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# When Expectations Become Aspirations

**Reference-Dependent Preferences and Liquidity Constraints** 

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# INTERNATIONAL FOOD POLICY RESEARCH INSTITUTE

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# ABSTRACT

A large body of literature suggests that consumers derive utility from gains and losses relative to a reference point. This paper shows that such reference dependence can affect savings in opposite directions depending on whether people face liquidity constraints. Existing models for wealth and intertemporal choice predict that reference dependence *reduces* savings but these models abstract from liquidity constraints. Introducing a liquidity constraint, I find that reference dependence *can increase* optimal savings for people without access to credit. Liquidity constraints force them to take part of an income loss in early periods, which may induce those who are reference dependent to concentrate the full loss in early periods and save in order to eliminate future losses. Further, anticipating a liquidity constraint raises the expected level of future consumption and thus the expectations-based reference point for future periods, creating a second savings motive. This underscores the impact that financial market imperfections can have when applying reference-dependent models in low-income settings.

# Keywords: loss aversion, imperfect financial markets, intertemporal choice

JEL Codes: D14, D81, D91, O12

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### **1. INTRODUCTION**

Imperfect access to credit hampers the poor's ability to smooth consumption. Introducing credit can therefore have significant welfare effects (Karlan and Zinman 2011; Banerjee et al. 2014; Angelucci, Karlan, and Zinman 2015), but quantifying these effects requires a better understanding of the poor's preferences and behavior (Harrison 2011). Most analyses of intertemporal choice in a context of imperfect credit markets assume that consumption *levels* determine well-being and that it is optimal to smooth consumption. Lower observed consumption is often interpreted as evidence of reference-dependent behavioral life-cycle models (Shefrin and Thaler 1988). These models typically include features of Kahneman and Tversky (1979)'s prospect theory, according to which *gains* and *losses* relative to a reference point determine well-being; losses are more painful than gains are pleasant ("loss aversion"); and marginal utility is decreasing in the size of a loss, meaning that utility from losses is convex ("diminishing sensitivity").<sup>1</sup>

To date, reference-dependent models for wealth and intertemporal choice have largely abstracted from liquidity constraints. It is hence unclear how reference dependence affects savings for the poor, who often do not have access to credit, and how credit impacts their well-being if they indeed do have reference-dependent preferences. This paper fills this gap by introducing a liquidity constraint in existing models for reference-dependent preferences, and finds that reference dependence can have qualitatively different effects depending on whether consumers face liquidity constraints. Relative to the reference-independent optimum, reference dependence *reduces* savings among those who can borrow but can *increase* savings when credit is unavailable. This poses a challenge to applying existing reference-dependent models for wealth and intertemporal choice in low-income settings.

The paper demonstrates this result using two distinct models that mainly differ in how consumers form their reference points. The first is a standard model for prospect theory in which gains and losses relative to an *exogenous* reference point determine well-being. This reference point can be interpreted in various ways, for instance as a predetermined expectation, a habit, the status quo, recent ownership, or the consumption level of a social reference group. Although the model is intuitive, it offers the discretion to select any reference point explaining the empirical fact in question. This comes at the cost of many degrees of freedom (Barberis 2013).

The second model builds on Kőszegi and Rabin (2006, 2009), who postulate that *endogenous* beliefs held prior to consumption shape the reference point and that updating these beliefs influences well-being. By loss aversion, bad news is more painful than good news is pleasant, and by diminishing

<sup>&</sup>lt;sup>1</sup>For reviews discussing the empirical evidence of prospect theory, see Edwards (1996) and Camerer (2004).

sensitivity, it is better to receive bad news in one go rather than in bits. A growing literature (for example Abeler et al. 2011; Crawford and Meng 2011; Ericson and Fuster 2011) support these assumptions. Moreover, the model can explain several anomalies in the literature on information preferences and intertemporal choice, in particular why people increase consumption immediately in response to good income news but delay cuts following bad news about their wealth or income (Kőszegi and Rabin 2009).

Neither of these reference-dependent models has been analyzed for a context in which consumers have limited access to credit. To assess whether missing credit markets affect the implications of reference dependence for optimal savings, this study introduces a liquidity constraint in both models. I consider a setting in which the sole savings motive for reference-independent people is to smooth consumption, meaning that reference-independent consumers save when present income is higher than future income, and either borrow (if unconstrained) or live hand-to-mouth (if liquidity constrained) when present income is lower. The main finding is that reference-dependent savings are often higher in the face of liquidity constraints, even though unconstrained savings are unambiguously lower than the reference-independent optimum.

Two mechanisms drive this asymmetry. First, liquidity constraints can prevent borrowing to avoid losses in early periods but cannot prevent saving to avoid losses in future periods, which may induce people to take the full loss immediately and save to eliminate future losses. This mechanism applies to reference-dependent consumers whose income drops below the reference point in early as well as future periods. They prefer to borrow and avoid losses in early periods, *reducing* optimal savings compared with the reference-independent optimum. However, this is not viable in the presence of liquidity constraints. By diminishing sensitivity, that is, convex loss utility, their second-best option is to take the full loss immediately and save to eliminate future losses, *increasing* optimal savings. In other words, rather than worsening beliefs regarding both imminent and future consumption, expectations regarding future consumption turn into an aspiration to save and maintain future consumption at the planned level.

Second, anticipating a binding liquidity constraint affects expected consumption and hence the expectations-based reference point. When income is lower in early periods than in future periods, liquidity-constrained people cannot smooth perfectly. They expect higher consumption in future periods, increasing their reference level and thus the marginal utility from future consumption. When income in early periods turns out not to be too bad, they may prefer to save, even if the reference-independent optimum is to live hand-to-mouth. This is a second aspirations-based savings motive in the presence of liquidity constraints.

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This paper contributes by highlighting novel savings motives from interacting reference dependence and liquidity constraints. The literatures on intertemporal choice in imperfect credit markets and reference-dependence have been fairly separated. The finding that liquidity constraints alter how reference dependence influences behavior is relevant for empirical studies aiming at identifying such preferences outside the laboratory (see Camerer 2004 and Barberis 2013 for reviews). So far, these studies have not considered how missing credit markets may confound estimates of loss aversion and diminishing sensitivity. I show that for people without access to credit markets, reference dependence can affect savings qualitatively differently from what we would expect on the basis of existing models. Moreover, if consumers act in line with the framework presented here, introducing credit will have different behavioral effects and welfare implications than previously assumed.

The remainder of this paper is organized as follows. The next section analyzes the first model with exogenous reference points. Section 3 discusses how to interpret this model and describes recent advances in modeling reference-dependence. Section 4 introduces a liquidity constraint in the model developed by Kőszegi and Rabin (2009), and predicts how savings respond to unanticipated income shocks. Using the same framework, Section 5 predicts savings when consumers know that income is stochastic while forming their beliefs. The final section concludes.

# 2. CASE 1: EXOGENOUS REFERENCE POINTS

## Model for Intertemporal Choice with Exogenous Reference Points

As a first step, consider a framework with exogenous reference points. This tractable model offers intuition for the results in more sophisticated models with endogenous reference points.

The model includes two periods  $t \in \{1,2\}$ : the present, t = 1, and the future, t = 2. In period t, a person earns deterministic income  $w_t$ , summarized in a  $2 \times 1$ -vector  $\mathbf{w} \equiv \{w_1, w_2\}$ . The person chooses how much to consume in the present,  $c_1$ , and how much to save in a risk-free asset with zero returns,  $w_1 - c_1$ . By a regular budget constraint, future consumption equals  $c_2 = w_1 + w_2 - c_1$ . To reflect the stylized fact that the poor often lack access to formal credit, the person faces a borrowing constraint,  $c_1 \leq w_1$ .<sup>2</sup>

Preferences are reference dependent, meaning that the person derives utility not from consumption,  $c_t$ , but from gains and losses compared with a reference point r > 0.3 Consumption  $c_t \ge r$ entails a gain. Lower consumption is considered a loss. Let  $\mu(x)$  be the utility from a gain or loss, x = c - r. A reference-dependent person optimizes lifetime utility  $U(c_1; \mathbf{w}, r)$ :

$$\max_{c_1 \le w_1} U(c_1; \mathbf{w}, r) = \max_{c_1 \le w_1} \mu(c_1 - r) + \mu(w_1 + w_2 - c_1 - r), \text{ and}$$
(1)  
$$c_{RD,1}^*(\mathbf{w}, r) = \arg\max_{c_1 \le w_1} U(c_1; \mathbf{w}, r),$$

where  $c_{RD,1}^*(\mathbf{w}, r)$  represents optimal consumption given income  $\mathbf{w}$  and a reference point r, and gain-loss utility  $\mu(x)$  is the standard value function often used in prospect theory. Figure 2.1 illustrates an example. Gain-loss utility is defined for all x > -r, continuous for all x, and twice differentiable for all  $x \neq 0$  and  $\mu(0)$  is normalized to zero,  $\mu(0) = 0$ . In addition, drawing on Bowman, Minehart, and Rabin (1999), gain-loss utility satisfies the following properties:

- (A1) Monotonicity:  $\mu(x)$  is strictly increasing.
- (A2) Loss aversion over small stakes:  $\lim_{x_l \downarrow 0} \mu'(x_l) / \lim_{x_g \uparrow 0} \mu'(x_g) \equiv \lambda > 1$ .
- (A3) Loss aversion over large stakes:  $\mu'(l)/\mu'(g) > 1$  for  $g > \varepsilon$  and l < 0, with  $\varepsilon/r = \lim_{x \downarrow 0} x$ .
- (A4) Diminishing sensitivity:  $\mu''(x) < 0$  and  $\mu''(-x) > 0$  for x > 0.

<sup>&</sup>lt;sup>2</sup>A more nuanced model for financial market imperfections would assume endogenous credit rationing, but to stay close to the literature, I follow seminal buffer stock savings models that have modeled financial market imperfections as an exogenous borrowing constraint (for example Deaton 1991; Carroll 1997). For the same reason, I model the savings technology as a buffer stock rather than a productive asset with positive returns.

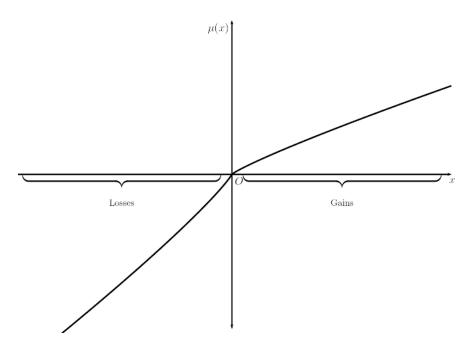
<sup>&</sup>lt;sup>3</sup>The reference point is time-invariant. Given a time-variant reference point, marginal utility is higher in the period with a higher reference level, and if this is the future period, reference-dependence increases savings independent of the presence of liquidity constraints. This section focuses on the novel contribution that reference-dependence may increase liquidity-constrained savings while it unambiguously reduces unconstrained savings.

Assumption (A1) simply says that increasing a gain (or reducing a loss) increases well-being and that reducing a gain (or increasing a loss) reduces well-being.

Assumption (A2) implies loss aversion over small stakes, meaning that a small loss is more painful than a small gain is pleasant. By loss aversion, gain-loss utility exhibits a kink at the reference point, explaining, for instance, why people prefer a lottery with an even chance of earning \$0 or \$50 above a safe option yielding \$20 with certainty, but at the same time prefer a safe option yielding \$0 with certainty above a lottery in which they either lose \$20 or gain \$30. Although the two choices are identical in terms of final outcomes, the second choice frames the worst outcome as a loss, inducing a preference for the safer option. Assumption (A3) generalizes loss aversion to larger stakes. A consumer will always prefer reducing a loss above increasing a gain  $g > \varepsilon$ , where  $\varepsilon$  is infinitesimal compared with the reference point r.

