WHAT LANGUAGE DEPENDENCE PROBLEM?
A REPLY FOR JOYCE TO FITELSON ON JOYCE

ARTHUR PAUL PEDERSEN AND CLARK GLYMOUR

ABSTRACT. In an essay recently published in this journal, Branden Fitelson argues that a variant of Miller's argument for the language dependence of the accuracy of predictions can be applied to Joyce's notion of accuracy of credences formulated in terms of scoring rules, resulting in a general potential problem for Joyce's argument for probabilism. We argue that no relevant problem of the sort Fitelson supposes arises, since his main theorem and his supporting arguments presuppose the validity of non-linear transformations of credence functions which Joyce's theory, charitably construed, would identify as invalid on the basis of the principle of simple dominance.

1. INTRODUCTION

In a recent essay addressing Joyce's proposed non-pragmatic justification of probabilism (2012), Branden Fitelson argues that a variant of Miller's argument for the language dependence of the accuracy of predictions can be applied to Joyce's notion of accuracy of credences formulated in terms of scoring rules, resulting in a potential problem for Joyce's argument for probabilism. Fitelson focuses on a particular scoring rule, the Brier score, which falls under Joyce's constraints on epistemic scoring rules. After presenting an illustrative example, Fitelson states a theorem for Brier scores that he believes to reveal a general potential problem of language dependence for Joyce's proposal. This note shows if Joyce's proposal has any potential problems, the one Fitelson raises is not among them. We contend that Fitelson's argument has not successfully posed such a problem for Joyce, as both his illustrative example and his main theorem presuppose the validity of transformations of credence functions which Joyce's theory, charitably construed, would identify as invalid on the basis of the principle of simple dominance. As we shall see, with the validity of the transformations undermined, no relevant problem of language dependence of the sort Fitelson supposes arises.

Since we intend for our note to be a direct response to Fitelson's essay, we advise the reader to consult Fitelson's article (Fitelson 2012) and Joyce's articles (Joyce

ARTHUR PAUL PEDERSEN, DEPARTMENT OF PHILOSOPHY, CARNEGIE MELLON UNIVERSITY, PITTSBURGH, PA 15213, USA, e-mail: apaulpedersen@cmu.edu.

CLARK GLYMOUR, DEPARTMENT OF PHILOSOPHY, CARNEGIE MELLON UNIVERSITY, PITTSBURGH, PA 15213, USA, e-mail: cg09@andrew.cmu.edu.

We wish to thank Branden Fitelson, Jim Joyce, Hailin Liu, Teddy Seidenfeld, and Greg Wheeler for valuable comments and discussion. We also wish to thank two anonymous referees for helpful suggestions.

1998, 2009) for details we have left out. Although we also intended for our note to be brief, and although we take our central point to be rather simple, we found it necessary to construct a rigorous explication of Fitelson's argument to avoid potential misunderstandings. Accordingly, the structure of this note is as follows. In §2 we reconstruct the illustrative example Fitelson uses to motivate the alleged potential problem of language dependence, calling attention to some points which require clarification. To assist with such clarification, in §3 we turn to a review of the notion of coherence pertinent to Fitelson's discussion of Brier score, emphasizing basic assumptions of Joyce's theory and in particular the principle of simple dominance. In §4 we state some basic facts regarding Fitelson's example in an effort to clarify the above mentioned points. Then in §5 we recall Fitelson's statement of his purported theorem, thereupon presenting a charitable interpretation of what it says. We argue that his theorem and illustrative example do not reveal a relevant problem of language dependence for Joyce's proposal. Finally, in §6, we conclude with some brief remarks.

2. FITELSON'S ILLUSTRATIVE EXAMPLE

To set the stage, we begin with Fitelson's example. His notation is somewhat confusing, and it is best to begin by quoting him:

> Now, following Joyce, we will associate the truth-value *True* with the number 1 and the truth-value *False* with the number 0. Let ϕ be the numerical value associated with P's truth-value, and let ψ be the numerical value associated with $\neg P$'s truth-value (of course, ϕ and ψ will vary in the obvious ways across the two salient possible worlds: w_1, in which P is false, and w_2, in which P is true). (169)

In other words, ϕ and ψ are functions from a set W of states, w_1 and w_2, in which P is assigned values in $\{1, 0\}$, coding *True* and *False*, respectively, subject to the constraint that $\psi = 1 - \phi$. Unfortunately, Fitelson does not clarify whether when he writes 'ϕ' he means a *functional quantity* or a *particular truth value*, and indeed he refers to ϕ and ψ interchangeably as truth values and quantities. Personal communication with Fitelson reveals that he intends for ϕ and ψ to be understood as real-valued functions on W.

