0.0170**	−0.0208**	0.6989
	(0.8726)	(0.0414)	(0.0746)			(0.8867)	(0.0425)	(0.0806)	
70 %	0.8832**	0.0058	0.0550**	0.7484	70 %	0.8311**	−0.0170**	−0.0161**	0.7063
	(0.8737)	(0.0414)	(0.0746)			(0.8878)	(0.0425)	(0.0805)	
80 %	0.8860**	0.0048	0.0607**	0.7508	80 %	0.8382**	−0.0174**	−0.0094	0.7147
	(0.8749)	(0.0414)	(0.0747)			(0.8890)	(0.0424)	(0.0804)	

Table 7 (*Continued*)

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
90 %	0.8896**	0.0023	0.0682**	0.7546	90 %	0.8467**	−0.0160**	0.0029	0.7214
	(0.8769)	(0.0418)	(0.0750)			(0.8904)	(0.0423)	(0.0802)	
95 %	0.8913**	−0.0011	0.0755**	0.7556	95 %	0.8426**	−0.0143**	0.0130*	0.7253
	(0.8782)	(0.0417)	(0.0750)			(0.8917)	(0.0422)	(0.0801)	
99 %	0.8903**	−0.0076*	0.0866**	0.7518	99 %	0.8621**	−0.0120*	0.0310**	0.7312
	(0.8806)	(0.0413)	(0.0757)			(0.8937)	(0.0421)	(0.0798)	
Panel C. Value of λ is set to 0.3					Panel D. Value of λ is set to 0.4				
0 %	0.7639**	−0.0162*	−0.0512**	0.6180	0 %	0.7455**	−0.0092	−0.0512**	0.5870
	(0.6156)	(0.0335)	(0.0935)			(0.5847)	(0.0295)	(0.0900)	
1 %	0.7641**	−0.0164**	−0.0513**	0.6185	1 %	0.7457**	−0.0095	−0.0514**	0.5874
	(0.8792)	(0.0429)	(0.0838)			(0.8814)	(0.0427)	(0.0847)	
10 %	0.7657**	−0.0190**	−0.0517**	0.6222	10 %	0.7466**	−0.0128	−0.0532**	0.5911
	(0.8852)	(0.0427)	(0.0829)			(0.8876)	(0.0426)	(0.0836)	
20 %	0.7678**	−0.0211**	−0.0523**	0.6267	20 %	0.7475**	−0.0164*	−0.0553**	0.5951
	(0.8874)	(0.0427)	(0.0825)			(0.8899)	(0.0425)	(0.0832)	
30 %	0.7702**	−0.0231**	−0.0523**	0.6313	30 %	0.7482**	−0.0201**	−0.0578**	0.5990
	(0.8888)	(0.0426)	(0.0823)			(0.8914)	(0.0424)	(0.0829)	
40 %	0.7734**	−0.0249**	−0.0518**	0.6367	40 %	0.7485**	−0.0237**	−0.0605**	0.6025
	(0.8900)	(0.0425)	(0.0821)			(0.8926)	(0.0424)	(0.0826)	
50 %	0.7771**	−0.0266**	−0.0511**	0.6427	50 %	0.7494**	−0.0271**	−0.0628**	0.6066
	(0.8911)	(0.0425)	(0.0819)			(0.8937)	(0.0423)	(0.0824)	
60 %	0.7819**	−0.0279**	−0.0492**	0.6499	60 %	0.7519**	−0.0294**	−0.0640**	0.6121
	(0.8921)	(0.0424)	(0.0817)			(0.8948)	(0.0423)	(0.0822)	
70 %	0.7879**	−0.0281**	−0.0467**	0.6581	70 %	0.7574**	−0.0302**	−0.0630**	0.6205
	(0.8931)	(0.0424)	(0.0815)			(0.8958)	(0.0422)	(0.0820)	
80 %	0.7949**	−0.0280**	−0.0430**	0.6670	80 %	0.7652**	−0.0301**	−0.0598**	0.6307
	(0.8943)	(0.0423)	(0.0813)			(0.8970)	(0.0421)	(0.0817)	
90 %	0.8053**	−0.0254**	−0.0353**	0.6783	90 %	0.7752**	−0.0289**	−0.0541**	0.6424
	(0.8958)	(0.0422)	(0.0811)			(0.8985)	(0.0420)	(0.0814)	
95 %	0.8144**	−0.0215**	−0.0259**	0.6862	95 %	0.7839**	−0.0255**	−0.0472**	0.6508
	(0.8970)	(0.0421)	(0.0809)			(0.8997)	(0.0419)	(0.0812)	
99 %	0.8269**	−0.0143*	−0.0012	0.6899	99 %	0.7992**	−0.0181**	−0.0219**	0.6575
	(0.8990)	(0.0419)	(0.0805)			(0.9017)	(0.0418)	(0.0807)	

For each confidence level, β and R^2 values are shown, and significance at the 1 % and 5 % levels are given by ** and *, respectively. R^2 value from simple linear regression representing the relationship between a single factor and portfolio returns is shown in parenthesis

followed by R1, and MV is the least correlated with the three factors. In fact, MV shows the lowest correlation and R2 shows the highest correlation in all 40 samples in both panels.

