# What is the Expected Return on the Market? 

Ian Martin

London School of Economics

## Returns on the stock market are predictable

$$
\text { return }_{t+1}=\frac{\text { price }_{t+1}+\text { dividend }_{t+1}}{\text { price }_{t}}=\underbrace{\frac{\text { price }_{t+1}}{\text { price }_{t}}}_{\text {capital gain }}+\underbrace{\frac{\text { dividend }_{t+1}}{\text { price }_{t}}}_{\text {dividend yield }}
$$

- Naive investor: If I buy when the dividend yield is high, I will have a high return on average
- 'Sophisticated' investor: No! The high dividend yield—that is, low price—is a sign that the market anticipates that future dividends will be disappointing. I therefore expect that a low capital gain will offset the high dividend yield
- Empirically, it appears that the naive investor is right

S\&P 500 Price / 10-Year Average of Earnings


## The equity premium

Figure from John Campbell's Princeton Lecture in Finance
Equity Premium -- US


## Motivation

- Find an asset price that forecasts expected returns
- without using accounting data
- without having to estimate any parameters
- imposing minimal theoretical structure
- and in real time


## A lower bound on the equity premium

1 year horizon, in \%


## A lower bound on the equity premium

1 month horizon, annualized, in \%


## Outline

(1) A volatility index, SVIX, gives a lower bound on the equity premium
(2) SVIX and VIX
(3) SVIX as a predictor variable
(4) What is the probability of a $20 \%$ decline in the market?

## Outline

(1) A volatility index, SVIX, gives a lower bound on the equity premium

## 3 SVIX as a predictor variable

## 44 What is the probability of a $20 \%$ decline in the market?

## Notation

- $S_{T}$ : level of S\&P 500 index at time $T$
- $R_{T}$ : gross return on the S\&P 500 from time $t$ to time $T$
- $R_{f, t}$ : riskless rate from time $t$ to time $T$
- $M_{T}$ : SDF that prices time- $T$ payoffs from the perspective of time $t$
- We can price any time- $T$ payoff $X_{T}$ either via the SDF or by computing expectations with risk-neutral probabilities:

$$
\text { time-t price of a claim to } X_{T}=\mathbb{E}_{t}\left(M_{T} X_{T}\right)=\frac{1}{R_{f, t}} \mathbb{E}_{t}^{*} X_{T}
$$

- Asterisks indicate risk-neutral quantities


## Risk-neutral variance and the risk premium

- As an example, we can write conditional risk-neutral variance as

$$
\begin{equation*}
\operatorname{var}_{t}^{*} R_{T}=\mathbb{E}_{t}^{*} R_{T}^{2}-\left(\mathbb{E}_{t}^{*} R_{T}\right)^{2}=R_{f, t} \mathbb{E}_{t}\left(M_{T} R_{T}^{2}\right)-R_{f, t}^{2} \tag{1}
\end{equation*}
$$

- We can decompose the equity premium into two components:

$$
\begin{aligned}
\mathbb{E}_{t} R_{T}-R_{f, t} & =\left[\mathbb{E}_{t}\left(M_{T} R_{T}^{2}\right)-R_{f, t}\right]-\left[\mathbb{E}_{t}\left(M_{T} R_{T}^{2}\right)-\mathbb{E}_{t} R_{T}\right] \\
& =\frac{1}{R_{f, t}} \operatorname{var}_{t}^{*} R_{T}-\operatorname{cov}_{t}\left(M_{T} R_{T}, R_{T}\right)
\end{aligned}
$$

- The first line adds and subtracts $\mathbb{E}_{t}\left(M_{T} R_{T}^{2}\right)$
- The second exploits equation (1) and the fact that $\mathbb{E}_{t} M_{T} R_{T}=1$


## Risk-neutral variance and the risk premium

$$
\mathbb{E}_{\mathrm{t}} R_{T}-R_{f, t}=\frac{1}{R_{f, t}} \operatorname{var}_{t}^{*} R_{T}-\underbrace{\operatorname{cov}_{t}\left(M_{T} R_{T}, R_{T}\right)}_{\leq 0, \text { under the NCC }}
$$

- The decomposition splits the risk premium into two pieces
- Risk-neutral variance can be computed from time-t asset prices
- The covariance term can be controlled: it is negative in theoretical models and in the data
- Formalize this key assumption as the negative correlation condition:

$$
\operatorname{cov}_{t}\left(M_{T} R_{T}, R_{T}\right) \leq 0
$$

## The NCC holds. . .

(1) ... in lognormal models in which the market's conditional Sharpe ratio exceeds its conditional volatility (Campbell-Cochrane 1999, Bansal-Yaron 2004, and many others).

