Strings 2008, CERN

What is the Simplest Quantum Field Theory?

Freddy Cachazo

Perimeter Institute for Theoretical Physics

Strings 2008, CERN

What is the Simplest Quantum Field Theory?

Freddy Cachazo
Perimeter Institute for Theoretical Physics

Based on N. Arkani-Hamed, F.C., J. Kaplan, arXiv:0808.1446

More Precise Formulation of the Question:

- QFT's in four dimensions.
- Simplicity at the level of the S-matrix.
- Perturbation theory.

More Precise Formulation of the Question:

- QFT's in four dimensions.
- Simplicity at the level of the S-matrix.
- Perturbation theory.

Answer: $\mathcal{N} = 8$ supergravity!

More Precise Formulation of the Question:

- QFT's in four dimensions.
- Simplicity at the level of the S-matrix.
- Perturbation theory.

Answer: $\mathcal{N} = 8$ supergravity!

Naive reason: Large amount of SUSY.

More Precise Formulation of the Question:

- QFT's in four dimensions.
- Simplicity at the level of the S-matrix.
- Perturbation theory.

Answer: $\mathcal{N} = 8$ supergravity!

Naive reason: Large amount of SUSY.

Goal of this talk:

To give evidence that there are many more surprises which are *not* a consequence of SUSY and are generic to theories with high spins. SUSY allows us to show that those properties extend to all other particles in the theory (even the wildest of all: scalars).

Main property: Amazing convergent behavior for infinite complex momenta.

Claim:

$\mathcal{N} = 8$ SUGRA has the nicest S-matrix in four dimensions.

- The entire tree-level S-matrix is determined recursively in terms of the three-particle one, which is completely fixed by Lorentz symmetry.
- The massless S-matrix exists in the whole moduli space and $E_{7(7)}$ has a simple action on the tree-level S-matrix.
- The one-loop S-matrix is determined by the most special and simplest of all singularities, called the leading singularity.

Claim:

$\mathcal{N}=8$ SUGRA has the nicest S-matrix in four dimensions.

- The entire tree-level S-matrix is determined recursively in terms of the three-particle one, which is completely fixed by Lorentz symmetry.
- The massless S-matrix exists in the whole moduli space and $E_{7(7)}$ has a simple action on the tree-level S-matrix.
- The one-loop S-matrix is determined by the most special and simplest of all singularities, called the leading singularity.

Conjecture:

The entire S-matrix is determined by its leading singularities. This is the nicest property a field theory can have (in perturbation theory). This implies among other things that the theory is finite (free of UV divergencies).

Of course, being finite is not very exciting by itself.

- Expansion parameter is E/M_{Pl}. Asymptotic expansion. $e^{-(M_{\mathrm{Pl}}/E)^p}$ corrections are large at super-Planckian energies.
- $E_{7(7)}$ must be broken down to a discrete subgroup by black holes.
- All this is consistent with the impossibility of decoupling it from M-theory. (Green, Ooguri, Schwarz 07/04.)
- The interesting features of quantum gravity are really a consequence of the breakdown of local field theory. (Holographic description).

Real Interest:

- All the structures I will explain today and the ones which are yet to be found hint towards the existence of a dual formulation.
- $\mathcal{N} = 8$ SUGRA seems to be the prototype where to fully test and develop the ideas of the analytic S-matrix program from the 60's. (One reason the program was so difficult was that it was applied to very difficult theories!)
- Perhaps a twistor string theory for gravity. More generally, a topological string theory. (This duality would imply finiteness while being non-perturbatively incomplete!).

(Witten 2003, Berkovits, Nair, Boels, Mason, Skinner, Wolf, Abou-Zeid, Hull, Mansfield, . . .)

Tree-Level S-Matrix

BCFW Deformation:

(Britto, F.C., Feng 12/04, with Witten 01/05)

Search for a one cplx. parameter family of deformations of M and compute it by using its physical singularities.

$$p_1 \to p_1(z) = p_1 + zq \quad \text{and} \quad p_2 \to p_2(z) = p_2 - zq$$

with

$$p_1 = (1,1,0,0), \quad p_2 = (1,-1,0,0), \quad q = (0,0,1,i)$$

BCFW Deformation:

(Britto, F.C., Feng 12/04, with Witten 01/05)

Search for a one cplx. parameter family of deformations of M and compute it by using its physical singularities.

$$p_1 \to p_1(z) = p_1 + zq \quad \text{and} \quad p_2 \to p_2(z) = p_2 - zq$$

with

$$p_1 = (1,1,0,0), \quad p_2 = (1,-1,0,0), \quad q = (0,0,1,i)$$

The amplitude becomes a rational function of z: $M_n \to M_n(z)$

$$M(z) = \oint_{\mathcal{C}_z} \frac{dz'}{z'-z} M(z')$$

BCFW Deformation:

(Britto, F.C., Feng 12/04, with Witten 01/05)

Search for a one cplx. parameter family of deformations of M and compute it by using its physical singularities.

$$p_1 \to p_1(z) = p_1 + zq \quad \text{and} \quad p_2 \to p_2(z) = p_2 - zq$$

with

$$p_1 = (1,1,0,0), \quad p_2 = (1,-1,0,0), \quad q = (0,0,1,i)$$

The amplitude becomes a rational function of z: $M_n \to M_n(z)$

$$M(z) = \oint_{\mathcal{C}_z} \frac{dz'}{z'-z} M(z')$$

$$M_{\text{scalar}} \to 1, \quad M_{\text{YM}}^{\text{anything}, -} \to \frac{1}{z}, \quad M_{\text{Grav}}^{\text{anything},-} \to \frac{1}{z^2}$$

(Arkani-Hamed, Kaplan 01/08, Cheung 08/08)

BCFW Deformation:

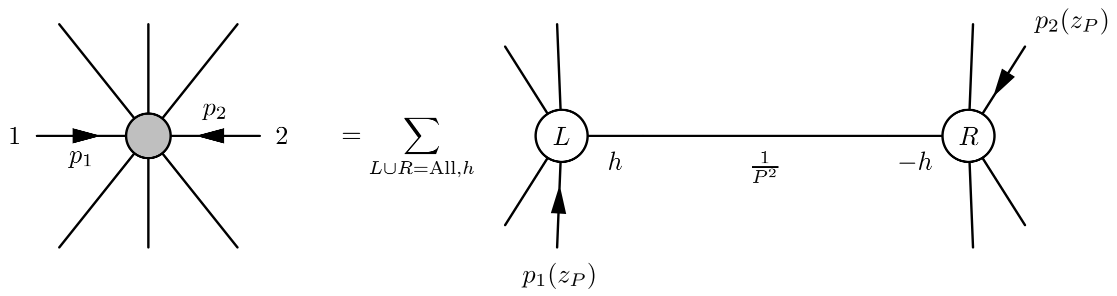
(Britto, F.C., Feng 12/04, with Witten 01/05)

Search for a one cplx. parameter family of deformations of M and compute it by using its physical singularities.

$$p_1 \to p_1(z) = p_1 + zq \quad \text{and} \quad p_2 \to p_2(z) = p_2 - zq$$

with

$$p_1 = (1,1,0,0), \quad p_2 = (1,-1,0,0), \quad q = (0,0,1,i)$$



Physics at Infinite Momentum

(Arkani-Hamed, Kaplan 01/08)

Background field method and a background q-light cone (or space cone) gauge. (Chalmers and Siegel 01/98)

Enhanced "Spin-symmetry":

For YM:

$$L = -\frac{1}{4}\mathrm{tr}\,\eta^{\mathrm{ab}}\mathrm{D}_\mu \mathrm{a}_\mathrm{a} \mathrm{D}^\mu \mathrm{a}_\mathrm{b} + \frac{\mathrm{i}}{2}\mathrm{tr}[\mathrm{a}_\mathrm{a}, \mathrm{a}_\mathrm{b}]\mathrm{F}^{\mathrm{ab}}.$$

$$M^{ab}_{s=1} = \left(cz\eta^{ab} + A^{ab} + \frac{B^{ab}}{z} + \cdots \right)$$

Pure gravity has two copies of the spin symmetry! This is why it is better behaved than YM.

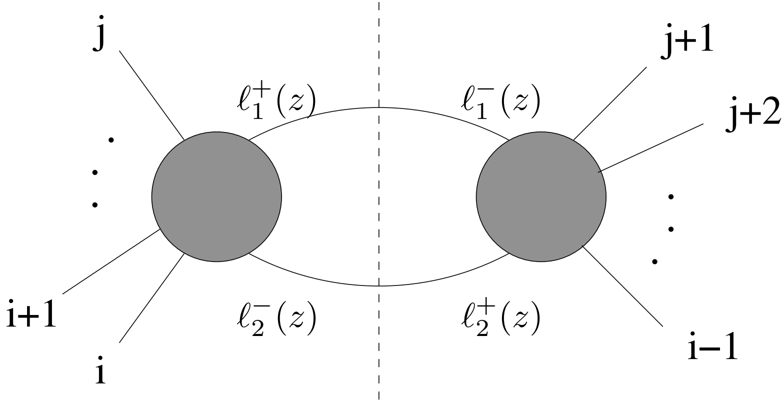
Number of copies of the Lorentz group is equal to the spin (i.e. zero for scalars!).

If pure gravity is so nice why bother make it SUSY (Maximal SUSY)!

If pure gravity is so nice why bother make it SUSY

(Maximal SUSY)!

Unitarity Cut =



$$M_{\text{Grav}}^{+,-} \to \frac{1}{z^2}, \qquad M_{\text{Grav}}^{-,+} \to z^2$$

CPT, Discrete vs. Smooth Objects

CPT forces us to add for every particle with helicity h a particle with helicity $-h$. This discreteness is the source of complications (a proliferations of unrelated objects $M(+--+--+)$, $M(--+++-)$)!

CPT, Discrete vs. Smooth Objects

CPT forces us to add for every particle with helicity h a particle with helicity $-h$. This discreteness is the source of complications (a proliferations of unrelated objects $M(+--+--+)$, $M(--+++-)$)!

Maximal SUSY fixes this problem and allows us to replace the mess by smooth objects. Multiplets are CPT self-conjugate.

$$|\bar{\eta}\rangle = e^{\bar{Q}^{I\dot{\alpha}}\bar{w}_{\dot{\alpha}}\bar{\eta}_I}|+s\rangle, \qquad |\eta\rangle = e^{Q_{I\alpha}w^{\alpha}\bar{\eta}_I}|-s\rangle$$

$$|\bar{\eta}\rangle = \int d^{\mathcal{N}}\eta e^{\eta\bar{\eta}}|\eta\rangle, \qquad |\eta\rangle = \int d^{\mathcal{N}}\bar{\eta}e^{\bar{\eta}\eta}|\bar{\eta}\rangle$$

$$M_n = M_n(\eta_1, \eta_2, \ldots, \eta_n)$$

Under Q-SUSY

$$M(\eta_i) \to M(\eta_i + \mu_i)$$

SWI becomes a simple translation in η.

What is $\mathcal{N} = 8$ SUGRA ?

Now I can define the object of study without writing a Lagrangian!

$\mathcal{N} = 8$ Supergravity S-matrix: $M_n = M_n(\eta_1, \eta_2, \ldots, \eta_n)$

One-particle states:

$$|\bar{\eta}\rangle = e^{\bar{Q}^{I\dot{\alpha}}\bar{w}_{\dot{\alpha}}\bar{\eta}_I}|+2\rangle, \qquad |\eta\rangle = e^{Q_{I\alpha}w^{\alpha}\bar{\eta}_I}|-2\rangle$$

with $I = 1, \ldots, 8$.

The index structure naturally gives rise to an $SU(8)$ R-symmetry group.

Obs: Scalars transforms in the four-index antisymmetric representation (70 of them).

What is $\mathcal{N} = 8$ SUGRA ?

