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Abstract

We decompose the Danish business cycle into ten structural shocks using an open-economy DSGE model with infrequent determination of prices and wages which we estimate with Bayesian techniques. Consistent with the Danish monetary policy regime, we formulate an imperfect peg on the foreign exchange rate and analyse the resulting monetary transmission mechanism.

We find that the Danish business cycle is dominated by stochastic movements in the labour supply in the long term, while demand shocks play a major role in the short term. Remarkably, the role of technology is negligible, and foreign factors only contribute little to the Danish business cycle, especially in the long term. With respect to the estimation, we generally find believable estimates although the degree of price stickiness is remarkably high.

Keywords: Open economy, Peg, Business cycles, Bayesian estimation

JEL Classifications: E3, E4, F4

1 Introduction

The aim of this paper is to analyse the determinants of business cycles in a small open economy with a fixed exchange rate. We formulate a dynamic stochastic general equilibrium (DSGE) model for a small open economy and estimate it on Danish data using Bayesian estimation techniques. Hence, our paper belongs to the new open-economy macroeconomics (NOEM) research programme (surveyed by Lane, 2001) which analyses open economies with well-specified microeconomic foundations. Since its inception with the seminal Obstfeld and Rogoff (1995) paper, the NOEM literature has offered many new insights and has proven popular with theorists and central bankers alike. However, empirical work has been relatively scarce within this framework, and thus a secondary aim of this paper is an empirical assessment of the NOEM framework's ability to adequately describe a Scandinavian economy.

We obtain three main conclusions regarding the determinants of the Danish business cycle: First, in the short run demand-side and supply-side shocks both contribute substantially to fluctuations, while long-run cycles are driven almost entirely by supply shocks. Second, even though supply shocks dominate fluctuations in the long run, the influence from technological shocks is almost negligible while shocks to labour supply are the main contributor to long-run volatility. Finally, a surprisingly large share of all cycles appears to be founded in domestic shocks rather than foreign ones. Our model allows for foreign shocks stemming from three channels; foreign prices, foreign demand and changes in the international interest rate level of which the latter channel appears to be the most potent foreign source of fluctuation.

We believe the Danish case to be particularly appealing for a structural estimation for three reasons: First, the dual assumption that the economy is small and open seems uncontroversial for the Danish economy, whereas we find the it more problematic for, e.g., the German, British and Japanese economies which have been considered in previous studies; second, the Danish economy has had a clear and unaltered monetary policy regime since 1987 which validates our empirical identification; and third, we have a reliable and consistent data set for the Danish economy covering the entire period (1987-2003) under consideration.

Since our focus is a small open economy, we base our theoretical model on that proposed by Kollmann (2001, 2002). Kollmann (2001) formulates and calibrates a model of a small open economy with imperfect competition and nominal rigidities in order to analyse the responses of nominal and real exchange rates to monetary policy shocks, while Kollmann (2002) elaborates on this calibrated model and analyses welfare consequences of different monetary policies. With respect to the monetary policy, which is of special interest to us, Kollmann (2001) considers a money-growth rule while Kollmann (2002) focuses on a generalised Taylor rule (although a perfect peg is also considered). However, as the Danish monetary policy has consisted of a peg on the euro (and the D-mark before 1999) with a constant parity since 1987, we introduce an imperfect peg regime to describe the monetary policy rule under the implicit assumption that the interest rate is the central bank instrument.[1]

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[1] As Kollmann (2002), we follow the current trend in this literature and consider the cashless limiting economy; that is, we consider an economy where money-based transactions are sufficiently unimportant for the utility of real consumption to be safely ignored. Thus, we ignore money and let the interest rate be the instrument for monetary policy. Woodford (2003) argues convincingly in favour of this approach which we consider to be the empirically relevant one.

We adopt the econometric framework of Smets and Wouters (2003), who successfully pioneered the application of Bayesian estimation techniques to a DSGE model. Thus, they estimated a variant of the complex model describing a closed economy originally constructed by Christiano et al. (2001, henceforth the CEE model) on data for the euro area. As Smets and Wouters, we structurally identify all volatility in the observed data which necessitates expansions of Kollmann's model. Thus, we include richer household preferences and a larger variety of structural shocks in our model.

The estimation of the structural model – the first of its kind that we are aware of on Danish data – yields plausible results. We do find a remarkably high degree of price stickiness, but as we discuss below this is a common feature of the emerging body of empirical evidence on DSGE models, and we discuss different possible explanations.

Bergin (2003) performs a related exercise; he estimates a variant of the Kollmann (2001) model on Australian, British and Canadian data and compares with reduced-form VAR models. In contrast to us, Bergin uses maximum-likehihood estimation and relies on a simple Choleski decomposition for identification of the structural shocks. He finds that the structural model provides a better fit than the VAR model, but is less successful at forecasting the paths of individual variables.

In another related paper, Lindé (2004) analyses the Swedish business cycle in an DSGE model. The focus is different from ours, however, as Lindé excludes preference shocks and nominal rigidities (and thus monetary policy) in his model and estimates it on annual data in accordance with his emphasis on the relative contributions of technology, fiscal policy and foreign factors to economic volatility. In contrast, we emphasise the short-term implications of the monetary policy regime and abstract from fiscal policy in our analysis.

The paper continues as follows; Section 2 presents the model, Section 3 describes the estimation methodology and the results, Section 4 analyses the properties of the estimated model and Section 5 concludes.

2   The Model

In this section we build a modified version of the open-economy DSGE model with staggered price setting presented in Kollmann (2002). Like him, we consider a small open economy that produces a continuum of intermediate goods which are aggregated and sold under imperfect competition to final-good producers at home and abroad. Producers of intermediaries only reoptimise prices infrequently a la Calvo (1983), but can differentiate fully between the domestic and foreign market and price their goods abroad in the local currency. It follows that prices are sticky in the currency of the buyer, an assumption that has been forcefully argued by, e.g., Betts and Devereux (1996, 2000). Recently, Bergin (2003, 2004) has compared local and producer currency pricing in estimated DSGE models and found strong empirical support for local currency pricing. Final goods are produced from aggregates of the intermediate goods from home and abroad and sold in a perfectly competitive market. Thus, all trade takes place in intermediary goods. McCallum and Nelson (1999, 2000) analyse a simpler model based on the same assumption and argue that it is empirically superior to one with trade in final goods.

We replace the homogenous and perfectly competitive labour market of Kollmann (2002) with one of differentiated labour services and rigid wage setting due to Erceg et al. (2000) and Kollmann (2001) which was also implemented in the CEE model. Furthermore, we follow Smets and Wouters (2003) and assume CRRA preferences and external habit formation; thus, the preferences analysed in Kollmann model are a special case of ours. We maintain, however, the quadratic investment adjustment costs in the relative level of capital, the debt premium on the interest earned on foreign bonds and the UIP shock from the Kollmann (2002) model. Finally, we introduce an imperfect peg regime for monetary policy with a persistent policy shock.

In this section we outline the various components of the rather rich model.[2]

2.1 Households

Like Erceg et al. (2000) we assume a continuum with unity mass of symmetric households with index j who obtain utility from consumption of the final good $C_t(j)$ and disutility from labour efforts $l_t(j)$. Thus, they are all characterized by the following preferences:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U\left(C_t^*(j), l_t(j)\right)\right],\tag{1}$$

$$U\left(C_t^*, l_t(j)\right) = \zeta_t^b\left[\frac{C_t^*(j)^{1-\sigma_C}}{1-\sigma_C} - \zeta_t^L\frac{l_t(j)^{1+\sigma_L}}{1+\sigma_L}\right], \quad \sigma_C, \sigma_L > 0$$

where ζ_t^b represents a shock to the discount rate and ζ_t^L represents a shock to the labour supply, while the coefficient of relative risk aversion σ_C is also the inverse intertemporal elasticity of substitution, and σ_L represents the inverse Frisch labour supply elasticity; finally, $j \in [0,1]$ signifies the household. We follow Smets and Wouters (2003) and assume external habit formation in consumption; that is, utility is obtained from

$$C_t^*(j) = C_t(j) - hC_{t-1}, \quad 0 \leq h \leq 1,\tag{2}$$

where hC_{t-1} is the habit stock at time t which is external in the sense that it is proportional to the past aggregate consumption level that is considered exogenous to the individual household. We further assume a security market where households completely diversify their individual income uncertainty, so that consumption is equalised across households; $C_t(j) = C_t, \ \forall j$.

Each household supplies an idiosyncratic variety of labour service $l_t(j)$. These labour services enter as a Dixit-Stiglitz aggregate in the intermediate-goods firm production; thus, letting $l_t(s,j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of labour services;

$$L_t(s) = \left[\int_0^1 l_t(s,j)^{\frac{1}{1+\gamma_t}}\, dj\right]^{1+\gamma_t}.\tag{3}$$

Here, γ_t is the net wage markup which is assumed to be an i.i.d. process with mean $\gamma > 0$.

Wage setting is staggered a la Calvo (1983). That is, in each period household j only

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[2]A technical appendix with a thorough derivation of the model and its steady state is available upon request.

optimises its wage $w_t(j)$ with probability $1 - D$. The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma_t}} dj \right]^{-(1+\gamma_t)}$ as given when it chooses its optimal wage $w_{t,t}$ and will meet any demand for the given type of labour;[3]

$$l_t(j) = \int_0^1 l_t(s,j) \, ds. \tag{4}$$

In addition to consumption, households can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows with δ measuring depreciation;

$$K_{t+1} = K_t(1-\delta) + I_t - \frac{\Phi}{2}\frac{(K_{t+1} - K_t)^2}{K_t}, \quad 0 < \delta < 1, \quad \Phi > 0, \tag{5}$$

where I_t is investment. Here, we have followed Kollmann (2002) and assumed quadratic adjustment costs. Domestic bonds A_t earns net interest i_t, while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the foreign interest level i_t^* as follows;

$$\left(1 + i_t^f \right) = \Omega_t^j \left(1 + i_t^* \right), \tag{6}$$

$$\Omega_t = \upsilon_t \exp\left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \quad \Xi = \frac{e P^x Q^x}{P}, \tag{7}$$

where e_t is the nominal exchange rate and P_t is the price of final goods, while Ξ is the steady-state value of export in units of the domestic final good. Thus, the interest on foreign bonds is growing in the foreign debt level which ensures the existence of a unique equilibrium, cf. Schmitt-Grohe and Uribe (2003), while υ_t is a stochastic shock which we motivate with the empirically observed departure from the uncovered interest parity. We style υ_t a UIP shock but abstain from a deeper explanation of its nature; Bergin (2004) offers a good discussion of UIP shocks in the NOEM literature.

Households own equal shares of domestic firms and thus earn profit from the intermediate-goods firms $(\Delta_t(j))$ in addition to rental rates R_t on the capital, wage income from their labour services and payments from their state-contingent securities $(S_t(j))$. Hence, the budget constraint of household j is

$$A_{t+1}(j) + e_t B_{t+1}(j) + P_t(C_t(j) + I_t(j)) = \tag{8}$$
$$A_t(j)(1 + i_{t-1}) + e_t B_t(j)\left(1 + i_{t-1}^f\right) + R_t K_t(j) + \Delta_t(j) + w_t(j) l_t(j) + S_t(j).$$

Thus, households decide their consumption, wages and investments in accordance with the solution to the following problem;

$$\max_{\{C_t(j), A_{t+1}(j), B_{t+1}(j), K_{t+1}(j), w_{t,t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t^*(j), l_t(j) \right) \right], \tag{9}$$
$$\text{s.t. (1)-(8).}$$

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[3] Note that the optimal wage in any period is identical across households, which is the reason why $w_{t,t}$ can be written without index j.