Likewise, Fig. 6 exhibits the same structure for the 40 samples under no-shorting constraints: the value of R^2 decreases in the order of R2, R1, and MV. Although the R^2 values of MV and R1 are lower than the outcomes under no constraints on portfolio weights, the

Table 8 β and R^2 values for R2 under no-shorting constraints using a 5-year rebalancing period

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
Panel A. Value of λ is set to 0.1					Panel B. Value of λ is set to 0.2				
0 %	0.8142**	−0.0069	0.0294**	0.6504	0 %	0.7520**	−0.0295**	−0.0483**	0.6027
	(0.6496)	(0.0355)	(0.0588)			(0.6002)	(0.0375)	(0.0890)	
1 %	0.9437**	0.0073	0.0441**	0.8618	1 %	0.9458**	0.0048	0.0308**	0.8740
	(0.8601)	(0.0415)	(0.0738)			(0.8732)	(0.0424)	(0.0816)	
10 %	0.9471**	0.0080*	0.0446**	0.8674	10 %	0.9500**	0.0049	0.0326**	0.8809
	(0.8656)	(0.0415)	(0.0742)			(0.8800)	(0.0428)	(0.0813)	
20 %	0.9483**	0.0083*	0.0447**	0.8695	20 %	0.9516**	0.0054	0.0336**	0.8830
	(0.8677)	(0.0415)	(0.0743)			(0.8820)	(0.0427)	(0.0811)	
30 %	0.9493**	0.0086*	0.0450**	0.8710	30 %	0.9527**	0.0058	0.0342**	0.8845
	(0.8692)	(0.0415)	(0.0744)			(0.8835)	(0.0427)	(0.0809)	
40 %	0.9499**	0.0087*	0.0450**	0.8721	40 %	0.9535**	0.0062	0.0347**	0.8857
	(0.8704)	(0.0415)	(0.0745)			(0.8846)	(0.0426)	(0.0808)	
50 %	0.9506**	0.0089*	0.0452**	0.8733	50 %	0.9543**	0.0065	0.0352**	0.8868
	(0.8715)	(0.0415)	(0.0745)			(0.8857)	(0.0426)	(0.0807)	
60 %	0.9513**	0.0091*	0.0452**	0.8744	60 %	0.9551**	0.0068*	0.0356**	0.8878
	(0.8726)	(0.0414)	(0.0746)			(0.8867)	(0.0425)	(0.0806)	
70 %	0.9520**	0.0093*	0.0455**	0.8755	70 %	0.9559**	0.0071*	0.0361**	0.8889
	(0.8737)	(0.0414)	(0.0746)			(0.8878)	(0.0425)	(0.0805)	
80 %	0.9527**	0.0094**	0.0456**	0.8768	80 %	0.9568**	0.0076*	0.0367**	0.8901
	(0.8749)	(0.0414)	(0.0747)			(0.8890)	(0.0424)	(0.0804)	
90 %	0.9535**	0.0087*	0.0452**	0.8787	90 %	0.9579**	0.0080*	0.0373**	0.8917
	(0.8769)	(0.0418)	(0.0750)			(0.8904)	(0.0423)	(0.0802)	
95 %	0.9543**	0.0090*	0.0455**	0.8800	95 %	0.9588**	0.0085*	0.0379**	0.8929
	(0.8782)	(0.0417)	(0.0750)			(0.8917)	(0.0422)	(0.0801)	
99 %	0.9557**	0.0105**	0.0448**	0.8823	99 %	0.9604**	0.0093**	0.0389**	0.8951
	(0.8806)	(0.0413)	(0.0757)			(0.8937)	(0.0421)	(0.0798)	
Panel C. Value of λ is set to 0.3					Panel D. Value of λ is set to 0.4				
0 %	0.7411**	−0.0236**	−0.0526**	0.5863	0 %	0.7262**	−0.0145*	−0.0554**	0.5622
	(0.5836)	(0.0343)	(0.0895)			(0.5595)	(0.0298)	(0.0885)	
1 %	0.9477**	0.0040	0.0275**	0.8799	1 %	0.9486**	0.0045	0.0262**	0.8820
	(0.8792)	(0.0429)	(0.0838)			(0.8814)	(0.0427)	(0.0847)	
10 %	0.9523**	0.0055	0.0307**	0.8860	10 %	0.9534**	0.0061	0.0298**	0.8884
	(0.8852)	(0.0427)	(0.0829)			(0.8876)	(0.0426)	(0.0836)	
20 %	0.9540**	0.0061	0.0319**	0.8882	20 %	0.9553**	0.0067*	0.0312**	0.8907
	(0.8874)	(0.0427)	(0.0825)			(0.8899)	(0.0425)	(0.0832)	
30 %	0.9551**	0.0065	0.0327**	0.8897	30 %	0.9565**	0.0072*	0.0322**	0.8923
	(0.8888)	(0.0426)	(0.0823)			(0.8914)	(0.0424)	(0.0829)	
40 %	0.9561**	0.0069*	0.0334**	0.8910	40 %	0.9575**	0.0076*	0.0329**	0.8936
	(0.8900)	(0.0425)	(0.0821)			(0.8926)	(0.0424)	(0.0826)	
50 %	0.9570**	0.0073*	0.0341**	0.8921	50 %	0.9584**	0.0079*	0.0336**	0.8947
	(0.8911)	(0.0425)	(0.0819)			(0.8937)	(0.0423)	(0.0824)	