## The NCC holds. . .

(1) .... in lognormal models in which the market's conditional Sharpe ratio exceeds its conditional volatility (Campbell-Cochrane 1999, Bansal-Yaron 2004, and many others).
(2) ... in a wide range of models with intertemporal investors, state variables, Epstein-Zin preferences, non-Normality, labor income.

## The NCC holds. . .

(1) ... in lognormal models in which the market's conditional Sharpe ratio exceeds its conditional volatility (Campbell-Cochrane 1999, Bansal-Yaron 2004, and many others).
(2) ... in a wide range of models with intertemporal investors, state variables, Epstein-Zin preferences, non-Normality, labor income.
(3)... if there is a one-period investor who maximizes expected utility, who is fully invested in the market, and whose relative risk aversion $\gamma(C) \equiv-\frac{C u^{\prime \prime}(C)}{u^{\prime}(C)} \geq 1$ (not necessarily constant).

## The NCC holds. . .

(1) ... in lognormal models in which the market's conditional Sharpe ratio exceeds its conditional volatility (Campbell-Cochrane 1999, Bansal-Yaron 2004, and many others).
(2) ... in a wide range of models with intertemporal investors, state variables, Epstein-Zin preferences, non-Normality, labor income.
(3)... if there is a one-period investor who maximizes expected utility, who is fully invested in the market, and whose relative risk aversion $\gamma(C) \equiv-\frac{C u^{\prime \prime}(C)}{u^{\prime}(C)} \geq 1$ (not necessarily constant).

- Proof. The given assumption implies that the SDF is proportional to $u^{\prime}\left(W_{t} R_{T}\right)$, so we must show that $\operatorname{cov}_{t}\left(R_{T} u^{\prime}\left(W_{t} R_{T}\right), R_{T}\right) \leq 0$.


## The NCC holds. . .

(1) ... in lognormal models in which the market's conditional Sharpe ratio exceeds its conditional volatility (Campbell-Cochrane 1999, Bansal-Yaron 2004, and many others).
C . . . in a wide range of models with intertemporal investors, state variables, Epstein-Zin preferences, non-Normality, labor income.

- ... if there is a one-period investor who maximizes expected utility, who is fully invested in the market, and whose relative risk aversion $\gamma(C) \equiv-\frac{C u^{\prime \prime}(C)}{u^{\prime}(C)} \geq 1$ (not necessarily constant).
- Proof. The given assumption implies that the SDF is proportional to $u^{\prime}\left(W_{t} R_{T}\right)$, so we must show that $\operatorname{cov}_{t}\left(R_{T} u^{\prime}\left(W_{t} R_{T}\right), R_{T}\right) \leq 0$.
- This holds because $R_{T} u^{\prime}\left(W_{t} R_{T}\right)$ is decreasing in $R_{T}$ : its derivative is $u^{\prime}\left(W_{t} R_{T}\right)+W_{t} R_{T} u^{\prime \prime}\left(W_{t} R_{T}\right)=-u^{\prime}\left(W_{t} R_{T}\right)\left[\gamma\left(W_{t} R_{T}\right)-1\right] \leq 0$.