The first-order conditions for domestic and foreign bonds yield regular Euler conditions;

$$(1 + i_t) E_t \left[\rho_{t,t+1} \right] = 1, \tag{10}$$

$$\left(1 + i_t^f \right) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t} \right] = 1, \tag{11}$$

$$\rho_{t,\tau} \equiv \beta^\tau \left(U_{C,\tau} / U_{C,t} \right) \left(P_t / P_\tau \right), \quad U_{C,t} \equiv \frac{\partial U \left(C_t^*, L_t \right)}{\partial C_t}, \tag{12}$$

where $\rho_{t,\tau}$ discounts profits at time τ. One should bear in mind, however, that in this case $U_{C,t}$ depends on C_{t-1} as well as C_t due to our assumption of external habits.

The optimal wage level $w_{t,t}$ is the solution to the following first-order condition;

$$\sum_{\tau=t}^{\infty} (\beta D)^{\tau-t} \frac{\chi_\tau}{\gamma_\tau} w_{t,t}^{-\frac{1+2\gamma_\tau}{\gamma_\tau}} E_t \left[\frac{U_{C,\tau}}{P_\tau} w_{t,t} - (1 + \gamma_\tau) U_{L,\tau} \right] = 0,$$

where $D^{\tau-t}$ is the probability that the chosen wage level $w_{t,t}$ is still in effect in period τ. Thus, the infrequent and stochastic reoptimisation implies that the household must weigh the marginal utility of consumption against the disutility of labour in all future periods when it sets its wage level. Finally, under the Calvo-like assumptions of the wage setting, the aggregate wage level evolves as follows;

$$W_t = \left[D \left(W_{t-1} \right)^{-\frac{1}{\gamma_t}} + (1 - D) \left(w_{t,t} \right)^{-\frac{1}{\gamma_t}} \right]^{-\gamma_t}.$$

2.2 Final Goods

Final goods Z_t are produced using intermediate-good bundles from home $\left(Q_t^d \right)$ and abroad $\left(Q_t^m \right)$ respectively. These intermediary aggregates are combined with a Cobb Douglas technology;[4]

$$Z_t = \left(\frac{Q_t^d}{\alpha^d} \right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m} \right)^{\alpha^m}, \quad \alpha^d, \alpha^m > 1 \quad \alpha^d + \alpha^m = 1.$$

Each bundle of intermediate goods is a Dixit-Stiglitz aggregate. Here, we follow the assumptions of the CEE model and let the net markup rate ν_t be an i.i.d. process with mean ν;

$$Q_t^i = \left[\int_0^1 q^i \left(s \right)^{\frac{1}{1+\nu_t}} ds \right]^{1+\nu_t}, \quad i = d, m.$$

Assuming that domestic firms face the problem of minimizing the cost of producing Z_t units of the final good, demands for goods produced domestically and abroad can be written as

$$Q_t^i = \alpha^i \frac{P_t}{P_t^i} Z_t, \quad i = d, m,$$

$$P_t = \left(P_t^d \right)^{\alpha^d} \left(P_t^m \right)^{\alpha^m},$$

where the appropriately defined price index P_t is the marginal cost of the final-goods producing firm. With perfect competition in the final-goods market, P_t is also the price of one unit of the

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[4]Bergin (2004) estimates a model where domestic and foreign intermediate goods are combined with the more flexible CES technology. He finds that the special Cobb-Douglas case is in accordance with the data.

final consumption good.

2.3 Intermediate Goods

Intermediate goods are produced from labour L_t and capital K_t using Cobb-Douglas technology. Thus, the production function of firm s is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1,$$

where θ_t is the aggregate level of technology. Producers operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set.

Firms rent capital at the rate R_t and compensate labour with wages W_t. Hence, any firm's marginal costs are

$$MC_t = \frac{1}{\theta_t} W_t^{1-\psi} R_t^\psi \psi^{-\psi} (1-\psi)^{-(1-\psi)}. \tag{13}$$

Producers sell their good variety to both domestic and foreign final-goods producers (that is, $y_t(s) = q_t^d(s) + q_t^m(s)$) and are able to price discriminate between the two markets. As is well-known from the Dixit-Stiglitz models, final-good producers demand individiual varieties of intermediaries as follows

$$q_t^i(s) = \left(\frac{p_t^i(s)}{P_t^i}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^i, \quad i = d, m,$$

and thereby firm profits can be written as

$$\pi^{dx}\left(p_t^d(s), p_t^x(s)\right) = \left(p_t^d(s) - MC_t\right) q_t^d(s) + (e_t p_t^x(s) - MC_t) q_t^x(s).$$

We furthermore assume that foreign exporters produce at unit costs equivalent to the aggregate foreign price level P_t^* and thus generate the following profits in the domestic market;

$$\pi^m(p_t^m(s)) = (p_t^m(s) - e_t P_t^*)\left(\frac{p_t^m(s)}{P_t^m}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^m.$$

Demands from foreign final-goods producers are assumed to be of the Dixit-Stiglitz form as well;

$$q_t^x(s) = \left(\frac{p_t^x(s)}{P_t^x}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^x, \quad Q_t^x = \left(\frac{P_t^x}{P_t^*}\right) Y_t^*,$$

where the foreign aggregates P_t^*, Y_t^* are exogenous.

As in the case of wages, we follow Calvo (1983) and assume that a firm only reoptimises its prices in any given period with probability $1-d$. Given that domestic firms seek to maximise profits discounted with a pricing kernel based on household utility (cf. equation (12)), a firm that reoptimises its domestic price faces the following problem;

$$p_{t,t}^d = \arg\max_\omega \sum_{\tau=t}^\infty d^{\tau-t} E_t\left[\rho_{t,\tau}\pi^{dx}\left(\omega, p_t^x(s)\right)\right].$$

As firms set prices in the domestic and foreign market separately, the constant marginal costs – cf. equation (13) – implies that the two price setting problems are independent. Hence, the optimal price $p_{t,t}^d$ is determined from the following first-order condition;

$$\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\left(p_{t,t}^d - (1 + \nu_\tau) MC_\tau \right) \rho_{t,\tau} \left(\frac{p_{t,t}^d}{P_\tau^d} \right)^{-\frac{1+\nu_\tau}{\nu_\tau}} \frac{Q_\tau^d}{p_{t,t}^d \nu_\tau} \right] = 0, \tag{14}$$

which illustrates an essential feature of Calvo pricing; because of the infrequent and stochastic price setting firms must consider expectations of all future levels of marginal costs and demands when calculating their optimal price. The optimal price in the export market $\left(p_{t,t}^x \right)$ is determined analogously.

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,\tau} \equiv \Pi_{s=t}^{\tau-1} (1 + i_s^*)^{-1}$. Thus, when they reoptimise, they set their prices in order to maximize discounted future profits measured in foreign units;

$$p_{t,t}^m = \arg\max_\omega \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} \pi^m (\omega) / e_\tau \right],$$

which again implies a condition for the optimal price $p_{t,t}^m$ similar to that for $p_{t,t}^d$.

Finally, the aggregate Dixit-Stiglitz prices of the intermediate goods are as follows;

$$P_t^i = \left[d \left(P_{t-1}^i \right)^{-\frac{1}{\nu_t}} + (1 - d) \left(p_{t,t}^i \right)^{-\frac{1}{\nu_t}} \right]^{-\nu_t}, \quad i = d, m, x.$$

2.4 Market Clearing

All intermediaries are demanded from either domestic or foreign final goods producers, while final goods can either be consumed or invested in capital. Hence, equilibria in the markets for intermediate and final goods require

$$\begin{aligned} Y_t &= Q_t^d + Q_d^x, \\ Z_t &= C_t + I_t. \end{aligned} \tag{15}$$

Equalising the supply and demand for capital implies

$$K_t = \int K_t (s) \, ds.$$

Finally, we assume that only domestic agents hold the domestic bond, implying that $A_t = \int A_t (j) \, dj = 0$ in equilibrium.

Aggregating and manipulating the household budget constraint (8) and using the final-good market equilibrium (15) yields the following equation which simply states that the net foreign assets position (NFA) changes with accruing interest and the net export.

$$\begin{aligned} e_t B_{t+1} + P_t (C_t + I_t) &= e_t B_t \left(1 + i_{t-1}^f \right) + R_t K_t + W_t L_t + P_t^d Q_t^d + e_t P_t^x Q_t^x - (R_t K_t + W_t L_t) \Rightarrow \\ B_{t+1} &= B_t \left(1 + i_{t-1}^f \right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m. \end{aligned}$$

2.5 Monetary Policy

We postulate an imperfect peg against the euro as the monetary policy; in our model the interest rate is the instrument, which is thus used to keep e_t constant up to an exogenous policy shock ξ_t with unity mean;

$$e_t = e\xi_t. \tag{16}$$

Log-linearizing equations (6) and (7) yields the following relation between the internal foreign interest rate and that paid to domestic holders of foreign bonds;

$$\hat{\imath}_t^f = \hat{\imath}_t^* + \hat{v}_t - \lambda\hat{B}_t.$$

Combining this relation with log-linearised versions of equations (10) and (11) yields

$$
\begin{aligned}
E_t\Delta\hat{e}_{t+1} &= \hat{\imath}_t - \hat{\imath}_t^f = \hat{\imath}_t - \hat{\imath}_t^* + \left(\lambda\hat{B}_t - \hat{v}_t\right), \\
\hat{\imath}_t &\equiv \log\left(\frac{1+i_t}{1+\bar{\imath}}\right), \quad \hat{v}_t \equiv \log\left(v_t/v\right), \quad \hat{B}_t \equiv \log\frac{(B_{t+1}/P_t^*)}{\Xi},
\end{aligned}
$$

which we can combine with (16) to obtain

$$\hat{\imath}_t = \hat{\imath}_t^* + \left(\hat{v}_t - \lambda\hat{B}_t\right) + E_t\Delta\hat{\xi}_{t+1},$$

that is, the interest rate responds (virtually) one-to-one with the foreign interest rate and the UIP shock and is additionally skewed by the spread and the policy shock.

2.6 Solving the model

We log-linearise the model around its deterministic steady state and solve the resulting linear rational expectation system with the Sims (2002) method. The log-linearised system is summarised in Appendix A, while the method used to solve it is described in Appendix B.

3 Estimation

We now consider the results and underlying assumptions of our estimation. Before we list our specific assumptions and report our estimation results, however, we briefly motivate the Bayesian methodology that we utilise.

3.1 Estimation Methodology

We seek suitable econometric tools to quantify and evaluate our postulated structural model of the Danish economy given our set of observed time series. Building on the seminal analysis in Smets and Wouters (2003), we follow what Geweke (1999) styled the strong econometric interpretation of our DSGE model. This implies that we postulate a full probabilistic characterisation of our observed data which allows us to estimate the structural parameters through classical maximum-likelihood methods; or alternatively–following Bayesian methodology–through

combining the likelihood function with prior distributions on the structural parameters and maximise the resulting posterior density.

In this paper we follow the Bayesian approach which allows us to formalise the use of any prior knowledge we may have on the structural parameters. On a more practical level it also helps stabilise the nonlinear minimization algorithm which we use for the estimation. Given the limited length of our sample, reasonable assumptions for the prior distributions (including restrictions on the support of certain parameters such as, e.g., standard deviations) are likely to be essential for obtaining plausible estimates. On the other hand, we utilise prior distributions we believe to be broad enough in order for the data to inform us on the structural parameters of the theoretical model.

Our model includes ten structural shocks and nine observed variables. Thus, we can proceed on the assumption that there is no measurement error in the data set without facing the problem of stochastic singularity. In other words, we attribute all stochastic volatility to identified structural shock processes. This approach was succesfully carried out in the Smets and Wouters (2003) analysis of a close variant of the CEE closed-economy model.