Assumption (A4) implies diminishing sensitivity, meaning that an additional gain is less pleasant as more has already been gained, and an additional loss hurts less as more has already been lost. Intuitively, under diminishing sensitivity, people can be risk averse when choosing between a safe option yielding \$25 with certainty and a lottery with an even chance of winning either \$0 or \$50, but they become risk seeking when choosing between a certain loss of \$25 and a gamble in which they lose either \$50 or \$0 with an even chance. This reflection effect requires the marginal value of both gains and losses to decrease in size. In other words, gain-loss utility is convex-concave around the reference point, as illustrated in Figure 2.1.

Figure 2.1 Gain-loss utility satisfying loss aversion and diminishing sensitivity



Source: Author's analyses based on Kahneman and Tversky (1979).

Note that without Assumption (A3), a person satisfying diminishing sensitivity could prefer increasing an already big loss at the benefit of incurring a small gain. Choice experiments do not always find evidence of convex loss utility (for example Levy and Levy 2002), and some of the evidence for convex loss utility results from nonincentivized decisions (Laury and Holt 2008). To ensure that loss aversion rather than diminishing sensitivity dominates behavior in tradeoffs involving both gains and losses, Assumption (A3) generalizes (A2) to large stakes. The convexity of loss utility shapes behavior only in trade-offs involving a sure loss.<sup>4</sup>

On a final note, the model abstracts from discounting. This improves tractability without major implications but leaves a few cases in which two consumption levels,  $\omega$  and  $\nu$ , both optimize utility. A person then prefers the solution that maximizes present consumption:

If 
$$U(\omega, \cdot) = U(\nu, \cdot) = \sup_{c_1} U(c_1, \cdot)$$
 and  $\omega > \nu$ , then  $c^*_{RD,1}(\cdot) = \omega$ . (3)

#### Solution of the Model with an Exogenous Reference Point

The main question is how reference-dependent savings deviate from the reference-independent optimum in a context of missing credit markets. This section addresses this question for the model presented above. Figure 2.2 summarizes optimal consumption in the present,  $c_1^*$ , as a function of income on hand,  $w_1$ . The solid line indicates reference-dependent consumption. As a benchmark, the dashed line indicates the reference-independent optimum, replacing  $\mu(c-r)$  with a strictly increasing and concave utility function m(c).

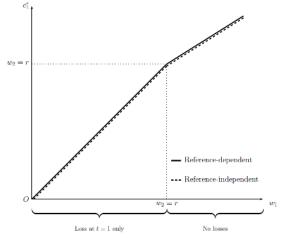
If 
$$\mu'(x_g) < \mu'(-r) \ \forall \ x_g > \varepsilon$$
, then  $\frac{\varepsilon}{r} = \lim_{x \downarrow 0} x.$  (2)

<sup>&</sup>lt;sup>4</sup>Bowman, Minehart, and Rabin (1999) adopted a weaker assumption for loss aversion over large stakes, but Assumption (A3) comes without too much loss of generality. To see why, define a threshold gain  $\varepsilon > 0$  such that the marginal utility from all gains  $x_g > \varepsilon$  is below feasible levels of marginal loss utility. Since marginal loss utility is minimized at x = -r, loss aversion generalizes to all stakes under the following condition:

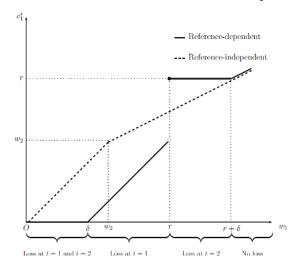
Loosely speaking, this condition requires  $\varepsilon$  to be a "small" gain relative to the reference point, which does not restrict the parameter set substantially. Take for instance the value function estimated in Tversky and Kahneman (1992). Using the value function  $\mu(x) = x^{\theta_G}$  for gains and  $\mu(x) = -\lambda(-x)^{\theta_L}$ , they estimate parameters  $\hat{\lambda} = 2.25$  and  $\hat{\theta}_L = \hat{\theta}_G = 0.88$ . Given these estimates, the ratio between the threshold gain and the reference point is  $\varepsilon/r = 0.0012$ , which is sufficiently small to generalize loss aversion to larger stakes.

# Figure 2.2 Consumption and period 1 income with an exogenous reference point

(a) Period 2 income equals the reference point  $(w_2 = r)$ 



(b) Period 2 income is below the reference point ( $w_2 = r - \delta$ )



Source: Author's analyses.

When preferences are reference independent (RI), the sole savings motive is to smooth consumption. Without discounting, a person prefers to smooth perfectly,  $c_1 = c_2 = \bar{w} \equiv (w_1 + w_2)/2$ . However, when  $w_1 < w_2$ , this entails borrowing, which is not possible in the presence of a liquidity constraint. By concavity, the feasible optimum is to consume income on hand,  $w_1$ , resulting in piecewise linear consumption:

$$c_{RL1}^*(\mathbf{w}) = \min\{\bar{w}, w_1\}, \ \bar{w} \equiv (w_1 + w_2)/2.$$

Reference-dependent (RD) savings depend on a person's ability to eliminate losses. In Panel (a), the reference point equals future income; that is,  $r = w_2$ . Consuming  $c_1 \le w_1$  in the present and  $c_2 \ge w_2$  in the future is hence sufficient to avoid a future loss. As a result, the solution depends only on the person's income, and savings are observationally equivalent to the reference-independent optimum. To see why, substitute *r* in Equation (1),

$$\max_{c_1 \le w_1} U(c_1; \mathbf{w}, w_2) = \max_{c_1 \le w_1} \mu(c_1 - w_2) + \mu(w_1 - c_1),$$

where  $c_1 - w_2$  represents a present gain or loss, and  $c_2 - w_2 = w_1 - c_1 \ge 0$  a future gain. Figure 2.2 distinguishes two cases, depending on whether the optimum results in a situation with a present loss,  $w_1 < w_2$ , or a situation without losses in either period,  $w_1 \ge w_2$ .

In the first situation, present income is relatively low,  $w_1 < w_2$ . The liquidity constraint,  $c_1 \le w_1$ , restricts consumption to a level below the reference level,  $w_2$ . Since future consumption is at or above the reference point, the liquidity constraint binds the person to a present loss and a future gain. By loss aversion for large stakes, Assumption (A3), it is optimal to minimize the present loss at the expense of the future gain and to consume present income,  $c_1 = w_1$ .

In the second situation, present income is relatively high,  $w_1 \ge w_2$ , and the liquidity constraint does not bind. By Assumptions (A2) and (A3), a loss-averse person opts to consume  $c_1 \in [w_2, w_1]$  and avoid a loss in both periods. In this interval, life-cycle utility is concave with an optimum that satisfies the first-order condition  $U'(c_1; \mathbf{w}, w_2) = 0$ . The present gain,  $c_1 - w_2$ , then equals the future gain,  $w_1 - c_1$ , which is equivalent to consumption smoothing,  $c_1 = (w_1 + w_2)/2 = \bar{w}$ . Thus, reference-dependent and -independent optima are equivalent:

$$c_{RD,1}^*(\mathbf{w}, w_2) = \min\{w_1, \bar{w}\} = c_{RI,1}^*(\mathbf{w}).$$

This result hinges upon the assumption that future income equals the reference point,  $w_2 = r$ , so that the person can be confronted with losses only in the early period. In Panel (b), future income is below the reference point,  $w_2 = r - \delta < r$ , and the consumer needs to save at least  $\delta$  to avoid a future loss. Equation (1) can now be written as

$$\max_{c_1 \le w_1} U(c_1; \mathbf{w}, w_2 + \delta) = \max_{c_1 \le w_1} \mu(c_1 - w_2 - \delta) + \mu(w_1 - \delta - c_1).$$

The solution now depends not only on income but also on the difference between the reference point and future income,  $\delta > 0$ . Panel (b) of Figure 2.2 distinguishes four cases that, depending on present income, according to whether a person avoids present or future losses.

First, when  $w_1 < \delta$ , the consumer cannot avoid a loss in either period. As a result, diminishing sensitivity implies that lifetime utility is convex. The optimum is a corner solution in which the loss is concentrated in one period and minimized in the other period. In other words, it is optimal to consume all wealth in one period and zero in the other. The liquidity constraint prevents the person from consuming all wealth in the present, making it optimal to consume all wealth in the future and take the full loss in the present,  $c_1 = 0.5$  Thus, it is optimal to save income on hand and minimize the future loss.<sup>6</sup>

Second, when  $w_1 \in [\delta, r)$ , the person cannot avoid an early loss, since present income is still below the reference point. However, she can now avoid a future loss by saving at least  $\delta$ . Due to loss aversion over large stakes, Assumption (A3), the person will not save more than  $\delta$ , as this would entail a future gain at the expense of a present loss. By diminishing sensitivity, Assumption (A4), she will also not save less than  $\delta$ , since it is optimal to take the full loss in one period. As a result, the solution is to save  $\delta$ in order to eliminate the future loss and to consume  $c_1 = w_1 - \delta$  in the present period.

In the third case, present income is at least as high as the reference point,  $w_1 \in [r, r + \delta)$ . It is possible to eliminate either the present loss and consume  $c_1 = r$ , or the future loss and consume  $c_1 = w_1 - \delta$ . Both options yield equal life-cycle utility and optimize the objective function. By Condition (3), the consumer prefers the solution with the highest level, and consumption jumps from  $c_1 = w_1 - \delta$  to  $c_1 = r$ .<sup>7</sup> Thus, the liquidity constraint no longer binds.

In the fourth case, present income is even higher and satisfies  $w_1 - \delta \ge w_2 + \delta$ . Now, it is possible to avoid losses in both periods. By diminishing sensitivity, utility for gains is concave, and the solution will satisfy a first-order condition such that the present gain,  $c_1 - w_2 - \delta$ , equals the future gain,  $w_1 - \delta - c_1$ . This means that optimal consumption is equivalent to the reference-independent solution,  $c_1 = \bar{w}$ .<sup>8</sup> Summarizing, if future income is below the reference point, reference-dependent consumption becomes

$$c_{RD,1}^{*}(\mathbf{w}, w_{2} + \boldsymbol{\delta}) = \begin{cases} \max\{0, w_{1} - \boldsymbol{\delta}\} < c_{RI,1}^{*}(\mathbf{w}) \ \forall \ w_{1} < w_{2} + \boldsymbol{\delta} \\\\ \max\{w_{2} + \boldsymbol{\delta}, \bar{w}\} \ge c_{RI,1}^{*}(\mathbf{w}) \ \forall \ w_{1} \ge w_{2} + \boldsymbol{\delta}. \end{cases}$$

 $U(0; \mathbf{w}, w_2 + \delta) - U(w_1; \mathbf{w}, w_2 + \delta) = \mu(-w_2 - \delta) + \mu(w_1 - \delta) - [\mu(w_1 - w_2 - \delta) + \mu(-\delta)] > 0 \Leftrightarrow w_2 + w_1 > w_2 - w_1, \ w_1 < \delta.$ 

<sup>&</sup>lt;sup>5</sup>The other corner solution,  $c_1 = w_1$ , yields strictly lower utility than  $c_1 = 0$  for all  $0 < w_1 < \delta$ :

<sup>&</sup>lt;sup>6</sup>Observations of zero consumption are rather uncommon, but one could interpret this result in terms of narrow bracketing. Intertemporal choice regarding permanent wealth may for instance still follow reference-independent consumption theory, ruling out zero consumption, while decisions regarding transitory income could be reference dependent.