Fitelson writes:

> Suppose that S's credence function (b) assigns the following values P and $\neg P$ (i.e., b entails the following numerical "estimates" of the quantities ϕ and ψ; see table 2).

FITELSON'S TABLE 2 (170)

	ϕ	ψ
b	$\frac{1}{2}$	$\frac{1}{4}$

Clearly b is incoherent as defined for the quantities ϕ and ψ, as can be seen from the fact that the values b "entails" for ϕ and ψ (using Fitelson's terminology) do not sum to 1. As Fitelson remarks, according to Joyce's theory there is a coherent b' defined for the quantities ϕ and ψ that dominates b with respect to Brier score (we clarify what this means in §3 and §4).

Fitelson then introduces two new functions defined on W (170):

$$\alpha := \frac{1}{2}\phi + \frac{1}{2}\psi + \frac{1}{16}(\frac{\phi+\psi}{\phi-\psi}); \tag{1}$$

$$\beta := \frac{1}{2}\phi + \frac{1}{2}\psi - \frac{1}{16}(\frac{\phi+\psi}{\phi-\psi}). \tag{2}$$

Note that $\alpha(w_2) = \frac{9}{16} = 1 - \alpha(w_1)$ and $\beta(w_1) = \frac{9}{16} = 1 - \beta(w_2)$. These are real-valued functions from W to $\{\frac{9}{16}, \frac{7}{16}\}$ whose values are determined uniquely in each world in W by the value of ϕ and ψ for that world, and $\alpha = 1 - \beta$.

Fitelson assumes the "same" functions, b and b', so α and β are "new" real-valued functions on which b and b' are defined. In fact, Fitelson produces an entire table of values for b and b' for all of $\phi, \psi, \alpha, \beta$ without explaining quite what it means or how it was obtained. We have reproduced the table below.

FITELSON'S TABLE 3 (Fitelson 2012, 171)

	ϕ	ψ	α	β
b	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{9}{16}$	$\frac{3}{16}$
b'	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{4}$
w_1	0	1	$\frac{7}{16}$	$\frac{9}{16}$
w_2	1	0	$\frac{9}{16}$	$\frac{7}{16}$

Summarizing, we understand Fitelson to have introduced four real-valued functions $\alpha, \beta, \phi, \psi : W \to \mathbb{R}$ by setting $\phi := I_{\{w_2\}}$ and $\psi := I_{\{w_1\}}$ (where I_A is the indicator for event A), and defining α and β as given in Eq. 1 and Eq. 2. In addition, we understand Fitelson to have introduced two functions $b, b' : \mathscr{X} \to \mathbb{R}$, where $\mathscr{X} := \{\alpha, \beta, \phi, \psi\}$, as follows:

$$b(\phi) := \tfrac{1}{2} \qquad b(\psi) := \tfrac{1}{4}$$
$$b(\alpha) := \tfrac{9}{16} \qquad b(\beta) := \tfrac{3}{16};$$

and

$$b'(\phi) := \tfrac{5}{8} \qquad b'(\psi) := \tfrac{3}{8}$$
$$b'(\alpha) := \tfrac{3}{4} \qquad b'(\beta) := \tfrac{1}{4}.$$

We point out that Fitelson may wish for the reader to consider independently b and b' as defined on $\{\phi, \psi\}$ on the one hand, and b and b' as defined on $\{\alpha, \beta\}$ on the other (thereby considering the restrictions of the above functions). Having said this, we claim that the present precise formulation, subject to the latter proviso, is a charitable formulation of the mathematical objects underlying Fitelson's example. In any case, in personal communication, Fitelson has indicated that our reading is correct.

We should mention now that Fitelson may have already stepped outside of Joyce's argument for probabilism, for Fitelson is considering functionals b and b' defined for simple real-valued functions (i.e., real-valued functions taking finitely many values), whereas Joyce restricts his attention to functionals defined on propositions, or real-valued functions taking the values 0 and 1. Nonetheless, for the sake of addressing Fitelson's purported potential problem, let us grant that Joyce can accommodate simple real-valued functions in a more general argument for probabilism (i.e., for finitely additive expectations), establishing that Brier score satisfies suitably reformulated constraints on scoring rules for functionals defined on simple real-valued functions (or even, say, bounded real-valued functions). Indeed, we understand Joyce to have such a larger goal in mind when discussing the special case of 0-1 valued quantities. Thus in Joyce's framework, one can evaluate the quality of a person's estimates $b(\chi)$ of various quantities χ in terms of the accuracy of the estimates with respect to their true values at each state. Indeed, in personal communication, Joyce has indicated that this understanding of his program is correct.