Now I can define the object of study without writing a Lagrangian!

$\mathcal{N} = 8$ Supergravity S-matrix: $M_n = M_n(\eta_1, \eta_2, \ldots, \eta_n)$

One-particle states:

$$|\bar{\eta}\rangle = e^{\bar{Q}^{I\dot{\alpha}}\bar{w}_{\dot{\alpha}}\bar{\eta}_I}|+2\rangle, \qquad |\eta\rangle = e^{Q_{I\alpha}w^{\alpha}\bar{\eta}_I}|-2\rangle$$

with $I = 1, \ldots, 8$.

The index structure naturally gives rise to an $SU(8)$ R-symmetry group.

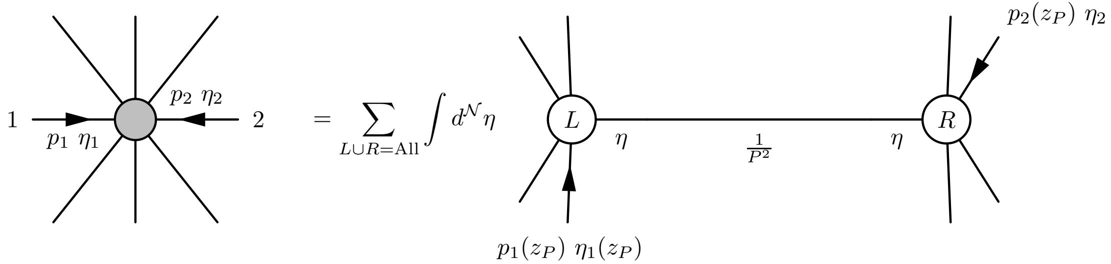
Obs: Scalars transforms in the four-index antisymmetric representation (70 of them).

(Cremmer, Julia, Scherk, Gaillard, Zumino, Nicolai, de Wit, Freedman (more recently: Kallosh, Soroush, Brink, Kim, Ramond, Green, Russo, Vanhove, Berkovits, Bern, Dixon, Kosower, Roiban, Carrasco, Johansson, Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager, Elvang, Brandhuber, Travaglini, Heslop, . . .))

BCFW Recursion Relations in Maximal SUSY

Naively Impossible! ($M(\phi_1, \ldots, \phi_n)$ does not vanish as $z \longrightarrow \infty$).

Key point: If one approaches infinity in a supersymmetric way then *all* amplitudes vanish at infinity (as $1/z^2$)!



$$p_1 \rightarrow p_1(z) = p_1 + zq \quad \text{and} \quad p_2 \rightarrow p_2(z) = p_2 - zq$$

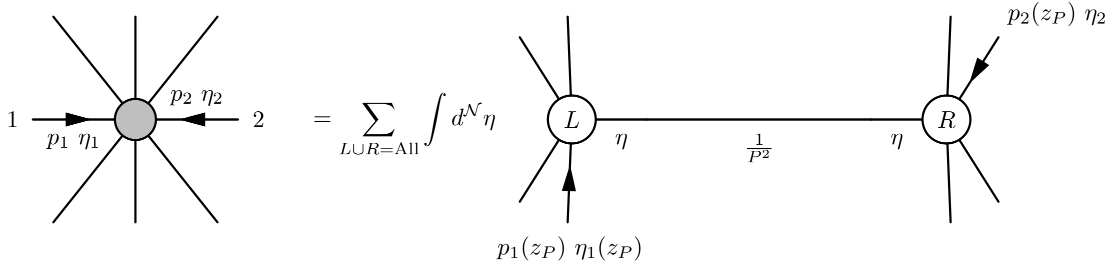
$$p_1 = (1, 1, 0, 0), \quad p_2 = (1, -1, 0, 0), \quad q = (0, 0, 1, i)$$

$$\eta_1(z) = \eta_1 + z\eta_2$$

BCFW Recursion Relations in Maximal SUSY

Naively Impossible! ($M(\phi_1, \ldots, \phi_n)$ does not vanish as $z \longrightarrow \infty$).

Key point: If one approaches infinity in a supersymmetric way then *all* amplitudes vanish at infinity (as $1/z^2$)!



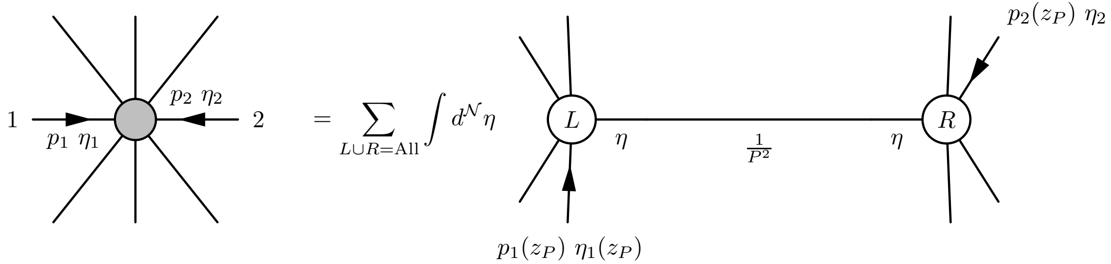
Proof:

Use the Q-SUSY to shift $\eta_1(z), \eta_2 \longrightarrow 0$ and use that $M^{-,-}(z) \longrightarrow 1/z^2$.

BCFW Recursion Relations in Maximal SUSY

Naively Impossible! ($M(\phi_1, \ldots, \phi_n)$ does not vanish as $z \longrightarrow \infty$).

Key point: If one approaches infinity in a supersymmetric way then *all* amplitudes vanish at infinity (as $1/z^2$)!

