

What is Bohmian Mechanics

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Abstract

Bohmian mechanics is a quantum theory with a clear ontology. To make clear what we mean by this, we shall proceed by recalling first what are the problems of quantum mechanics. We shall then briefly sketch the basics of Bohmian mechanics and indicate how Bohmian mechanics solves these problems and clarifies the status and the role of of the quantum formalism.

1 What is quantum mechanics about?

The basic problem of quantum mechanics is that it is not clear what quantum mechanics is about—what quantum mechanics describes—as repeatedly stressed by John Bell [5], and, more recently, by Shelly Goldstein [22].

It might seem that quantum mechanics is basically about the behavior of wave functions. However, as Schrödinger has effectively shown with his famous cat paradox [28], it turns out that the complete state description cannot be given by the wave function—obeying Schrödinger's equation—as this would lead to paradoxical conclusions like, for example, a superposition between a dead and an alive cat. Bell has rephrased this mental experiment in a less cruel way as follows: consider a cat in a perfectly isolated room. Together with the cat,

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the experimenter has put in the room a radioactive source and a complicated mechanics. If a radioactive nucleus decays, the mechanism opens a source of milk such that it fills a cup and the cat can drink. The room has no window so that what happens inside is completely hidden to the experimenter: she doesn't know whether the cat is still hungry or if she enjoyed her meal. In this way the radioactive decay, a microscopic event, influences directly a macroscopic event, like the presence or not of some milk molecules in the stomach of the cat.

From the mathematical rules of quantum mechanics it follows that, given that the wave function of the radioactive nucleus is in a superposition of decayed–non decayed wave function, the cat is nor hungry nor filled up but it is a superposition of both states. However, from ordinary experience, we know that macroscopic object cannot be in such a superposition of states with macroscopically disjoint supports, so somewhere, somehow, quantum mechanics gives the wrong answer. Note that, if the experimenter opens the door of the room, finds out the cat always or in one or in the other state: as a consequence of observation (measurement), the wave function has collapsed in one of the two possibilities.

The Schrödinger's cat paradox poses several questions: What is the role of the observer? Which observer is entitled to reduce the wave function? Where is the border between the microscopic world, in which superpositions can exist, and the macroscopic world, in which they cannot? Nobody has ever found satisfactory answers to these questions within the standard framework of quantum mechanics. Indeed, Bell has drawn the conclusion that there are only two ways out: or to add something to the wave function for the description of the state of the system, or to modify Schrödinger's equation. While this latter path is that taken by the so called theories of spontaneous localization, or shortly GRW theories [20], Bohmian mechanics is a theory that follows the first direction.¹

¹Bohmian mechanics (also called “pilot-wave theory” or “causal interpretation”) was discovered in 1927 by De Broglie [29] and soon abandoned. It was rediscovered, extensively extended, and for the first time fully understood, in 1952 by David Bohm [6]. During the sixties, seventies and eighties, John Bell was his principal proponent; his book [5] contains yet unsurpassable introductions to Bohmian mechanics. Other standard references are the books of Bohm and Hiley [7] and that of Holland [25]. The approach we are following here is that of the “Rutgers-München-Genova” group (quite in line with the approach of Bell), see, e.g., [13], [14], [22], [23], [12].

2 Bohmian Mechanics

The first step in the construction of a physical theory is to establish what are the mathematical entities (particles, fields, strings, ...) with which one intends to describe physical reality. These mathematical entities are what the theory is about and they are often called the ontology of the theory—a rather complicated way of expressing a simple, even though deep, physical notion.

In nonrelativistic Bohmian mechanics the world is described by point-like particles which follow trajectories determined by a law of motion. The evolution of the positions of these particles is guided by the wave function which itself evolves according to Schrödinger's equation. In other words, in Bohmian mechanics the complete description of the state of an N point-like particle system is the couple (Ψ, Q) , where $\Psi = \Psi(q)$ and $Q = (Q_1(t), \dots, Q_N(t))$ are respectively the wave function and the *actual* configuration of the system, with Q_k denoting the position of the k -th particle in ordinary three dimensional space.

