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What is the Color Glass Condensate?

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Abstract: I describe the Color Glass Condensate and its importance for a variety of problems related to small-x physics.

1 What is the Small-x Problem?

We imagine some hadron in a frame where it has a very large momentum which we take along the z (longitudinal) axis. We define x as the ratio of the plus components of light cone momenta:

$$x = p^+_{constituent}/P^+_{hadron} \tag{1}$$

The rapidity of the constituent is

$$y = y_{hadron} - ln(1/x) \tag{2}$$

and the rapidity density is related to the structure function as

$$\frac{dN}{dy} = xG(x, Q^2) \tag{3}$$

where Q^2 is the momentum squared at which we resolve the constituent. Most of the constituents at small x should be gluons.

The small x distributions for gluons are illustrated in Figure 1.[1] The small-x problem is the observation that these distributions are not flat in rapidity for small x. In fact they seem to grow, perhaps like an exponential $dN/dy \sim e^{\kappa|y-y_{proj}|}$

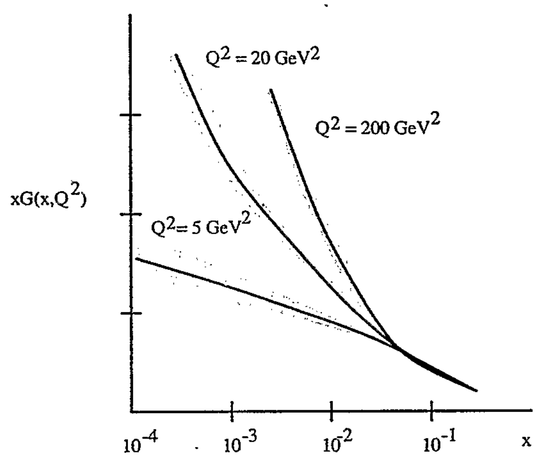

Figure 1: The Zeus data for the gluon structure functions.

This rapid growth in the phase space density of gluons implies the existence of a scale in the problem, the saturation momenta.[2]-[4] It is proportional to the density of gluons per unit area per unit rapidity. If the phase space density becomes really large, then the saturation momentum can become larger than Λ_{QCD},

$$\Lambda^2 = \frac{1}{\pi R^2}\frac{dN}{dy} >> \Lambda^2_{QCD} \tag{4}$$

At some very small x, we are led to believe the system becomes weakly coupled since

$$\alpha_s(\Lambda) << 1 \tag{5}$$

(We will later identify $\Lambda^2 \sim Q^2_{sat}/\alpha_s(Q_{sat})$).

This does not necessarily mean that the system is trivial. The high density of gluons can make the typical phase space density of gluons very large,

$$\frac{1}{\pi R^2}\frac{dN}{dy} \sim \frac{1}{\alpha_s} \tag{6}$$

so that even though the intrinsic interactions are weak, the gluon field can interact coherently and produce large effects.

2 Space-Time Distribution of Glue

It is useful to develop a space-time picture of the small x gluon distribution. The ordinary rapidity of produced hadrons is

$$y = \frac{1}{2} ln(p^+/p^-) = ln(p^+/m_T) \tag{7}$$

where m_T is the transverse mass. This is also approximately

$$y = ln(p^+_{hadron}/m_T) + ln(p^+/p^+_{hadron}) \sim y_{hadron} - ln(1/x) \tag{8}$$

Using that $p \cdot x = p_t \cdot x_T - p^+x^- - p^-x^+$ so that the uncertainty principle is $x^\pm \sim 1/x^\mp$, we see that

$$y \sim y_H - ln(x^- p^+_{hadron}) \sim \frac{1}{2} ln(x^+/x^-) \tag{9}$$

These exhaust all the standard definitions of momentum space and coordinate space rapidity. They are all equal to within a unit of rapidity. This implies that particles born in a momentum space rapidity range arise from more or less the same range in space time rapidity. It also means that localization in momentum space rapidity implies a localization in space time rapidity. Therefore, the increase in dN/dy comes in a spatially localized region of longitudinal phase space, that is the densities become large in a comoving frame.

In Fig. 2, a distribution of particles is shown inside a hadron. The rapidity variable along the lower axis may be taken to be both the momentum space and coordinate space rapidity. In Fig. 3, the same region of longitudinal phase space is shown in real coordinate variable. Here, the longitudinal phase space is Lorentz contracted. The region of the tube which intersects the nucleus is large. Viewed in the second frame, all the charges of the individual quarks and gluons which are intersected by the tube must add together incoherently. The density of charge is large in the small x limit at fixed resolution scale dx.