Alternative ways of estimating DSGE models do exist; Christiano et al. (2001), e.g., estimate their model using GMM techniques based on a loss criterion measuring the distance from impulse-response functions of a monetary policy shock generated by an identified VAR and the parameterized DSGE model, respectively. This, however, does not appeal to us; we acknowledge that consistency between the predictions of a VAR and a DSGE model is a strong indication that the predicted outcome of a given experiment is robust; we believe, however, that these predictions should be obtained apart from each other. In particular, the sketched method of estimation depends critically on the right identificatin of the VAR model – a controversial issue on its own that will have significant effects on the estimation. By maximising a likelihood function combined with priors, we link the DSGE model and its way of propagating shocks directly to the observed patterns in the data, thus avoiding controversial assumptions of the identification of a VAR model.

3.2 Data

We treat Denmark as the home country and a weigthed average of a Germany, France and the Netherlands as the foreign country. For Denmark we include observations of real GDP, total real consumption, the GDP deflator, total employment adjusted for variations in hours worked, and a three-month money-market interest rate, corresponding to the theoretical variables Y, C, P, L and i. We are unable to find a satisfactory measure for wages; although a suitable measure is availabe in the MONA databank, is only observed annually and thus unsuited for our analysis. We use quarterly data for the period 1987-2003 as the last adjustment of the parity between the Danish krone and the D-mark occured in January 1987.

Due to the data break in German GDP implied by the unification of East and West Germany in 1991, we have manipulated the German real GDP series; specifically we ran an OLS regression including only a linear time trend and a unification step dummy taking the value of one from 1991 onwards. Using the obtained coefficient we shifted the level of GDP and obtained our measure of German real GDP.

Since our model assumes that the home country is pegging the foreign country, and the Danish krone was effectively pegged to the D-mark before the current peg on the euro, our foreign aggregate should at the same time be broad enough to cover as much as possible of the Danish trade and narrow enough that we can plausibly claim that the relevant exchange rate for the foreign area was historically the D-mark. We settled on Germany, France and the Netherlands which constituted 28 percent of Danish exports in 2003. We used their relative weights from the current effective exchange rate for the Danish krone as calculated by Danmarks Nationalbank which are 69, 17 and 14 percent for Germany, France and the Netherlands, respectively. For this EU aggregate we include observations of geometric averages of real GDP and the GDP deflator, and of the D-mark/euro exchange rate vis-à-vis the Danish krone and a German three-month money-market interest rate, matching the theoretical variables Y^*, P^*, e and i^*. The data and their sources are further detailed in Appendix C.

Our log-linearized model describes stationary deviations from a steady state, so we follow Smets and Wouters (2003) and remove a linear trend from the log of our GDP, consumption and labour supply series. We further adjust the price series for a nominal trend in inflation and remove the same trend from the interest rates.

3.3 Prior Distributions

We fix a subset of key parameters which are likely to be poorly determined in a model that only considers deviations from the steady state. In a Bayesian sense, we assume very fixed prior distributions, namely ones with no variance. Thus, the discount factor β is fixed at 0.99, implying an approximate quarterly return of 1 percent, while the depreciation rate of capital δ is set at 0.025. The capital share ψ is set at 0.33, while the share of domestic intermediates in final production α^d is fixed at 0.7, and the net steady-state mark-up rate ν is fixed at 0.2. Furthermore, we follow Kollmann (2002) and set the capital mobility parameter λ at 0.0019 in accordance with the empirical findings of Lane and Milesi-Ferretti (2002). Finally, we fix η at unity corresponding to a foreign technology equal to that assumed for the home country.

This leaves us with six structural parameters capturing nominal rigidities, preferences and capital adjustment costs, as well as 17 parameters defining the structural shock processes. We assume beta distributions for parameters restricted to the range between 0 and 1, inverse gamma distributions for the standard deviations of the shock innovations, and gamma distributions for the remaining parameters. Thus, all parameters are restricted to take on positive values.

The Calvo parameters d and D are assumed to follow beta destributions with mean 0.75, which would imply that prices and wages are reoptimised every year on average. We keep the distributions tight (standard deviations are set at 0.03) since we consider price and wage contracts lasting more than 2 years on average implausible.

We assume that the inverse intertemporal elasticity of substition σ_C follows a gamma distribution with mean 1, corresponding to log preferences, while the inverse elasticity of work effort σ_L is assumed to be gamma distributed with mean 2 and a variance broad enough to cover the lower values obtained in the microeconometric studies and the higher values used in the RBC literature.

We further assume that the autocorrelation parameters ρ for the persistent structural shocks

all follow a gamma distribution with mean 0.85 and a standard devation of 0.06; the tight distributions around these high autocorrelations ensure that the persistent structural shocks are distinguishable from the i.i.d. shocks. All variances are assumed to follow inverse gamma distributions (which are the conjugate prior distributions in this case), and we have drawn on the assumptions in Smets and Wouters (2003), the calibrations in Kollmann (2002) and regressions on our data set to determine the mean of each distribution.

All the assumptions on prior distributions are summarised in Table 1.

Table 1: Parameter Estimates

Parameter		Prior distribution			Posterior estimate		Posterior distribution[3]		
		Type	Mean	Std. dev.[1]	Mode	Std. error[2]	5%	Median	95%
d	Calvo, intermediaries	Beta	0.75	0.03	0.941	0.003	0.933	0.940	0.946
D	Calvo, wages	Beta	0.75	0.03	0.770	0.017	0.733	0.781	0.824
h	Habit persistence	Beta	0.70	0.10	0.433	0.061	0.310	0.487	0.656
σ_C	Household inverse IES	Gamma	1	10	1.709	0.127	1.143	1.583	2.183
σ_L	Inverse Frisch elasticity	Gamma	2	10	1.032	0.064	1.140	1.357	1.729
Φ	Capital adj. cost	Gamma	15	4	14.422	0.133	14.435	14.907	15.222
Shocks, persistence									
ϱ^b	Discount rate	Beta	0.85	0.06	0.825	0.077	0.668	0.785	0.874
ϱ^l	Labor supply	Beta	0.85	0.06	0.962	0.013	0.894	0.950	0.979
ϱ^t	Technology	Beta	0.85	0.06	0.824	0.015	0.753	0.823	0.882
ϱ^m	Peg	Beta	0.85	0.06	0.899	0.030	0.856	0.898	0.935
ϱ^i	Foreign interest rate	Beta	0.85	0.06	0.877	0.123	0.838	0.876	0.906
ϱ^P	Foreign price level	Beta	0.85	0.06	0.925	0.012	0.878	0.921	0.952
ϱ^Y	Foreign GDP	Beta	0.85	0.06	0.912	0.025	0.855	0.914	0.959
Shocks, volatility									
σ^b	Discount rate	Inv. gamma	0.01	4	0.041	0.010	0.033	0.044	0.060
σ^l	Labor supply	Inv. gamma	0.01	4	0.161	0.016	0.154	0.199	0.336
σ^t	Technology $\times 100$	Inv. gamma	0.70	4	1.073	0.090	0.953	1.094	1.266
σ^U	UIP $\times 100$	Inv. gamma	0.05	4	0.342	0.029	0.303	0.350	0.407
σ^{pm}	Price markup	Inv. gamma	0.15	4	2.095	0.018	1.896	2.070	2.184
σ^{wm}	Wage markup	Inv. gamma	0.15	4	0.002	0.029	0.001	0.003	0.008
σ^m	Peg $\times 100$	Inv. gamma	0.80	4	0.739	0.059	0.656	0.749	0.863
σ^i	Foreign int. rate $\times 100$	Inv. gamma	0.10	4	0.102	0.050	0.090	0.104	0.122
σ^P	Foreign price level $\times 100$	Inv. gamma	0.40	4	0.337	0.029	0.300	0.342	0.399
σ^Y	Foreign GDP $\times 100$	Inv. gamma	0.80	4	0.786	0.064	0.693	0.797	0.911

Footnote: [1]For the gamma and the inverse gamma distributions, we have cited the shape parameter α rather than the standard deviation. [2]The standard errors of the posterior-mode estimates are based on the numerically calculated Hessian matrix. [3]The posterior distribution is based on a Metropolis-Hastings sampling of 60,000 draws.

3.4   Posterior Estimates

We show our estimation results in Table 1. First, we report the mode of the posterior distribution using a numerical minimisation routine. These estimates are shown along with standard errors derived from the numerically calculated Hessian. Secondly, we report the median and the 5th and 95th percentiles from the posterior distributions. These distributions were simulated with Markov-chain Monte-Carlo methods. In particular, we ran a Metropolis-Hastings algorithm with 60,000 draws from a multivariate Gaussian jumping distribution.[5]

The prior and posterior distributions of the 23 estimated parameters are also illustrated in Figures 1 to 3. Most parameters are estimated to be significantly different from zero. The exceptions are the standard deviations of the innovations to the foreign interest rate and of the wage markup shock. Since we did not have observations of the wage level, it is not too surprising that the wage markup shock is estimated to be insignificant. The autocorrelations of the persistent structural shocks lie in the range from 0.82 (the level of technology) to 0.96 (labour supply); thus, the data have not caused them to diverge much from our prior assumptions.

Our estimates of the preference parameters seem plausible. The labor supply elasticity is approximately one and the intertemporal elasticity of substitution is a half, so that both belong to the range of values regularly applied in similar analyses. The estimate for the external habit stock h lies between a third and a half; this is on the lower side compared with the literature at large, but should be equally uncontroversial.

The estimated nominal rigidities are more problematic. The first thing that stands out is the large discrepancy between our prior beliefs and the posterior distribution of the Calvo price parameter d. The posterior mode is 0.94, corresponding to an average duration of intermediary price contracts of four years. Given a strict interpretation of the Calvo pricing model, our estimate is blatantly implausible. Obviously, our implementation of Calvo pricing is the simplest one possible, and a first attempt to improve it would be to add indexation to past inflation as in the CEE model.[6] However, when Smets and Wouters (2003) estimate this more elaborate model, they obtain results that are very similar to ours; hence, we are skeptical that this expansion of the pricing model will yield an estimated Calvo parameter in the range we consider plausible. Another obvious extension is to consider separate Calvo parameters for import and export prices, although one would have to include observed time series of these prices to improve the estimation this way.

Smets and Wouters speculate that the high price stickiness is in part due to the assumption of constant returns to scale in the intermediary-goods sector – as confirmed empirically by Galí et al. (2001), this assumption implies an upward bias in the price rigidity if the returns to scale are in fact decreasing. Alternatively, Altig et al. (2005) show that if one replaces the assumption of an economy-wide rental market for capital with one of firm-specific capital, the implied Calvo price parameter is reduced significantly; thus, in their benchmark model (a

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[5]More accurately, we made 61,000 draws but discarded to first thousand in order to avoid problems with the initial point. We used the inverse Hessian calculated at the posterior mode scaled down by a factor eight as covariance matrix in the jumping distribution.

[6]By indexation to past inflation we mean that prices and wages which are not reoptimized in a given period are adjusted for past aggregate inflation instead. We refer the reader to Woodford (2003, ch. 3) for a thorough discussion of indexation in the Calvo pricing model.

variant of the closed-economy CEE model) the average time between price reoptimisations is reduced from 5.6 to 1.5 quarters, as they change the assumption regarding the capital market. Both arguments indicate that the cause of the implausibly high estimate of the Calvo parameter may just as well lie outside the specific formulation of the pricing model.

The other troubling result of our estimation is the large variance of the price markup shock. We suspect the high volatility is caused by two factors; first, we have neither public spending nor investment shocks in the model to explain the stochastic nature of demand; second, we believe that the large variance is compensating for the restrictive version of the Calvo pricing model we have implemented.

According to our estimation, the capital adjustment cost parameter Φ follows a tight posterior distribution located very close to the prior mean. Our experience with the estimation algorithm and the impulse-response functions indicate that the model properties change markedly with even small changes in Φ.

We find plausible estimates for the remaining parameters which define the structural shock processes.

The observed time series (solid lines) and the one-step-ahead predictions (dashed lines) from the Kalman filter are shown in Figure 4. Given the simplicity of the presented model, we find the fit to be overall satisfactory, although the restrictive Calvo model yields a somewhat problematic fit for prices, and the model is not quite capable of explaining the large and persistent swings in the observed labour supply.