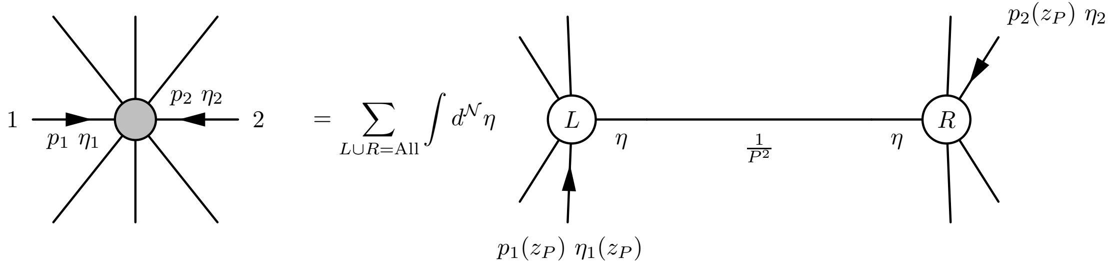


Summary: *All amplitudes can be computed using recursion relations down to three-particle amplitudes which are determined by Lorentz invariance!*

BCFW Recursion Relations in Maximal SUSY

Naively Impossible! ($M(\phi_1, \ldots, \phi_n)$ does not vanish as $z \longrightarrow \infty$).

Key point: If one approaches infinity in a supersymmetric way then *all* amplitudes vanish at infinity (as $1/z^2$)!



Summary: *All amplitudes can be computed using recursion relations down to three-particle amplitudes which are determined by Lorentz invariance!*

This is why I do not need to write a Lagrangian!

Vacuum Structure, Soft Emission and $E_{7(7)}$

$\mathcal{N} = 4$ SYM: Nice massless S-matrix only at the origin of moduli space.

$\mathcal{N} = 8$ SUGRA: Nice massless S-matrix everywhere in the moduli space.

Moduli space: 70 scalars in the $(4 - \mathrm{index})_A$ of $SU(8)$ $R-$symmetry.

$$-i\,[X_{I_1,\dots,I_4}, X_{I_5,\dots,I_8}] = \epsilon_{JI_2\dots I_8} T^J_{I_1} + \cdots + \epsilon_{I_1,\dots,I_7 J} T^J_{I_8}$$

Non-Linearly realized symmetry, $63 + 70 = 133$, $E_{7(7)}$.

$$[T, T] \sim T, \quad [X, T] \sim X, \quad [X, X] \sim T$$

Vacuum Structure, Soft Emission and $E_{7(7)}$

$\mathcal{N} = 4$ SYM: Nice massless S-matrix only at the origin of moduli space.

$\mathcal{N} = 8$ SUGRA: Nice massless S-matrix everywhere in the moduli space.

Moduli space: 70 scalars in the $(4 - \mathrm{index})_A$ of $SU(8)$ $R-$symmetry.

$$-i\left[X_{I_1,\cdots,I_4}, X_{I_5,\cdots,I_8}\right] = \epsilon_{JI_2\cdots I_8} T^J_{I_1} + \cdots + \epsilon_{I_1,\cdots,I_7 J} T^J_{I_8}$$

Non-Linearly realized symmetry, $63 + 70 = 133$, $E_{7(7)}$.

$$[T, T] \sim T, \quad [X, T] \sim X, \quad [X, X] \sim T$$

Single-Soft Emission

Using the RRs one can show that gravitons give a divergence (Weinberg), graviphotons gives a finite answer, scalars vanish! This is the indication of a moduli space.

Double Soft Emission and $E_{7(7)}$

Naively: If we give a vev to $\langle \phi^\alpha \rangle = \theta^\alpha$ then one can map the Hilbert spaces $|\psi\rangle_\theta = e^{iQ^\alpha \theta_\alpha} |\psi\rangle_0$. Then $M^{\psi_\theta} = M^\psi$.

Expanding $|\psi\rangle_\theta = |\psi\rangle + \theta_\alpha |\psi\rangle^\alpha + 1/2\theta_\alpha\theta_\beta|\psi\rangle^{\alpha\beta} + \dots$

Invariance tells us that $M^{\psi^\alpha} = 0$, $M^{\psi^{\alpha\beta}} = 0, \dots$

Double Soft Emission and $E_{7(7)}$

Naively: If we give a vev to $\langle\phi^\alpha\rangle = \theta^\alpha$ then one can map the Hilbert spaces $|\psi\rangle_\theta = e^{iQ^\alpha\theta_\alpha}|\psi\rangle_0$. Then $M^{\psi_\theta} = M^\psi$.

Expanding $|\psi\rangle_\theta = |\psi\rangle + \theta_\alpha|\psi\rangle^\alpha + 1/2\theta_\alpha\theta_\beta|\psi\rangle^{\alpha\beta} + \ldots$

Invariance tells us that $M^{\psi^\alpha} = 0$, $M^{\psi^{\alpha\beta}} = 0, \ldots$

Puzzle: $[X_\alpha, X_\beta] = f^j{}_{\alpha\beta}T_j$ while Bose statistics says that $M^{\alpha\beta}(0,0) = M^{\beta\alpha}(0,0)$.

Double Soft Emission and $E_{7(7)}$

Naively: If we give a vev to $\langle\phi^\alpha\rangle = \theta^\alpha$ then one can map the Hilbert spaces $|\psi\rangle_\theta = e^{iQ^\alpha\theta_\alpha}|\psi\rangle_0$. Then $M^{\psi_\theta} = M^\psi$.

Expanding $|\psi\rangle_\theta = |\psi\rangle + \theta_\alpha|\psi\rangle^\alpha + 1/2\theta_\alpha\theta_\beta|\psi\rangle^{\alpha\beta} + \ldots$

Invariance tells us that $M^{\psi^\alpha} = 0$, $M^{\psi^{\alpha\beta}} = 0, \ldots$

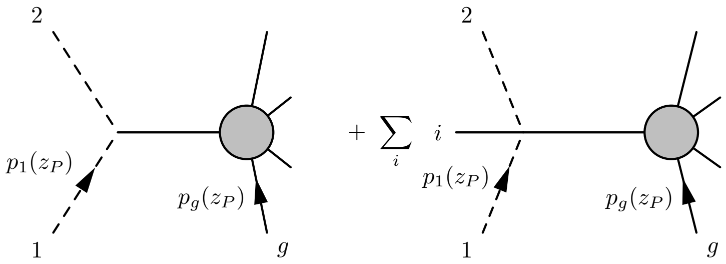
Puzzle: $[X_\alpha, X_\beta] = f^j{}_{\alpha\beta}T_j$ while Bose statistics says that $M^{\alpha\beta}(0,0) = M^{\beta\alpha}(0,0)$.