This point aside, it is easy to see how Fitelson has arrived at the values for b and b' for α and β, given the values of b and b' for ϕ and ψ. Indeed, treating b as an operator preserving scalar multiplication, addition, and *division*, one obtains the values in Table 3. Thus, in effect, Fitelson presupposes that the following equations hold:

$$\begin{aligned} b(f^+(\phi,\psi)) &= f^+(b(\phi), b(\psi)); \\ b'(f^+(\phi,\psi)) &= f^+(b'(\phi), b'(\psi)); \end{aligned}$$

and

$$\begin{aligned} b(f^-(\phi,\psi)) &= f^-(b(\phi), b(\psi)); \\ b'(f^-(\phi,\psi)) &= f^-(b'(\phi), b'(\psi)), \end{aligned}$$

where $f^+(x,y) = \frac{1}{2}x + \frac{1}{2}y + \frac{1}{16}\frac{x+y}{x-y}$ and $f^-(x,y) = \frac{1}{2}x + \frac{1}{2}y - \frac{1}{16}\frac{x+y}{x-y}$. Again, in personal communication, Fitelson acknowledges that he presupposes the aforementioned equations. Thus, again, we claim that our exposition of Fitelson's example is charitable.

Now to the important bit: Fitelson claims that b', a coherent function, Brier dominates b "with respect to ϕ and ψ." He further claims that b, an incoherent function, dominates the coherent function b', using Brier score, "with respect to α and β."

Fitelson announces that these claims constitute an illustration of his main theorem (170), asserting that such a "reversal" reveals a potential "Milleresque" problem of language dependence of the accuracy of credences. However, the content of his claims is unclear. In particular, it is unclear what we he means by the locutions beginning with "with respect to..."

3. Coherence and Simple Dominance

In this section we briefly review two notions of coherence connected to finitely additive probabilities and more generally finitely additive expectations. Doing so will help to clarify the content of Fitelson's claims and some basic assumptions underlying Joyce's argument, accordingly being a natural place to introduce Brier score. In addition, it will help us to understand the statement of Fitelson's theorem. It is illuminating to first review the notion of coherence for betting odds due to de Finetti.

Given a collection $\mathscr{X}$ of bounded real-valued functions, an agent posts his fair prices $P(f)$ for each $f \in \mathscr{X}$ subject to the understanding that he is ready to accept any finite combination of gambles of the form $G(c, f) = c(f - P(f))$, where $c \in \mathbb{R}$. The payoff to any finite combination of gambles is given by the sum of the gambles. Thus, the value of $P(f)$ is the number that leaves the agent indifferent as to whether c is positive, negative, or zero. The notion of coherence due to de Finetti demands that no finite combination of gambles leads to a sure loss.

Definition 3.1 (Coherence$_1$)**.** Let W be a set of states, let $\mathscr{X} \subseteq \mathbb{R}^W$ be a collection of bounded real-valued functions, and let $P : \mathscr{X} \to \mathbb{R}$ be a real-valued function. We say that P is *coherent*$_1$ if there are no $f_0, \ldots, f_{n-1} \in \mathscr{X}$, $c_0, \ldots, c_{n-1} \in \mathbb{R}$, and $\epsilon > 0$ such that for every $w \in W$:

$$\sum_{i<n} c_i(f_i(w) - P(f_i)) \quad < \quad -\epsilon.$$

Otherwise, P is said to be *incoherent*$_1$.

Observe that when each real-valued function in $\mathscr{X}$ is simple, as in Fitelson's and Joyce's papers, then P is coherent$_1$ just in case there are no $f_0, \ldots, f_{n-1} \in \mathscr{X}$ and $c_0, \ldots, c_{n-1} \in \mathbb{R}$ such that for every $w \in W$:

$$\sum_{i<n} c_i(f_i(w) - P(f_i)) \quad < \quad 0.$$

Also observe that $\mathscr{X}$ may be an *arbitrary* class of bounded real-valued functions and in particular is not required to satisfy any measurability conditions with respect to an underlying algebra or σ-algebra.

As we have indicated, Fitelson's discussion is couched in terms of Brier score. There is a well known notion of coherence formulated in terms of this score due to de Finetti and explored by Joyce. Let $L(f) = (f - P(f))^2$ stand for the loss, or score, suffered by an agent who evaluates $P(f)$ as the estimate of f. The loss given to a finite set of such evaluations $P(f_0), \dots, P(f_{n-1})$ is the sum of the losses for each evaluation. The Brier score is one of the "measures of epistemic disutility" satisfying Joyce's underlying philosophical assumptions about epistemic scoring rules. We recall de Finetti's notion of coherence for Brier score.