One might think of Bohmian mechanics as a dynamical system and from this point of view it can be compared with classical mechanics. While in Newtonian mechanics the dynamics of the point particles is determined by a *second* order differential equation

$$\frac{d^2 Q_t}{dt^2} = \frac{1}{m} F(Q_t), \quad (1)$$

in which $F(Q)$ is a force field, e.g, derived from a potential field V as $F(Q) = -\nabla V$, in Bohmian mechanics the point particles dynamics is given by a *first* order differential equation

$$\frac{dQ_t}{dt} = v^\Psi(Q_t), \quad (2)$$

where $v^\Psi = (v_1^\Psi, \dots, v_N^\Psi)$ is a velocity field on the configuration space. This field is generated by the wave function Ψ which itself evolves according to Schrödinger's equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi, \quad (3)$$

where H is the Hamiltonian, e.g., given, for non relativistic spinless particles, by

$$H = - \sum_{k=1}^N \frac{\hbar^2}{2m_k} \nabla_k^2 + V \quad (4)$$

The velocity field is determined by reasons of simplicity and symmetry [13],

$$v_k^\Psi = \frac{\hbar}{m_k} \text{Im} \left[\frac{\nabla_k \Psi}{\Psi} \right] \quad (5)$$

(with the wave function playing somehow the role of “potential field” for the velocity field). The factor $\frac{\hbar}{m}$ comes from the requirement of Galilei invariance, the imaginary part is a consequence of invariance for time reversal, the gradient is from rotational invariance and the fact that one has to divide for Ψ derives from the homogeneity of degree zero of the velocity field—the fact that a quantum state is a ray in Hilbert space. If there is a magnetic field $B = \text{rot}A$, ∇_k should represent the covariant derivative associated with the vector potential A . If the wave function is a spinor, we should rewrite the velocity field as

$$v_k^\Psi = \frac{\hbar}{m_k} \text{Im} \left[\frac{\Psi^* \nabla_k \Psi}{\Psi^* \Psi} \right], \quad (6)$$

where now in the numerator and denominator appears the scalar product in the spinor space.

The global existence of Bohmian dynamics has been proven with full mathematical rigor in [8] where it has been shown that for a large class of Schrödinger Hamiltonians (4), including Coulomb potential V with arbitrary charges and masses, and sufficiently regular initial datum ψ_0 of (3) the solution of (2) exists uniquely and globally in time for $|\psi_0|^2$ -almost all initial configurations Q_0 .

Equations (2) and (3) (together with (4) and (5)) form a complete specification of the theory. Without any other axiom, all the phenomena governed by nonrelativistic quantum mechanics, from spectral lines and quantum interference experiments to scattering theory, superconductivity and quantum computation follow from the analysis of the dynamical system defined by (2) and (3).

3 Experimental Predictions

Bohmian mechanics makes the same predictions as does non relativistic ordinary quantum mechanics for the results of any experiment, provided that we assume a random distribution for the configuration of the system and the apparatus at the beginning of the experiment given by $\rho(q, t) = |\Psi(q, t)|^2$. In fact, consider the quantum continuity equation

$$\frac{\partial \rho}{\partial t} + \text{div } J^\Psi = 0, \quad (7)$$

which is, by himself, a simple consequence of Schrödinger’s equation. Here $J^\Psi = (J_1^\Psi, \dots, J_N^\Psi)$ is the quantum probability current

$$J_k^\Psi = \frac{\hbar}{m_k} \text{Im} [\Psi^* \nabla_k \Psi] = |\Psi|^2 v_k^\Psi. \quad (8)$$

Equation (7) becomes the classical continuity equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho v^\Psi = 0 \tag{9}$$

for the system $dQ_t/dt = v^\Psi$ and it governs the evolution of the probability density ρ under the motion defined by the guiding equation (2) for the particular choice $\rho = |\Psi|^2$. In other words, if the probability density for the configuration satisfies $\rho(q, t_0) = |\Psi(q, t_0)|^2$ at some time t_0 , then the density to which this is carried by the motion (7) at any time t is also given by $\rho(q, t) = |\Psi(q, t)|^2$. This is an extremely important property of any Bohmian system. In fact it expresses a compatibility between the two equations of motion defining the dynamics, which we call *equivariance* of $|\Psi|^2$.