<sup>&</sup>lt;sup>7</sup>This jump could still occur in the presence of discounting, albeit at a lower income level.

<sup>&</sup>lt;sup>8</sup>Smoothing of gains means that the following first-order condition is satisfied:  $\mu'(c_1 - w_2 - \delta) = \mu'(w_1 - \delta - c)$ . Rearranging  $c_1 - w_2 - \delta = w_1 - \delta - c_1$  yields the result that smoothing gains is equivalent to smoothing consumption,  $c_1 = \bar{w}$ .

To conclude, reference dependence does not affect optimal savings when future income is sufficiently high to avoid future losses, as in Panel (a). In contrast, when future income is insufficient to avoid future losses, as in Panel (b), reference dependence affects decisions differently depending on whether consumers face a binding liquidity constraint. A liquidity constraint forces a person with low income on hand,  $w_1 < r$ , to take part of the loss in the early period, so that it is optimal to concentrate the loss in that period and save *more* than reference-independent consumers. A person with higher income,  $w_1 \in [r, r + \delta)$ , does not face a binding liquidity constraint and can postpone taking the full loss to the future. Thus, she saves *less* than reference-independent consumers.

# 3. DISCUSSION OF THE MODEL WITH EXOGENOUS REFERENCE POINTS

#### Interpretation of the Model's Solution for Exogenous Reference Points

The model with exogenous reference points illustrates a mechanism through which a binding liquidity constraint affects the difference between reference-dependent and -independent savings. To interpret this mechanism, it is important to specify how people form their reference points. Previous literature on reference-dependent savings has hypothesized that the reference point is determined by changes in wealth (Kahneman and Tversky 1979); by past consumption levels that lead to habit formation (Bowman, Minehart, and Rabin 1999), by a comparison with neighbors' or the society's consumption, creating a preference for "keeping up with the Joneses" (Bogliacino and Ortoleva 2013), or by rational expectations of consumption (Kőszegi and Rabin 2006).

The findings presented in the previous section can be interpreted in terms of each of these alternative specifications. For instance, if reference points are shaped by average consumption in the society, low income might induce a person to save in the early period in order to catch up with others in the future. Alternatively, in the presence of habit formation, someone may get used to a consumption level that is no longer affordable given her present income. Diminishing sensitivity can motivate this person to reduce present consumption below the reference point and save to return to her habits in the future. Finally, when expected consumption determines the reference point and present income is lower than anticipated, a person may prefer to tighten her belt and save enough to consume expected levels in the future.

At the same time, it is difficult to empirically test this framework. Since the model does not specify a unique reference point, it can explain any consumption pattern simply by adjusting the reference point to match the observed behavior. More sophisticated models specify how the reference point is being formed, reducing the degrees of freedom. Another weakness is that consumption levels do not matter at all in this framework. Since it is unlikely that utility is strictly convex when consumption reaches zero, the gain-loss frame discussed above may apply to small or moderate gains and losses, but should not be used to analyze behavior among consumers whose incomes drop below subsistence levels.

#### Advances in Theories on Reference Dependence and Intertemporal Choice

The last decade has seen substantial progress in modeling reference-dependent preferences. One reference-dependent model addressing the caveats discussed above is Kőszegi and Rabin (2009)'s framework for wealth and intertemporal choice (henceforth KR). This framework builds on the Kőszegi and Rabin (2006) model for reference-dependent preferences, a "significant attempt to clarify how people think about gains and losses, ... both disciplined and portable across different contexts" (Barberis 2013).

The framework specifies expectations prior to consumption, that is, consumption plans, as the reference point, and models how consumers make these plans. As a result, the framework specifies a unique reference point, eliminating the degrees of freedom that hampered applications of prospect theory outside the lab. To my best knowledge, this is the most advanced theory of reference dependence in intertemporal choice.

The theory makes three key assumptions. To start, not only gains and losses but also consumption levels affect well-being. Reference-independent preferences are nested within the model, which has two benefits (Barberis 2013). First, it is unlikely that absolute levels of consumption do not matter at all or that a decision maker concentrates all consumption in one period. Second, the nested model allows analyzing whether the additional parameters embedded in reference-dependent models are necessary to describe consumption. Ultimately, the question is not whether we should *replace* consumption utility by gain-loss utility, but whether *adding* gain-loss utility significantly improves behavioral predictions.

A second assumption in the KR model for intertemporal choice is that rational expectations of consumption serve as a reference point and that gain-loss utility is derived from information (that is, news or changes in beliefs) regarding consumption. This assumption unifies stylized facts from laboratory experiments on information preferences, effort provision, and trading decisions. In terms of information preferences, participants bet significantly less in treatments with higher feedback frequency (Gneezy and Potters 1997; Haigh and List 2005; Bellemare et al. 2005), and investors monitored their portfolios more frequently in rising markets than in flat or falling markets (Karlsson, Loewenstein, and Seppi 2009). In terms of effort provision, Abeler et al. (2011) found that manipulating rational expectations of earnings influences effort.<sup>9</sup> Ericson and Fuster (2011) showed that the *probability* of being allowed to trade or obtain an item affects the valuation of that item, suggesting that beliefs held before the trade drive participants' decisions.

Given this interpretation, changes in wealth affect well-being by creating news about consumption, and expectations are adjusted as soon as the news arrives. In line with this idea, Bronchetti et al. (2013) argued that tax filers rejecting a default savings plan for their income tax refund did so because they already anticipated the refund.

A final assumption is that news regarding present consumption resonates more than news affecting future consumption. In other words, changing beliefs regarding present consumption carries a higher weight than changing beliefs regarding future consumption. This assumption can explain why people prefer receiving the same piece of good information sooner rather than later. Combined with loss aversion,

<sup>&</sup>lt;sup>9</sup>Crawford and Meng (2011) used this theory to explain negative wage elasticities in New York City taxi drivers' labor supply. Dupas and Robinson (2013) found evidence that the labor supply among a sample of Kenyan bicycle taxi drivers was target based and that targets were partly determined by expectations.

this also explains an asymmetry in how people respond to good versus bad income news. Shea (1995) and Bowman, Minehart, and Rabin (1999) found that union workers receiving bad news regarding future income took the full loss in the future and did not reduce present consumption, while those receiving positive news took the full gain immediately and left future consumption unaffected.

In sum, the KR framework addresses some of the key challenges associated with earlier models of reference dependence, and its key features have been supported by recent experimental and observational studies. By endogenizing the reference point as a rational belief about consumption, it defines a unique prediction of expected and actual consumption, yielding a framework that can be tested empirically outside the lab.

# 4. CASE 2: ENDOGENOUS REFERENCE POINTS AND DETERMINISTIC INCOME

#### Model for Intertemporal Choice with Expectations-Based Reference Points

The second model to be analyzed builds on these advances in reference-dependent theories. Specifically, it introduces a liquidity constraint in a two-period version of the KR framework. Prior to the present period, consumers form beliefs regarding both present and future consumption, serving as a reference point for actual consumption. This section assumes that people have perfect foresight of consumption when forming their beliefs. Taking these beliefs as given, this section analyzes how savings respond to unanticipated income shocks.

Formally, a person believes her income will be  $\mathbf{w} = (w_1, w_2)$ . Let  $\mathbf{c} = (c_1, c_2)$  and  $\mathbf{b} = (b_1, b_2)$ represent actual and anticipated consumption paths, respectively. Actual consumption solves the following problem:<sup>10</sup>

$$\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{b}, \gamma) = \max_{c_1 \le w_1, c_2} \log(c_1) + \mu(x_1) + \gamma \mu(x_2) + \log(c_2),$$
(4)

$$x_t = \log(c_t) - \log(b_t), \tag{5}$$

$$c_2 = w_1 + w_2 - c_1, b_2 = w_1 + w_2 - b_1, \text{ and}$$
 (6)

$$\mathbf{c}_{RD}^{*}(\mathbf{w}, \mathbf{b}, \gamma) = \arg \max_{c_{1} \leq w_{1}, c_{2}} U(\mathbf{c}; \mathbf{w}, \mathbf{b}, \gamma);$$
(7)

Here,  $\log(c_t)$  represents consumption utility in period t;  $\mu(x_t)$  gain-loss utility; and  $\gamma < 1$  the utility weight of prospective gains or losses, that is the information that future consumption will be higher or lower than anticipated (for tractability, future consumption utility is not discounted). In this two-period setting, gains and losses result from comparing actual consumption,  $\log(c_t)$ , with the anticipated–or planned–level of utility,  $\log(b_t)$ . A regular budget constraint defines actual and anticipated–or planned–consumption for the second period.

KR assumes that two mechanisms jointly determine a consumption plan, **b**. First, consumers make only plans to which they can commit. In other words, the solution satisfies an equilibrium condition  $\mathbf{c}_{RD}^*(\mathbf{w}; \mathbf{b}, \gamma) = \mathbf{b}$ . Plans satisfying this criterion are labeled a "personal equilibrium." Second, out of all personal equilibria, the consumer chooses the plan that optimizes utility ex ante. This so-called preferred personal equilibrium provides a unique reference point. Appendix A discusses the formation of plans in more detail and derives the preferred personal equilibrium for the model presented in this section.

<sup>&</sup>lt;sup>10</sup>The two-period framework covers the main dynamics of the model. A longer time horizon will not generate additional insights and is computationally intense. Because anticipated consumption is endogenous, an additional time period increases the dimension of the problem exponentially.

Gain-loss utility  $\mu(x)$  is a standard gain-loss utility function satisfying Assumptions (A1) to (A4). Because consumers derive log utility from gains and losses relative to planned log utility,  $x_t = \log(c_t) - \log(b_t)$ , we can interpret small gains and losses as the relative change in consumption, with  $x \approx (c-b)/b$  for a gain c > b, and  $-x \approx (b-c)/c$  for a loss c < b.<sup>11</sup> Combined, consumption utility  $\log(c)$ , gain-loss utility  $\mu(x)$ , and weights  $\gamma_1 = 1$  and  $\gamma_2 = \gamma$  yield the instantaneous utility function  $u(c_t, x_t; \gamma_t)$  for period *t*:

$$u(c_t, x_t; \gamma_t) \equiv \log(c_t) + \gamma_t \mu(x_t).$$

To pose more structure on instantaneous utility, the model imposes two additional conditions:

(C1) Intertemporal loss aversion:  $\gamma \lambda > 1$ 

(C2) Declining diminishing sensitivity to losses:  $\mu'''(x)\mu'(x) > \mu''(x)^2$  for x < 0

Condition (C1) says that future losses carry a higher marginal utility weight than early gains,  $\gamma \lambda > 1$ , which we may refer to as "intertemporal loss aversion." This means that future losses are more painful than present gains are pleasant. The endogenous consumption plan,  $\mathbf{b} = \mathbf{c}_{RD}^*(\mathbf{w}, \mathbf{b}, \gamma)$ , is equivalent to the reference-independent optimum if and only if this condition is satisfied:

$$\mathbf{c}_{RD}^{*}(\mathbf{w}, \mathbf{c}_{RI}^{*}(\mathbf{w}), \gamma) = \mathbf{c}_{RI}^{*}(\mathbf{w}).$$
(8)

Appendix A proves this statement in a setting with liquidity constraints, and Kőszegi and Rabin (2009) derived a similar result for the case without liquidity constraints. They also show that without intertemporal loss aversion, the endogenous plan is to consume more in the present than in the future. This is because someone planning to smooth consumption has an incentive to surprise herself with an immediate gain at the expense of a future loss. As this paper is mainly interested in analyzing how a reference-dependent person planning to consume the reference-independent optimum responds to unanticipated income shocks, the analyses impose Condition (C1).