Resolution: Q does not exist without a regulator and while the double soft limit is finite it is ambiguous, i.e., it depends on the way it is approached. (This is good. Precisely the non-abelian structure comes from the presence of curvature!)

Double Soft Emission and $E_{7(7)}$

Consider an amplitude with two scalars ϕ_1^{abcd} and $\phi_{2;efgh}$ in the double soft limit.

Using the SUSY BCFW recursion relations:



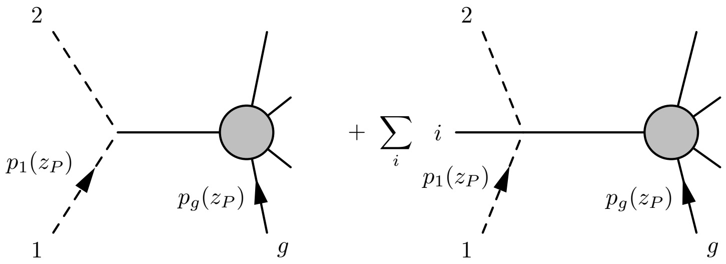
$$M_{n+2}(1,2,\ldots) \longrightarrow \sum_{i=3}^{n+2} \frac{1}{2}\frac{p_i\cdot(p_1-p_2)}{p_i\cdot(p_1+p_2)} R(\bar{\eta}_i) M_n(\bar{\eta}_i,\ldots)$$

$$R(\bar{\eta}_i) = \epsilon^{abcdK}_{efghJ} \times \left(\bar{\eta}_{i;K}\partial_{\bar{\eta}_{i:J}}\right).$$

Double Soft Emission and $E_{7(7)}$

Consider an amplitude with two scalars ϕ_1^{abcd} and $\phi_{2;efgh}$ in the double soft limit.

Using the SUSY BCFW recursion relations:



$$M_{n+2}(1,2,\ldots) \longrightarrow \sum_{i=3}^{n+2} \frac{1}{2} \frac{p_i \cdot (p_1 - p_2)}{p_i \cdot (p_1 + p_2)} R(\bar{\eta}_i) M_n(\bar{\eta}_i, \ldots)$$

$$R(\bar{\eta}_i) = \epsilon^{abcdK}_{efghJ} \times (\bar{\eta}_{i;K} \partial_{\bar{\eta}_{i:J}}) \, .$$

Obs: $\mathcal{N} = 4$ SYM diverges!

One-Loop S-Matrix

.

One-Loop S-Matrix

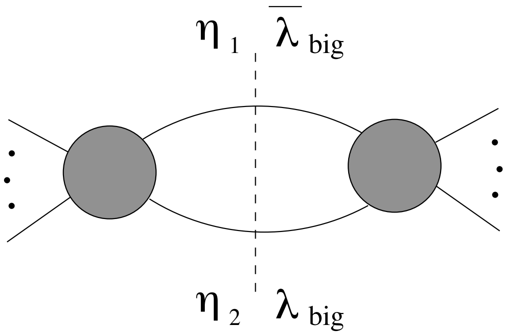
(D. Forde 04/07. Berger, Dixon, Kosower, Bern, Ellis, Giele, Kunszt, Melnikov, Badger, Ossola, Papadopoulos, Pittau, Mastrolia, Britto, Feng, Carrasco, Johansson, Buchbinder, Skinner, . . .)

Eden, Landshoff, Olive, Polkinghorne, “The Analytic S-Matrix”.

Singularities of the S-matrix

At tree-level we found poles. At one-loop level one also finds branch cuts.

Discontinuity across a branch cut (branch point $s = 0$)



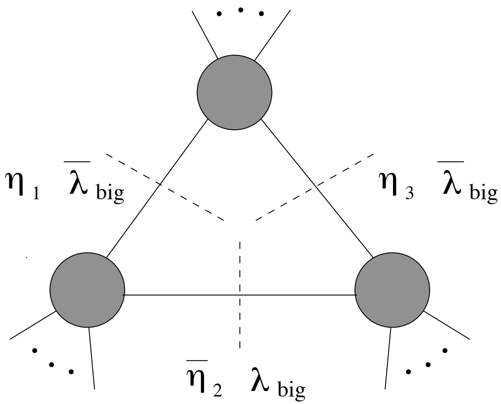
Double Cut.

E.g: $\Delta(\log^2(s)) \sim \log(s)$.

Singularities of the S-matrix

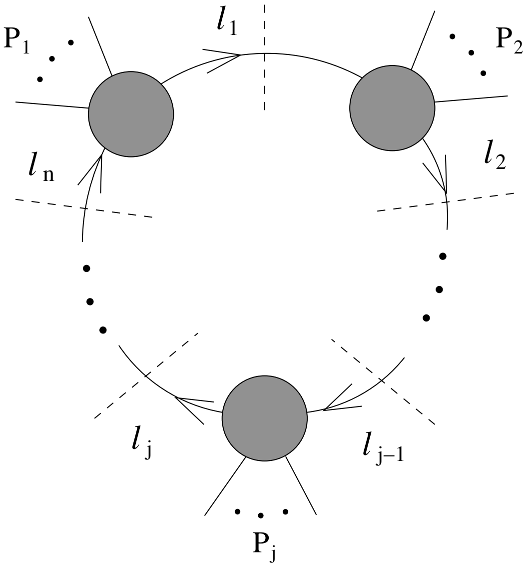
At tree-level we found poles. At one-loop level one also finds branch cuts.

Discontinuity across a branch cut (branch point $s = 0$)



Triple Cut: Discontinuity of a discontinuity.

Singularities of the S-matrix



In D-dim: D-cut computes the discontinuity across the:

Leading Singularity.