## Whose equity premium?

$$
\mathbb{E}_{t} R_{T}-R_{f, t} \geq \frac{1}{R_{f, t}} \operatorname{var}_{t}^{*} R_{T}
$$

- Does not require that everyone holds the market
- Does not assume that all economic wealth is invested in the market
- Simply ask: What is the equity premium perceived by a rational one-period investor who holds the market and whose risk aversion is at least 1 ?
- This question is a sensible benchmark even in the presence of constrained and/or irrational investors


## Comparison to Merton (1980)

- Merton (1980) suggested estimating the equity premium from equity premium $=$ risk aversion $\times$ return variance
- Holds if marginal investor has power utility and the market follows a geometric Brownian motion
- No distinction between risk-neutral and real-world variance in a diffusion-based model (Girsanov's theorem)
- The appropriate generalization relates the equity premium to risk-neutral variance
- Bonus: Risk-neutral variance is directly measurable from asset prices


## Comparison to Hansen-Jagannathan (1991)

$$
\frac{1}{R_{f, t}} \operatorname{var}_{t}^{*} R_{T} \leq \mathbb{E}_{t} R_{T}-R_{f, t} \leq R_{f, t} \cdot \sigma_{t}\left(M_{T}\right) \cdot \sigma_{t}\left(R_{T}\right)
$$

- Left-hand inequality is the new result
- Good: relates unobservable equity premium to an observable quantity
- Bad: requires the negative correlation condition
- Right-hand inequality is the Hansen-Jagannathan bound
- Good: no assumptions
- Bad: neither side is observable


## How to measure risk-neutral variance

- We want to measure $\frac{1}{R_{f, t}} \operatorname{var}_{t}^{*} R_{T}=\frac{1}{R_{f, t}} \mathbb{E}_{t}^{*} R_{T}^{2}-\frac{1}{R_{f, t}}\left(\mathbb{E}_{t}^{*} R_{T}\right)^{2}$
- Since $\mathbb{E}_{t}^{*} R_{T}=R_{f, t}$, this boils down to calculating $\frac{1}{R_{f, t}} \mathbb{E}_{t}^{*} S_{T}^{2}$
- That is: how can we price the 'squared contract' with payoff $S_{T}^{2}$ ?


## How to measure risk-neutral variance

- How can we price the 'squared contract' with payoff $S_{T}^{2}$ ?
- Suppose you buy:
- 2 calls with strike $K=0.5$
- 2 calls with strike $K=1.5$
- 2 calls with strike $K=2.5$
- 2 calls with strike $K=3.5$
- etc...


## How to measure risk-neutral variance

payoff


- So, $\frac{1}{R_{f, t}} \mathbb{E}_{t}^{*} S_{T}^{2} \approx 2 \sum_{K} \operatorname{call}_{t, T}(K)$
- In fact, $\frac{1}{R_{f, t}} \mathbb{E}_{t}^{*} S_{T}^{2}=2 \int_{0}^{\infty} \operatorname{call}_{t, T}(K) d K$


## How to measure risk-neutral variance

option prices


- Using put-call parity, we end up with a simple formula:

$$
\frac{1}{R_{f, t}} \operatorname{var}_{t}^{*} R_{T}=\frac{2}{S_{t}^{2}}\left\{\int_{0}^{F_{t, T}} \operatorname{put}_{t, T}(K) d K+\int_{F_{t, T}}^{\infty} \operatorname{call}_{t, T}(K) d K\right\}
$$

- $F_{t, T}$ is the forward price of the underlying, which is known at time $t$


## A lower bound on the equity premium

1mo horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red


## A lower bound on the equity premium

3mo horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red


## A lower bound on the equity premium

1yr horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red


## Robustness

- Can't observe deep-OTM option prices
option prices



## Robustness

- Even near-the-money, can't observe a continuum of strikes



## Robustness

- Both these effects mean that the true lower bound is even higher
- By ignoring deep-OTM options, we underestimate the true area under the curve
- Discretization in strike also leads to underestimating the true area, because call $l_{t, T}(K)$ and $\operatorname{put}_{t, T}(K)$ are both convex in $K$
- Maybe option markets were totally illiquid in November 2008 ?
- If so, we should expect to see wide bid-ask spread
- Is lower bound much lower if bid prices are used for options, rather than mid prices? No. And volume was high