Condition (C2), combined with Assumptions (A1) and (A4), implies that marginal gain-loss utility is convex. This is a very general condition typically used to obtain decreasing absolute risk aversion (DARA) in concave utility functions. By analogy, the analyses impose Condition (C2) to obtain declining diminishing sensitivity of instantaneous utility to losses, where I define the degree of diminishing sensitivity for period t as

<sup>&</sup>lt;sup>11</sup>The original KR framework assumes a more flexible consumption utility function m(c) rather than  $\log(c)$ . The results in this paper generalize to any function m(c) satisfying decreasing absolute risk aversion (DARA), meaning that marginal consumption utility is convex. The more flexible form is, however, notationally more complex and more difficult to interpret.

$$DS(x_t; \gamma_t) \equiv \frac{\gamma_t |\mu''(x_t)|}{1 + \gamma_t \mu'(x_t)} \text{ for } x_t \neq 0.$$
(9)

The degree of diminishing sensitivity in period *t* depends on the size of the gain or loss in that period,  $x_t$ , and on the weight of this gain or loss,  $\gamma_t$ . It is indicative of how strong a person's preferences for spreading gains or concentrating losses are. By Condition (C2), diminishing sensitivity declines as a loss increases in size.<sup>12</sup> In other words, the preference for concentrating losses is strongest when losses are small. As losses become larger, diminishing sensitivity has a smaller influence on behavior.

This is relevant since instantaneous utility is not strictly convex for losses c < b, in contrast to the previous section, where instantaneous utility did not include concave consumption utility. Utility will be concave for relatively large losses because marginal consumption utility will exceed marginal gain-loss utility for very small consumption levels,  $\lim_{c\downarrow 0} (\log(c_t))' = \infty$ . Binging consumption into one period is therefore never optimal. Nevertheless, since diminishing sensitivity becomes stronger as a loss becomes smaller, instantaneous utility can be convex in consumption, c, when losses are not too large. The following lemma formalizes this result.

**LEMMA 1.** Assume existence of a loss  $L_{\gamma^*} < 0$  and a gain-loss utility weight  $\gamma^* < 1$  such that  $DS(L_{\gamma^*}; \gamma^*) = 1$ . If Condition (C2) holds, then

$$\frac{\partial^2 u(c_t, x_t; \gamma_t)}{\partial c_t^2} > 0 \ \forall \ x \in (L_{\gamma^*}, 0), \gamma_t \geq \gamma^*.$$

For a proof, see Appendix B. This lemma says that by Condition (C2), parameters  $L_{\gamma^*}$  and  $\gamma^*$  exist such that if the weight attached to gain-loss utility is at least  $\gamma^*$ , instantaneous utility is convex as long as the loss is sufficiently small. If a lower weight is attached to gain-loss utility,  $\gamma_t < \gamma^*$ , or if losses are more extreme,  $x_t < L_{\gamma^*}$ , the concavity of consumption utility will dominate the convexity of loss utility. As an example, consider the common gain-loss utility function,  $\mu(x) = \eta x^{\theta}$  for  $x \ge 0$  and  $\mu(x) = -\eta \lambda (-x)^{\theta}$  for x < 0, with parameters  $\gamma^* = 1$ ,  $\eta = 10$ ,  $\lambda = 2.25$ , and  $\theta = 0.88$  (the latter two based on estimates from Tversky and Kahneman 1992). Given this example, utility is convex for all losses such that the planned level is at most 11.5 percent above actual consumption,  $-x < L_{\gamma^*} \approx 0.115$ .

$$DS'(x_t; \gamma_t) = \mu'''(x_t)(1 + \gamma_t \mu'(x_t)) - \gamma_t \mu''(x_t)^2 > \gamma(\mu'''(x_t)\mu'(x_t) - \mu''(x_t)^2) > 0.$$

<sup>&</sup>lt;sup>12</sup>To see this, note that the first derivative of the degree of diminishing sensitivity for  $x_t < 0$  is proportional to

where the latter inequalities follow from  $\gamma_t \leq 1$  and Condition (C2).

#### **Response to Unanticipated Income Shocks in a Context of Liquidity Constraints**

This section analyzes how within this framework exogenous income shocks affect intertemporal choice. By Condition (C1), intertemporal loss aversion is satisfied, and planned consumption equals the reference-independent optimum. Shocks induce consumers to revise their plan. Kőszegi and Rabin (2009) showed that they do so in an asymmetric way. Consumers facing income losses do not cut spending until the future, while they spend income gains immediately. This reduces savings compared with the reference-independent optimum. However, their analyses do not consider the existence of a liquidity constraint.

To analyze how a liquidity constraint can influence the response to unanticipated income shocks, consider a consumer who plans to earn  $\mathbf{w} = (w_1, w_2)$  but learns that her earnings will be  $\mathbf{w}' = (w'_1, w_2)$ , with  $w'_1 \neq w_1$ . The reference-independent optimum is to smooth consumption unless the liquidity constraint is binding,  $w'_1 < w_2$ , preventing perfect smoothing. In that case, the feasible optimum is  $w'_1$ . Thus,

$$\mathbf{c}_{RI,1}^*(\mathbf{w}') = \min\{w_1', \bar{w}'\}, \ \bar{w}' = (w_1' + w_2)/2.$$

In contrast, the reference-dependent optimum depends on the prior consumption plan, which is the reference-independent optimum given the anticipated income path,  $\mathbf{c}_{RI}^*(\mathbf{w})$ , under Condition (C1). The revised plan does not need to satisfy the personal equilibrium conditions and solves the following problem:

$$\max_{c_1 \le w'_1, c_2} U(\mathbf{c}; \mathbf{w}', \mathbf{c}_{RI}^*(\mathbf{w}), \gamma) = \max_{c_1 \le w'_1, c_2} \log(c_1) + \mu(x_1) + \gamma \mu(x_2) + \log(c_2),$$
(10)

$$c_2 = w_1' + w_2 - c_1, \tag{11}$$

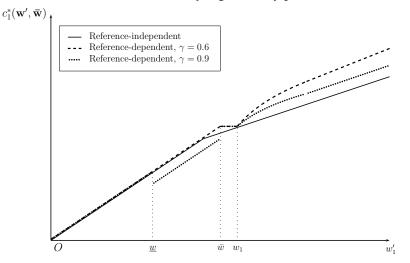
$$x_t = \log(c_t) - \log(c_{RI,t}^*(\mathbf{w})), \text{ and}$$
(12)

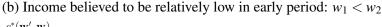
$$\mathbf{c}_{RD}^{*}(\mathbf{w}',\mathbf{c}_{RI}^{*}(\mathbf{w}),\boldsymbol{\gamma}) = \arg\max_{c_{1} \leq w_{1}',c_{2}} U(\mathbf{c};\mathbf{w}',\mathbf{c}_{RI}^{*}(\mathbf{w}),\boldsymbol{\gamma}).$$
(13)

Figure 4.1 presents the solution to this problem, illustrating revised consumption in the first period as a function of actual income on hand. The figure distinguishes between reference-independent consumers–the solid line–and two types of reference-dependent consumers–the dashed and dotted lines. These two types vary in the weight they attach to prospective gain-loss utility,  $\gamma$ , with the dashed line assuming a lower weight than the dotted line.

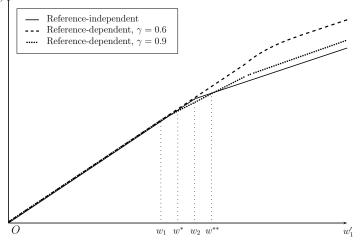
#### Figure 4.1 Unanticipated income shocks with diminishing sensitivity

(a) Income believed to be relatively high in early period:  $w_1 > w_2$ 









Source: Author's analyses.

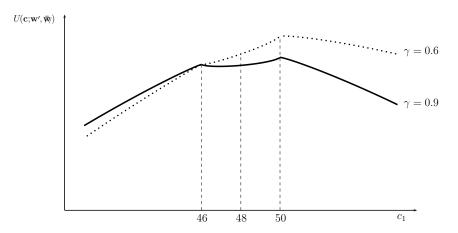
This figure illustrates the solution to Problems (10)–(13) using convex-concave gain-loss utility  $\mu(x) = \eta x^{\theta}$  for  $x \ge 0$  and Note:  $\mu(x) = -\eta \lambda (-x)^{\theta}$  for x < 0 with  $x_t = \log(c_t/b_t)$ . Income is  $w_1 = 52$  and  $w_2 = 48$  in Panel (a), and  $w_1 = 48$  and  $w_2 = 52$  in Panel (b). Parameter values are  $\eta = 10$ ,  $\lambda = 2.25$ , and  $\theta = 0.88$ .

The figure distinguishes two mechanisms through which reference dependence may increase savings under liquidity constraints. Panel (a) presents a first mechanism. Here, consumers anticipate higher income in the first period than in the second period,  $w_1 > w_2$ , and plan to consume  $\mathbf{c}_{RI}^*(\mathbf{w}) = (\bar{w}, \bar{w}) \equiv \bar{\mathbf{w}}$ . For income gains and relatively small losses,  $w_1' \ge \bar{w}$ , consumption responds asymmetrically to shocks. This is similar to the asymmetry KR obtained in the case without liquidity constraints: consumers do not cut spending when facing small income losses, but they spend windfall gains immediately. This increases consumption (and reduces savings) compared with the reference-independent optimum. Further, if the revised level of income on hand is very small,  $w'_1 \le w$ , marginal consumption utility in the first period,  $1/c_1 \ge 1/w'_1$ , will be large enough for life-cycle utility to be increasing in  $c_1$ . In that case, it is optimal to spend income on hand as in the reference-independent case.

However, for moderately sized income losses,  $w'_1 \in (\underline{w}, \overline{w})$ , the two reference-dependent types diverge. Consumers need to take a loss in at least one period. By Lemma 1, instantaneous utility is convex if losses are not too large,  $-x > L_{\gamma^*}$ . It is then optimal to concentrate losses in one period. The liquidity constraint prevents consumers from taking the full loss in the future period. Instead, if prospective gains and losses carry a sufficiently high weight, it is optimal to take the full loss in the early period and save the amount they planned to set aside for future consumption. The dotted line is therefore strictly below the reference-independent optimum for all  $w'_1 \in (\underline{w}, \overline{w})$ . In contrast, the type represented by the dashed line attaches a lower weight to future losses and prefers minimizing the early loss by consuming income on hand.

As an example, Figure 4.2 plots lifetime utility if the consumer plans to earn  $\mathbf{w} = (52, 48)$  but learns that she earns  $w'_1 = 48$  rather than  $w_1 = 52$ . The figure distinguishes again between a high weight and a low weight of prospective gain-loss utility,  $\gamma = 0.9$  and  $\gamma = 0.6$ , respectively. Consumption levels  $c_1 > 46$  are associated with bad news regarding future periods. Since the relative disutility from such news is minimized when consumers do not attach much weight to prospective gains and losses, lifetime utility is higher for  $\gamma = 0.6$  than for  $\gamma = 0.9$ .





Source: Author's analyses.

Note: Anticipated and actual income paths are  $\mathbf{w} = (52,48)$  and  $\mathbf{w}' = (48,48)$ , respectively. Gain-loss utility satisfies  $\mu(x) = \eta x^{\theta}$  for all  $x \ge 0$  and  $\mu(x) = -\eta \lambda (-x)^{\theta}$  for all x < 0, with  $\eta = 10$ ,  $\lambda = 2.25$ , and  $\theta = 0.88$ .