Table 8 (*Continued*)

Confidence	β_M	β_{SMB}	β_{HML}	R^2	Confidence	β_M	β_{SMB}	β_{HML}	R^2
60 %	0.9578**	0.0076*	0.0346**	0.8931	60 %	0.9592**	0.0083*	0.0343**	0.8958
	(0.8921)	(0.0424)	(0.0817)			(0.8948)	(0.0423)	(0.0822)	
70 %	0.9586**	0.0080*	0.0352**	0.8942	70 %	0.9601**	0.0087**	0.0350**	0.8969
	(0.8931)	(0.0424)	(0.0815)			(0.8958)	(0.0422)	(0.0820)	
80 %	0.9595**	0.0084*	0.0359**	0.8955	80 %	0.9611**	0.0091**	0.0358**	0.8981
	(0.8943)	(0.0423)	(0.0813)			(0.8970)	(0.0421)	(0.0817)	
90 %	0.9608**	0.0090**	0.0368**	0.8970	90 %	0.9623**	0.0097**	0.0368**	0.8997
	(0.8958)	(0.0422)	(0.0811)			(0.8985)	(0.0420)	(0.0814)	
95 %	0.9617**	0.0094**	0.0375**	0.8982	95 %	0.9633**	0.0102**	0.0375**	0.9009
	(0.8970)	(0.0421)	(0.0809)			(0.8997)	(0.0419)	(0.0812)	
99 %	0.9634**	0.0103**	0.0387**	0.9004	99 %	0.9650**	0.0110**	0.0389**	0.9030
	(0.8990)	(0.0419)	(0.0805)			(0.9017)	(0.0418)	(0.0807)	

For each confidence level, β and R^2 values are shown, and significance at the 1 % and 5 % levels are given by ** and *, respectively. R^2 value from simple linear regression representing the relationship between a single factor and portfolio returns is shown in parenthesis

R^2 values for R2 are at least 0.7 for all samples under no-shorting constraints. This pattern of MV having lower correlation with Fama-French factors than R1 and R2 is detected in all values of λ for both no constraints and no-shorting constraints.

Furthermore, Fig. 7 presents R^2 values for several confidence levels for R1 and R2 under no constraints when the value of λ is set to 0.05. Similarly, Fig. 8 presents the same information under no-shorting constraints when the value of λ is set to 0.3. Observe that the correlation increases as the portfolio's robustness is increased; robust portfolios with a confidence level of 90 % have the highest R^2 values, followed by 50 %, and then 10 %. Particularly for R2, all samples show a strict increase in correlation with the Fama-French factors as the robustness increases. Although we utilize a sample of 200 stocks, our results provide strong evidence that our findings at the industry-level are also observed with individual securities.

6 Further analysis

We further analyze portfolios MV, R1, and R2 in order to validate our findings on robust portfolios: we confirm that the robust portfolios in our empirical tests are indeed more robust, and also look into the diversification levels to show that our results are not an effect of diversification. Therefore, we first confirm whether R1 and R2 form portfolios that are relatively more robust compared to MV, and also look for patterns in performance among the returns of the three portfolios. Second, we examine diversification levels of MV, R1, and R2 using two diversification measures to check if diversification has any influence on our empirical results reported in Sect. 5.

6.1 Robustness

Out-of-sample tests are performed on industry-level portfolios to compare the robustness of MV, R1, and R2. For year t, we create 10 portfolios using one-year data each from year