Definition 3.2 (Coherence$_2$)**.** Let W be a set of states, let $\mathscr{X} \subseteq \mathbb{R}^W$ be a collection of bounded real-valued functions, and let $P : \mathscr{X} \to \mathbb{R}$ be a real-valued function. We say that P is *coherent*$_2$ if there are no $f_0, \dots, f_{n-1} \in \mathscr{X}$, $F(f_0), \dots, F(f_{n-1}) \in \mathbb{R}$, and $\epsilon > 0$ such that for every $w \in W$:

$$\sum_{i<n}(f_i(w) - F(f_i))^2 + \epsilon \quad < \quad \sum_{i<n}(f_i(w) - P(f_i))^2.$$

Otherwise P is said to be *incoherent*$_2$.

Much like above, observe that when each real-valued function from $\mathscr{X}$ is simple, then P is coherent$_2$ just in case there are no $f_0, \dots, f_{n-1} \in \mathscr{X}$ and $F(f_0), \dots, F(f_{n-1}) \in \mathbb{R}$ such that for every $w \in W$:

$$\sum_{i<n}(f_i(w) - F(f_i))^2 \quad < \quad \sum_{i<n}(f_i(w) - P(f_i))^2.$$

Again, $\mathscr{X}$ may be an arbitrary class of bounded real-valued functions.

De Finetti has established the equivalence of coherence$_1$ and coherence$_2$. In fact, he has shown the following:

Theorem 3.3 (de Finetti 1974a, 1974b)**.** *Let W be a set of states, let $\mathscr{X} \subseteq \mathbb{R}^W$ be a collection of bounded real-valued functions, and let $P : \mathscr{X} \to \mathbb{R}$ be a real-valued function. Then the following are equivalent:*

(i) *P is coherent$_1$.*
(ii) *P is coherent$_2$.*
(iii) *There exists a finitely additive probability p on W such that under the expectation functional $\mathbb{E}_p$ under p, for every $f \in \mathscr{X}$, $\mathbb{E}_p(f) = P(f)$.*

This result holds generally and so in particular for 0-1 valued quantities (i.e., events). In addition, when $\mathscr{X}$ is finite, consisting of, say, n bounded real-valued functions $f_0, \dots, f_{n-1}$, it can be shown that if and only if F is incoherent$_2$ (incoherent$_1$), there is a coherent$_2$ (coherent$_1$) P such that $\sum_{i<n}(f_i(w)-P(f_i))^2 < \sum_{i<n}(f_i(w)-F(f_i))^2$ for every $w \in W$. Following Fitelson's choice of terminology, given P and F defined on $\mathscr{X} = \{f_0, \dots, f_{n-1}\}$, let us say that P *Brier dominates* F if $\sum_{i<n}(f_i(w) - P(f_i))^2 < \sum_{i<n}(f_i(w) - F(f_i))^2$ for every $w \in W$. Again, Fitelson, like Joyce, restricts his attention to a finite collection of simple real-valued functions $\mathscr{X}$ (as mentioned in §2, Joyce focuses in particular on 0-1 valued quantities). For the sake of discussion, we do the same.

We emphasize that when $\mathscr{X}$ consists of simple real-valued functions, both coherence$_1$ and coherence$_2$, respectively formulated in terms of sums of gambles or sums of losses, follow from the principle of *simple dominance*, which states that if an option o_1 dominates an option o_2 in each state (i.e., $o_1(w) > o_2(w)$ for each state w), then o_2 is inadmissible for choice in any decision problem in which o_1 is feasible. In particular, regarding coherence$_2$, any function F Brier dominated by another function P is inadmissible whenever P is a feasible function. While perhaps formulated using different language, we understand Joyce to subscribe to (at least) the principle of simple dominance for Brier score as well as any other scoring rule falling under his theory. As such, when Brier score is adopted as a scoring rule, coherence$_2$ follows.

4. A Clarification of Fitelson's Claims

As indicated at the end of §2, it is unclear what Fitelson means by his claim that (i) b' Brier dominates b "with respect to ϕ and ψ" and his claim that (ii) b Brier dominates the coherent measure b' "with respect α and β." The purpose of this section is to state some elementary facts regarding Fitelson's example and then to attempt to explicate Fitelson's claims. As mentioned in passing in the last section, doing so will serve to support the interpretation of Fitelson's theorem given in the next section.

Fact 4.1. *Let $b, b' : \mathscr{X} \to \mathbb{R}$ be defined as in §2. Then for every $w \in W$:*

$$(\phi(w) - b'(\phi))^2 + (\psi(w) - b'(\psi))^2 \quad < \quad (\phi(w) - b(\phi))^2 + (\psi(w) - b(\psi))^2.$$

Hence, b is incoherent$_2$.