When $D = 4$, the leading singularities are computed using quadruple cuts.

When $D = 4$, the leading singularities are computed using quadruple cuts.

Claim: One-loop amplitudes in $\mathcal{N} = 8$ SUGRA are determined by their leading singularities.

When $D = 4$, the leading singularities are computed using quadruple cuts.

Claim: One-loop amplitudes in $\mathcal{N} = 8$ SUGRA are determined by their leading singularities.

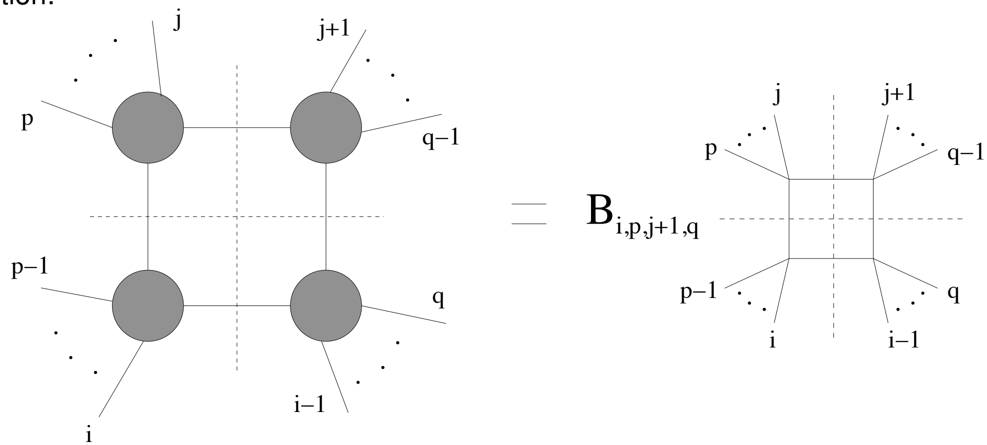
Naively this is impossible! How can a function with many branch cut singularities be determined by only knowing the discontinuity across the highest codimension branch cuts? If true this is a property of very special functions.

When $D = 4$, the leading singularities are computed using quadruple cuts.

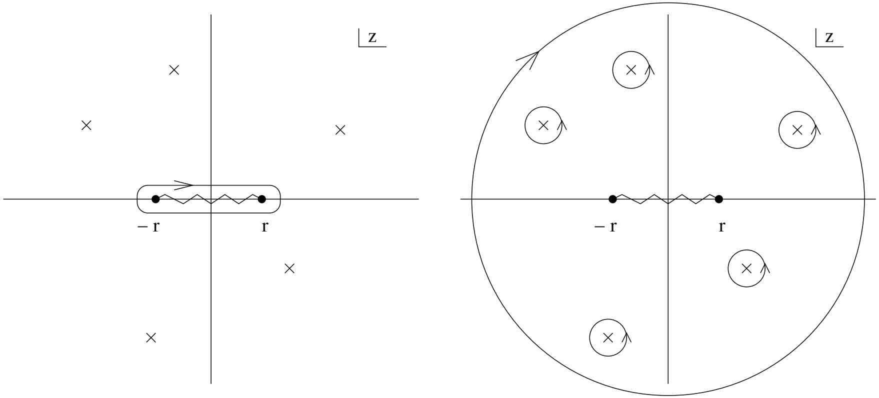
Claim: One-loop amplitudes in $\mathcal{N} = 8$ SUGRA are determined by their leading singularities.

Naively this is impossible! How can a function with many branch cut singularities be determined by only knowing the discontinuity across the highest codimension branch cuts? If true this is a property of very special functions.

Construction:



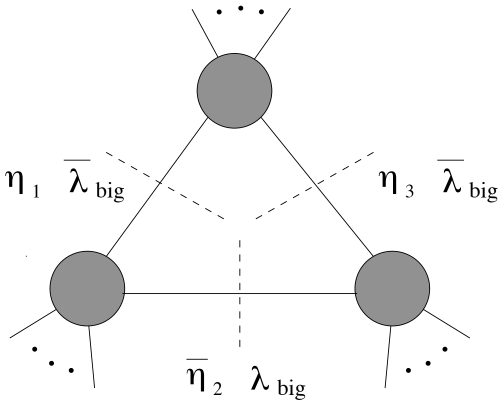
Triple Cuts: ($\int dLIPS \to \int_{\mathcal{C}} dz$)



Lorentz invariant phase space integral can be written as

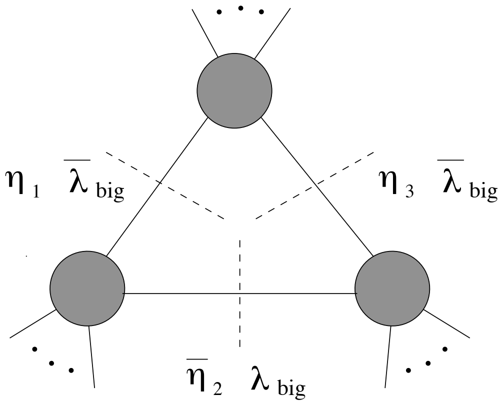
$$\Delta_3 = \sum_i \Delta_4^{(i)} + \Delta_{3;\infty}$$

If $\Delta_{3;\infty} = 0$ then triple cuts are determined in term of quadruple cuts!



$$\Delta_{3;\infty} = \int \frac{dz}{z} \int d^8\eta_1 d^8\bar{\eta}_2 d^8\eta_3 \;\; M_1(z) M_2(z) M_3(z)$$

If $\Delta_{3;\infty} = 0$ then triple cuts are determined in term of quadruple cuts!



$$\Delta_{3;\infty} = \int \frac{dz}{z} \int d^8\eta_1 d^8\bar{\eta}_2 d^8\eta_3 \ \ M_1(z)M_2(z)M_3(z)$$

In $\mathcal{N} = 8$ SUGRA $\Delta_{3;\infty}$ vanishes!