## A lower bound on the equity premium

| horizon | mean | s.d. | $\min$ | $1 \%$ | $10 \%$ | $25 \%$ | $50 \%$ | $75 \%$ | $90 \%$ | $99 \%$ | $\max$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 mo | 5.00 | 4.60 | 0.83 | 1.03 | 1.54 | 2.44 | 3.91 | 5.74 | 8.98 | 25.7 | 55.0 |
| 2 mo | 5.00 | 3.99 | 1.01 | 1.20 | 1.65 | 2.61 | 4.11 | 5.91 | 8.54 | 23.5 | 46.1 |
| 3 mo | 4.96 | 3.60 | 1.07 | 1.29 | 1.75 | 2.69 | 4.24 | 5.95 | 8.17 | 21.4 | 39.1 |
| 6 mo | 4.89 | 2.97 | 1.30 | 1.53 | 1.95 | 2.88 | 4.39 | 6.00 | 7.69 | 16.9 | 29.0 |
| 1 yr | 4.64 | 2.43 | 1.47 | 1.64 | 2.07 | 2.81 | 4.35 | 5.72 | 7.19 | 13.9 | 21.5 |

Table: Mean, standard deviation, and quantiles of EP bound (in \%)

- The time series average of the lower bound is about $5 \%$
- It is volatile and right-skewed, particularly at short horizons


## Outline

(1) A volatility index, SVIX, gives a lower bound on the equity premium
(2) SVIX and VIX

3 SVIX as a predictor variable
4. What is the probability of a $20 \%$ decline in the market?

## SVIX and VIX

- By analogy with VIX, define

$$
\mathrm{SVIX}_{t}^{2}=\frac{2 R_{f, t}}{(T-t) \cdot F_{t, T}^{2}}\left\{\int_{0}^{F_{t, T}} \mathrm{put}_{t, T}(K) d K+\int_{F_{t, T}}^{\infty} \operatorname{call}_{t, T}(K) d K\right\}
$$

- In this notation, equity premium $\geq R_{f, t} \cdot \mathrm{SVIX}_{t}^{2}$
- Compare SVIX with

$$
\mathrm{VIX}_{t}^{2}=\frac{2 R_{f, t}}{T-t}\left\{\int_{0}^{F_{t, T}} \frac{1}{K^{2}} \operatorname{put}_{t, T}(K) d K+\int_{F_{t, T}}^{\infty} \frac{1}{K^{2}} \operatorname{call}_{t, T}(K) d K\right\}
$$

- These are definitions, not statements about pricing


## SVIX and VIX

- VIX is similar to SVIX, but is more sensitive to left tail events
- SVIX measures risk-neutral variance, SVIX $^{2}=\operatorname{var}_{t}^{*}\left(R_{T} / R_{f, t}\right)$
- VIX measures risk-neutral entropy,
$\mathrm{VIX}^{2}=\log \mathbb{E}_{t}^{*}\left(R_{T} / R_{f, t}\right)-\mathbb{E}_{t}^{*} \log \left(R_{T} / R_{f, t}\right)$
- What VIX does not measure: VIX ${ }^{2} \neq \frac{1}{T-t} \mathbb{E}_{t}^{*}\left[\int_{t}^{T} \sigma_{\tau}^{2} d \tau\right]$


## VIX and SVIX



Figure: VIX (dotted) and SVIX (solid). Jan 4, 1996-Jan 31, 2012
Figure shows 10 -day moving average. $T=1$ month

## VIX minus SVIX



Figure: VIX minus SVIX. Jan 4, 1996-Jan 31, 2012
Figure shows 10 -day moving average. $T=1$ month

## No conditionally lognormal model fits option prices

- If returns and the SDF are conditionally lognormal with return volatility $\sigma_{R, t}$ then we can calculate VIX and SVIX in closed form:

$$
\begin{aligned}
\operatorname{SVIX}_{t}^{2} & =\frac{1}{T-t}\left(e^{\sigma_{R, t}^{2}(T-t)}-1\right) \\
\mathrm{VIX}_{t}^{2} & =\sigma_{R, t}^{2}
\end{aligned}
$$

- VIX would be lower than SVIX—which it never is in my sample
- No conditionally lognormal model is consistent with option prices


## Outline

(1) A volatility index, SVIX, gives a lower bound on the equity premium
(2) SVIX and VIX
(3) SVIX as a predictor variable