These sources of color charge produce a color field. For a field measured at a rapidity much less than the rapidity of the particles in the sheet, the variations in x^+ are small due to Lorentz time dilation. On the other hand, variations in x^- are large, since the charges sit on a spatially localized sheet

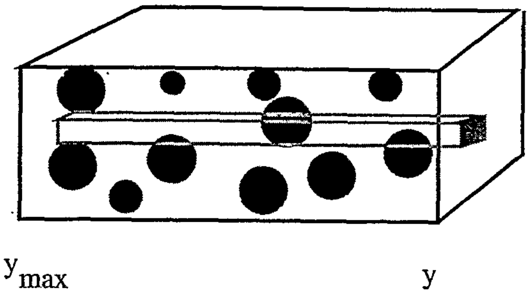

Figure 2: A single nucleus shown in terms of the space-time rapidity. The red circles indicate partons.

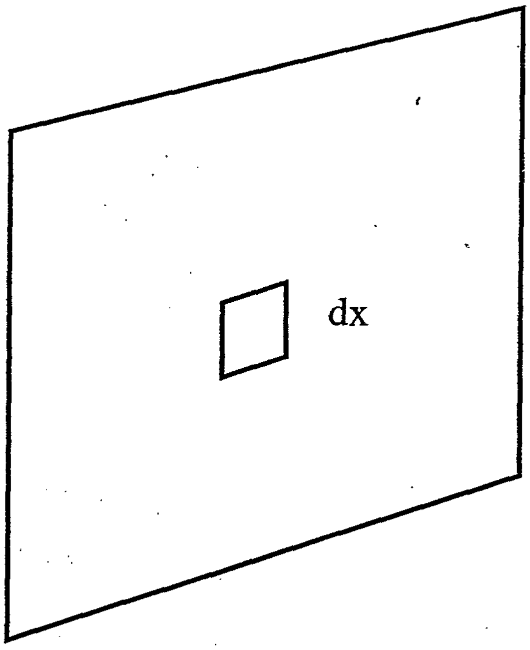

Figure 3: A single nucleus in the infinite momentum frame as seen by a small x probe.

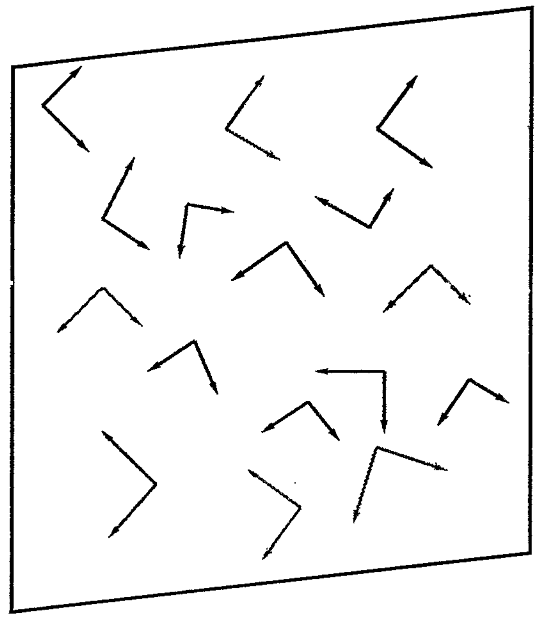

Figure 4: The non-abelian Lienard-Wiechert potentials which form the Color Glass Condensate.

of small size. The field strength F^{i+} is big, F^{i-} is small, and the F^{ij} are of intermediate strength. Since this implies $F^{i0} \sim F^{iz}$ are the big components, the fields are $\vec{E} \perp \vec{B} \perp \vec{z}$. The color orientations are random, as the color charges which generate the surface charge density are longitudinally far separated in the limit dx is small. This is shown in Fig. 4.

3 The Color Glass Condensate

The distribution of fields shown in Fig. 4 is called the Color Glass Condensate.[5] It's name is derived from:

- Color: The matter is composed of colored gluons.
- Glass: The sources of the fields comes from higher rapidities. These sources are time dilated. The fields evolve slowly compared to their natural time scales.