Since the person has received bad news regarding present income, she has to give up her plan of consuming  $\bar{w} = 50$  in both periods. Since  $\gamma < 1$ , it is optimal to delay the loss and consume  $\mathbf{c} = (50, 46)$ . But this entails borrowing, which is not feasible. Because utility is convex for a sure loss,  $c_1 \in (46, 50)$ , the consumer prefers concentrating the loss in one period if  $\gamma$  is sufficiently high, as is the case on the solid line, with  $\gamma = 0.9$ . This type takes the full loss immediately,  $c'_1 = 46$ , and saves enough to consume  $c'_2 = \bar{w} = 50$  in the second period. In other words, the aspiration to carry through future plans enhances savings for the liquidity-constrained consumer.

Panel (b) in Figure 4.1 presents a second mechanism through which reference dependence can increase savings in the presence of a liquidity constraint. It shows the optimal revision for consumers who anticipate lower income in the first period than in the second period,  $w_1 < w_2$ . They plan to consume  $\mathbf{c}_{RD}^*(\mathbf{w}, \mathbf{c}_{RI}^*(\mathbf{w}), \gamma) = \mathbf{w} = (w_1, w_2)$ . The dashed line is associated with a relatively low weight attached to future gains and losses. As in Panel (a), the liquidity constraint binds and the optimal revision is to take losses and modest income gains immediately, resulting in hand-to-mouth consumption,  $c_1 = w'_1$ , even when gains are large enough to raise first-period income above second-period income,  $w'_1 > w_2$ . The consumer saves some of the windfall for future periods only when the gain is very large. As a result, reference-dependent consumption is strictly larger than reference-independent consumption for all  $w'_1 > w_2$ .

This is not the case for the dotted line, with a relatively high weight attached to prospective gains and losses. If  $\gamma$  is high, the income level above which it is optimal to spread the gain over time is lower than it is for reference-independent consumers. Intuitively, because lower consumption was planned for the first period than for the second period, the reference level for first-period consumption is lower as well. It is therefore optimal to save when facing a small windfall, even when the first period is associated with lower consumption than the second period. For larger windfalls, gains are large enough to render differences in gain-loss utility from the two periods negligible. In that case, since  $\gamma < 1$  and gains in the first period have a higher weight than gains in the second period, consumers save less than reference-independent consumers.

The next two propositions formalize these results. The first proposition considers a person who anticipates present income to be higher than future income,  $w_1 > w_2$ . As illustrated in Panel (a) of Figure 4.1, if she attaches sufficient weight to prospective gain-loss utility so that utility is convex for all losses  $x \in (L_{\gamma}, 0)$ , and faces a moderate income loss,  $w'_1 \in (\underline{w}, \overline{w})$ , she will save the amount she anticipated to save, increasing savings compared with the reference-independent case:

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**PROPOSITION 1.** Assume existence of a loss  $-1 < L_{\gamma} < 0$  such that  $DS(L_{\gamma}, \gamma) = 1$ . Suppose anticipated income in the present period was  $w_1 > w_2$ , but that actual income is  $w'_1 \neq w_1$ . If future income satisfies  $w_2 \ge \phi w_1$ , with  $\phi \equiv (2 - L_{\gamma})/(2 - 3L_{\gamma}) < 1$ , then income levels  $\underline{w} < w_2$  and  $\overline{w} = (w_1 + w_2)/2 > w_2$  exist such that the revised plan  $\mathbf{c}_{RD}^*(\mathbf{w}', \mathbf{\bar{w}}, \gamma)$  solving Equations (10)–(13) satisfies

$$c_{RD,1}^{*}(\mathbf{w}', \mathbf{\bar{w}}, \gamma) \begin{cases} = c_{RI,1}^{*}(\mathbf{w}') \forall w_{1}' \leq \underline{w} \\ < c_{RI,1}^{*}(\mathbf{w}') \forall w_{1}' \in (\underline{w}, \overline{w}) \\ > c_{RI,1}^{*}(\mathbf{w}') \forall w_{1}' \geq \overline{w}. \end{cases}$$

The second proposition considers a person who anticipates present income to be lower than future income,  $w_1 < w_2$ , and hence plans to consume less in the present. As illustrated in Panel (b) of Figure 4.1, if the weight attached to prospective gain-loss utility is sufficiently large, this person may prefer to spread income gains over the two periods and save. She will do so at income levels for which reference-independent consumers prefer to live hand-to-mouth, again increasing savings compared to the reference-independent case:

**PROPOSITION 2.** Suppose anticipated income in the present period was  $w_1 < w_2$ , but that actual income is  $w'_1 \neq w_1$ . If  $\gamma > \mu'((w_2 - w_1)/w_1)/\mu'_+$ , with  $\mu'_+ = \lim_{x\downarrow 0} \mu'(x)$ , then income levels  $\{w^*, w^{**}\}$  satisfying  $w_1 < w^* < w_2 < w^{**}$  exist such that the revised plan  $\mathbf{c}'_{RD}(\mathbf{w}', \mathbf{w}, \gamma)$  solving Equations (10)–(13) satisfies

$$c_{RD,1}^{*}(\mathbf{w}', \mathbf{w}, \gamma) \begin{cases} = c_{RI,1}^{*}(\mathbf{w}') \ \forall \ w_{1}' \leq w^{*} \\ < c_{RI,1}^{*}(\mathbf{w}') \ \forall \ w_{1}' \in (w^{*}, w^{**}) \\ > c_{RI,1}^{*}(\mathbf{w}') \ \forall \ w_{1}' > w^{**}. \end{cases}$$

See Appendix B for a proof of both propositions.

# 5. CASE 3: ENDOGENOUS REFERENCE POINTS AND STOCHASTIC INCOME

#### **Expectations-Based Reference Points when Consumption Is Stochastic**

The previous section assumed that consumers do not anticipate the possibility of an income shock when making their plans. This assumption might be somewhat disputable, in part because borrowing restrictions will apply in particular for people with volatile incomes, as banks may fear more defaults among this group. This section therefore analyzes optimal savings when consumers anticipate the income risk. Specifically, when making their plans, consumers do not know yet how much they will earn in the two periods, and they find out just prior to the actual consumption-saving decision. Thus, there is risk in the planning stage, but at the time of actual consumption, all risk will have resolved.

For reference-independent consumers, this means that the optimum is equivalent to the reference-independent optimum in previous sections. These consumers' sole savings motive is to spread consumption over time. Since there is no risk regarding future income when choosing how much to save, precautionary savings motives do not play a role. Assuming perfect financial markets, KR shows that reference-dependent consumers save *less* than the reference-independent optimum, since news about immediate consumption carries a higher utility weight. I will later show that incorporating liquidity constraints yields the opposite prediction: reference-dependent consumers may save *more* than reference-independent consumers, because the anticipation of a liquidity constraint increases expected consumption–and hence the reference point–for future compared to early periods.

Henceforth, consumers' disposable income in period  $t \in \{1,2\}$ ,  $W_t$ , is independent and identically distributed (i.i.d.) with a cumulative distribution  $F(\cdot)$  and density  $f(\cdot)$ . Upper cases refer to the stochastic variable before the realization of the risk, and lower cases refer to realized values. Income is stochastic due to temporary price shocks, unpredictable expenditures, or income shocks, and these shocks do not affect the marginal productivity of capital. Both buffer stock models and the KR framework model risk in this way. To stay close to this literature, this paper does not analyze other types of risk such as asset or capital income risk.<sup>13</sup>

The consumer makes her plans without perfect foresight of income during an initial period t = 0. At that stage, she only knows the income *distribution* in periods  $t \in \{1,2\}$ . Let  $\mathbf{W} = (W_1, W_2)$  represent income before the realization of risk. The consumption plan made at t = 0 is represented by a distribution of beliefs,  $\mathbf{B}(\cdot) = (B_1(\cdot), B_2(\cdot))$ , meaning a contingency plan in which  $B_1(\mathbf{w}) \le w_1$  and

<sup>&</sup>lt;sup>13</sup>This is not without loss of generality. In expected utility frameworks, prudent consumers (that is, those with convex marginal consumption utility) have a precautionary motive to save. These precautionary savings are a self-insurance strategy to cope with income risk. They require the availability of a risk-free buffer stock. When asset or capital income risk dominates risk in income  $W_t$ , a mean-preserving spread of risk induces consumers to save less rather than more (Gunning 2010).

 $B_2(\mathbf{w}) = w_1 + w_2 - B(\cdot)$  are defined as the planned consumption level in the first and second periods, given realized incomes  $\mathbf{w} = (w_1, w_2)$ . To isolate the effect of risk at the planning stage, the income risk regarding both periods is resolved at t = 1.<sup>14</sup>

Once the consumer knows how much she will earn in both periods, she decides to consume  $c_1 \le w_1$  in the first period, save  $w_1 - c_1$  in a risk-free asset with zero returns for the second period, and consume  $c_2 = w_1 + w_2 - c_1$  in the second period. Comparing first-period consumption,  $c_1$ , with the contingency plan,  $B_1(\cdot)$ , yields contemporaneous gain-loss utility. Comparing second-period consumption,  $c_2$ , with the contingency plan for future consumption,  $B_2(\cdot)$ , yields prospective gain-loss utility. There will be no updating of beliefs, and hence no gain-loss utility, in the final period, t = 2.

For tractability, let us abstract from diminishing sensitivity, Assumption (A4), and assume a linear gain-loss utility function with the following functional form:  $\mu(x) = \eta x$  for  $x \ge 0$  and  $\mu(x) = \eta \lambda x$  for x < 0, where  $\eta$  represents the weight attached to gain-loss utility, and  $\lambda > 1$  the degree of loss aversion. Assumptions (A1)–(A3) for monotonicity and loss aversion over small and large stakes are still satisfied under this specification.

Assuming that the integral below exists, gain-loss utility from changing a plan is defined as

$$N(c;B(\cdot)) = \int \mu(\log(c) - \log(B(W)))dF(W).$$
(14)

To illustrate this definition of gain-loss utility, consider a consumer who expects to consume either 50 or 100, both with probability 1/2. The gain or loss from comparing these two outcomes is log(100) - log(50) = log(2). If she learns in period 1 that she will consume 50, she experiences a loss relative to the good outcome of consuming 100, which would have occurred with probability 1/2. Gain-loss utility is hence  $-1/2\eta\lambda \log(2)$ . If instead she learns that consumption will be 100, she experiences a gain relative to the worst-case scenario of consuming 50, which would have occurred with probability 1/2 as well. Gain-loss utility is  $1/2\eta \log(2)$ . Hence, the probability that a gain or loss occurs determines its decision weight.<sup>15</sup>

<sup>&</sup>lt;sup>14</sup>This study does not discuss the effect of risk resolved in period 2. KR show that without liquidity constraints, period 2 income risk induces precautionary savings. In contrast to buffer stock models (Deaton 1991; Carroll 1997), this is not because consumers are prudent, resulting in a second-order savings motive, but because they are loss averse, creating a first-order savings motive. Introducing liquidity constraints would not yield new insights here. As in the deterministic case, consumption at relatively low levels of period 1 income is hand-to-mouth. This result is qualitatively similar to predictions by the buffer stock model with reference-independent preferences and a liquidity constraint.

<sup>&</sup>lt;sup>15</sup>Gain-loss utility could always include a probability weight  $\pi(p)$ . Probability weighting is a core feature of rank-dependent utility and cumulative prospect theory (Tversky and Kahneman 1992). If people overweight the probability that negative events will occur, they may become more risk averse, while if they underweight this probability, they may become more risk seeking. Probability weighting may therefore confound the effects of loss aversion. Therefore, this paper studies the effect of reference dependence, and in particular loss aversion, in isolation of probability weighting.