4   Analysing the Properties of the Estimated Model

4.1   Impulse Response Analysis

We now consider the effects of shocking the exogenous processes of the model. First, we focus on the effects of an expansionary monetary policy shock and a drop in the foreign interest rate level, respectively. Subsequently, we briefly consider the effects of an uanticipated rise in the technology level and in the foreign demand. The impulse-response functions to innovations in these four shocks are shown in Figures 5-8; we depict the median (solid line) and the 5th and 95th percentiles (dashed lines) as calculated from 1,000 draws from the posterior distributions.

We begin with an analysis of the monetary transmission mechanism, by which we mean the endogenous responses to a change in the domestic interest rate which is the instrument of the central bank in our model. As discussed in section 2.5, an expansionary monetary policy shock under our peg regime implies setting the interest rate below the level required to keep the exchange rate constant. Thus, technically, the expansive policy shock is equivalent to a temporary devaluation, and we implement it through a positive innovation to ξ_t.

The effects of an expansionary monetary policy shock are shown in Figure 5. We see that a devaluation implies a positive response in consumption and investments and, hence, output. The expansive effect of a devaluation on output peaks one quarter after the devaluation with an increase in output of .8 percent following a 1 percent devaluation. Investments respond more strongly and peaks instantaneously with a 2.9 percent response. The transmission mechanism is as follows; the opportunity cost of consumption decreases with the lower interest rate, as it gets
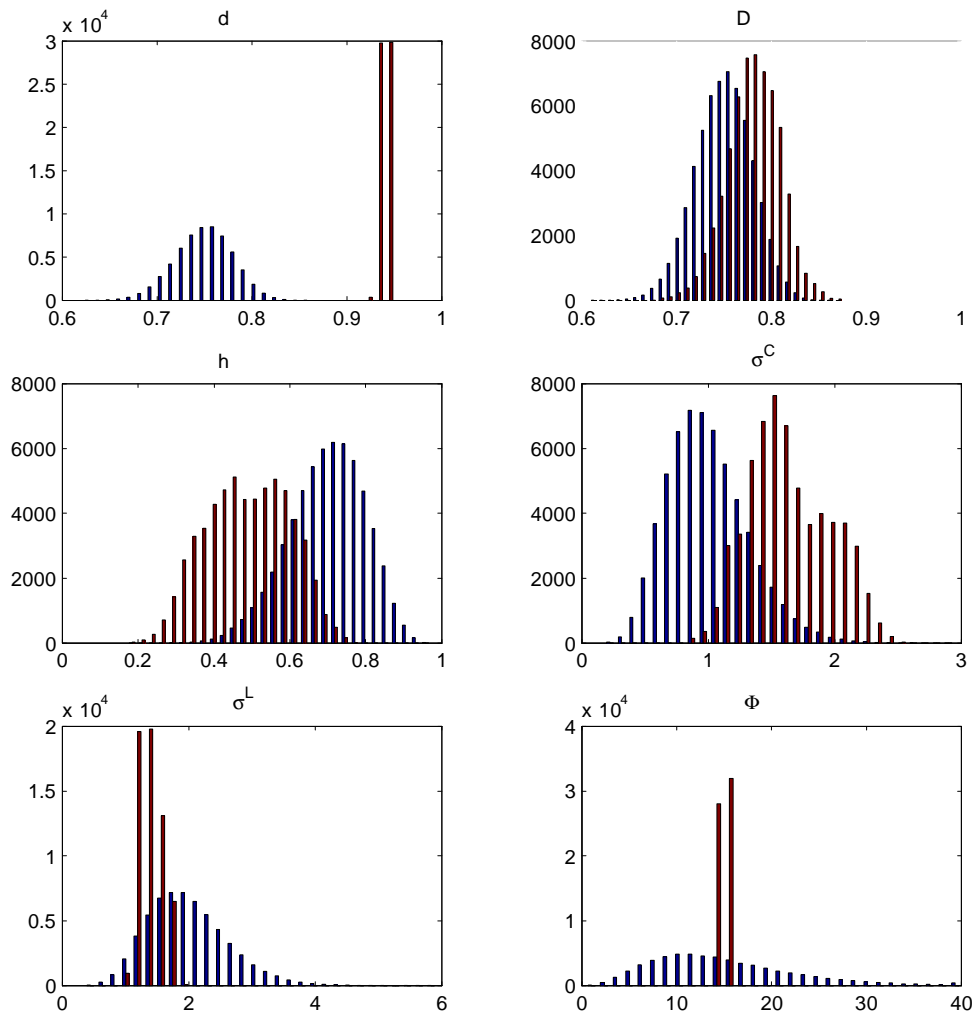
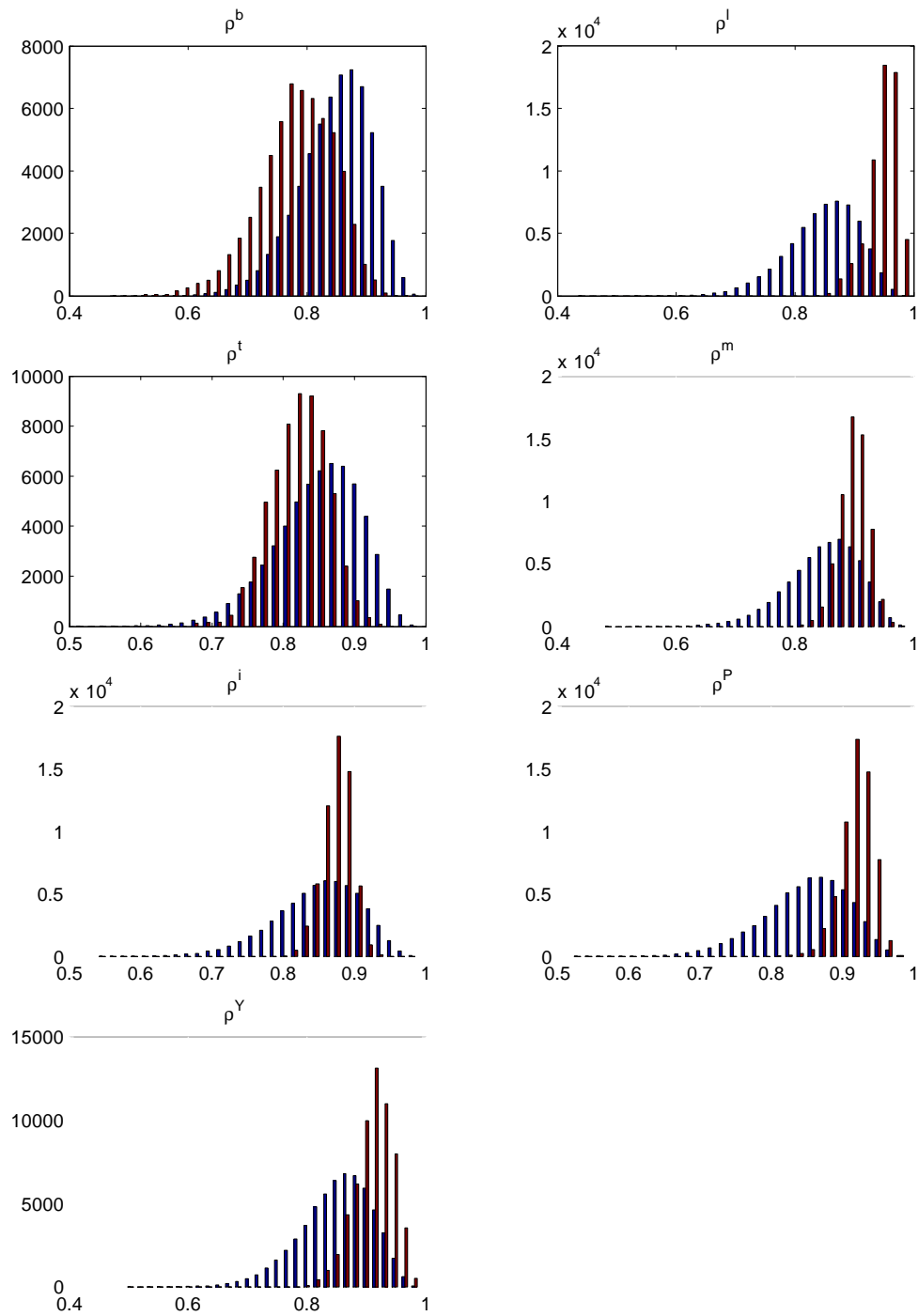Figure 1: Preference and rigidity parameters

Figure 2: Persistence of structural shocks
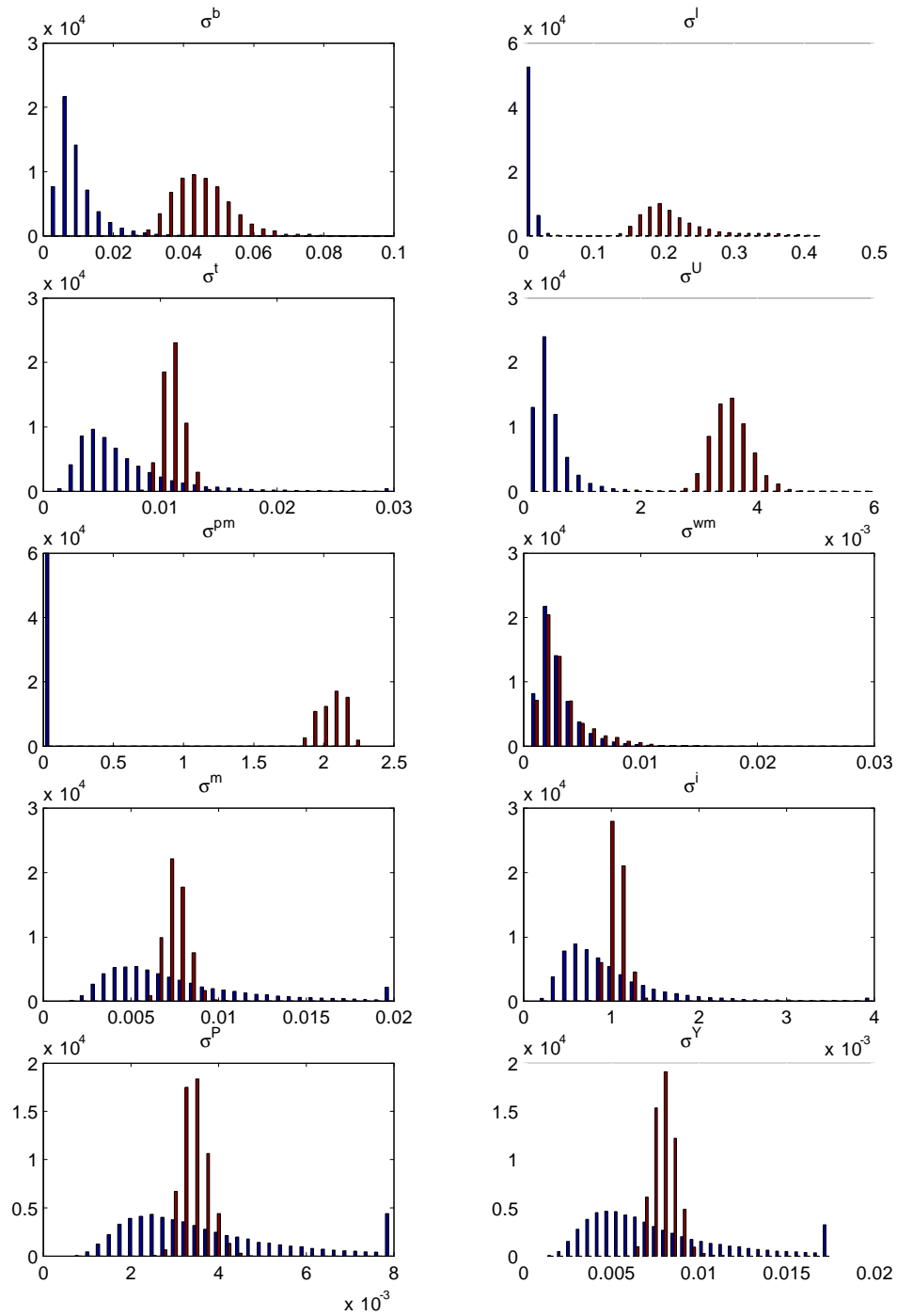
Figure 3: Volatility of structural shocks