Panel A. Value of λ is set to 0.03

Panel B. Value of λ is set to 0.07

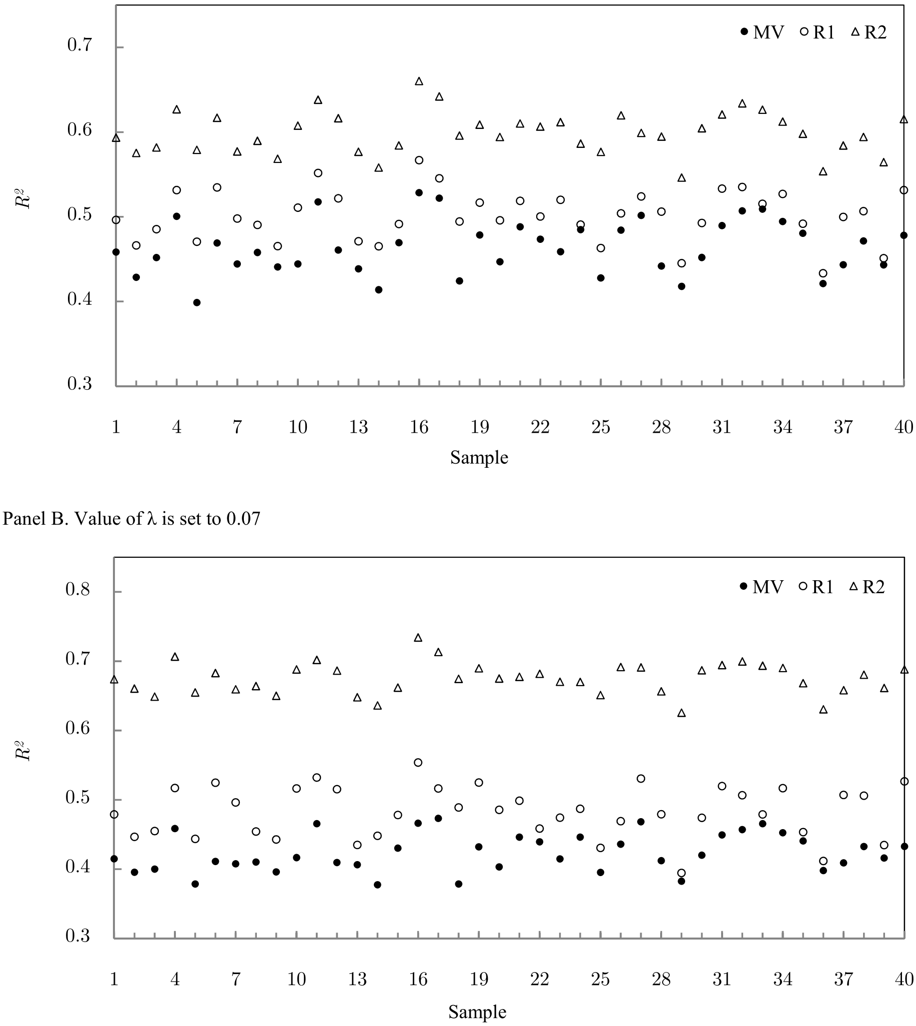
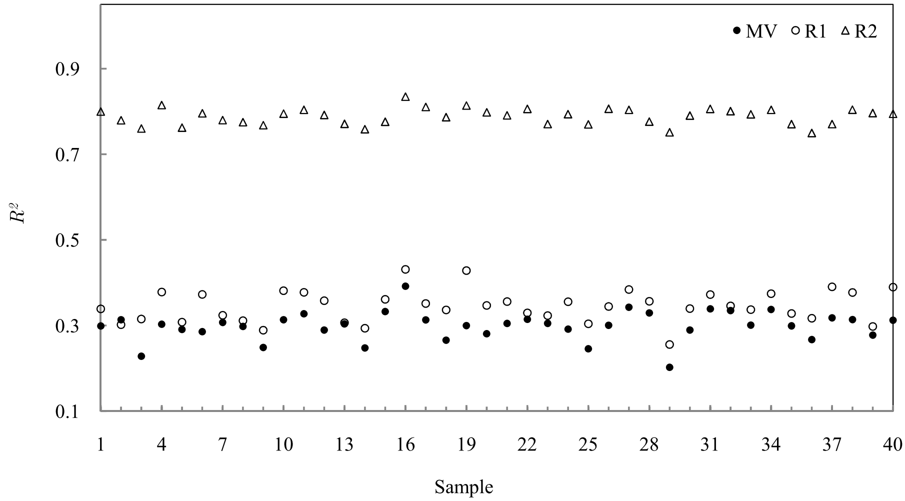


Fig. 5 Security-level results: R^2 values of 40 samples under no constraints with 95 % confidence

$t-10$ to year $t-1$. The returns of these 10 portfolios at year t are used to calculate the robustness and performance for each portfolio. Figures 9 and 10 plot the results when the value of λ is set to 0.03 and 0.07 with a 95 % confidence level under no constraints on portfolio weights. Although no clear dominance is observed in the mean return and mean Sharpe ratio among MV, R1, and R2, the standard deviation of portfolio returns and mean volatility of portfolios for MV are almost always higher than for both R1 and R2. In fact, higher values for MV are witnessed for all values of λ. The exact same pattern is observed under no-shorting constraints—that is, portfolios formed using the mean-variance model produce more volatile results. Lower standard deviation of returns and mean portfolio volatility indi-

Panel A. Value of λ is set to 0.2



Panel B. Value of λ is set to 0.4

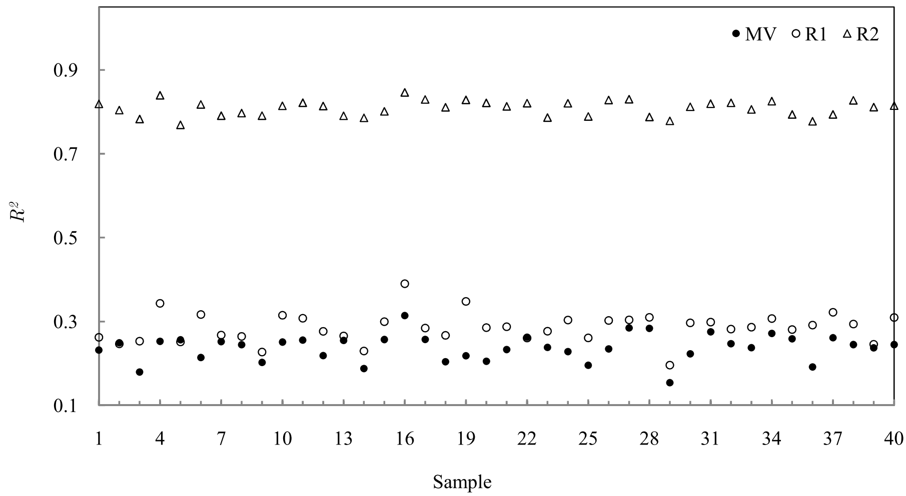


Fig. 6 Security-level results: R^2 values of 40 samples under no-shorting constraints with 95 % confidence

cate that R1 and R2 are less sensitive to changes in the inputs than MV, demonstrating that they are indeed more robust.