Regarding the formation of plans, this section considers two types of reference-dependent consumers. The first type is a "naive" consumer who plans at t = 0 for the reference-independent optimum,  $c_{RI,1}^*(\mathbf{w}) = \min\{w_1, \bar{w}\}$ , where  $\bar{w}$  is average income from the two periods. Formally, optimal consumption for a naive reference-dependent (RD,N) consumer is defined by the following optimization problem:

$$\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{c}_{RI}^*(\cdot), \gamma) = \max_{c_1 \le w_1, c_2} m(c_1) + N(c_1; c_{RI,1}^*(\cdot)) + \gamma N(c_2; c_{RI,2}^*(\cdot)) + m(c_2),$$
  

$$c_2 = w_1 + w_2 - c_1, \text{ and}$$
  

$$\mathbf{c}_{RD,N}^*(\mathbf{w}, \mathbf{c}_{RI}^*(\cdot), \gamma) \equiv \arg\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{c}_{RI}^*(\dot{)}, \gamma).$$

Optimal consumption given income **w** is not necessarily equivalent to the planned level given that income level,  $\mathbf{c}_{RI}^*(\mathbf{w})$ . The second type therefore concerns a "sophisticated" consumer (RD,S) who can make only plans that she is committed to carry through. In other words, optimal consumption solves the following problem:

$$\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{B}(\cdot), \gamma) = \max_{c_1 \le w_1, c_2} m(c_1) + N(c_1; B_1(\cdot)) + \gamma N(c_2; B_2(\cdot)) + m(c_2)$$
$$c_2 = w_1 + w_2 - c_1, \text{ where the solution satisfies}$$
$$\mathbf{c}^*_{RD,S}(\mathbf{w}, \mathbf{B}(\cdot), \gamma) \equiv \arg\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{B}(\cdot), \gamma) = \mathbf{B}(\mathbf{w}) \ \forall \ \mathbf{w} \in \mathbf{W}.$$

A sophisticated consumer optimizes  $U(\mathbf{c}; \mathbf{w}, \mathbf{B}(\cdot), \gamma)$ , life-cycle utility, given the contingency plan to

consume  $\mathbf{B}(\cdot)$ , and this optimum does not deviate from that contingency plan given realized income w. KR uses this latter condition to define a personal equilibrium (see Appendix A and Kőszegi and Rabin (2006) for a more detailed discussion of this equilibrium concept).

#### Implications of Liquidity Constraints for Stochastic Reference Points

This section compares reference-dependent and reference-independent consumption plans. Risk at the planning stage does not affect reference-independent behavior, since the risk is resolved prior to the consumption-savings decision. Without liquidity constraints, the reference-independent optimum is to smooth consumption,  $\mathbf{c}_{RI}^*(\mathbf{w}) = \bar{\mathbf{w}}$ . KR shows that without a liquidity constraint, reference dependence leads to higher consumption in early periods. Given the plan to consume  $\mathbf{c}_{RI}^*(\mathbf{w}) = \bar{\mathbf{w}}$  for all  $\mathbf{w} \in \mathbf{W}$ , marginal lifetime utility evaluated at  $\mathbf{c} = \bar{\mathbf{w}}$  is

$$m'(\bar{w}) + N'(\bar{w}; c^*_{RI,1}(\cdot)) - m'(\bar{w}) - \gamma N'(\bar{w}; c^*_{RI,2}(\cdot)) = N'(\bar{w}; c^*_{RI,1}(\cdot))(1-\gamma) > 0,$$

for all  $\gamma < 1$ , where we can write the gain-loss utilities from both periods as one term since  $c_{RI,2}^*(\cdot) = c_{RI,1}^*(\cdot)$ . Thus, when consumers are able to borrow and attach more weight to changes in plans regarding immediate consumption, they have an incentive to increase  $c_1$  above the planned level,  $\bar{w}$ . Naive consumers without liquidity constraints will hence overconsume relative to the reference-independent optimum.

A sophisticated consumer anticipates her incentive to overconsume in the first period and plans for higher consumption, because she will only make plans that she is committed to carry through. Assuming, like KR, that plans regarding both first-period and second-period consumption,  $B_1(\mathbf{w})$  and  $B_2(\mathbf{w})$ respectively, are strictly increasing in total income, the solution is characterized by the following set of first-order conditions:

$$m'(B_1(\mathbf{w})) + N'(B_1(\mathbf{w}); B_1(\cdot)) = m'(B_2(\mathbf{w})) + \gamma N'(B_2(\mathbf{w}); B_2(\cdot)).$$

Because  $\gamma < 1$ , it is optimal to consume more in period 1 than in period 2. Thus, without liquidity constraints, also sophisticated reference-dependent consumers will save *less* than reference-independent consumers.

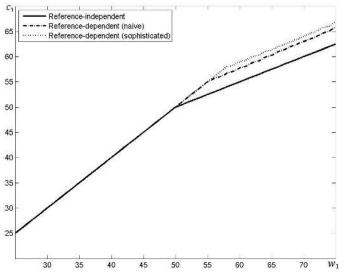
This result changes when introducing a liquidity constraint. Figure 5.1 illustrates optimal consumption in the first period as a function of realized income in the first period, assuming uniformly distributed income  $W_t \sim U[25,75]$  for  $t \in \{1,2\}$  and realized income for the second period  $w_2 = 50$ . The figure shows the reference-independent optimum (the solid line), the naive reference-dependent optimum (the dashed line), and the sophisticated optimum (the dotted line). Panel (a) is based on a relatively low weight attached to future gain-loss utility, whereas Panel (b) attaches a relatively high weight to future gains and losses.

In both panels, the reference-independent optimum is to consume  $c_{RI,1}^*(\mathbf{w}) = \min\{w_1, \bar{w}\}$ . At low levels of early income, the reference-dependent optimum is equivalent as both types consume their income  $w_1$ . At higher levels of early income, reference-dependent consumption is either higher or lower, depending on the weight attached to future gains and losses. In Panel (a),  $\gamma$  is relatively low, and reference dependence increases consumption, reducing optimal savings. In Panel (b),  $\gamma$  is relatively high, and reference dependence reduces optimal consumption, increasing the optimal level of savings.

Intuitively, in the presence of a liquidity constraint, a consumer anticipates income realizations for which she cannot smooth consumption since the liquidity constraint will bind. For these income realizations, consumption is higher in the second period than in the first period, raising the expected level of future consumption. This increases consumers' marginal gain-loss utility for the future period, creating an incentive to consume less in the first period and save more for future consumption if the weight attached

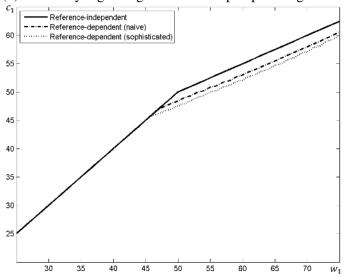
to future gain-loss utility,  $\gamma$ , is sufficiently high. Differences are more pronounced for sophisticated compared with naive consumers, because the former anticipate the deviation from the reference-independent optimum, which influences their expectations and hence optimal consumption.

Figure 5.1 First-period consumption as a function of first-period income without perfect foresight  $W_t \in [25,75]$ ,  $w_2 = 50$ ,  $\mu(x) = \eta x$  for x > 0 and  $\mu(x) = -\eta \lambda x$  with  $\eta = 10$  and  $\lambda = 2.25$ 



(a) Relatively low weight attached to prospective gains and losses,  $\gamma = 0.65$ 

(b) Relatively high weight attached to prospective gains and losses,  $\gamma = 0.95$ 



Source: Author's analyses.

The following proposition generalizes these results to any continuous i.i.d. distribution with a sufficiently high weight attached to future gain-loss utility,  $\gamma > \gamma^*$ .

**PROPOSITION 3.** Suppose that income **W** is continuous and independent and identically distributed (*i.i.d.*), and that a consumer learns at t = 1 that she will earn **w**. Parameters  $\gamma^* < 1$  and  $\varepsilon^* > 0$  exist such that

$$c_{RD,S,1}^{*}(\mathbf{w}, c_{RD,S,1}^{*}(\cdot), \gamma) < c_{RD,N,1}^{*}(\mathbf{w}, c_{RI,1}^{*}(\cdot), \gamma) < c_{RI,1}^{*}(\mathbf{w})$$

*for all*  $\gamma \geq \gamma^*$  *and*  $w_1 \geq \bar{w} - \varepsilon^*$ *.* 

### 6. DISCUSSION

Many laboratory experiments and observational studies provide empirical support of the theory that preferences are reference dependent. These studies are, however, restricted to settings with high bank penetration, where consumers have more access to credit and other insurance strategies than consumers in low-income countries. The literature on reference-dependent preferences has implicitly assumed perfect access to credit, making it less applicable to millions of consumers with limited access to formal and informal insurance strategies. As a result, it is unclear how reference dependence affects intertemporal choice in settings without well-functioning financial markets, and how improved access to credit and other risk-coping strategies affects reference-dependent consumption patterns.

This paper therefore investigated how reference dependence affects intertemporal choice in a context of liquidity constraints. Two different models of reference-dependent preferences were analyzed: a reference-dependent model for intertemporal choice in which gains and losses compared with exogenous reference points influence well-being, and the Kőszegi and Rabin (2009) model with endogenous expectations-based reference points in which utility is derived from both consumption and changes in beliefs. The former has the advantage that it is tractable and easy to interpret as long as it is applied to small stakes, but it misses one key ingredient, namely, how people form their reference points. The latter model with expectations-based reference points has the advantage that it specifies a unique reference point, thereby unifying a growing literature on prospect theory and other models for reference dependence. Moreover, it integrates some key features of reference-dependent models in a standard framework for intertemporal choice. Thus, the framework can be applied to and tested for behavior outside the lab.

The study introduced a liquidity constraint in both models to predict consumption-savings behavior in the presence of missing credit markets. Analyses of both models demonstrated that reference dependence can increase liquidity-constrained savings while having the opposite effect in the absence of liquidity constraints. Two mechanisms explain this asymmetry. First, when liquidity constraints force consumers to take part of a loss in early periods, they may decide to take the full loss and increase savings compared with the reference-independent case. For instance, if a consumer plans to save and smooth consumption, but faces a moderate income loss in early periods, it is optimal to consume the planned level in early periods and borrow to concentrate the full loss in future periods. However, liquidity constraints prevent the consumer from doing so. By diminishing sensitivity, the best feasible alternative is then to take the full loss in early periods and save the planned amount for future periods.

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Second, reference dependence can increase liquidity-constrained savings since anticipating a liquidity constraint itself may influence expected consumption and thus an expectations-based reference point. A person who knows that a binding liquidity constraint will prevent her from smoothing consumption expects lower income in early periods than in future periods. As a result, the planned level–and hence the reference point–is relatively high in future periods. This increases the marginal utility from future consumption, which in turn creates a savings motive among consumers without access to credit. In a framework with deterministic income, diminishing sensitivity is necessary to obtain this result. However, when forming their beliefs, if people anticipate that income is stochastic and that they will learn how much their income will be just prior to consumption in the early period, diminishing sensitivity is not necessary to acquire this result.