4 What is the probability of a $20 \%$ decline in the market?

## Might the lower bound hold with equality?

- Time-series average of lower bound in recent data is around $5 \%$
- Fama and French (2002) estimate unconditional equity premium of $3.83 \%$ (from dividend growth) or $4.78 \%$ (from earnings growth)
- Fama interviewed by Roll: "I always think of the number, the equity premium, as five per cent."
- Estimates of $\operatorname{cov}\left(M_{T} R_{T}, R_{T}\right)$ in linear factor models are statistically and economically close to zero


## $\widehat{\operatorname{cov}}\left(M_{T} R_{T}, R_{T}\right)$ is negative and close to zero

|  | constant | $R_{M}-R_{f}$ | $S M B$ | $H M L$ | $M O M$ | $\widehat{\operatorname{cov}}\left(M_{T} R_{T}, R_{T}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Full sample | 1.072 | -2.375 | -0.648 | -5.489 | -5.572 | -0.0018 |
|  | $(0.020)$ | $(0.746)$ | $(1.011)$ | $(1.131)$ | $(1.033)$ | $(0.0020)$ |
| Jan '27-Dec '62 | 1.071 | -2.355 | -0.587 | -3.882 | -5.552 | -0.0021 |
|  | $(0.029)$ | $(1.034)$ | $(1.747)$ | $(2.163)$ | $(1.565)$ | $(0.0041)$ |
| Jan '63-Dec '13 | 1.092 | -3.922 | -2.400 | -9.020 | -5.152 | -0.0020 |
|  | $(0.029)$ | $(1.272)$ | $(1.475)$ | $(1.795)$ | $(1.427)$ | $(0.0022)$ |
| Jan '96-Dec '13 | 1.047 | -3.231 | -2.327 | -5.789 | -2.548 | -0.0017 |
|  | $(0.034)$ | $(1.981)$ | $(2.224)$ | $(2.491)$ | $(1.637)$ | $(0.0036)$ |

Table: Estimates of coefficients in the 4-factor model, and of $\operatorname{cov}\left(M_{T} R_{T}, R_{T}\right)$.

- Test assets: market, riskless asset, $5 \times 5$ portfolios sorted on size and $B / M, 10$ momentum portfolios; monthly data from Ken French's website
- Estimate $M$ and $\operatorname{cov}\left(M_{T} R_{T}, R_{T}\right)$ by GMM


## Forecasting returns with risk-neutral variance

- We want to test the null hypothesis that $\mathbb{E}_{t} R_{T}-R_{f, t}=R_{f, t} \cdot \operatorname{SVIX}_{t}^{2}$
- Run regressions

$$
R_{T}-R_{f, t}=\alpha+\beta \times R_{f, t} \cdot \mathrm{SVIX}_{t}^{2}+\varepsilon_{T}
$$

- Sample period: January 1996-January 2012
- Robust Hansen-Hodrick standard errors account for heteroskedasticity and overlapping observations


## Forecasting returns with risk-neutral variance

| horizon | $\widehat{\alpha}$ | s.e. | $\widehat{\beta}$ | s.e. | $R^{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 mo | 0.012 | $[0.064]$ | 0.779 | $[1.386]$ | $0.34 \%$ |
| 2 mo | -0.002 | $[0.068]$ | 0.993 | $[1.458]$ | $0.86 \%$ |
| 3 mo | -0.003 | $[0.075]$ | 1.013 | $[1.631]$ | $1.10 \%$ |
| 6 mo | -0.056 | $[0.058]$ | 2.104 | $[0.855]$ | $5.72 \%$ |
| 1 yr | -0.029 | $[0.093]$ | 1.665 | $[1.263]$ | $4.20 \%$ |

Table: Coefficient estimates for the forecasting regression.

- Cannot reject the null at any horizon


## Forecasting returns with risk-neutral variance

| horizon | $\widehat{\alpha}$ | s.e. | $\widehat{\beta}$ | s.e. | $R^{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 mo | -0.095 | $[0.061]$ | 3.705 | $[1.258]$ | $3.36 \%$ |
| 2 mo | -0.081 | $[0.062]$ | 3.279 | $[1.181]$ | $4.83 \%$ |
| 3 mo | -0.076 | $[0.067]$ | 3.147 | $[1.258]$ | $5.98 \%$ |
| 6 mo | -0.043 | $[0.072]$ | 2.319 | $[1.276]$ | $4.94 \%$ |
| 1 yr | 0.045 | $[0.088]$ | 0.473 | $[1.731]$ | $0.27 \%$ |