Fact 4.2. *Let $b, b' : \mathscr{X} \to \mathbb{R}$ be defined as §2. Then for every $w \in W$:*

$$(\alpha(w) - b(\alpha))^2 + (\beta(w) - b(\beta))^2 \quad < \quad (\alpha(w) - b'(\alpha))^2 + (\beta(w) - b'(\beta))^2.$$

Hence, b' is incoherent$_2$.

Thus, we see that both b and b' are incoherent$_2$.

Let us now turn to a clarification of Fitelson's claims (i) and (ii). Where $b'|_{\{\phi,\psi\}} : \{\phi, \psi\} \to \mathbb{R}$ is defined by setting $b'|_{\{\phi,\psi\}} := b'(\theta)$ for every $\theta \in \{\phi, \psi\}$, i.e., where $b'|_{\{\phi,\psi\}}$ is the *restriction* of b' to $\{\phi, \psi\}$, the statement that b' Brier dominates b with respect to ϕ and ψ is given meaning in Fact 4.1. Similarly, where $b|_{\{\alpha,\beta\}} : \{\alpha, \beta\} \to \mathbb{R}$ is the restriction of b to $\{\alpha, \beta\}$, the statement that b Brier dominates b' with respect to α and β is given meaning in Fact 4.2. We mention that it trivially follows from Fact 4.2 that $b'|_{\{\alpha,\beta\}}$ is incoherent$_2$.

We also have the following:

Fact 4.3. *The restriction $b'|_{\{\phi,\psi\}}$ is coherent$_2$. That is, with respect to the collection $\mathscr{X}_{\phi,\psi} := \{\phi, \psi\}$, $b'|_{\{\phi,\psi\}}$ is coherent$_2$.*

Fact 4.4. *The restriction $b|_{\{\alpha,\beta\}}$ is incoherent$_2$. That is, with respect to the collection $\mathscr{X}_{\alpha,\beta} := \{\alpha, \beta\}$, $b|_{\{\alpha,\beta\}}$ is incoherent$_2$.*

Fact 4.3 gives meaning to Fitelson's claim that b' is coherent with respect to ϕ and ψ, while Fact 4.4 explains Fitelson's apparent recognition that b is incoherent with respect to α and β. Yet whether we are considering $\mathscr{X}$ or $\mathscr{X}_{\alpha,\beta}$, Fitelson's claim (172) that his example illustrates that b' is a *coherent* function Brier dominated by the incoherent b is plainly false or meaningless, as witnessed by the aforementioned facts.

5. The Theorem and Its Relevance to A Potential Problem of Language Dependence

We now turn to Fitelson's theorem and the (potential) lesson he wishes to draw from it.

Theorem 5.1 (Fitelson 2012, 170-171)**.** *For any coherent function b' that Brier dominates S's credence function b with respect to ϕ and ψ, there exist quantities α and β that are symmetrically interdefinable with respect to ϕ and ψ, via the following specific intertranslations:*

$$\begin{aligned}\alpha &= \frac{1}{2}\phi + \frac{1}{2}\psi + \frac{1}{16}(\frac{\phi+\psi}{\phi-\psi});\\ \beta &= \frac{1}{2}\phi + \frac{1}{2}\psi - \frac{1}{16}(\frac{\phi+\psi}{\phi-\psi});\\ \phi &= \frac{1}{2}\alpha + \frac{1}{2}\beta + \frac{1}{16}(\frac{\alpha+\beta}{\alpha-\beta});\\ \psi &= \frac{1}{2}\alpha + \frac{1}{2}\beta - \frac{1}{16}(\frac{\alpha+\beta}{\alpha-\beta}),\end{aligned}$$

where b Brier dominates b' with respect to α and β.

Fitelson believes it is "noteworthy that the true values of α and β 'behave like truth values,' in the sense that (1) the true value of α (β) in w_1 (w_2) is identical to the true value of β (α) in w_2 (w_1), and (2) the values of α and β always sum to one" (p171-172). He continues, "Indeed, these transformations are guaranteed to preserve coherence of all dominating b''s, and the "truth vectors" (171).

The statement of the theorem is unclear, and at first sight it appears that Fitelson's theorem contradicts de Finetti's result (Theorem 3.3). Unfortunately, Fitelson does not furnish a proof of the theorem for possible clarification, instead pointing to a web address from which a *Mathematica* notebook containing verifications of all formal claims in his essay is said to be available for download. As of the writing of this note, the linked web address is broken.