For this anomaly to occur in practice, prospective gains and losses need to carry a sufficiently high utility weight. A direction for future research is to empirically test the propositions in this paper and verify how much attention consumers in low-income settings pay to future gains and losses. Having said that, the main aim of this paper was to demonstrate that one cannot directly apply existing models for reference dependence to intertemporal choice in low-income settings, where consumers have limited access to credit and other risk-coping strategies. One should account for the finding that reference dependence can affect savings in opposite directions depending on whether consumers face liquidity constraints. The propositions in this study can serve as a first guide toward testing models of reference-dependent preferences outside the laboratory, in settings where such models have rarely been tested.

## **APPENDIX A: SOLUTION CONCEPT FOR ENDOGENOUS REFERENCE POINTS**

This appendix derives endogenous reference points for the model presented in Section 4. This model specifies the reference point as a rational expectation or belief held prior to consumption. In this way, the reference point can be interpreted as an endogenous consumption plan. The appendix will first discuss how Kőszegi and Rabin (2009) have modeled the formation of these consumption plans, and will then derive endogenous consumption plans in a context of liquidity constraints.

As discussed in the main text, two mechanisms jointly determine a plan. To start, consumers cannot make plans ex ante to which they cannot commit ex post. Plans satisfying this criterion are labeled a "personal equilibrium." Next, out of all personal equilibria, the consumer chooses the plan that optimizes utility ex-ante. This "preferred personal equilibrium" provides a unique reference point and is equivalent to the reference-independent optimum given the preference structure in Section 4.

Formally, in a personal equilibrium, it is optimal to consume the initially planned level. This rules out any dynamic inconsistencies in which the consumer sets overly pessimistic plans to "surprise" herself with windfall utility from higher consumption. For instance, if a consumer plans zero consumption, any outcome would be considered a gain and create utility.<sup>16</sup> Therefore, a personal equilibrium optimizes total lifetime utility at t = 1 and is consistent with a consumer's initial plan,  $\mathbf{c}^*(\mathbf{w}, \mathbf{b}, \gamma) = \mathbf{b}$ :

**Personal Equilibrium** A personal equilibrium plan  $\mathbf{b} \in PE$  solves

$$\max_{\mathbf{c}} U(\mathbf{c}; \mathbf{w}, \mathbf{b}, \gamma), where$$
$$\mathbf{c}^{*}(\mathbf{w}, \mathbf{b}, \gamma) \equiv \arg\max_{\mathbf{c}} U(\mathbf{c}; \mathbf{w}, \mathbf{b}, \gamma) \text{ satisfies } \mathbf{c}^{*}(\mathbf{w}, \mathbf{b}, \gamma) = \mathbf{b}.$$

Figure A.1 illustrates how the personal equilibrium condition restricts a consumer's plans. In Panel (a), the vertical axis indicates  $c_1^*(\mathbf{w}, \mathbf{b}, \gamma)$ , the optimal level of period 1 consumption. This optimum depends on a consumer's initial plan,  $b_1$ , which is indicated on the horizontal axis and is a personal equilibrium if and only if it equals the planned level,  $c_1^*(\cdot) = b_1$ . Thus, in a personal equilibrium, the optimum intersects the 45-degree line. This imposes an upper and lower bound for consumption plans. When the plan is to consume more (less) than the upper bound  $\overline{b}$  (lower bound  $\underline{b}$ ), optimal consumption  $c_1^*(\cdot)$  is lower (higher).<sup>17</sup> As a result, the range of personal equilibrium plans is restricted to  $b_1 \in (\underline{b}, \overline{b})$ .

$$\lim_{c_1\uparrow \overline{b}} U'(\mathbf{c};\mathbf{w},\overline{\mathbf{b}},\gamma) = 0 \Leftrightarrow m'(\overline{b})(1+\eta\lambda) = m'(w_1+w_2-\overline{b})(1+\gamma\eta).$$

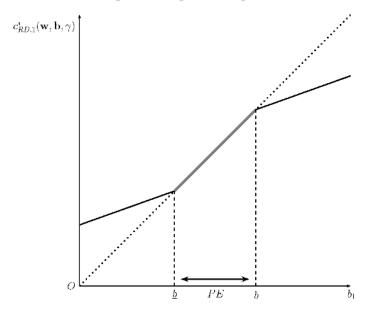
Consuming less than planned creates a loss in period 1 - with weight  $\eta\lambda$  - and an equally-sized gain in period 2 - with weight  $\gamma\eta$ .

<sup>&</sup>lt;sup>16</sup>To my knowledge, the literature does not document evidence of such pessimism. In fact, consumers are often overconfident (for a review of the literature, see Caliendo and Huang 2008).

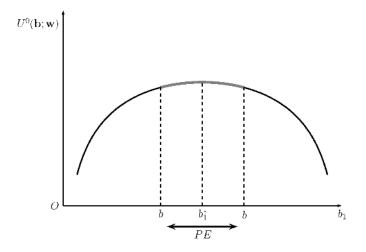
<sup>&</sup>lt;sup>17</sup>Formally,  $\mathbf{\overline{b}} = (\overline{b}, w_1 + w_2 - \overline{b})$  is the plan at which marginal utility is zero when consumption is an infinitesimal amount lower:

### Figure A.1 Solution concept with an endogenous reference point

(a) Illustration of personal equilibrium plans



(b) Illustration of a preferred personal equilibrium (PPE)



Source: Author's analyses.

Given the preference structure in Section 4, this set of personal equilibrium plans is nonempty and convex since <u>b</u> is always lower than  $\overline{b}$ . The personal equilibrium is not necessarily unique. KR therefore restricts the solution to a "*preferred* personal equilibrium" (PPE). Indicating  $U^0(\mathbf{b}; \mathbf{w})$  as lifetime utility evaluated at t = 0, an ex ante period in which a consumer makes her plans, the PPE is defined as follows:

Likewise,  $\underline{\mathbf{b}} = (\underline{b}, w_1 + w_2 - \underline{b})$  is defined as the planned level at which marginal utility is zero for infinitesimal higher consumption:  $\lim_{c_1 \downarrow \underline{b}} U'(\mathbf{c}; \mathbf{w}, \underline{\mathbf{b}}, \gamma) = 0 \Leftrightarrow m'(\underline{b})(1 + \eta) = m'(w_1 + w_2 - \underline{b})(1 + \gamma \eta \lambda).$ 

Consuming more than planned creates a gain in period 1-with weight  $\eta$ -and an equal-sized loss in period 2-with weight  $\gamma\eta\lambda$ .

#### **Preferred personal equilibrium (PPE)** The PPE $\mathbf{b}^*(\mathbf{y})$ solves

$$\max_{b_1 \in [\underline{b}, \overline{b}]} U^0(\mathbf{b}; \mathbf{w}) = \max_{b_1 \in [\underline{b}, \overline{b}]} m(b_1) + m(w_1 + w_2 - b_1).$$

Ex ante utility does not include gain-loss utility since the consumer has perfect foresight.

Panel (b) of Figure A.1 indicates the PPE. The figure illustrates ex ante utility,  $U^0(\mathbf{b}; \mathbf{w})$ , as a function of the initial plan for period 1 consumption,  $b_1$ , and highlights the range of personal equilibrium plans from Panel (a). The personal equilibrium plan  $\mathbf{b}^*$  optimizes ex-ante utility and is therefore the PPE. KR shows that Condition (8) for intertemporal loss aversion implies perfect smoothing,  $\mathbf{b}^* = \bar{\mathbf{w}}$ .<sup>18</sup> Thus, without liquidity constraints, reference-dependent consumption plans are equivalent to the reference-independent optimum.

I find the same result when introducing a liquidity constraint,  $c_1 \le w_1$ , as long as a consumer is aware of this liquidity constraint when making plans. In that case, the liquidity constraint restricts the set of feasible personal equilibrium plans to  $b_1 \le w_1$ . Defining <u>b</u> and <u>b</u> as above, with <u>b</u> < <u>b</u>, the range of personal equilibrium plans is

$$b_1 \in PE \Leftrightarrow b_1 \in \left[\min\{w_1, \underline{b}\}, \min\{w_1, \overline{b}\}\right].$$

A consumer with relatively high income in the first period,  $w_1 \ge \bar{w}$ , can carry through the unconstrained PPE plan,  $\bar{w}$ , but this plan is unfeasible for consumers with lower income,  $w_1 < \bar{w}$ . For them, the maximum feasible level optimizes ex ante utility and is hence the PPE,  $w_1$ , yielding the reference-independent optimum

$$\mathbf{c}_{RD}^*(\mathbf{w},\mathbf{b}^*,\boldsymbol{\gamma}) = \mathbf{b}^* = \mathbf{c}_{RI}^*(\mathbf{w})$$

where  $c_{RI,1}^*(\mathbf{w}) = \min\{w_1, \bar{w}\}$  and  $c_{RI,2}^*(\mathbf{w}) = \max\{w_2, \bar{w}\}$ . Thus, if  $\gamma \ge 1/\lambda$ , the reference-dependent plan is to consume the reference-independent optimum.

<sup>&</sup>lt;sup>18</sup>If Condition (8) is satisfied, consumers attach relatively high weight to future losses,  $\gamma \ge 1/\lambda$  and  $\overline{b} \ge \overline{w}$ . As a result,  $\overline{w}$  is a personal equilibrium. If  $\gamma < 1/\lambda$ , then  $\overline{b} < \overline{w}$  and  $\overline{w}$  is not a personal equilibrium.

#### **APPENDIX B: PROOFS**

### **PROOF LEMMA 1**

To see whether instantaneous utility for a loss x < 0 is convex, note that

$$\frac{\partial^2 u(c_t, x_t; \gamma_t)}{\partial c_t^2} = (\log(c_t))''(1 + \gamma_t \mu'(x_t)) + [(\log(c_t))']^2 \gamma_t \mu''(x_t),$$
  

$$\propto \gamma_t \mu''(x_t) - (1 + \gamma_t \mu'(x_t)) > 0,$$
  

$$\Leftrightarrow 1 + \gamma_t \mu'(x_t) < \gamma_t \mu''(x_t),$$
  

$$\Leftrightarrow DS(x_t; \gamma_t) > 1,$$

which is implied by  $(\log(c_t))'' = -((\log(c_t))')^2 < 0$  and the definition for the degree of diminishing sensitivity,  $DS(x_t; \gamma_t)$ , in Equation (9).

Declining diminishing sensitivity to losses, Condition (C2), implies that  $DS(x_t; \gamma^*) > DS(L_{\gamma^*}; \gamma^*) = 1$  if and only if  $L_{\gamma^*} < x_t < 0$ . Further, note that the partial derivative of diminishing sensitivity with respect to  $\gamma_t$  is proportional to

$$|\mu''(x_t)|(1+\gamma_t\mu'(x_t))-\gamma_t|\mu''(x_t)|\mu'(x_t)|=|\mu''(x_t)|,$$

which is strictly positive. As a result,  $DS(x_t; \gamma_t) \ge DS(x_t; \gamma^*) > DS(L_{\gamma^*}; \gamma^*) = 1$  for all  $\gamma_t \ge \gamma^*$  and  $x_t \in (L_{\gamma^*}, 0)$ . This implies convexity and completes the proof.<sup>19</sup>

$$\frac{\partial^2 u(c_t, x_t; \gamma_t)}{\partial c_t^2} = m''(c_t)(1 + \gamma_t \mu'(x_t)) + m'(c_t)^2 \gamma_t \mu''(x_t) \propto m'(c_t)^2 \gamma_t \mu''(x_t) + m''(c_t)(1 + \gamma_t \mu'(x_t)) > 0,$$
  
$$\Leftrightarrow -m''(c_t)(1 + \gamma_t \mu'(x_t)) < m'(c_t)^2 \gamma_t \mu''(x_t) \Leftrightarrow DS(x_t; \gamma_t) > \frac{-m''(c_t)}{m'(c_t)^2}.$$

<sup>&</sup>lt;sup>19</sup>Relaxing the assumption of log utility and instead assuming some concave consumption utility m(c) would imply the following:

The right-hand side of the above inequality is decreasing in  $c_t$  under the decreasing absolute risk aversion (DARA) condition that  $m'''(c_t)m'(c_t) < m''(c_t)^2$ . As a result, if a combination  $\{L_{\gamma^*}, \gamma^*\}$  exists such that  $DS(L_{\gamma^*}; \gamma^*) = -m''(c^*)/m'(c^*)^2$ , then  $DS(x_t; \gamma_t) > m''(c_t)/m'(c_t)^2$  for all  $x_t \in (L_{\gamma}, 0)$  and  $\gamma \ge \gamma^*$ . Findings hence generalize to other utility functions satisfying DARA.