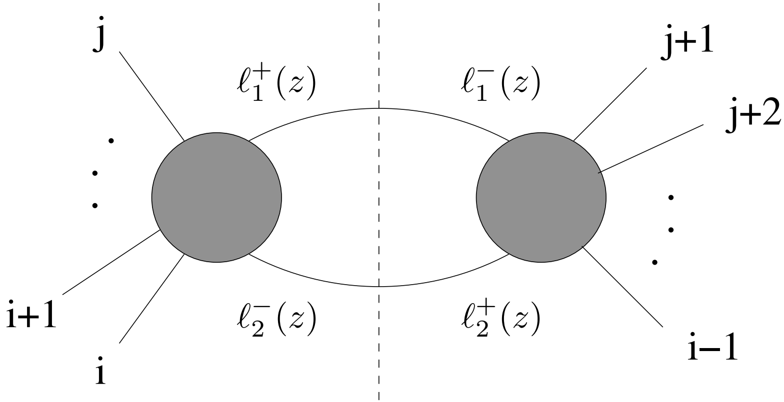
This means that no scalar triangles are needed to reproduce the singularities of Feynman diagrams.

Double Cuts:

Lorentz invariant phase space integral is written as

$$\Delta_2 = \sum_i \Delta_3^{(i)} + \Delta_{2;\infty}$$

$$\Delta_{2;\infty} = \int_{z=\infty} \frac{dz}{z}$$



In $\mathcal{N} = 8$ SUGRA $\Delta_{2;\infty}$ vanishes!

No bubbles are needed to reproduce the singularities of Feynman diagrams.

An additional step using Dim. Reg. is possible. A single cut. This signals the presence of rational functions which are not determined by the cuts in four-dimensions. Work in progress (Arkani-Hamed, F.C., J. Kaplan)

An additional step using Dim. Reg. is possible. A single cut. This signals the presence of rational functions which are not determined by the cuts in four-dimensions. Work in progress (Arkani-Hamed, F.C., J. Kaplan)

We argued the absence of rational terms in $\mathcal{N} = 8$ SUGRA by using some number theoretic properties of the functions as kinematical invariants are taken to be algebraic numbers!

An additional step using Dim. Reg. is possible. A single cut. This signals the presence of rational functions which are not determined by the cuts in four-dimensions. Work in progress (Arkani-Hamed, F.C., J. Kaplan)

We argued the absence of rational terms in $\mathcal{N} = 8$ SUGRA by using some number theoretic properties of the functions as kinematical invariants are taken to be algebraic numbers!

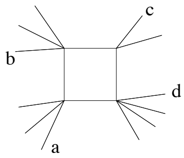
Summary:

One-loop amplitudes in $\mathcal{N} = 8$ SUGRA are completely determined by their leading singularities.

In other words, all one-loop amplitudes can written as a sum over scalar box integrals in $D = 4 - 2\epsilon$ with coefficients which are rational functions of the external kinematical invariants. (This is the no-triangle hypothesis) (Very recently proven by N. Bjerrum-Bohr and P. Vanhove using string-based methods for reduction procedures. arXiv:0805.3682)

One-Loop Solution

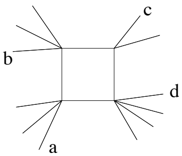
$$M_n^{1\text{-loop}} = \sum_{\mathcal{I}_{abcd}} B_{abcd} \times$$



$\mathcal{I}_{abcd}$ are partitions of $\{1, \ldots, n\}$ into four non-empty sets.

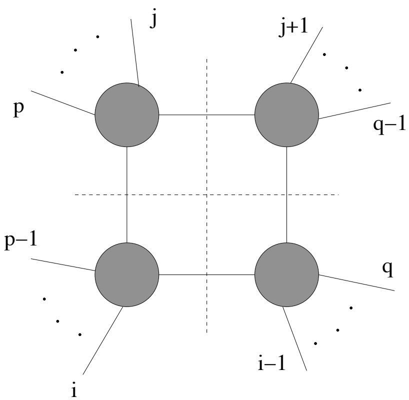
One-Loop Solution

$$M_n^{\text{1-loop}} = \sum_{\mathcal{I}_{abcd}} B_{abcd} \times$$

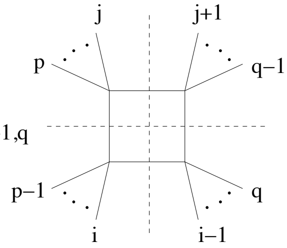


$\mathcal{I}_{abcd}$ are partitions of $\{1, \ldots, n\}$ into four non-empty sets.

The coefficients are completely determined in terms of tree-amplitude. All tree amplitudes can be computed recursively.



$$= \mathrm{B}_{i,p,j+1,q}$$



Leading Singularity Conjecture

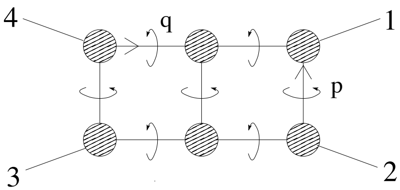
The S-matrix of $\mathcal{N} = 8$ SUGRA is completely determined by its leading singularities.

At L-loop level, the leading singularity is computed by a $4L$-cut which completely localizes the loop integration variables.

Leading Singularity Conjecture

The S-matrix of $\mathcal{N} = 8$ SUGRA is completely determined by its leading singularities.