6.2 Diversification

One of the shortcomings of the mean-variance model is its tendency to put much weight on only a few assets since it searches through corner cases. Therefore, if robust portfolios are investing in more assets, the higher correlation with fundamental factors that we observe could be due to diversification. Although Tütüncü and Koenig (2004) find that robust port-

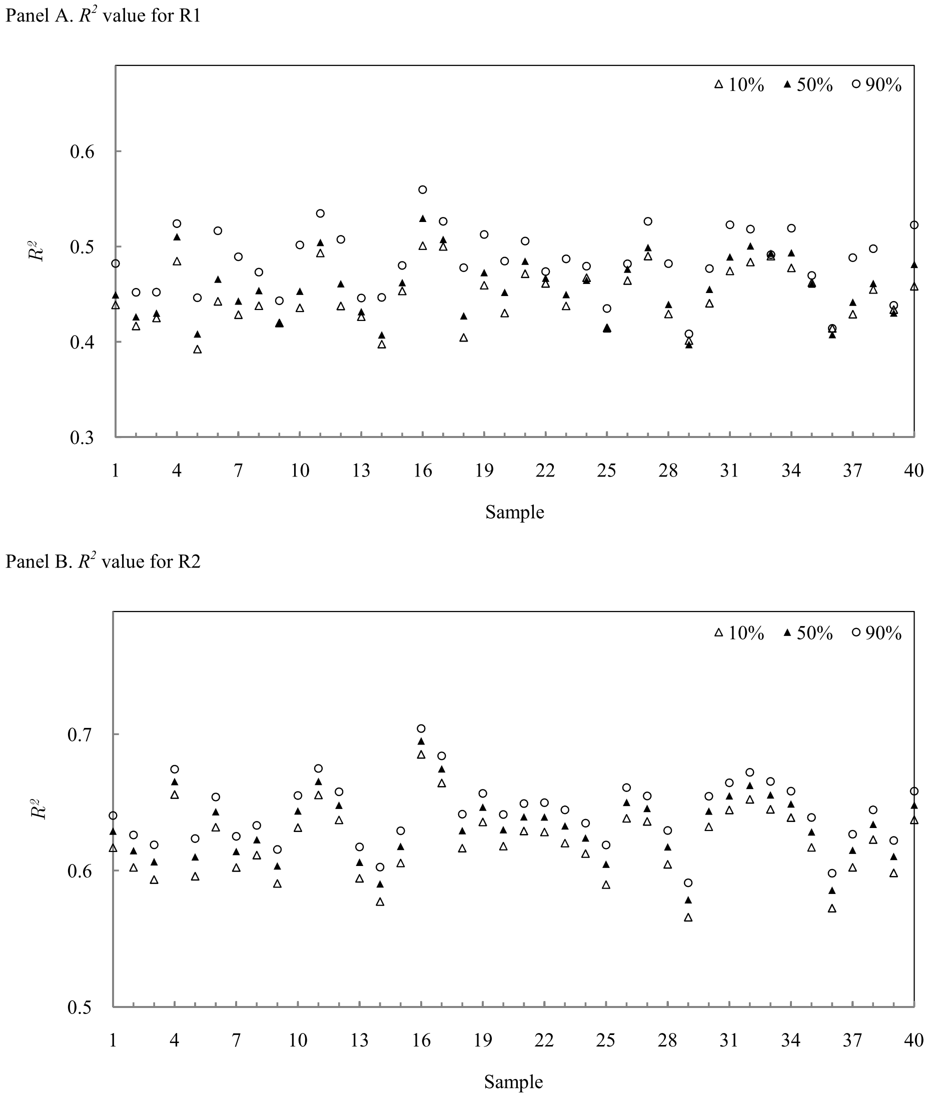


Fig. 7 Security-level results: R^2 values of 40 samples under no constraints for $\lambda = 0.05$

folios concentrate on a small set of asset classes, they conclude that a comparison based on diversification requires an appropriate metric. In order to eliminate the possibility that our observed patterns are caused by diversification, we compare the level of diversification among MV, R1, and R2.