The above assumption, which guarantees agreement between Bohmian mechanics and quantum mechanics regarding the results of any experiment, is what has been called [13] the *quantum equilibrium hypothesis*: when a system has a wave function ψ , its configuration Q is random with probability distribution given by

$$\rho(q) = |\psi(q)|^2. \tag{10}$$

While the meaning and justification of this hypothesis—which should be regarded as a local manifestation of a global *quantum* equilibrium state of our universe—is a delicate matter, which has been discussed at large elsewhere [13], it is important to recognize that, merely as a consequence of (9) and (10), Bohmian mechanics is a counterexample to all of the claims to the effect that a deterministic theory cannot account for quantum randomness in the familiar statistical mechanical way, as arising from averaging over ignorance: Bohmian mechanics is clearly a deterministic theory, and, as we have just explained, it does account for quantum randomness as arising from averaging over ignorance given by $|\psi(q)|^2$.

It is important to realize that non simply Bohmian mechanics makes the same predictions as does orthodox quantum theory for the results of any experiment, but that *the quantum formalism of operators as observables emerges naturally and simply from it as the very expression of the empirical import of Bohmian mechanics*. More precisely, it turns out that in Bohmian mechanics self-adjoint operators arise in association with *specific* experiments as a tool to compactly express and represent the relevant data—the results and their statistical distributions—of these experiments. The key ingredient to understand how this comes about is to recall [14] that a completely general experiment is described by:

- a unitary map U transforming the initial state of system and apparatus $\psi_0(x) \otimes \phi_0(y)$ into a final state $\Psi(x, y) = U(\psi_0(x) \otimes \phi_0(y))$ (x refers to the configurations of the system and y those of the apparatus);
- a pointer variable $Z = F(Y)$ representing the pointer orientation in terms of the microscopic configurations Y of the apparatus.

It is a direct consequence of quantum equilibrium and linearity of Schrödinger's equation [14], that the probability distribution of the pointer variable Z is a measured-valued quadratic form on the Hilbert space of wave function, and, a such, mathematically equivalent to a positive-operator-valued measure (POVM). It turn out that self-adjoint operators (which are, by the spectral theorem, in one to one correspondence with projector-valued measures) represent quantum observables associated with the special class of repeatable experiments [16].

Note that as a byproduct of the foregoing considerations one obtains a very general notion of measurability:

A physical quantity is measurable only if its probability distribution is a measure-valued quadratic form on the Hilbert space of wave functions. (11)

Sometimes it is claimed that it is possible to experimentally discriminate between Bohmian mechanics and quantum mechanics. This claim is however totally unfounded: there *must* be experimental agreement as a consequence of quantum equilibrium. The experimental equivalence of Bohmian mechanics with quantum mechanics might appear, somehow, a little frustrating fact: while, on one hand, all the experimental evidence confirms Bohmian mechanics as well as quantum mechanics, on the other hand it would be easier if the experimental prediction were different. In fact, if there were a crucial experiment able to discriminate between the two theories, there would be something *objective* to establish which is the correct theory. It must be made clear, however, that the experimental equivalence of Bohmian mechanics with quantum mechanics holds as long as the predictions of quantum mechanics are not ambiguous. There are in fact a variety of experimental issues that don't fit comfortably within the standard operator quantum formalism, such as dwell and tunneling times [27], escape

times and escape positions [11], scattering theory [15], but are easily handled by Bohmian mechanics.

Actually, after the discussion of the previous sections, it should be clear that the comparison shouldn't be made only on the level of experimental prediction but, on the contrary, the decision of what is the right theory should be taken on the deeper level of the ontology of the theory—what the theory is about.