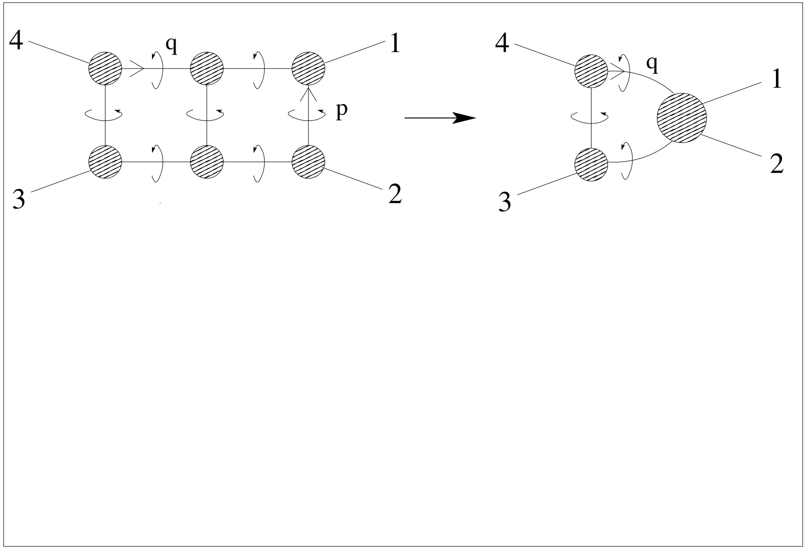
Naive problem: At two-loop $n = 4$ one has seven propagators and eight integration variables. (F.C. and E. Buchbinder 06/05, with Skinner 01/08.)



Leading Singularity Conjecture

The S-matrix of $\mathcal{N} = 8$ SUGRA is completely determined by its leading singularities.

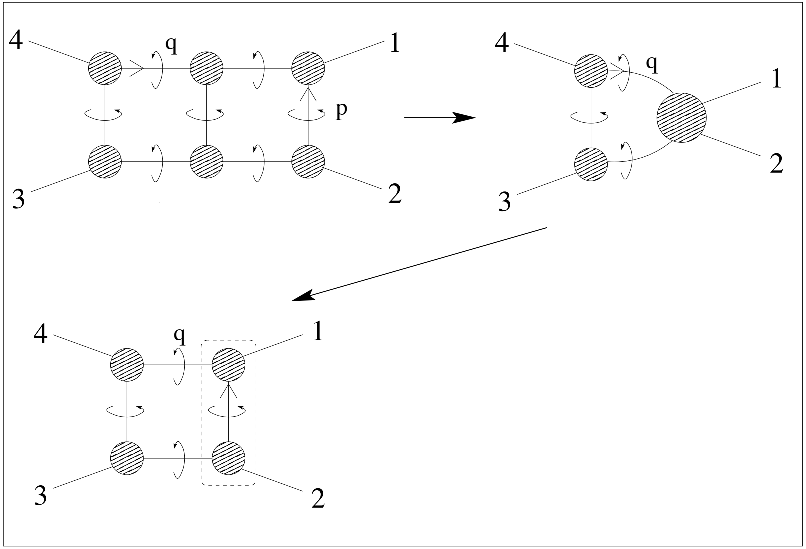
Naive problem: At two-loop $n = 4$ one has seven propagators and eight integration variables. (F.C. and E. Buchbinder 06/05, with Skinner 01/08.)



Leading Singularity Conjecture

The S-matrix of $\mathcal{N} = 8$ SUGRA is completely determined by its leading singularities.

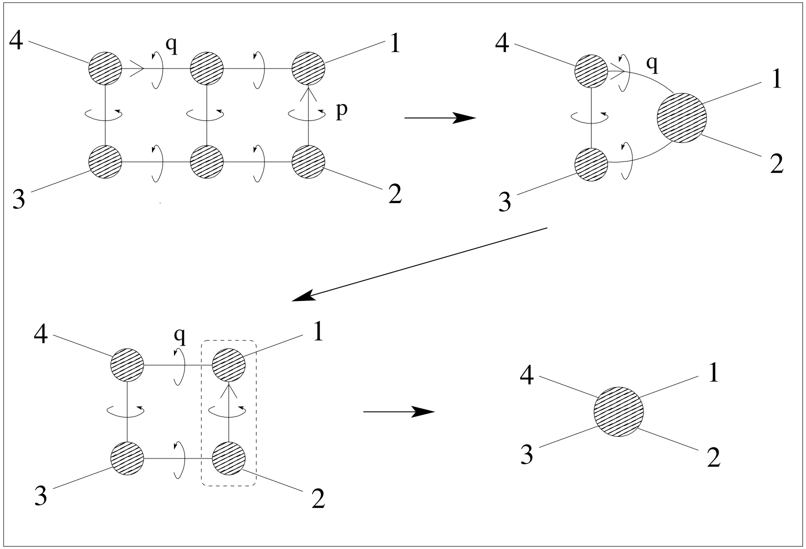
Naive problem: At two-loop $n = 4$ one has seven propagators and eight integration variables. (F.C. and E. Buchbinder 06/05, with Skinner 01/08.)



Leading Singularity Conjecture

The S-matrix of $\mathcal{N}=8$ SUGRA is completely determined by its leading singularities.

Naive problem: At two-loop $n=4$ one has seven propagators and eight integration variables. (F.C. and E. Buchbinder 06/05, with Skinner 01/08.)



Leading Singularity Conjecture

The S-matrix of $\mathcal{N} = 8$ SUGRA is completely determined by its leading singularities.

Ingredients:

- Generalization to include Dim. Reg.
- Careful study of $d\mathrm{LIPS}$ in higher loop amplitudes.
- Maximal SUSY to relate the infinite cplx. momentum limit to that of well behaved gravitons.
- Good behavior at infinite complex momentum of graviton amplitude (Not related to SUSY).

Leading Singularity Conjecture

The S-matrix of $\mathcal{N} = 8$ SUGRA is completely determined by its leading singularities.

Consequences:

- All the S-matrix is determined algebraically in terms of tree-level amplitude which are determined by Lorentz symmetry.
- If at any loop level the S-matrix fails to be determined by the leading singularity it will be a sign of an UV divergence, e.g., at three-loop in pure gravity the $(+++)$ and $(---)$ vertices, which are not there at tree-level, are needed. These are generated by the two-loop counterterm.

Future Directions

- Prove the Leading Singularity Conjecture.
- Find a dual theory.
- Construct the finite action of $E_{7(7)}$ on the S-matrix.
- Understand the connection between the presence of $E_{7(7)}$ and the other properties of the S-matrix.
- Start the exploration of higher dimensional field theories.