Table: Coefficient estimates excluding Aug '08-Jul '09

- Predictability is not driven by the crisis


## Realized variance doesn't predict reliably

| horizon | $\widehat{\alpha}$ | s.e. | $\widehat{\beta}$ | s.e. | $R^{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 mo | 0.049 | $[0.045]$ | -0.462 | $[0.784]$ | $0.27 \%$ |
| 2 mo | 0.044 | $[0.043]$ | -0.341 | $[0.586]$ | $0.26 \%$ |
| 3 mo | 0.035 | $[0.046]$ | -0.173 | $[0.722]$ | $0.09 \%$ |
| 6 mo | -0.025 | $[0.050]$ | 1.182 | $[0.430]$ | $5.45 \%$ |
| 1 yr | -0.042 | $[0.068]$ | 1.293 | $[0.499]$ | $8.13 \%$ |

Table: Regression $R_{T}-R_{f, t}=\alpha+\beta \times S V A R_{t}+\varepsilon_{T}$, full sample.

## Realized variance doesn't predict reliably

| horizon | $\widehat{\alpha}$ | s.e. | $\widehat{\beta}$ | s.e. | $R^{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 mo | -0.007 | $[0.049]$ | 1.478 | $[1.125]$ | $0.71 \%$ |
| 2 mo | -0.006 | $[0.050]$ | 1.429 | $[1.272]$ | $1.13 \%$ |
| 3 mo | -0.004 | $[0.049]$ | 1.342 | $[1.265]$ | $1.32 \%$ |
| 6 mo | 0.028 | $[0.049]$ | 0.299 | $[1.424]$ | $0.09 \%$ |
| 1 yr | 0.034 | $[0.064]$ | -0.348 | $[2.469]$ | $0.15 \%$ |

Table: Regression $R_{T}-R_{f, t}=\alpha+\beta \times S V A R_{t}+\varepsilon_{T}$, excluding Aug '08-Jul '09.

## Forecasting returns with valuation ratios

- Goyal-Welch (2008): Conventional predictor variables fail out-of-sample
- Campbell-Thompson (2008) response: Gordon growth model suggests a forecast

$$
\mathbb{E}_{t} R_{T}=D / P_{t}+G
$$

- Important: coefficient on $D / P_{t}$ is not estimated but fixed a priori
- A good comparison for the risk-neutral variance approach