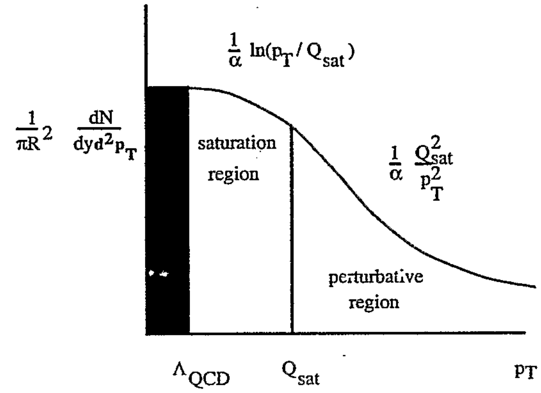

Figure 5: The gluon distribution function.

- Condensate: The phase space density of gluons is of order $1/\alpha_s$. The occupation number for the condensed mode scales as the transverse area times $1/\alpha_s$ and is big.

One can construct a theory of the Color Glass Condensate as classical Yang-Mills theory in the presence of a stochastic source. The classical approximation is justified because the occupation number of the gluon Fock space states is large. We shall describe this construction later in this lecture.

The result of a simple computation of the gluon distribution function is shown in Fig. 5.[6]-[9] ¿From the curve, we identify $\Lambda^2 \sim Q^2_{sat}/\alpha_s$. We see that at large p_T, the phase space density, which must be dimensionless, goes as $Q^2_{sat}/(p_T^2\alpha_s)$. At low p_T of order the saturation momentum, the curve flattens and up to logarithms is of order $1/\alpha_s$. This softening of the curve is due to a cancellation of field strengths when we try to resolve the fields on a scale size larger than their typical separation.

In terms of a saturation picture, it means that gluons cannot be piled up in phase space, due to repulsive gluon interactions. As one adds more glue to the system, it must be added at a momentum larger than that of saturation scale. This means the saturation scale itself is energy dependent. We expect the saturation scale will grow with energy like some power of x. There is never a problem with unitarity, since at fixed Q^2, at some arbitrarily large energy, the Q^2 will become less than Q^2_{sat}, an the gluon distribution function

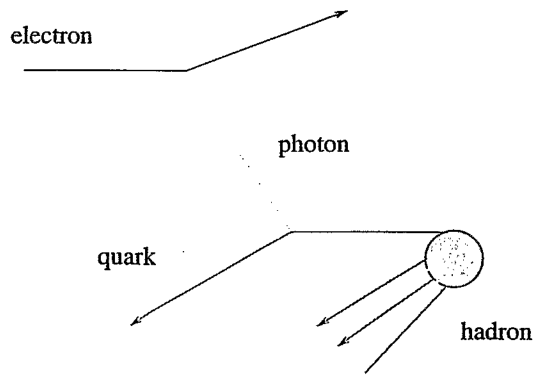

Figure 6: Deep inelastic scattering of an electron on a hadron.

at that resolution scale will cease growing. More quantitatively,

$$xG(x,Q^2) \sim \int_0^{Q^2} d^2p_T \frac{dN}{d^2p_T dy} \tag{10}$$

implies

$$G \sim Q_{sat}^2/\alpha_s \tag{11}$$

for $Q^2 << Q_{sat}^2$ and

$$G \sim ln(Q^2)/\alpha_s \tag{12}$$

for $Q^2 >> Q_{sat}^2$.

4 What Can One Compute with the Color Glass Condensate?

The Color Glass Condensate description allows for a computation of deep inelastic scattering from hadrons and nuclei at very small x as is shown in Fig. 6.[10] One can also compute diffractive processes and there is a relation between diffractive and inclusive structure functions. Basically, inelastic structure functions arise from computing the square of an amplitude in

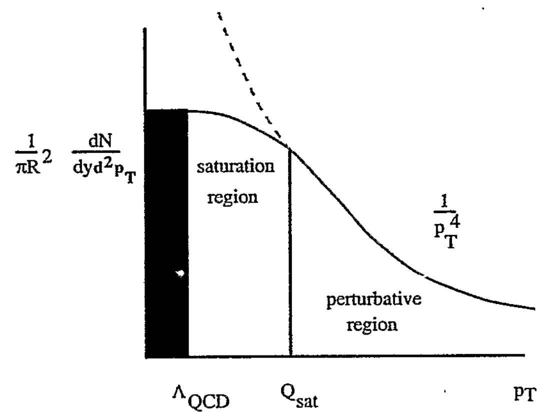

Figure 7: The p_T distribution for mini-jets produced by a Color Glass Condensate.