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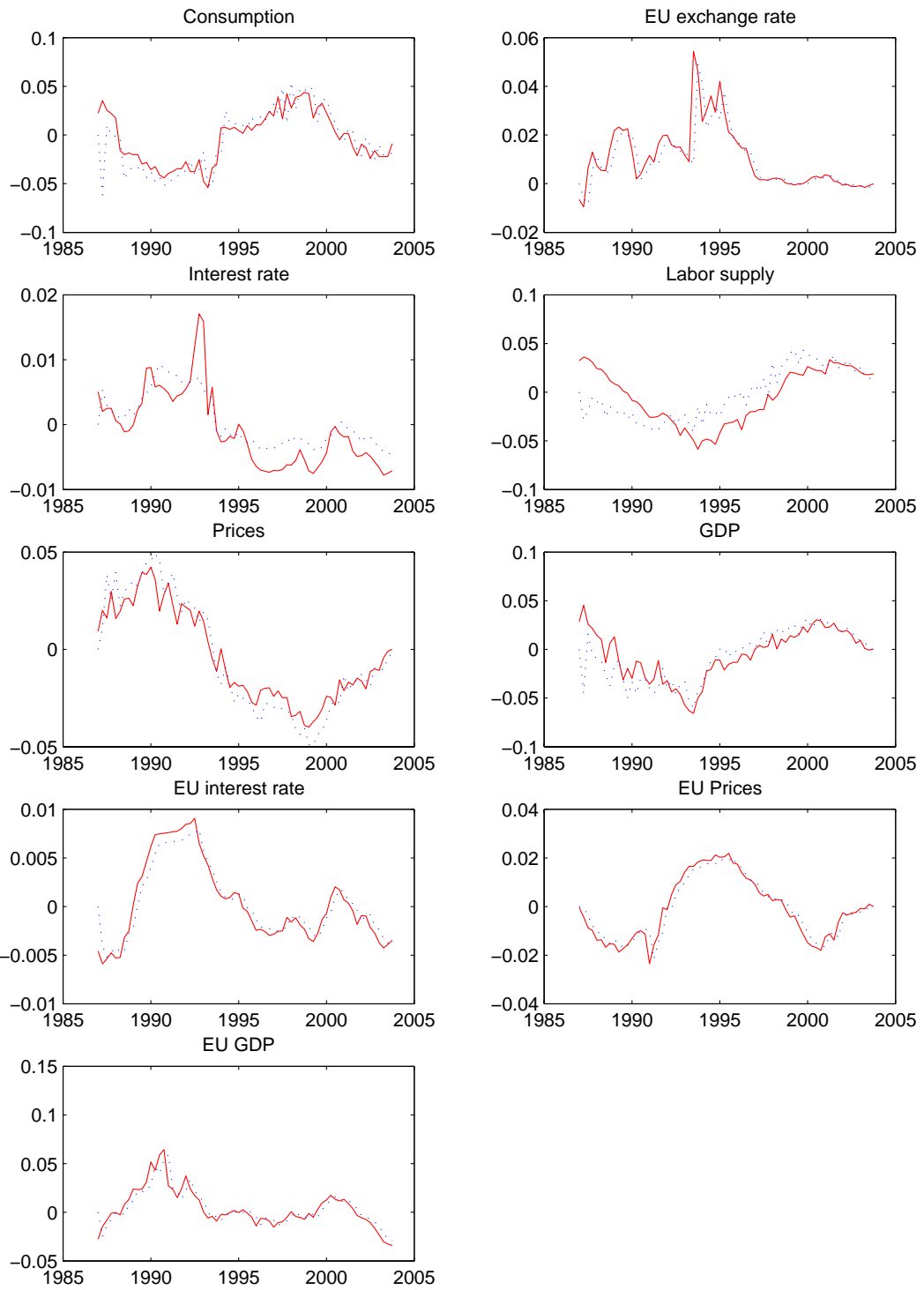
Figure 4: Observed (solid lines) and predicted data
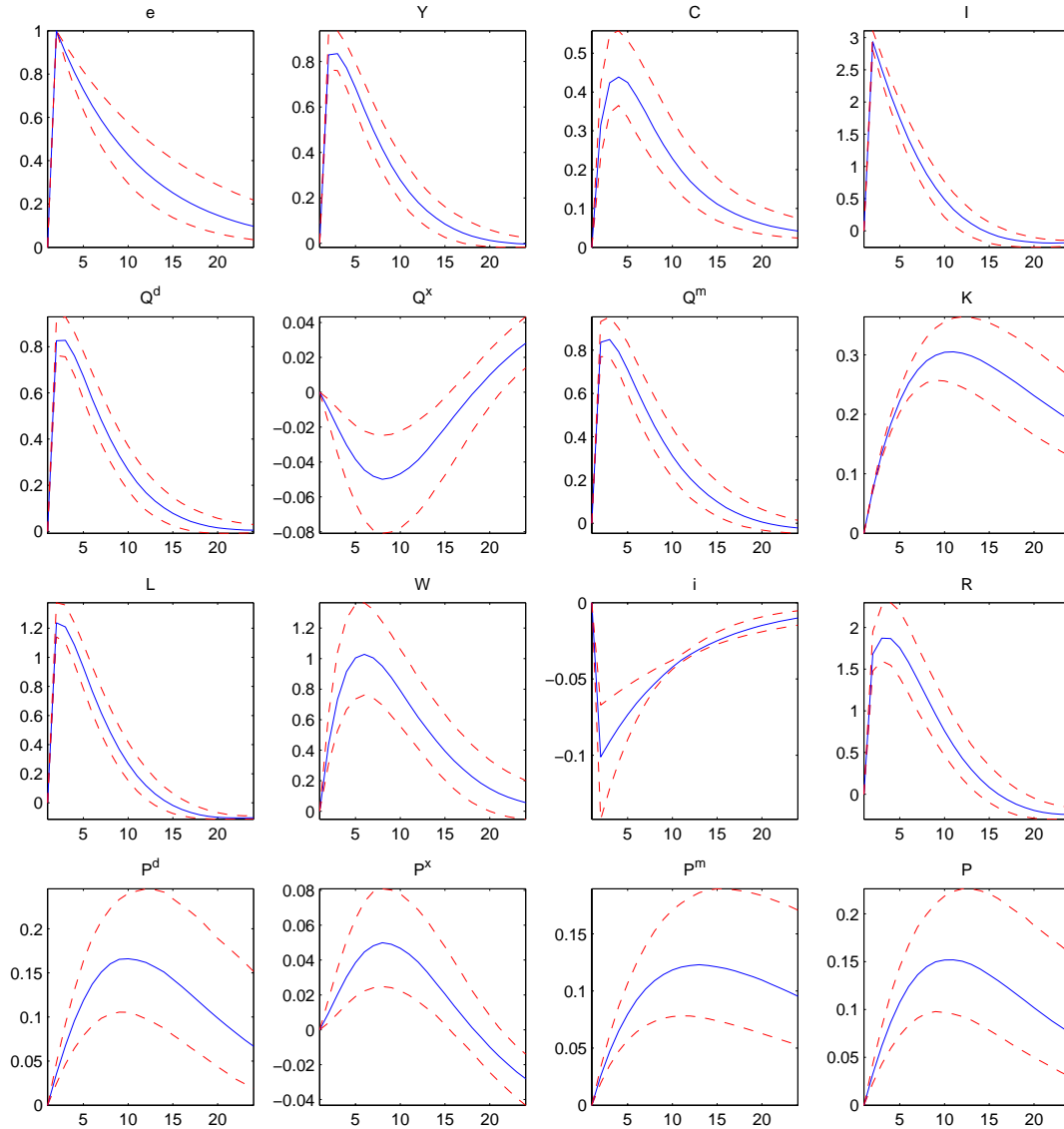
Figure 5: Responses to a monetary policy shock

less beneficial to hold domestic bonds. Hence, aggregate demand rises which increases demand for labour and capital. This stimulates investments and pushes up marginal costs as wages and the rental price of capital increase. Since prices on the intermediary goods are determined as a markup on marginal costs, they increase correspondingly. Note, however, that the rise in the price on exported intermediaries (measured in the foreign currency) is less than a third of the rise the in domestic price level. The explanation is that by assumption the firm sets the price in the foreign currency, but seeks to maximise profits measured in local currency; thus, as the devaluation itself provides a sizeable increase in profits from exported intermediaries, maintaining the optimal markup requires a relatively smaller rise in the export price. In total, the devaluation creates inflation, and prices peak after 10 quarters with an estimated rise of .15 percent.

In Figure 6 we consider the effects of shocking the level of the foreign interest rate. The peg regime implies that the central bank lowers the domestic interest rate one-for-one with the
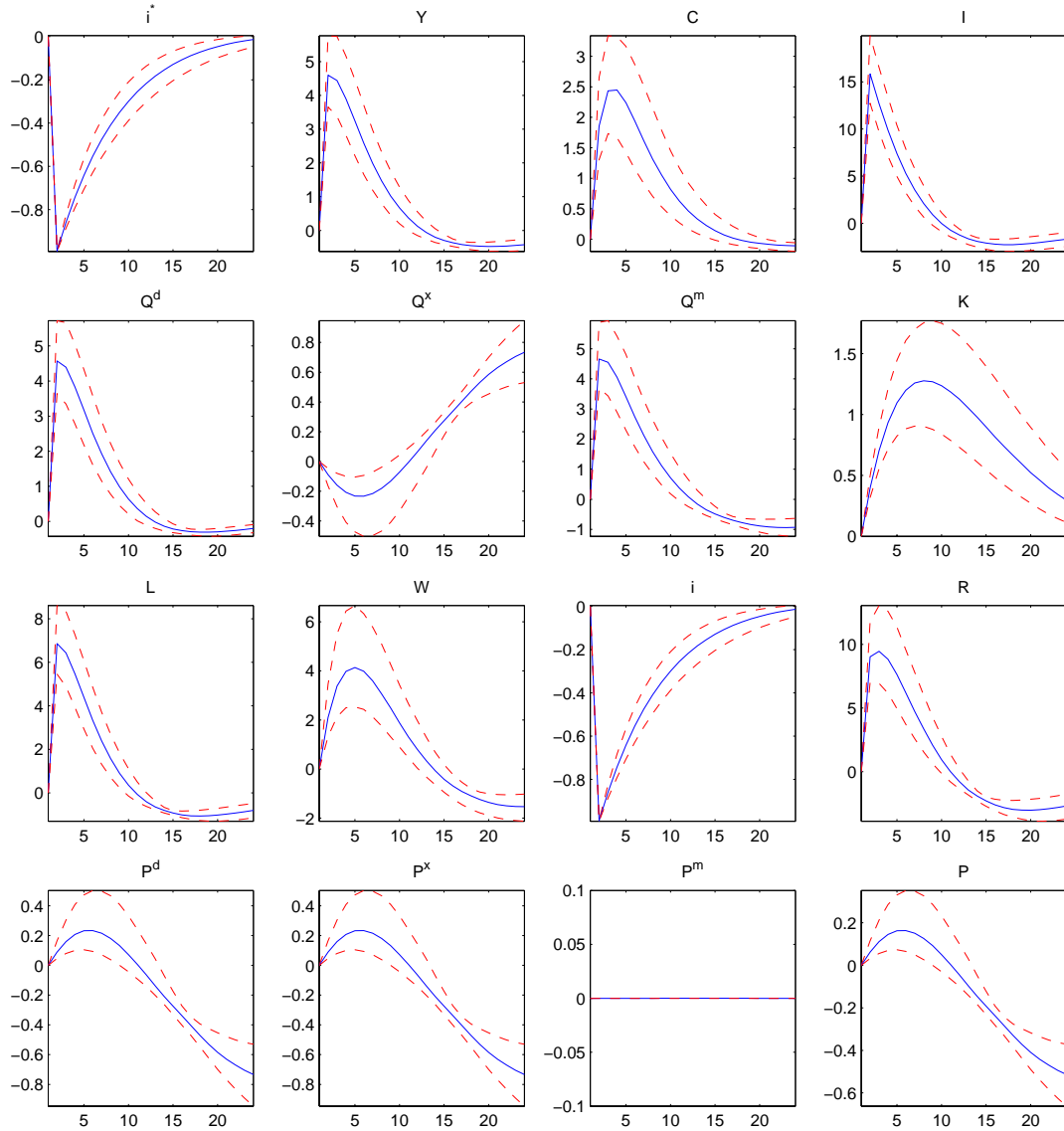
Figure 6: Responses of a shock to the foreign interest rate