4 The Collapse of the wave function

The existence of configurations in Bohmian mechanics allows for a natural and clear notion of wave function of a subsystem. In fact, consider a composite system composed by a subsystem and by its environment. If $Q_t = (X_t, Y_t)$, where X_t is the actual (i.e., what really is) configuration of the sub-system at time t , and Y_t is that its environment at the same time, we can define the *conditional* wave function for the x -system at time t as

$$\psi_t(x) = \Psi_t(x, Y_t), \tag{12}$$

that is, the wave function of the whole universe (the biggest system of all) Ψ_t calculated in the actual configuration of the environment. Under appropriate conditions $\psi(x)$, satisfies Schrödinger's equation in x . In this case it is indeed the *effective* wave function for the x -system, that is, the collapsed wave function that the ordinary quantum formalism assigns to the subsystem after a quantum measurement. In fact, suppose Ψ has the structure occurring in a measurement situation

$$\Psi_t(x, y) = \psi_t(x)\phi_t(y) + \Psi_t^\perp(x, y), \tag{13}$$

where $\phi_t(y)$ and $\Psi_t^\perp(x, y)$ (the part of Ψ_t which is not $\psi_t(x)\phi_t(y)$) have macroscopically disjoint y -supports. If Y_t belongs to the support of $\phi_t(y)$, $\psi_t(x)$ is the effective wave function of the x -system at time t . (For a clear exposition of this, see [14] or [13].)

Thus, the collapse of the wave function can be deduced from Bohmian mechanics without introducing any active role to the observer. Consider, again, the cat paradox in the original version, were the two superposing states are dead and alive cat. In Bohmian mechanics at any time t the cat is something real, she is or dead or alive, independently on who is looking at her. Note that she *can* be in a superposition state because the wave function evolves according

to Schrödinger’s equation, but in Bohmian mechanics the state of the system is given by the couple (Ψ, Q) of the wave function and the configurations $Q = (Q_1, \dots, Q_n)$ of all the particles composing the system (the cat). Thus, according to which support Q belongs to (to those of the wave function Ψ_{dead} describing the dead cat or to those of the wave function Ψ_{alive} describing the alive cat), the cat is actually dead or alive. Note that superpositions exist on all scales (from micro to macro) but don’t influence at all the fact that the cat is this or that. At this point a question could arise: due to the presence of a superposition wave function, could it be possible that the cat, who at some time is dead, returns alive? The cat has an actual configuration, belonging (in our example) to the support of Ψ_{dead} , and its evolution is guided by the wave function. There seems to be nothing to prevent to Q to be guided in the support of Ψ_{alive} , making the dead–alive transition possible. Actually, this is very unlikely to happen, in fact the supports of the two wave functions are macroscopically distinguishable. By this we mean that the macroscopic variables, like, e.g., the temperature, assume different values in the two states, even if the microscopic quantities from which they have been derived might be similar. The temperature of an dead cat and of an alive cat are, in general, different. Thus, if Q at some time belongs to the support of Ψ_{dead} , the effect of Ψ_{alive} is completely negligible: we can forget of it for the dynamics of Q . The dead–alive transition could be possible if we would be able to bring the two wave functions close to each other again. But the probability of having success in this would be even less probable than the fact that all the molecules of perfume we have sprayed in a room would come back spontaneously in the neighborhood of the bottle!

5 Bohmian Mechanics and Newtonian Mechanics

To point out some interesting features of Bohmian mechanics, it can be useful to write the wave function Ψ in the polar form

$$\Psi = R e^{\frac{i}{\hbar} S} \tag{14}$$

and then rewrite Schrödinger’s equation in terms of these new variables. This is indeed what Bohm originally did in his 1952 paper [6]. In this way one obtains from (3) a pair of coupled equations: the continuity equation for R^2 ,

$$\frac{\partial R^2}{\partial t} + \operatorname{div} \left(\frac{\nabla_k S}{m} \right) R^2 = 0, \tag{15}$$

which suggests that $\rho = R^2$ can be interpreted as a probability density, and a modified Hamilton-Jacobi equation for S

$$\frac{\partial S}{\partial t} + \frac{(\nabla_k S)^2}{2m} + V - \sum_k \frac{\hbar^2}{2m_k} \frac{\nabla_k^2 R}{R} = 0 \quad (16)$$