In the absence of such assistance—yet in light of the discussion in the previous section—we take the above theorem to state something like the following: If $b, b' : \{\phi, \psi\} \to \mathbb{R}$ are such that b' is coherent and Brier dominates b (in Fitelson's language, b' Brier dominates b with respect to ϕ and ψ), then considering the quantities α and β as above:

(I) b and b' can be extended to $\{\phi, \psi, \alpha, \beta\}$ by setting:

$$\begin{aligned} b(\alpha) &:= f^+(b(\phi), b(\psi)); \\ b'(\alpha) &:= f^+(b'(\phi), b'(\psi)); \end{aligned}$$

and

$$\begin{aligned} b(\beta) &:= f^-(b(\phi), b(\psi)); \\ b'(\beta) &:= f^-(b'(\phi), b'(\psi)), \end{aligned}$$

where, as in §2:

$$\begin{aligned} f^+(x, y) &= \frac{1}{2}x + \frac{1}{2}y + \frac{1}{16}\frac{x+y}{x-y}; \\ f^-(x, y) &= \frac{1}{2}x + \frac{1}{2}y - \frac{1}{16}\frac{x+y}{x-y}; \end{aligned}$$

and

(II) $b|_{\{\alpha,\beta\}}$ Brier dominates $b'|_{\{\alpha,\beta\}}$ (in Fitelson's language, b Brier dominates b' with respect to α and β).

We claim that this, or a slight variation of it, is a charitable formulation of his theorem. In (I), for example, Fitelson might wish for us to consider the "transformations" b and b' defined on $\{\alpha, \beta\}$ in terms of the values assigned by b and b' to ϕ and ψ rather b and b' defined on all of $\{\phi, \psi, \alpha, \beta\}$. In either case, the appropriate way to dispute our claim to a charitable formulation would be to furnish an equally rigorous formulation that is more charitable. Our remarks apply to either variation.

We note that Fitelson's theorem is apparently intended to hold for ϕ taking different values than ψ at each state and, depending on its precise formulation, for b and b' assigning distinct values to ϕ and ψ. (Indeed, Fitelson's theorem is false if credence functions may take values outside of $(0, 1)$.) As illustrated by Fitelson's example, while $b'|_{\{\phi,\psi\}}$ may be coherent$_2$, in general $b|_{\{\alpha,\beta\}}$ will not be coherent$_2$. Furthermore, as Fitelson's own example illustrates, the coherence of b' is *not* preserved by the transformations (contrary to his assertion; see the remarks at the end of the last section). While we do not wish to dispute claims (1) and (2) following the statement of his theorem, we do not know what Fitelson means by his claim that coherence is preserved. In any case, the theorem is unsurprising, as it is well-known that coherence is not generally preserved by non-linear transformations.

Having stated the theorem, Fitelson writes:

> So while it is true that there are some aspects of "the truth" with respect to which S's credence function b is bound to be less accurate than (various) coherent b''s, it also seems to be the case that (for any such b') there will be specifiable, symmetrically interdefinable aspects of "the truth" on which the opposite is true (i.e., with respect to which b' is bound to be less accurate). (171)

Fitelson does not clarify what "aspects" he has in mind. In any case, he concludes, "I think the present phenomenon challenges us to get clearer on the precise content of the *accuracy norm*(*s*) that are applicable to (or constitutive of) the Joycean cognitive act of "estimation of the (numerical) truth-value of a proposition" (171).

Later Fitelson elaborates upon this challenge. Using Fitelson's notation, where $\mathcal{E}(x, y) = \langle p, q \rangle$ is understood to be the claim that agent S is committed to the values of $\langle p, q \rangle$ as their "estimates" of the quantities $\langle x, y \rangle$, Fitelson thinks that "we need to know...the conditions under which the following principle...is acceptable, relative to Joyce's notion of 'estimation' $\mathcal{E}$":

(†) If $\mathcal{E}(\phi, \psi) = \langle p, q \rangle$, then $\mathcal{E}(\alpha, \beta) = f(p, q)$, where f is a symmetric intertranslation function that maps values of $\langle \phi, \psi \rangle$ to/from values of $\langle \alpha, \beta \rangle$. (173)

That is, Fitelson explains, he would like to know which translations f are acceptable. He presumes that Joyce would want to reject the translation as given in the theorem, but he thinks "it is natural to ask precisely what grounds Joyce might have for such a rejection" (174). Apparently Fitelson thinks that answering the aforementioned question would go toward a promising and useful response to his "reversal argument" (as he calls it):

> I think the most promising (and useful) response to the phenomenon is to argue (i) that there are crucial disanalogies between "estimation" (in Joyce's sense) and "prediction" (in the sense presupposed by Popper and Miller), and (ii) these disanalogies imply that my "reversal argument" is presupposing something incorrect regarding the norms appropriate to "estimation." (172)

Unfortunately, Fitelson does not present a clear argument indicating why such a response would be promising or useful. Nor does he supply a clear argument showing that Joyce must answer his general question about (†). In the absence of such arguments, we think that for Joyce it is sufficient to show why Fitelson's theorem and illustrative example fail to present a "Milleresque" problem for his theory; he may oblige Fitelson with a partial answer to his questions about (†) and (i) and (ii) (or their subquestions) in the course of his response if doing so is expedient. We propose to respond on Joyce's behalf.