foreign decrease in order to keep the nominal exchange rate fixed, and thus the main differences between the former experiment and the present are (i) the positive co-movement in the domestic and foreign interest rate levels; and (ii) the fact that the in the present experiment the exchange rate is fixed. The real effects of this shock are similar to the ones above; the nominal effects, however, differ. Notably, the determinants of the import price are unaffected, while the prices set domestically experience long-lasting swings. Domestic producers initially increase their prices because of the higher demand. As capital accumulates and pushes the rental rate down, while the interest rates on bonds return to towards the steady-state level, investment and output demand drop below their initial levels, and firms respond by lowering their prices. Thus, while prices rise at first, they begin to fall after approximately 6 quarters and fall below the initial level after three years; only after some eight years do they begin a slow return towards the long-run equilibrium.

With regards to the technology shock depicted in Figure 7, we see that a positive shock to

technology results in an initial drop in output. This is in sharp contrast to a standard RBC model. The reason is the estimated high degree of price stickiness in intermediaries. Thus, even though the positive shock to technology shifts the supply curve of the firms to the right, the price inertia causes the short-run supply curve to be almost horizontal, and thus the direct supply-side effect on output is small. Furthermore, a given level of production can now be reached using fewer production resources due to the higher level of productivity, causing employment as well as capital demand to decrease. In turn, households wish to hold less capital stock and disinvest. Thus, total demand for final goods has fallen, and in equilibrium this effect dominates the positive supply effect, implying a lower output equilibrium than before the shock. Over time, however, prices do fall because of the highly persistent technology shock that has decreased marginal costs, and as demand responds to the lower prices, capital is accumulated and investments rise.

The initial negative response in output seems implausible; we should expect to see output rise, and very possibly the introduction of variable capital utilisation would secure this result. In the presence of variable utilisation, firms have the option of holding idle capital rather than eliminating capital through depreciation, so that initial investment is optimal in order to spread out the convex costs of the medium-term increase in the capital stock, even though demand for capital services has momentarily gone down. Finally, we confirm the nice hump-shaped behaviour of Y and I which is considered an empirically relevant response, cf. Cogley and Nason (1995).

The effects of shocking the foreign demand are found in Figure 8. No surprises here; an increased demand from abroad increases demands for intermediaries; this raises demand for capital and labour pulling the rental rate and wages up, hence increasing marginal costs. Domestic producers of intermediary goods respond by increasing their prices, and consequently the price of domestic goodsdomestic The raises P_d and P_x, and hence P goes up as well.

4.2   Variance Decomposition

In Table 2 and 3 we present a variance decomposition of the forecast mean squared errors (MSE) at various time horizons measured in quarters which is based on 1,000 draws from the posterior distribution. Decomposing the contribution of the individual shocks to the movements in the endogenous variables yields some interesting conclusions. First, we note that technological innovations do not contribute much to the volatility in GDP in neither the short run nor in the long run. This result stands in sharp contrast to the baseline RBC model that relies on technological shocks as the sole driving force behind business cycles. Thus, our results strongly indicate that a model with a more elaborate set of structural shocks is important for understanding the forces that drive the economy. We note that Smets and Wouters (2003) find that technology plays a minor role in the business cycle of the euro area and regard our results to be consistent with theirs. In contrast, Bergin (2003) finds that output volatility is dominated by technology shocks, but it is worth nothing that he does not consider shocks to labour supply in his analysis.

Second, demand-side shocks – mainly preferences – have a sizeable impact in the short run, while supply shocks drive cycles in the long run; despite the apparently limited role of technological shocks, we note that the supply side do contribute substantially to fluctuations in
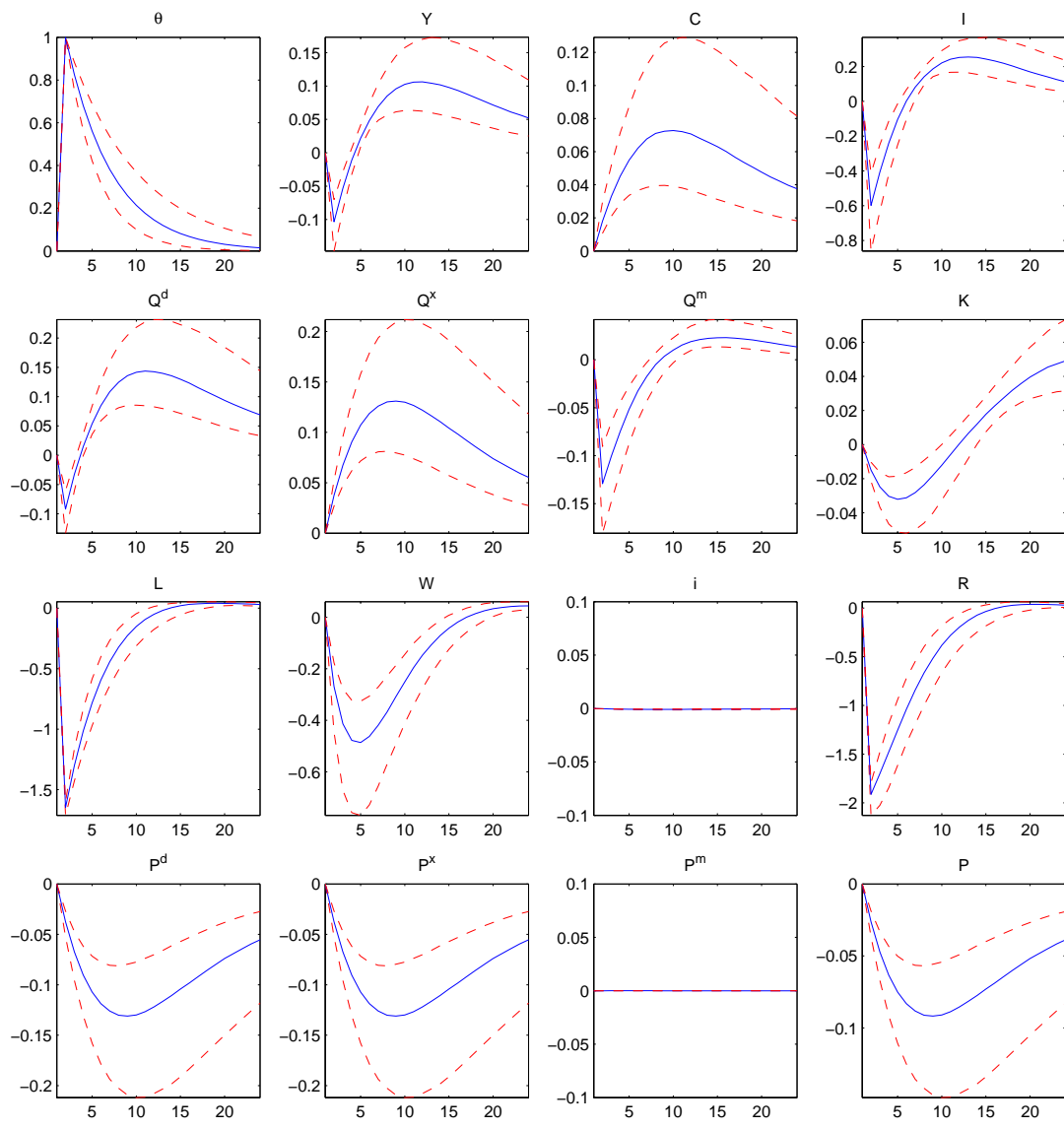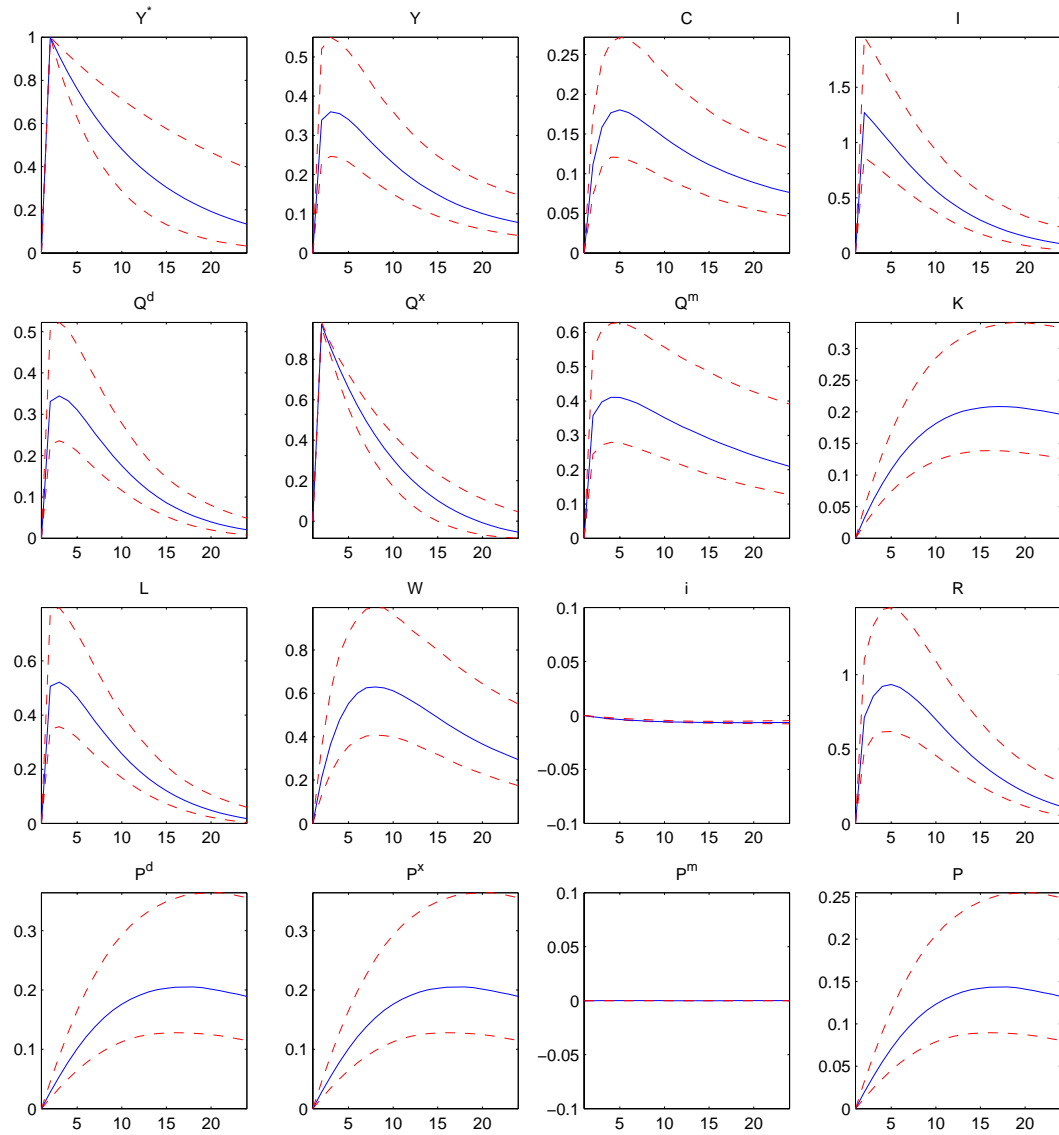
Figure 7: Responses to a shock to technology

Figure 8: Responses to a foreign demand shock

GDP. Initially, shifts in labour supply account for 23 percent of the volatility in GDP and more than two-thirds of the volatility in investments. In the long run labour supply shocks account for between 80 and 90 percent of all volatility in private consumption, investments and, hence, GDP. In the short run, however, we see that shocks to the discount rate yield the largest impact on GDP. Also, varying price markups contribute heavily. This pattern does not change much in the medium-term.