## $R^{2}$ from Campbell and Thompson (2008)

|  | Sample: 1927-1956 |  |  | Sample: 1956-1980 |  |  | Sample: 1980-2005 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unconstrained | Pos. Intercept, Bounded Slope | Fixed Coefs | Unconstrained | Pos. Intercept, Bounded Slope | Fixed Coefs | Unconstrained | Pos. Intercept, Bounded Slope | Fixed Coefs |
|  | A: Monthly Returns |  |  |  |  |  |  |  |  |
| Dividend/price | -0.86\% | 0.21\% | 0.63\% | 0.88\% | 0.57\% | 0.67\% | -1.30\% | -0.21\% | -0.54\% |
| Earnings/price | 0.16 | 0.28 | 1.04 | 0.56 | 0.45 | 0.30 | -0.53 | -0.09 | 0.07 |
| Smooth carnings/price | 0.56 | 0.53 | 1.33 | 0.80 | 0.48 | 0.51 | $-1.06$ | $-0.06$ | 0.01 |
| Dividend/price + growth | -0.15 | 0.18 | 0.78 | 0.18 | 0.18 | 0.59 | 0.11 | 0.11 | 0.14 |
| Earnings/price + growth | -0.06 | 0.12 | 0.73 | -0.12 | -0.12 | 0.33 | 0.05 | 0.05 | 0.16 |
| Smooth earnings/price + growth | 0.19 | 0.25 | 0.93 | 0.19 | 0.19 | 0.47 | 0.06 | 0.06 | 0.16 |
| Book-to-market + growth |  |  |  | -0.62 | $-0.73$ | 0.73 | -0.12 | $-0.02$ | 0.00 |
| Dividend/price + growth - real rate | $-0.01$ | 0.30 | 0.45 | -0.24 | -0.24 | 0.76 | 0.11 | 0.11 | -0.08 |
| Earnings/price + growth - real rate | 0.06 | 0.20 | 0.41 | -0.34 | -0.34 | 0.66 | 0.06 | 0.06 | 0.03 |
| Smooth earnings/price + growth - real rate | 0.27 | 0.39 | 0.60 | -0.28 | -0.28 | 0.74 | 0.04 | 0.04 | 0.02 |
| Book-to-market + growth - real rate |  |  |  | -0.82 | -0.91 | 0.89 | $-0.14$ | $-0.02$ | -0.27 |
|  |  |  |  |  |  |  |  |  |  |
| Dividend/price | 9.95 | 4.53 | 3.67 | 9.46 | 5.99 | 6.88 | -16.19 | -1.38 | -7.98 |
| Earnings/price | 7.45 | 5.34 | 7.58 | 5.08 | 3.25 | 2.56 | -6.06 | 0.88 | 1.47 |
| Smooth eamings/price | 12.51 | 8.22 | 10.49 | 4.93 | 3.71 | 3.71 | -8.86 | 1.33 | 1.33 |
| Dividend/price + growth | 2.77 | 3.05 | 4.83 | 1.76 | 1.74 | 6.61 | 1.87 | 1.82 | 0.28 |
| Eamings/price + growth | 2.21 | 2.38 | 4.37 | $-0.85$ | -0.85 | 3.97 | 1.63 | 1.63 | 1.60 |
| Smooth earningsiprice + growth | 3.73 | 3.87 | 6.38 | 1.27 | 1.19 | 4.65 | 2.30 | 2.23 | 1.81 |
| Book-to-market + growth |  |  |  | $-7.09$ | -5.16 | 10.34 | -0.24 | 0.14 | $-2.43$ |
| Dividend/price + growth - real rate | 4.40 | 4.51 | 1.67 | $-3.57$ | $-3.56$ | 8.28 | 2.19 | 2.19 | -2.95 |
| Eamings/price + growth - real rate | 3.44 | 3.49 | 1.25 | -4.68 | -4.68 | 7.16 | 1.88 | 1.88 | -0.64 |
| Smooth earnings/price + growth - real rate | 5.34 | 5.37 | 3.19 | -4.91 | -4.84 | 7.32 11.85 | 2.36 | 2.36 | $-0.47$ |
| Book-to-market + growth - real rate |  |  |  | -3.36 | -4.22 | 11.85 | -0.25 | 0.35 | -6.20 |

## Out-of-sample $R^{2}$

Fixed coefficients $\alpha=0, \beta=1$

| horizon | $R_{O S}^{2}$ |
| :--- | :---: |
| 1 mo | $0.42 \%$ |
| 2 mo | $1.11 \%$ |
| 3 mo | $1.49 \%$ |
| 6 mo | $4.86 \%$ |
| 1 yr | $4.73 \%$ |

Table: $R^{2}$ using SVIX $_{t}^{2}$ as predictor variable with $\alpha=0, \beta=1$

## Are the $R^{2}$ too low?

No. Small $R^{2} \longrightarrow$ high Sharpe ratios

- We can use the predictor in a market-timing strategy
- On day $t$, invest $\alpha_{t}$ in the S\&P 500 index and $1-\alpha_{t}$ in cash
- Choose $\alpha_{t}$ proportional to 1-mo SVIX ${ }_{t}^{2}$
- Earns a daily Sharpe ratio of $1.97 \%$ in sample
- For comparison, the daily Sharpe ratio of the index is $1.35 \%$
- The point is not that Sharpe ratios are necessarily the right metric, but that apparently small $R^{2}$ can make a big difference


## The value of a dollar invested

In cash (yellow), in the S\&P 500 (red), and in the market-timing strategy (blue)


- Mean: 35\% S\&P 500, 65\% cash. Median: 27\% S\&P 500, 73\% cash.