Two measures have been used in other empirical studies as a measure of diversification: (1) the total number of assets in the portfolio and (2) deviation of a portfolio from the market portfolio. Goetzmann and Kumar (2008) use the first portfolio diversification measure, total number of assets in the portfolio, as a crude measure of diversification. Since we are using only 49 industries, for this diversification measure we count the number of industries with

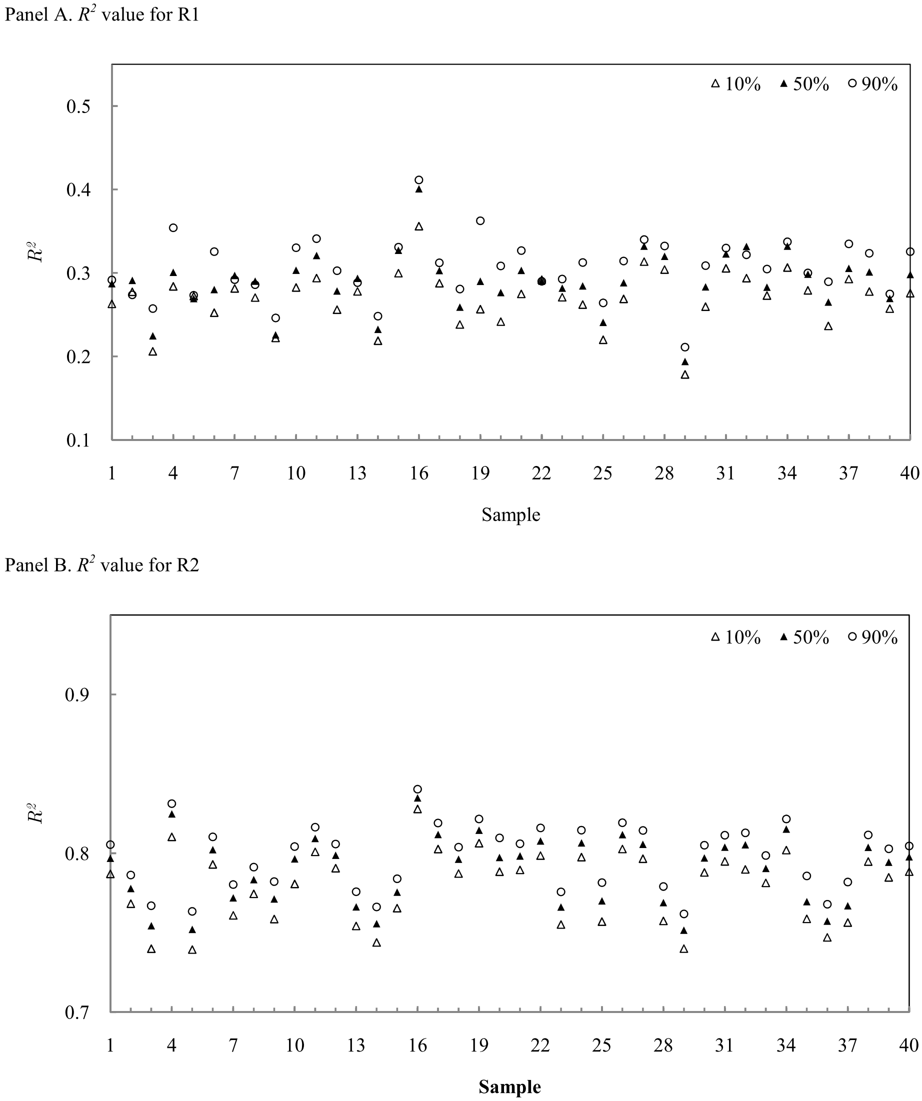


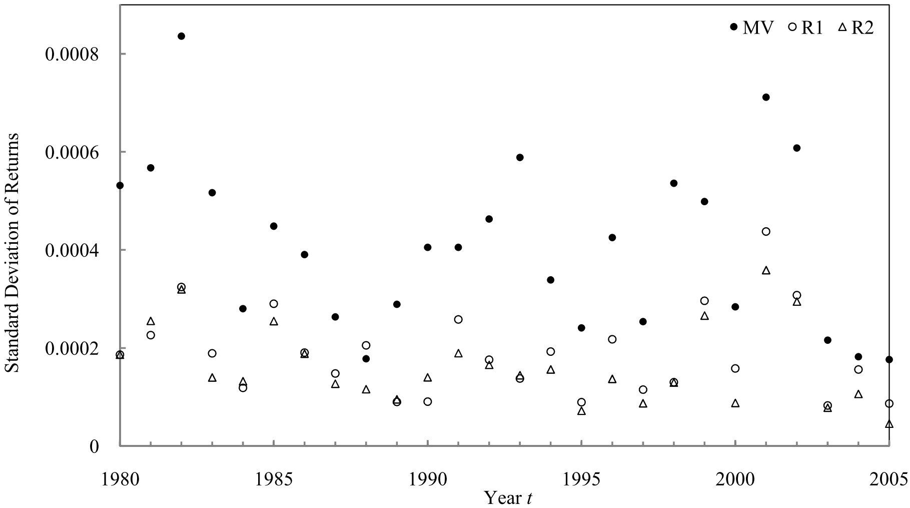
Fig. 8 Security-level results: R^2 values of 40 samples under no-shorting constraints for $\lambda = 0.3$

weights having an absolute value of at least 1 %; portfolios with a diversification level of 49 are considered fully diversified.