Consider a Joycean rational agent for whom Brier score measures the epistemic disutility of estimates of quantities.[1] Among other things, such an agent at least respects the principle of simple dominance for Brier score. Thus, in considering feasible functions b and b' defined for ϕ and ψ and the quantities α and β defined in terms of ϕ and ψ as in the statement of the theorem or the illustrative example, the agent would identify b as inadmissible because it can be Brier dominated by a feasible function F. The agent would also rule the extensions of b and b' to $\{\phi, \psi, \alpha, \beta\}$ as given by the translation in (I) as inadmissible, since each can be Brier dominated by an alternative feasible function F defined on $\{\phi, \psi, \alpha, \beta\}$. Alternatively, in considering the "jump" (i.e., transformations) from b and b' defined on $\{\phi, \psi\}$ *to* b and b' defined on $\{\alpha, \beta\}$ by way of the translations in (I), the agent would rule b and b' so defined on $\{\alpha, \beta\}$ as inadmissible on the basis of simple dominance, since each can be Brier dominated by an alternative feasible function F defined on

[1] Our remarks here apply to other scoring rules, although, again, Joyce has not yet offered an argument for probabilism more generally for, say, bounded real-valued quantities. For the sake of addressing Fitelson's alleged potential problem, it is suffices to frame our discussion in terms of Brier score, the focus of Fitelson's essay. See §2.

$\{\alpha, \beta\}$. In either case, a Joycean rational agent has the resources to evaluate the inadmissibility of various functions b^* on the basis of simple dominance, and the extensions/transformations of b and b' are inadmissible.

In addition, although α and β as *defined* take values depending on ϕ and ψ (and vice versa), and although b and b' as *defined* "reverse" the direction of Brier dominance when first considering ϕ and ψ and then considering α and β, no relevant Miller-Popper reversal has taken place, since, by the above reasoning, the rational agent is in no way committed to the transformations giving rise to the numerical evaluations b and b' which lead to a "reversal." By contrast, we take the significance of the Miller-type transformations to depend on the logico-mathematical commitments following from quantitative theories being compared, where these commitments lead to genuine evaluatory reversals depending upon which quantities are taken as more "basic" in determining accuracy. For example, in the case of a Miller-type reversal, the values of quantities η and ξ are entailed by hypotheses H_1 and H_2, and *consequently* the values of the symmetrically interdefinable transforms η' and ξ' are also entailed by H_1 and H_2. Accordingly, depending upon whether the pair η and ξ or the pair η' and ξ' is taken as more "basic," H_1 may be judged closer to the truth than H_2 is, and vice versa. In Fitelson's illustrative example and theorem, b and b', the presumed analogues of H_1 and H_2 (recall Fitelson's suggestive term "entail" from §2; see (Fitelson 2012, 169-170)), are such that b' Brier dominates b with respect to ϕ and ψ, and b Brier dominates b' with respect to α and β (again, using Fitelson's language). But unlike a Miller-type reversal, the values of b and b' for α and β *do not follow* from those of ϕ and ψ. They follow *if it is assumed* that (I) holds, a premise to which the agent is decidedly *not* committed. Of course, if the claim is that there is a reversal when considering the restrictions of b and b' to $\{\phi, \psi\}$ and then to $\{\alpha, \beta\}$, clearly the Brier score has been misapplied in determining the (in)accuracy of b and b' on $\{\phi, \psi, \alpha, \beta\}$, since the evaluation of (in)accuracy is to be applied at once to all quantities in $\{\phi, \psi, \alpha, \beta\}$ (and here neither is more or less inaccurate than the other). In short, the relevant analogue following the adverb "consequently" above is missing. The mere fact that the assumed translations "reverse" the direction of dominance relations (or "epistemic disutility") does not show that the agent's judgments are, or could, also be relevantly reversed, depending on whether the agent is considering on the one hand, ϕ and ψ, or on the other hand, α and β. And this is so in spite of the translations having properties (1) and (2) mentioned above. The supposed analogy with a Miller-type reversal does not stand to reason.