Note that this equation differs from the usual classical Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} + \frac{(\nabla_k S)^2}{2m} + V = 0 \quad (17)$$

only by the appearance of an extra term, the *quantum potential*

$$U \equiv - \sum_k \frac{\hbar^2}{2m_k} \frac{\nabla_k^2 R}{R}. \quad (18)$$

This modified Hamilton-Jacobi equation can be used, together with the continuity equation for R , to define particle trajectories identifying the velocity with $v_k = \frac{\nabla_k S}{m}$. In this way the resulting motion is precisely what would have been obtained classically if the particles were subjected by the force generated by the quantum potential in addition to the usual forces.

It should be noted, however, that the rewriting of Schrödinger's equation through the polar variables (R, S) is somehow misleading. In fact, first of all, there is an increase in complexity: Schrödinger's equation is a linear equation while the modified Hamilton-Jacobi equation is highly nonlinear and still requires the continuity equation for its closure. Note that, since in Bohmian mechanics the dynamics is completely defined by Schrödinger's equation (3) and the guiding equation (2), there is no need of any further *axioms* involving the quantum potential and thus it should not be regarded as the most basic structure defining Bohmian mechanics.

Bohmian mechanics is not a rephrasing of quantum mechanics in classical terms. It is not simply classical mechanics with an additional force term. In Bohmian mechanics the velocities are not independent of positions, as they are classically, but are constrained by the guiding equation (2). The correct way of regarding to Bohmian mechanics is as a first-order theory, in which the fundamental quantity is the position of particles, whose dynamics is specified directly and simply by the velocity field (6). In Bohmian mechanics the second-order (Newtonian) concepts of acceleration and force, work and energy play no *fundamental* role. Rather, they are fundamental to the theory to which Bohmian mechanics converges in the classical limit, namely Newtonian mechanics. In fact, regardless of whether or not we think to the quantum potential as fundamental, it can be useful: equation (16), or, equivalently,

$$\frac{d^2 Q_t}{dt^2} = -\nabla[V(Q_t) + U(Q_t)], \quad (19)$$

show that all the *deviations* from classicality are embodied in the quantum force $-\nabla U$, so that, whenever this force is negligible, there is classical motion [25], [7]. This observation is the starting point for a complete derivation of the classical limit of quantum mechanics. And the crucial step for such a derivation is to characterize the physical conditions that guarantee smallness of the quantum force (see [1], [2], [3]).

6 Nonlocality and Hidden Variables

In the literature it is common to refer to Bohmian mechanics as a theory of *hidden variables*. This is a consequence of the famous EPR paper [19] in which Einstein, Podolsky and Rosen argued that quantum mechanics might be incomplete. Their proposal was to look for some non measurable variables (somehow hidden) to complete the theory.

It should be stressed that the problems faced by Einstein, Podolsky and Rosen in their paper was about the locality of quantum theory: they assumed that reality is local, i.e. action at distance is impossible, and proposed a mental experiment (that we shall not recall here). Their conclusions were that, if reality is local, quantum mechanics is incomplete and there is need of some extra variables to take this into account. From the violation of Bell's inequality (see [5], [4]) it followed that their assumption was wrong: reality *is* non local and therefore from their reasoning we cannot conclude anything concerning the existence of hidden variables.

We should emphasize that the reason for introducing the configuration of the particles as an extra variable in quantum mechanics has nothing to do with nonlocality. This has created and indeed still creates a lot of confusion in understanding which are the consequences of the violation of Bell's inequality—that reality is nonlocal and that any completion of quantum mechanics with *local* hidden variable is impossible. This is not the case of Bohmian mechanics, in which nonlocality follows directly from the fact that the wave function is a function in configuration space, not in ordinary space. This means that the velocity of each particle of a system composed by N particles, independently on how far are they. The degree of *action at distance* depends on the degree of entanglement. It must be stressed that nonlocality is not—by any means—a peculiarity of Bohmian mechanics: *nonlocality has turned out to be a fact of nature: nonlocality must be a feature of any physical theory accounting for the observed violations of Bell's inequality* [5].