Blume and Friend (1975) introduced the second portfolio diversification measure: the deviation of a portfolio from the market portfolio. Since the weight of each security in the market portfolio would be very small, they approximated this measure with the sum of the squares of the proportions invested in each stock.

$$\sum_{i=1}^{N}(w_i - w_m)^2 = \sum_{i=1}^{N}\left(w_i - \frac{1}{N_m}\right)^2 \approx \sum_{i=1}^{N} w_i^2$$

Panel A. Value of λ is set to 0.03



Panel B. Value of λ is set to 0.07

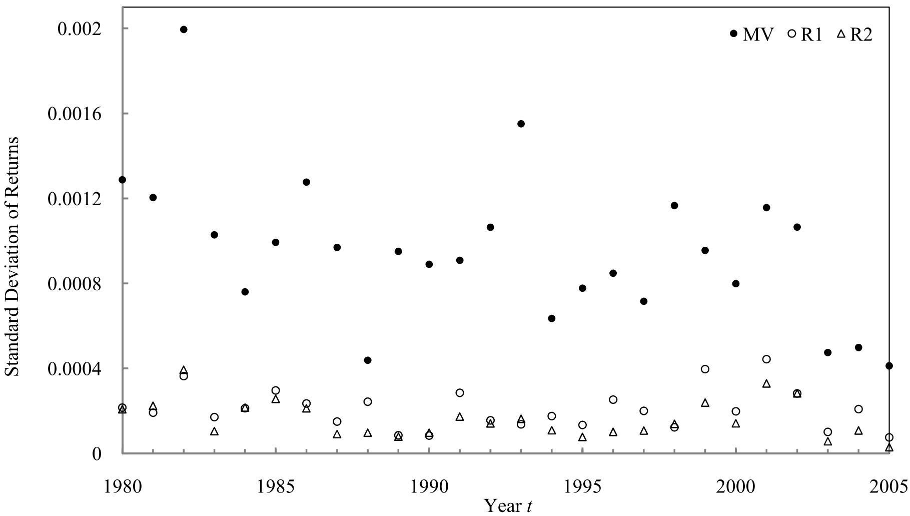


Fig. 9 Standard deviation of returns with 95 % confidence. Each point represents standard deviation of returns of 10 portfolios using one-year data each from year $t-10$ to year $t-1$ year, respectively

where N is the number of stocks in the portfolio, N_m is the number of stocks in the market portfolio, w_i is the weight given to security i in the portfolio, and w_m is the weight given to a security in the market portfolio. Since we are only using 49 industry-level portfolios in our case, we approximate this measure of diversification with the deviation from the equal-weighted portfolio as our second diversification measure,

$$\sum_{i=1}^{N}(w_i - w_m)^2 = \sum_{i=1}^{49}\left(w_i - \frac{1}{49}\right)^2$$