Third, somewhat surprisingly we see that business cycles in Denmark to a large degree appear to be founded domestically. Although Denmark is a highly open economy, less than 10 percent of the volatility in the short and medium term can be directly traced to foreign sources, by which we mean foreign GDP, prices and interest rates. However, shocks to price mark-ups also have a component that is determined abroad, as we have not distinguished between markup shocks from domestic and foreign producers. Taking this into consideration, at most 40 percent of the short run volatility can be attributed to foreign impulses dropping to 5-6 percent in the long run. Interestingly, this result conforms with Lindé (2004) who finds that foreign factors play a minor role in the Swedish business cycles, especially in the long run. This weak link to the foreign economies might explain why Denmark seems to have been left relatively unaffected by the international slowdown since 2001.

Recently, there has been a heated debate about the response of labour supply (measured in hours worked) to technological innovations as well as the contribution of these innnovations to volatility in activity. On the one hand, Galí (2004) and Galí and Rabanal (2004) argue strongly in favour of a very limited role of technology in this respect, while McGrattan (2004) defends the technology driven business cycle and the RBC model. With regards to the response of hours to technological innovations, Galí (1999) argues that hours fall after a positive shock to technology, while Christiano et al. (2004) find that hours increase. And recently, Uhlig (2004) concluded that the response is slightly positive, but insignificant. On both accounts, our findings are in accordance with those of Galí.

5   Conclusion

In this paper we formulated and estimated a DSGE model of a small open economy with several rigidities in order to fascilitate a structural decomposition of the Danish business cycle. We identified ten structural shocks and quantified their relative contributions to the volatility of six central variables; consumption, investments, output, interest rates, wages and capital gains.

Somewhat surprisingly, we find that fluctuations in the Danish economy stem predominantly from domestic shocks, despite the fact that the Danish economy by most standards is considered to be an open economy. In the short run, output cycles are mainly driven by demand shocks,

Table 2: Variance Decomposition

		C			I			Y		
$t = 0$	Preferences	0.67	**0.82**	0.91	0.00	**0.00**	0.01	0.25	**0.32**	0.41
	Labour supply	0.02	**0.07**	0.17	0.58	**0.67**	0.75	0.16	**0.26**	0.37
	Technology	0.00	**0.00**	0.00	0.00	**0.01**	0.01	0.00	**0.00**	0.01
	UIP	0.00	**0.01**	0.01	0.01	**0.01**	0.02	0.01	**0.02**	0.03
	Price mark-up	0.02	**0.04**	0.08	0.11	**0.16**	0.22	0.12	**0.19**	0.27
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.02	**0.03**	0.06	0.06	**0.08**	0.11	0.08	**0.11**	0.15
	Foreign int. rate	0.01	**0.02**	0.04	0.03	**0.05**	0.07	0.04	**0.06**	0.1
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.00**	0.01
	Foreign demand	0.00	**0.00**	0.01	0.01	**0.02**	0.04	0.01	**0.02**	0.05
	MSE (in pct.)	0.01	**0.02**	0.02	0.50	**0.61**	0.77	0.03	**0.04**	0.04
$t = 4$	Preferences	0.47	**0.63**	0.78	0.00	**0.01**	0.01	0.28	**0.39**	0.53
	Labour supply	0.12	**0.24**	0.39	0.54	**0.64**	0.73	0.08	**0.12**	0.18
	Technology	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.00**	0.00
	UIP	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.01**	0.01
	Price mark-up	0.03	**0.06**	0.09	0.13	**0.19**	0.26	0.16	**0.24**	0.34
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.02	**0.04**	0.06	0.06	**0.08**	0.12	0.08	**0.12**	0.18
	Foreign int. rate	0.01	**0.02**	0.04	0.02	**0.04**	0.06	0.04	**0.06**	0.1
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.01**	0.01
	Foreign demand	0.00	**0.01**	0.02	0.01	**0.02**	0.06	0.01	**0.03**	0.08
	MSE (in pct.)	0.08	**0.13**	0.19	1.37	**1.68**	2.09	0.1	**0.13**	0.18
$t = 12$	Preferences	0.19	**0.31**	0.45	0.02	**0.04**	0.08	0.12	**0.19**	0.29
	Labour supply	0.44	**0.59**	0.73	0.51	**0.6**	0.69	0.4	**0.49**	0.59
	Technology	0.00	**0.00**	0.01	0.00	**0.01**	0.01	0.00	**0.00**	0.01
	UIP	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.00**	0.00
	Price mark-up	0.03	**0.05**	0.08	0.14	**0.2**	0.27	0.12	**0.17**	0.22
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.01	**0.02**	0.04	0.05	**0.07**	0.11	0.04	**0.07**	0.11
	Foreign int. rate	0.01	**0.01**	0.02	0.02	**0.03**	0.05	0.02	**0.03**	0.05
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.00**	0.01
	Foreign demand	0.00	**0.01**	0.02	0.01	**0.03**	0.07	0.01	**0.02**	0.06
	MSE (in pct.)	0.17	**0.29**	0.48	1.73	**2.1**	2.61	0.2	**0.28**	0.4
$t = 100$	Preferences	0.04	**0.11**	0.23	0.01	**0.03**	0.07	0.03	**0.06**	0.11
	Labour supply	0.67	**0.84**	0.94	0.7	**0.78**	0.84	0.72	**0.84**	0.93
	Technology	0.00	**0.00**	0.01	0.00	**0.00**	0.01	0.00	**0.00**	0.01
	UIP	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Price mark-up	0.01	**0.03**	0.06	0.07	**0.10**	0.15	0.02	**0.05**	0.1
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.00	**0.01**	0.02	0.02	**0.04**	0.06	0.01	**0.02**	0.04
	Foreign int. rate	0.00	**0.00**	0.01	0.01	**0.02**	0.03	0.00	**0.01**	0.02
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Foreign demand	0.00	**0.00**	0.01	0.01	**0.01**	0.04	0.00	**0.01**	0.03
	MSE (in pct.)	0.36	**0.84**	2.48	3.03	**4.12**	5.86	0.53	**1.03**	2.55

The variance decomposition calculation is based on 1,000 draws from the posterior distribution. In the first and third column for each enogenous variable we report the 5th and 95th percentiles respectively, while the middle column in bold states the median.

Table 3: Variance Decomposition, II

			i			W			R	
$t=0$	Preferences	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.08	**0.14**	0.21
	Labour supply	0.00	**0.00**	0.00	0.86	**0.92**	0.97	0.48	**0.61**	0.75
	Technology	0.00	**0.00**	0.00	0.00	**0.01**	0.01	0.07	**0.11**	0.16
	UIP	0.83	**0.88**	0.92	0.00	**0.00**	0.00	0.00	**0.00**	0.01
	Price mark-up	0.00	**0.00**	0.00	0.00	**0.01**	0.01	0.03	**0.06**	0.1
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.02	**0.04**	0.09	0.00	**0.01**	0.02	0.02	**0.04**	0.06
	Foreign int. rate	0.05	**0.08**	0.11	0.00	**0.00**	0.01	0.01	**0.02**	0.04
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Foreign demand	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.01**	0.02
	MSE (in pct.)	0.00	**0.00**	0.00	0.07	**0.14**	0.28	0.28	**0.41**	0.64
$t=4$	Preferences	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.05	**0.11**	0.23
	Labour supply	0.00	**0.00**	0.00	0.89	**0.94**	0.98	0.61	**0.75**	0.86
	Technology	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.02	**0.04**	0.07
	UIP	0.61	**0.7**	0.77	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Price mark-up	0.00	**0.00**	0.00	0.00	**0.01**	0.01	0.02	**0.04**	0.06
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.05	**0.11**	0.2	0.00	**0.01**	0.01	0.02	**0.03**	0.05
	Foreign int. rate	0.13	**0.18**	0.26	0.00	**0.00**	0.01	0.01	**0.01**	0.02
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Foreign demand	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.01**	0.02
	MSE (in pct.)	0.00	**0.00**	0.00	1.65	**2.80**	4.84	2.08	**3.33**	5.53
$t=12$	Preferences	0.00	**0.01**	0.02	0.00	**0.00**	0.00	0.03	**0.07**	0.16
	Labour supply	0.00	**0.01**	0.01	0.93	**0.96**	0.98	0.72	**0.83**	0.9
	Technology	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.01	**0.03**	0.04
	UIP	0.54	**0.63**	0.71	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Price mark-up	0.00	**0.00**	0.00	0.00	**0.01**	0.01	0.02	**0.03**	0.05
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.08	**0.14**	0.22	0.00	**0.01**	0.01	0.01	**0.02**	0.03
	Foreign int. rate	0.15	**0.21**	0.29	0.00	**0.00**	0.00	0.00	**0.01**	0.01
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Foreign demand	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.00	**0.01**	0.02
	MSE (in pct.)	0.00	**0.00**	0.00	5.90	**8.18**	11.93	4.14	**6.19**	9.47
$t=100$	Preferences	0.01	**0.02**	0.04	0.00	**0.00**	0.00	0.03	**0.06**	0.12
	Labour supply	0.01	**0.03**	0.06	0.94	**0.97**	0.98	0.78	**0.85**	0.9
	Technology	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.01	**0.02**	0.04
	UIP	0.5	**0.59**	0.68	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Price mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.01	0.02	**0.03**	0.04
	Wage mark-up	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Monetary policy	0.09	**0.14**	0.22	0.00	**0.00**	0.01	0.01	**0.02**	0.03
	Foreign int. rate	0.15	**0.2**	0.28	0.00	**0.00**	0.00	0.00	**0.01**	0.01
	Foreign price	0.00	**0.00**	0.00	0.00	**0.00**	0.00	0.00	**0.00**	0.00
	Foreign demand	0.00	**0.01**	0.01	0.00	**0.00**	0.01	0.00	**0.01**	0.02
	MSE (in pct.)	0.00	**0.00**	0.00	8.42	**11.3**	16.58	5.66	**7.41**	10.22

The variance decomposition calculation is based on 1,000 draws from the posterior distribution. In the first and third column for each enogenous variable we report the 5th and 95th percentiles respectively, while the middle column in bold states the median.

with shocks to preferences being the largest, yet supply-side shocks also play an important role; thus, shocks to labour supply account for one-fourth of the short-term fluctuations, whereas the contribution from technology shocks is almost negligible.

In the longer run we find that cycles are mainly driven by supply shocks. Specifically, labour supply accounts for 85 percent of all output volatility after 25 years. Technology shocks do not contribute at all. The finding that demand factors matter greatly for the short-term cycles, while long-run cycles are driven by the supply side is consistent with the traditional distinction between Keynesian models for short-run modelling and classical model long-run modelling.

We paid special attention to the monetary transmission mechanism when we analysed the properties of our estimated model. We find that a one percent devaluation implies a 0.8 percent rise in output, peaking after two quarters. This experiment leads to a similar short-run stimulus of the real economy as a negative shock to the international interest rate. In both scenarios the central bank lowers the domestic interest rate; in the first scenario the bank deviates temporarily from the peg with an expansionary monetary policy shock, while it responds to the foreign interest shock in the second scenario in order to maintain the exchange rate peg.