the Color Glass background field and then averaging over sources of color. Diffraction corresponds to computing the amplitude, averaging over color and then squaring. The result of this computation are formulae very close in functional form to that of the Golec-Biernat-Wüstolf model, and the data seem to be in accord with these results.[11]-[12]

One can also consider hadron-hadron scattering, as shown in Fig. 7.[13] - [15] At large p_T, the Color Glass description reproduces the high p_T distributions computed for jets. At lower p_T, the coherence of the Color Glass cuts off the singular p_T dependence at $p_T \sim Q_{sat}$. Here the phase space distribution scales as $1/\alpha_s$. In AA collisions, this predicts a total multiplicity for central collisions which scales as N_{part}, but at $p_T >> Q_{sat}$, the distributions scale as $N_{part}^{4/3}$ in accord with data.

One can also compute properties of pA scattering. This turns out to be a very useful paradigm, because for the AA scattering problem, or more generally equal size hadrons at very small x, and at $y = 0$, one is forced to solve the problem numerically to compute multiplicities and transverse energies. In pA collisions, the computations can be done analytically.[16]

It also turns out that diffractive and non-diffractive photoproduction of heavy quarks from large nuclei can give information about the saturation scale Q_{sat}. This might be studied in peripheral heavy ion collisions where the Coulomb field of one nucleus excites the hadronic components of the other.

5 Recent Theoretical Developments

The theory which describes the Color Glass Condensate is classical Yang-Mills theory in the presence of a stochastic source

$$J^+ = \rho(y, x_T) \tag{13}$$

where the averaging is done with a measure

$$\int [d\rho]\ e^{-F[\rho]} \tag{14}$$

In the simplest version of this theory, F is Gaussian

$$F = \frac{1}{2}\int_{y_0}^{y_{proj}} dy d^2x_T\ \rho(y, x_T)^2/\mu(y)^2 \tag{15}$$

The sources are at space-time rapidities greater than a cutoff y_0 and the fields exist for $y < y_0$.

What determines y_0? It is an arbitrary scale. In fact it is determined by renormalization group. [6],[17]-[19] If one computes the first quantum correction to the above theory, one gets corrections of order $\alpha_s(y-y_0)$ so that when $\Delta y >> 1/\alpha_s$, the quantum corrections are big and must be resummed. This can be done by succesive iterations and yields a renormalization group. If we change the cutoff scale for y_0 to $y_0 - dy$, then we find that the action for the gluon fields is unchanged, but the functional form of F is modified.

If we define

$$Z = e^{-F} \tag{16}$$

we find that this renormalization group equations can be written as a Euclidean functional Schrodinger equation

$$\frac{d}{dy}Z = -\ H(\rho, \delta/\delta\rho)Z \tag{17}$$

It turns out that H is quadratic in $\delta/\delta\rho$, and has no potential for $V(\rho, \partial\rho)$. This equation is a non-linear quantum diffusion equation.[5],[20]-[21]

For example, if we have the ordinary diffusion equation,

$$\frac{d}{dy}Z = -\frac{1}{2}p^2 Z \tag{18}$$

the solution is

$$Z = \frac{1}{\sqrt{2\pi y}} e^{-x^2/2y} \tag{19}$$

and the wavepacket spreads as y get larger. This is unlike the case where there would be a potential. In this latter case, the system would go to the minimum of the potential and oscillate around. Expectation values with a potential would become time independent, but those without contain non-trivial time evolution. This non-trivial time evolution is at the heart of the small x problem.

One can find approximate solutions of the above equation. For large transverse momentum scales, one recovers the Gaussian ansatz of the MV model.[4] For small transverse momentum scales, but still large compared to Λ_{QCD}, the solution for F becomes a scale invariant Gaussian, and this region is universal in that its behavior is independent of the saturation scale.[22]

The equations above also reproduce all the known evolution equations for structure functions. In the linear limit where the Color Glass field is weak, the DGLAP and BFKL equations can be generated.[23]-[24] The first non-linear corrections also follow,[2]-[3] and in the large N_c limit, the Balitsky-Kovchegov equation to all orders in density for F_2 is recovered.[25]-[26]

One might hope that this equation might be solvable exactly, at least in large N_c. At least one should be able to formulate a reasonable non-perturbative algorithm for its numerical solution.

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