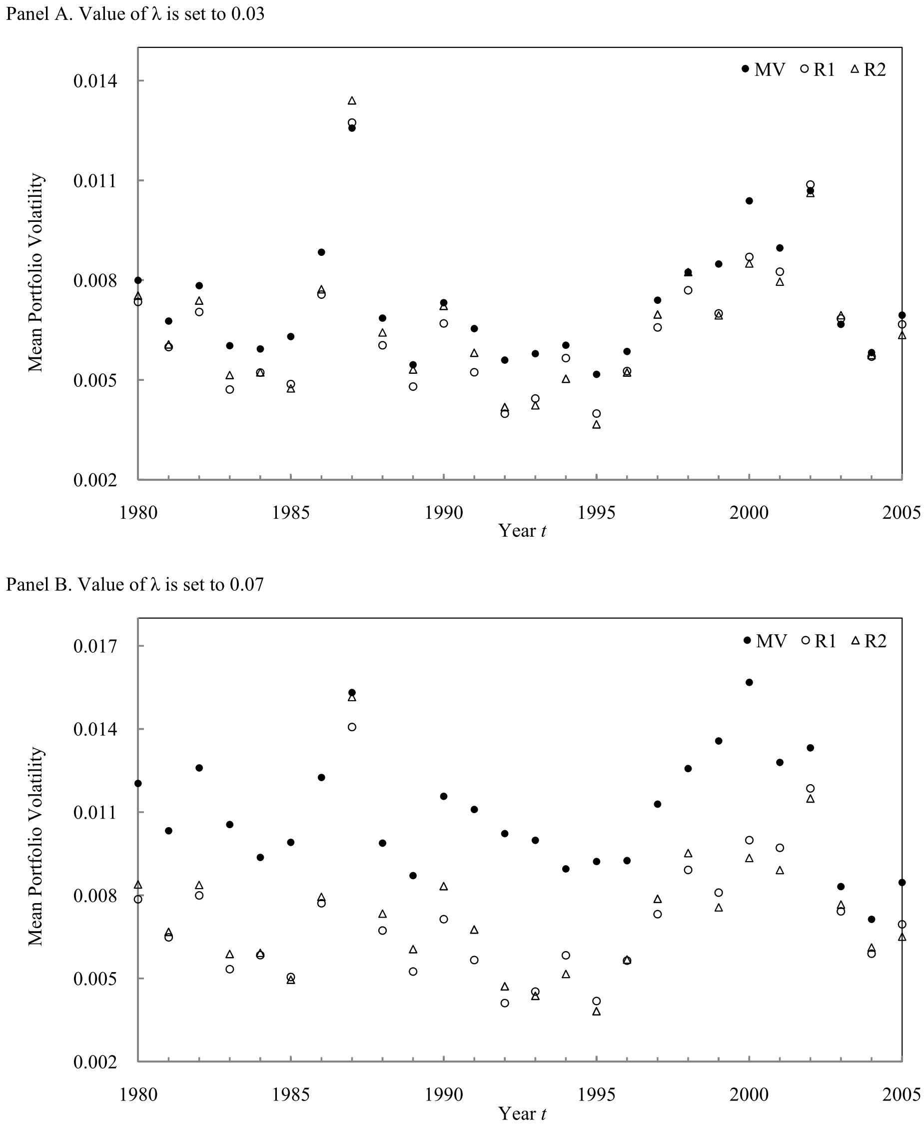


Fig. 10 Mean volatility of portfolios with 95 % confidence. Each point represents mean volatility of returns of 10 portfolios using one-year data each from year $t-10$ to year $t-1$ year, respectively

where w_i is the weight assigned to industry i. In contrast to the first measure, a sum of squared differences close to zero indicates well diversified portfolios.

Diversification levels for MV, R1 and R2 with 95 % confidence levels using a 5-year rebalancing period are summarized in Table 9. In terms of the number of assets in the portfolio, there is no consistent evidence showing that MV is less diversified than the other two robust portfolios. In fact, MV portfolios under no constraints have a greater number of securities in their portfolios than the portfolios generated from the two robust models. When no-shorting constraints are included, MV portfolios invest in a lot less number of securities

Table 9 Diversification levels for 95 % confidence level using a 5-year rebalancing period

	λ	Total number of assets			Sum of squared deviation		
		MV	R1	R2	MV	R1	R2
No constraints	0.01	40.8	37.1	38.5	0.5602	0.4168	0.3080
	0.03	44.9	26.8	37.1	0.7763	0.3657	0.1678
	0.05	46.3	20.0	36.3	1.2396	0.3417	0.1131
	0.07	47.1	15.9	36.4	1.9538	0.3219	0.0862
	0.09	47.1	14.1	36.6	2.9131	0.3085	0.0714
No-shorting constraints	0.1	11.8	9.8	31.8	0.1163	0.2653	0.0224
	0.2	11.1	9.4	34.0	0.1364	0.2372	0.0147
	0.3	8.3	7.6	34.3	0.1914	0.2799	0.0134
	0.4	6.3	6.0	34.6	0.2628	0.3537	0.0129
	0.5	4.8	5.3	34.5	0.3155	0.4178	0.0128

than the no constraints case, but R1 portfolios seem to have similar diversification levels as MV portfolios. Furthermore, when looking at the second measure of diversification, robust portfolios seem to be more diversified than MV under no constraint. However, MV is clearly more diversified than R1 under no-shorting constraints, making it difficult to conclude that robust portfolios are more diversified than mean-variance portfolios. Deviation for R2 is much lower than MV and R1, which could be one of the reasons why R2 has a much higher correlation with the Fama-French factors, especially under no-shorting constraints. Nevertheless, evidence that robust portfolios are not always more diversified allows us to discard the possibility that diversification is the main reason why robust portfolios show more dependence with fundamental factors.

7 Conclusion

Numerous studies have been conducted on solving the sensitivity problem of the classical mean-variance portfolio optimization. However, there has been relatively little effort to understand the characteristics of robust models. One of the notable works on finding the characteristics of robust portfolios is the mathematical explanation by Kim et al. (2012) to show how an increase in robustness leads a portfolio to become closer to the portfolio whose variance is most dependent on factors. In this study, we provide comprehensive support for their analytic findings and look for characteristics of robust equity portfolios that could further explain the behavior of robust models. Our main approach is to perform regression analysis to investigate the correlation between portfolio returns and fundamental factors as the robustness of a portfolio increases.

Using the Fama-French three-factor model, we find that as the robustness of a portfolio increases, the explanatory power for the return series of the three factors in that model increases. First, both robust models which assume the uncertainty set for the expected return as a box and an ellipsoid show higher R^2 values compared with the classical mean-variance model. Second, as the confidence level increases within the same robust portfolio optimization model, the R^2 value also increases, providing evidence that an increase in robustness also increases its dependency to the three fundamental factors. The plot of portfolios with various confidence levels in the mean-variance framework looks similar to the conventional shape of efficient frontiers; the portfolios with higher confidence level are plotted towards the left-bottom in the mean-variance plane, indicating that more robust portfolios move towards the risk-averse section. Furthermore, we find that an increase in robustness results in portfolios betting more on the market return. Particularly under no constraints on portfolio weights, it appears that robust equity portfolios bet on the market and large factors.

From these results, we see that robust portfolio optimization not only reduces its sensitivity to estimate errors but also puts more weight in factors as a side-effect of the process. We confirm this behavior at an industry level as well as at the individual security level, and also eliminate the likelihood that diversification of robust models is the sole reason for the observed patterns. Being more dependent on certain factors means that robust equity portfolios bet less on individual asset risks. Hence, we carefully suggest that robust equity portfolios could be more robust than classical mean-variance portfolios partly because they consistently bet on fundamental factors.

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