Overall, we consider the estimation to be satisfactory. However, we do acknowledge three critical aspects in relation to the estimation of the model. First, we obtain a very high degree of price stickiness in the intermediate sector, corresponding to firms only being able to re-optimise prices only once every fourth year on average. This is, however, a well-known problem in this literature, and *ex ante* we did not expect to solve it. We did, however, discuss some of the recent explanations that have been suggested recently; of these changing the assumption regarding the capital market seems particularly fruitful, and we should like to implement this in future work.

The second problem concerns the contemporaneous negative reaction in output following a technology shock, and is related to the first problem. Since the short-run supply is almost horizontal, the immediate impact from a technology shock is small. When firms' production technology at the same time have improved, demands for capital decrease *ceteris paribus*. Hence investments fall and more than outweigh the rise in consumption causing output to fall. This implausible property could probably be avoided by introducing variable capital utilisation in the model.

Thirdly, the price markup shock is estimated to be implausibly volatile. We suspect two reasons for this result; (i) we have chosen the most simple and thus inflexible version of the Calvo pricing model; and (ii) we have ignored government spending and investment shock on the demand side. Thus, we leave much of the price dynamics to be accounted for by the markup shock. This result suggests that future work within this framework should consider a more

flexible model for the price setting of domestic and foreign firms and include more demand components in the final goods market.

To the best of our knowledge, this is the first attempt to estimate a DSGE model on Danish data. Despite the problems just mentioned, we consider the estimated model to be a major step forward in establishing a suitable framework for the analysis of the Danish business cycle. Not only do we believe to have captured essential features of the Danish economy, we also have a utility-based metric for evaluating the welfare effects of different policies. One obvious question that begs to be answered is the consequences of alternative monetary policy regimes to the current peg. We are currently seeking to implement a generalised Taylor rule in a close variant of the estimated model presented in this paper and quantifying the implied changes in welfare.

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A   Log-linearised Model

$$\hat{Q}_t^d = \hat{P}_t + \hat{Z}_t - \hat{P}_t^d, \tag{17}$$

$$\hat{Q}_t^m = \hat{P}_t + \hat{Z}_t - \hat{P}_t^m, \tag{18}$$

$$\hat{Q}_t^x = -\eta \hat{P}_t^x + \eta \hat{P}_t^* + \hat{Y}_t^*, \tag{19}$$

$$\hat{P}_t = \alpha^d \hat{P}_t^d + \left(1 - \alpha^d\right) \hat{P}_t^m, \tag{20}$$

$$\hat{L}_t = \hat{R}_t - \hat{W}_t + \hat{K}_t, \tag{21}$$

$$\hat{K}_t = -\hat{\theta}_t - (1 - \psi)\hat{R}_t + (1 - \psi)\hat{W}_t + \hat{Y}_t, \tag{22}$$

$$\widehat{MC}_t = -\hat{\theta}_t + (1 - \psi)\hat{W}_t + \psi\hat{R}_t, \tag{23}$$

$$\hat{\rho}_{t+1} = \hat{U}_{C_{t+1}} - \hat{U}_{C_t} + \hat{P}_t - \hat{P}_{t+1}, \tag{24}$$

$$\hat{P}_t^d - d\hat{P}_{t-1}^d = (1 - d)(1 - d\beta)\left[\widehat{MC}_t + \hat{\nu}_t\right] + d\beta E_t\left[P_{t+1}^d - dP_t^d\right], \tag{25}$$

$$\hat{P}_t^x - d\hat{P}_{t-1}^x = (1 - d)(1 - d\beta)\left(\widehat{MC}_t - \hat{e}_t + \hat{\nu}_t\right) + d\beta E_t\left[\hat{P}_{t+1}^x - d\hat{P}_t^x\right], \tag{26}$$

$$\hat{P}_t^m - d\hat{P}_{t-1}^m = (1 - d)(1 - d\beta)\left(\hat{e}_t + \hat{P}_t^* + \hat{\nu}_t\right) + d\beta E_t\left[\hat{P}_{t+1}^m - d\hat{P}_t^m\right], \tag{27}$$

$$\hat{W}_t - D\hat{W}_{t-1} = (1 - D)(1 - D\beta)\left(\hat{P}_t + \hat{U}_{L,t} - \hat{U}_{C,t} + \hat{\gamma}_t\right) + D\beta E_t\left[\hat{W}_{t+1} - D\hat{W}_t\right], \tag{28}$$

$$\hat{K}_{t+1} = (1 - \delta)\hat{K}_t + \delta\hat{I}_t, \tag{29}$$

$$\hat{B}_{t+1} = (1 + \bar{\imath})\hat{B}_t + \hat{P}_t^x + \hat{Q}_t^x - \hat{P}_t^m + \hat{e}_t - \hat{Q}_t^m, \ \hat{B}_t \equiv \frac{B_t}{P^x Q^x}, \tag{30}$$

$$\hat{U}_{C,t} = \hat{\zeta}_t^b - \frac{\sigma_C}{(1 - h)}\hat{C}_t + \frac{h\sigma_C}{(1 - h)}\hat{C}_{t-1}, \tag{31}$$

$$\hat{U}_{L,t} = \hat{\zeta}_t^b + \hat{\zeta}_t^L + \sigma_L\hat{L}_t, \tag{32}$$

$$\Phi(1 + \beta)\hat{K}_{t+1} = E_t\hat{\rho}_{t+1} - \hat{P}_t + \beta(1 - \delta)E_t\hat{P}_{t+1} + [1 - \beta(1 - \delta)]E_t\hat{R}_{t+1} + \Phi\hat{K}_t + \beta\Phi E_t\hat{K}_{t+2}, \tag{33}$$

$$\hat{\imath}_t = -E_t\hat{\rho}_{t+1}, \tag{34}$$

$$\hat{\imath}_t^f = -E_t\hat{\rho}_{t+1} - E_t\hat{e}_{t+1} + \hat{e}_t, \tag{35}$$

$$\hat{\imath}_t^f = \hat{\imath}_t^* + \hat{v}_t - \lambda\hat{B}_{t+1}, \tag{36}$$

$$\hat{e}_t = \hat{\xi}_t, \tag{37}$$

$$\hat{Y}_t = \alpha^d\hat{Q}_t^d + \left(1 - \alpha^d\right)\hat{Q}_t^m, \tag{38}$$

$$\hat{Z}_t = \frac{C}{Z}\hat{C}_t + \frac{I}{Z}\hat{I}_t. \tag{39}$$

The system has 24 endogenous and 10 exogenous variables. Of the latter we assume that the markup shocks and the UIP shock $(\nu_t, \gamma_t, \upsilon_t)$ are i.i.d. and the remaining seven are AR(1) processes;

$$\hat{\zeta}_t^b = \varrho^b \hat{\zeta}_{t-1}^b + \varepsilon_t^b, \tag{40}$$

$$\hat{\zeta}_t^l = \varrho^l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \tag{41}$$

$$\hat{\theta}_t = \varrho^t \hat{\theta}_{t-1} + \varepsilon_t^t, \tag{42}$$

$$\hat{\xi}_t = \varrho^m \hat{\xi}_{t-1} + \varepsilon_t^m, \tag{43}$$

$$\hat{\imath}_t^* = \varrho^i \hat{\imath}_{t-1}^* + \varepsilon_t^i, \tag{44}$$

$$\hat{P}_t^* = \varrho^P \hat{P}_{t-1}^* + \varepsilon_t^P, \tag{45}$$

$$\hat{Y}_t^* = \varrho^Y \hat{Y}_{t-1}^* + \varepsilon_t^Y. \tag{46}$$

B Solving the Log-linearised Model with gensys

We solve the log-linearised system (17)-(46) with the `gensys` method developed by Sims (2002). For this purpose we collect the 23 endogenous variables with 6 lagged variables and 9 exogenous processes (excluding the policy shock ξ_t) in the (38×1) vector Υ_t;[7]

$$\Upsilon_t \quad : \quad \hat{B}_t, \hat{C}_t, \hat{e}_t, \hat{\imath}_t, \hat{\imath}_t^f, \hat{I}_t, \hat{K}_t, \hat{L}_t, \widehat{MC}_t, \hat{P}_t, \hat{P}_t^d, \hat{P}_t^x, \hat{P}_t^m, \hat{Q}_t^d, \hat{Q}_t^x, \hat{Q}_t^m, \hat{R}_t, \hat{\rho}_{t+1}, \hat{U}_{C,t}, \hat{U}_{L,t}, \hat{W}_t, \hat{Y}_t, \hat{Z}_t,$$
$$\hat{K}_{t-1}, \hat{P}_{t-1}^d, \hat{P}_{t-1}^x, \hat{P}_{t-1}^m, \hat{W}_{t-1},$$
$$\hat{\zeta}_t^b, \hat{\zeta}_t^l, \hat{\theta}_t, \hat{\nu}_t, \hat{\gamma}_t, \hat{\xi}_t, \hat{\imath}_t^*, \hat{P}_t^*, \hat{Y}_t^*.$$

The i.i.d. shocks are included in the vector $\varepsilon_t \equiv \left(\varepsilon_t^b, \varepsilon_t^l, \varepsilon_t^t, \hat{\upsilon}_t, \varepsilon_t^m, \varepsilon_t^{pm}, \varepsilon_t^{wm}, \varepsilon_t^i, \varepsilon_t^P, \varepsilon_t^Y\right)$ includes the set of i.i.d. shocks, and the seven expectational errors are included in the vector $\eta_t = \left(\eta_t^d, \eta_t^x, \eta_t^m, \eta_t^W, \eta_t^K, \eta_t^A, \eta_t^B\right)$ so that we can write the model in the canonical VAR(1) `gensys` form;

$$\Gamma_0 \Upsilon_t = \Gamma_1 \Upsilon_{t-1} + \Psi \varepsilon_t + \Pi \eta_t.$$

Applying the `gensys` method recasts the system in the solved form

$$\Upsilon_t = \Theta_1 \Upsilon_{t-1} + \Theta_z Z_t.$$

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[7]Hence, we add six identity equations to the system (17)-(46), corresponding to the six lagged endogenous variables included in Υ_t, and two definitions of the mark-up shocks ($\hat{\nu}_t = \varepsilon_t^{mp}, \hat{\gamma}_t = \varepsilon_t^{mw}$).

C   Data

We use quarterly data for the period 1987-2003. The sources of our data set are specified in Table 4.

Table 4: Data Sources

	Variable	Database	Mnenomic
	GDP		
[1]	Denmark	MONA	fy
[2]	Germany	EO	deu_gdpvq
[3]	France	MEI	fra_expgdp_vnbqrsaq
[4]	Netherlands	MEI	nld_expgdp_lnbqrsaq
	Prices		
[5]	Denmark	MONA	pfy
[6]	Germany	EO	deu_pgdpq
[7]	France	MEI	fra_expgdp_dnbsaq
[8]	Netherlands	MEI	nld_expgdp_dnbsaq
	Interest rates		
[9]	Denmark	EW	dnk14010
[10]	Germany	IFS	s13460c00zfq
	Exchange rate		
[11]	DEM/DKK	EW	dnk19008
	Miscellaneous Danish variables		
[12]	Consumption	MONA	fcp
[13]	Private employment	MONA	qp
[14]	Public employment	MONA	qo
[15]	Self-employed	MONA	qs
[16]	Working hours pr. week (max)	MONA	maxtid
[17]	Employment		$([13] + [14] + [15]) \times [16]^{0.7}$

MONA: Model of the Danish Central Bank (cf. Christensen and Knudsen, 1992); EO: OECD Economic Outlook; MEI: OECD Main Economic Indicators; IFS: IMF International Financial Statistics. Data from the latter three sources were provided by EcoWin.