To oblige Fitelson, if indeed $\mathcal{E}(\phi, \psi) = \langle p, q \rangle$ holds, on Joyce's theory it does not generally follow that $\mathcal{E}(\alpha, \beta) = f(p, q)$ also holds—in particular when these translated estimates can be dominated by a rival feasible function F. To the extent that an agent is committed to transformations f as in (†), the presupposition of Fitelson's theorem—a particular such non-linear transformation given in condition (I)—does not respect the principle of simple dominance for Brier score. In particular, if in fact a Joycean coherent agent evaluates ϕ and ψ according to b', and b' Brier dominates b with respect to these quantities, it unequivocally does not follow that he is therefore committed to evaluate b' as defined for α and β, leading to a "reversal" of the sort Fitelson wants. Charitably understood, Joyce's proposal supplies normative standards which conflict with an unbridled (†).

6. Concluding Remarks

As we have indicated, we believe that a charitable understanding of Joyce's work would recognize that his basic ideas can be applied to quantities taking values in $\{0, 1\}$ as well as any other values. Brier score satisfies his constraints on scoring rules for 0-1 valued quantities, and given his predilection for Brier score, we think that Joyce would endorse suitably reformulated constraints on scoring rules for simple or even, say, bounded real-valued functions admitting Brier score as considered here and in Fitelson's essay, leading to a more general argument for probabilism with respect to which scoring rules respect the principle of simple dominance for simple real-valued quantities. As we have mentioned in §2, Joyce has confirmed that this understanding is correct. To be sure, Joyce has hinted at such a possibility:

> Graded beliefs help us estimate quantities of interest. These can be almost anything: the fair price of a bet, the proportion of balls in an urn, the average velocity of stars in a distant galaxy, the truth-value of a proposition, the frequency of a disease in a population, and so on. Since the values of such quantities often depend on unknown factors, we imagine the believer being uncertain about which member of a given set $\mathbf{w}$ of total contingencies (= possible worlds) actually obtains, and we think of the quantity of interest as a function, or 'random variable,' f that assigns each world W in $\mathbf{w}$ a unique real number $f(W)$. The objective in estimation is to come up with an anticipated value f^* for f that is, in some sense, the best possible given the information at hand...[O]ne can use a gradational, or closeness counts, scale that assigns estimates higher degrees of accuracy the closer they are to the actual value of quantity being estimated. (Joyce 2005, 155)[2]

As we have seen, in the special case of Brier score for simple real-valued quantities, Joyce, simply on the basis of a respect for the principle of simple dominance for Brier score, can identify feasible functions b as incoherent.[3] The presupposed translations of Fitelson's theorem and illustrative example conflict with this requirement within Joyce's theory, thereby undermining Fitelson's claim to challenging Joyce with a relevant problem of language dependence. If Fitelson's portrayal of Joyce's arguments does not give proper place to the principle of simple dominance within his theory, then we fail to see how Fitelson has charitably represented Joyce's arguments.

---

[2]See (Joyce 2009, 264) for Joyce's formulation of the laws of probability and the associated footnote 2. See also (Joyce 2009, 268-269) and footnote 7 and footnote 8.

[3]Indeed, in this simple context, coherence$_2$ respects weak dominance insofar as an agent's forecasts are coherent$_2$ just in case no finite subcollection of the agent's forecasts can be *weakly dominated* by a rival set of forecasts. This is an instance of a more general result proved by Predd et al. (2009, 4788), who established that given a continuous strictly proper scoring rule, an agent's forecasts are coherent if and only if no finite subcollection of the agent's forecasts can be weakly dominated by a rival set of forecasts under the scoring rule. Schervish et al. (2009) extend the results of Predd et al. (2009).

References

de Finetti, B. 1974a. *Theory of Probability* (1990 ed.), Volume I. Wiley.

de Finetti, B. 1974b. *Theory of Probability* (1990 ed.), Volume II. Wiley.

Fitelson, B. 2012. "Accuracy, Language Dependence and Joyce's Argument for Probabilism." *Philosophy of Science 79*(1), 167–174.

Joyce, J. M. 1998. "A Nonpragmatic Vindication of Probabilism." *Philososophy of Science 65*(4), 575–608.

Joyce, J. M. 2005. "How Probabilities Reflect Evidence." *Philosophical Perspectives 19*(1), 153–178.

Joyce, J. M. 2009. "Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial Belief." In F. Huber and C. Schmidt-Petri (Eds.), *Degrees of Belief*, Volume 342 of *Synthese Library*, 263–297. Synthese.

Predd, J., R. Seiringer, E. Lieb, D. Osherson, V. Poor, and S. Kulkami. 2009. "Probabiistic Coherence and Proper Scoring Rules." *IEEE Transactions on Information Theory 55*(10), 4786–4792.

Schervish, M. J., T. Seidenfeld, and J. B. Kadane. 2009. "Proper Scoring Rules, Dominated Forecasts, and Coherence." *Decision Analysis 6*(4), 202–221.