The so called “no-go” theorems for hidden variables (von Neumann [30], Gleason [21],

Kochen-Speker [26]) show that there is no “good” map from operators to random variables (on the space of “hidden variables”), where by “good” we mean in the sense that the joint distributions of the random variables are consistent with the corresponding quantum mechanical distributions whenever the latter are defined. As commonly understood, these theorems involve a certain irony: They conclude with the impossibility of a deterministic description, or more generally of any sort of realist description, only by in effect themselves assuming a “realism” of a most implausible variety, namely, naive realism about operators [17]. There is in fact no reason to expect there to be such a map: the fact that the same operator plays a role in different experiments does not imply that these experiments have much else in common, and certainly not that they involve measurements of the same thing. It is thus with detailed experiments, and not with the associated operators, that random variables might reasonably be expected to be associated [5], [17], [14].

Finally, it is interesting to note, as a side remark, that the true “hidden” variable is actually the wave function. In fact, it is not stressed sufficiently that it is indeed the wave function that cannot be measured! If the wave function were measurable, it would exist an experimental device revealing the the actual wave function ψ_0 of the system prior to the measurement and the statistics of the pointer measuring the wave function would be formally given by $\delta(\psi - \psi_0)$, which, however, is not a quadratic form of ψ_0 , so that (11) is violated and and thus the wave function is not measurable!

7 What about Relativity?

Bohmian mechanics, the theory defined by eqs. (2) and (3) (together with (4) and (5)), is not Lorentz invariant, since (3) is a nonrelativistic equation, and, more importantly, since the right hand side of (5) involves the positions of the particles at a common (absolute) time. It is also frequently asserted that Bohmian mechanics cannot be made Lorentz invariant, by which it is presumably meant that no Bohmian theory—no theory that could be regarded somehow as a natural extension of Bohmian mechanics—can be found that is Lorentz invariant. In this regard, we wish to make some remarks.

1. The main reason for the belief that Bohmian mechanics cannot be made Lorentz is the manifest nonlocality of Bohmian mechanics, but nonlocality, as we have stressed in the previous section, is a fact of nature.

2. Concerning the other (somehow related) widespread belief, that standard quantum theories have no problems incorporating relativity while Bohmian mechanics does, we completely agree with the assessment of Jean Bricmont: “Indeed, whatever the Copenhagen interpretation really means, it must somewhere introduce a collapse or reduction of the state vector or an intervention of the observer or some—usually verbal—substitute for that. And, to my knowledge the Lorentz invariance of that part of the “theory” is never discussed. And, if it was, problems caused by EPR-type experiments, that are the source of the difficulty in obtaining a Lorentz invariant Bohmian theory, would soon become evident. [10]”
3. Indeed, the Bohmian state description (Ψ, Q) has been extended to (bosonic) field theories with Q representing the instantaneous configuration of the field (see, e.g., [6], [7], [25]). Though this might be the appropriate ontology for relativistic physics, it should be stressed that Bell ([5], 173–180) has proposed a Bohmian model for a quantum field theory involving both bosonic and fermionic quantum fields in which the primitive ontology is associated *only* with fermions.
4. While the above extensions agree with the predictions of quantum field theory—and thus they are relativistically invariant on the phenomenological level—they seem to lack “*serious*” Lorentz invariance (as Bell has put it [5]) on the level of the basic dynamical laws (for a discussion of this point see [9]).
5. Finally, we’d like to stress that it is indeed possible to construct “*serious*” Lorentz invariant Bohmian models, i.e., models for which Lorentz invariance holds, non only on the phenomenological level, but also on the microscopic level of the basic dynamical laws [9], [18], [24].

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