## Risk-neutral variance vs. valuation ratios

Blue: earnings yield (Campbell and Thompson (2008)). Red: risk-neutral variance


## Black Monday, 1987

- It is interesting to identify points at which my claims contrast most starkly with the conventional view based on valuation ratios
- In particular: what happened to the equity premium during and immediately after Black Monday in 1987, which was by far the worst day in stock market history?
- Valuation ratios: it moved from about 5\% to about 6\%
- Suppose $D / P=2 \%$ and then market halves in value. $D / P$ only increases to 4\%
- Options: it exploded
- Implied risk premium about twice as high as in the recent crisis


## Risk-neutral variance exploded on Black Monday

1mo horizon, annualized and using VXO as a proxy for true measure


## Risk-neutral variance vs. valuation ratios

- Campbell-Shiller: $d_{t}-p_{t}=k+\mathbb{E}_{t} \sum_{j=0}^{\infty} \rho^{j}\left(r_{t+1+j}-\Delta d_{t+1+j}\right)$
- If dividend growth is unforecastable,

$$
d_{t}-p_{t}=k+\sum_{j=0}^{\infty} \rho^{j} \mathbb{E}_{t} r_{t+1+j}
$$

- Dividend yield measures expected returns over the very long run
- Difference between SVIX $_{t}^{2}$ and $d_{t}-p_{t} \approx$ gap between short-run expected returns and long-run expected returns
- Consider the late 1990s: 1-year expected returns $\left(\mathrm{SVIX}_{t}^{2}\right)$ were high, very long-run expected returns $(D / P)$ were low


## The term structure of the equity premium


$-6 \mathrm{mo} \rightarrow 12 \mathrm{mo}$
$-3 \mathrm{mo} \rightarrow 6 \mathrm{mo}$
$-2 \mathrm{mo} \rightarrow 3 \mathrm{mo}$
$-1 \mathrm{mo} \rightarrow 2 \mathrm{mo}$
$-0 \mathrm{mo} \rightarrow 1 \mathrm{mo}$

- In bad times, high equity premia can mostly be attributed to very high short-run premia


## What's the equity premium right now?

 Delayed: 10:11AM CST INDEXCBOE deta delayed by 15 mins - Disciaimer
Compare: Enler tickerhere Add

Compare: Enter bickerhere Add

 $\qquad$ $\therefore$ 0n 04, 20.5 - Fct 01, $2016+2.56(14.66 \% \%$


- Annualized 1-month equity premium $\approx 20.77 \%^{2}=4.3 \%$


## Outline

(1) A volatility index, SVIX, gives a lower bound on the equity premium
(2) SVIX and VIX

3 SVIX as a predictor variable
4) What is the probability of a $20 \%$ decline in the market?

## What is the probability of a $20 \%$ decline?

- Take the perspective of an investor with log utility whose portfolio is fully invested in the market
- Expectations of such an investor obey the following relationship:

$$
\widetilde{\mathbb{E}}_{t} X_{T}=\frac{1}{R_{f, t}} \mathbb{E}_{t}^{*}\left[X_{T} R_{T}\right]
$$

- So if we can price a claim to $X_{T} R_{T}$ then we know the log investor's expectation of $X_{T}$
- Interpretation: "What a log investor would have to believe about $X_{T}$ to make him or her happy to hold the market"


## What is the probability of a $20 \%$ decline?



## What is the probability of a $20 \%$ decline?

$T=1 \mathrm{mo}$


## What is the probability of a $20 \%$ decline?

$T=2 \mathrm{mo}$


## What is the probability of a $20 \%$ decline?

$T=3 \mathrm{mo}$


## What is the probability of a $20 \%$ decline?

$T=6 \mathrm{mo}$


## What is the probability of a $20 \%$ decline?

$T=1 \mathrm{yr}$


## New directions



- What is the expected return on an individual stock? (joint work with Christian Wagner, Copenhagen Business School)
- Our approach outperforms conventional predictors


## Conclusions

- Have shown how to measure the equity premium in real time
- The results point to a new view of the equity premium
- Extremely volatile, at faster-than-business-cycle frequency
- Right-skewed, with occasional opportunities to earn exceptionally high expected excess returns in the short run
- Black Monday, October 19, 1987, provides the starkest illustration
- $D / P$ : annual equity premium moved from $4 \%$ to $5 \%$
- SVIX: equity premium was $\sim 8 \%$ over the next one month