#### **PROOF PROPOSITION 1**

The consumer anticipates earning  $w_1 > w_2$ , which means that the liquidity constraint is not binding and that the planned consumption path is  $\mathbf{b}^* = (\bar{w}, \bar{w}) \equiv \bar{\mathbf{w}}$ . The revised reference-independent optimum does not depend on this planned consumption path and is  $\mathbf{c}_{RI}^*(\mathbf{w}') = \bar{\mathbf{w}}'$  for all  $w'_1 \ge w_2$  and  $\mathbf{c}_{RI}^*(\mathbf{w}') = \mathbf{w}'$  for all  $w'_1 < w_2$ . Reference-dependent lifetime utility depends on the prior plan and reduces to:

$$U(c_1; \mathbf{w}', \bar{\mathbf{w}}) = \log(c_1) + \mu(x_1) + \gamma \mu(x_2) + \log(c_2)$$
 with  $x_t = \log(c_t/\bar{w})$  and  $c_2 = w_1' + w_2 - c_1$ .

First, the proof rules out solutions in which a person combines a loss in one period with a gain in the other period. For  $w'_1 + w_2 \ge \bar{w}$ , the person could take a loss in the first period and a gain in the second period by consuming  $c_1 < \min\{\bar{w}, 2\bar{w}' - \bar{w}\}$ , or a gain in the first period and a loss in the second period by consuming  $c_1 > \max\{\bar{w}, 2\bar{w}' - \bar{w}\}$ .

Regarding the first case, consider a person combining a loss in the first period,  $l_1 = \log(c_1/\bar{w}) < 0$ , with a gain in the second period,  $g_2 = \log(c_2/\bar{w}) > 0$ . Her lifetime utility will be strictly increasing in consumption:

$$U'(c_1; \mathbf{w}', \bar{\mathbf{w}}) = \frac{1 + \mu'(l_1)}{c_1} - \frac{1 + \gamma \mu'(g_2)}{c_2} > \frac{\mu'(l_1) - \mu'(g_2)}{c_1} > 0 \ \forall \ c_1 < \min\{\bar{w}, 2\bar{w}' - \bar{w}\}.$$
(15)

Here, the first inequality follows from  $c_1 < c_2$  and  $\gamma < 1$ . The second inequality follows from Assumption (A3), loss aversion for large stakes, which says that  $\mu'(g) < \mu'(l)$  for all feasible gains  $g > \varepsilon$  and losses l < 0. We can hence rule out any solution satisfying  $c_1 < \min\{\bar{w}, 2\bar{w}' - \bar{w}\}$  if  $w'_1 + w_2 \ge \bar{w}$ .<sup>20</sup>

For the same reason, the revised optimum cannot combine a gain in the first period,  $g_1 = \log(c_1/\bar{w}) > 0$ , and a loss in the second period,  $l_2 = \log(c_2/\bar{w}) < 0$ , if  $\gamma = 1$ :

$$U'(c_1; \mathbf{w}', \bar{\mathbf{w}}) = \frac{1 + \mu'(g_1)}{c_1} - \frac{1 + \mu'(l_2)}{c_2} < \frac{\mu'(g_1) - \mu'(l_2)}{c_1} < 0 \ \forall \ c_1 > \max\{\bar{w}, 2\bar{w}' - \bar{w}\}.$$
(16)

By continuity, a parameter  $\gamma^* < 1$  exists such that  $U'(c_1; \mathbf{w}', \bar{\mathbf{w}}) \le 0$  for all  $\gamma \ge \gamma^*$  and  $c_1 > \max\{\bar{w}, 2\bar{w}' - \bar{w}\}$ . Intertemporal loss aversion combined with loss aversion over large stakes is sufficient to make this assumption. We can hence also rule out any solution satisfying  $c_1 > \max\{\bar{w}, 2\bar{w}' - \bar{w}\}$  if  $w'_1 + w_2 \ge \bar{w}$ .

As a next step, the proof derives the optimal solution at different levels of present income. First, consider what happens in case present income is higher than anticipated,  $w'_1 > w_1$ , so that  $\bar{w} < 2\bar{w}' - \bar{w}$ . The two inequalities above, (15) and (16), imply that  $\bar{w} \le c_1 \le 2\bar{w}' - \bar{w}$  and that consumption exceeds the

<sup>&</sup>lt;sup>20</sup>Because anticipated present income was higher than anticipated future income, the consumer can avoid gains in the future period while incurring a present loss by saving less.

planned level in both periods. By Assumption (A4), lifetime utility is strictly concave. Hence, the optimum satisfies the first-order condition

$$U'(c_1; \mathbf{w}', \bar{\mathbf{w}}) = 0 \Leftrightarrow \frac{1 + \mu'(x_1)}{c_1} = \frac{1 + \gamma \mu'(x_2)}{c_2} \Leftrightarrow c'_{RD,1}(\mathbf{w}', \bar{\mathbf{w}}) > c^*_{RI,1}(\mathbf{w}') = \bar{w}' \,\forall \, w'_1 > w_1.$$

Revised consumption is higher than the reference-independent optimum because future gains weigh less than present gains,  $\gamma < 1$ . This increases marginal utility from present consumption compared with marginal utility from future consumption.

Second, consider what happens in case present income is lower than what the person anticipated, but still higher than future income,  $w_2 \le w'_1 < w_1$ , meaning that a reference-independent consumer smooths consumption. The inequalities (15) and (16) combined with the liquidity constraint imply that  $\max\{2\bar{w}' - \bar{w}, 0\} \le c_1 \le \min\{\bar{w}, w'_1\}$ , that is, the consumer takes a loss in both periods.<sup>21</sup> With consumption in both periods being at least  $2\bar{w}' - \bar{w}$ , and present income satisfying  $w'_1 \ge w_2$ , the most extreme value of this loss is  $-x_t = \log(\bar{w}) - \log(2w_2 - \bar{w})$ . By a Taylor approximation,  $\log(1 + a) \approx a$  for 0 < a < 1, this loss is less extreme than  $L_{\gamma}$ ,

$$|x_t| = \log\left(\frac{\bar{w}}{2w_2 - \bar{w}}\right) = \log\left(1 + \frac{2(w_1 - w_2)}{3w_2 - w_1}\right) \approx \frac{2(w_1 - w_2)}{3w_2 - w_1} \le \frac{2(1 - \phi)}{3\phi - 1} = |L_\gamma|,$$

where the inequality follows from the condition that  $w_2 \ge \phi w_1$ . Substituting its definition for  $\phi$  yields the last equality. Since  $x_t \in (L_{\gamma}, 0)$ , Lemma 1 implies that utility from both periods–and hence lifetime utility–is convex. It is therefore optimal to concentrate the full loss in one period.

If  $\bar{w} \le w'_1 < w_1$ , the optimum is either  $c_1 = 2\bar{w}' - \bar{w}$  (and  $c_2 = \bar{w}$ ) or  $c_1 = \bar{w}$  (and  $c_2 = 2\bar{w}' - \bar{w}$ ). Since  $2\bar{w}' - \bar{w} < \bar{w}$  and  $\gamma < 1$ , it is optimal to consume  $c_1 = \bar{w} > c^*_{RI,1}(\mathbf{w}') = \bar{w}'$  for all  $\bar{w} \le w'_1 < w_1$ .

However, if  $w_2 \le w'_1 < \bar{w}$ , present income drops below the planned consumption level,  $\bar{w}$ , forcing the person to take a present loss. The corner solutions in this case are to consume either  $c_1 = 2\bar{w}' - \bar{w}$  (and  $c_2 = \bar{w}$ ) or  $c_1 = w'_1$  (and  $c_2 = w_2$ ). By convexity, the utility-maximizing consumption level is the option in which the loss is concentrated in one period–in this case, the future. By continuity, we can define some  $\underline{w} < w_2$  for which lifetime utility is convex, so that this is satisfied for all  $\underline{w} < w'_1 < \bar{w}$ , with optimal consumption being  $c_1 = 2\bar{w}' - \bar{w} < c^*_{Rl,1}(\mathbf{w}') = w'_1$ .

Finally, if  $w'_1 < \underline{w}$ , lifetime utility is not convex but concave, and strictly increasing for all  $c_1 < w'_1$ . The parameter at which the second derivative of lifetime utility changes sign,  $\underline{w}$ , will always be strictly above zero since  $\lim_{c\downarrow 0} m'(c) = \infty$ . Therefore, the revised optimum satisfies  $c'_{RD,1}(\mathbf{w}', \mathbf{w}) = w'_1$  for all  $w'_1 \le \underline{w}$ . This completes the proof.

<sup>&</sup>lt;sup>21</sup>Because  $w_1 > w_2$ , revised income will always exceed the left-hand side of this inequality, since  $2\bar{w}' - \bar{w} = w'_1 + w_2 - \bar{w} < w'_1$  if and only if  $w_2 < \bar{w}$ , that is,  $w_2 < w_1$ . Thus, the liquidity constraint will not prevent the person from consuming  $c_1 \ge 2\bar{w}' - \bar{w}$ .

#### **PROOF PROPOSITION 2**

Since the consumer anticipates higher income in the second than in the first period,  $w_2 > w_1$ , she plans to consume  $c_1 = w_1$  and  $c_2 = w_2$ . Thus, reference-dependent lifetime utility reduces to

$$U(c_1; \mathbf{w}', \mathbf{w}, \gamma) = \log(c_1) + \mu(x_1) + \gamma \mu(x_2) + \log(c_2) \text{ with } x_t = \log(c_t/y_t) \text{ and } c_2 = w_1' + w_2 - c_1.$$

If  $w'_1 \le w_1$  and first-period consumption satisfies  $c_1 < w'_1$ , the consumer takes a loss in the first period, and the first derivative is

$$U'(c_1; \mathbf{w}', \mathbf{w}, \gamma) = \frac{1 + \mu'(x_1)}{c_1} - \frac{1 + \gamma \mu'(x_2)}{c_2} > \frac{\mu'(x_1) - \mu'(x_2)}{c_1} \ge 0 \ \forall \ c_1 < w_1' \le w_1,$$

where the first inequality follows from  $\gamma < 1$  and  $c_1 < c_2$ , and the second inequality from global loss aversion. Thus, it is optimal to consume  $c_1 = w'_1$ .

If  $w'_1 = w_2$ , the first derivative when consuming  $c_1 = w'_1$  is

$$U'(c_1; \mathbf{w}', \mathbf{w}, \gamma) = \frac{1 + \mu'(x_1)}{w_2} - \frac{1 + \gamma \mu'_+}{w_2} = \frac{\mu'(x_1) - \gamma \mu'_+}{w_2} < 0$$
(17)

because  $x_1 = (w_2 - w_1)/w_1$  and  $\gamma > \mu'((w_2 - w_1)/w_1)/\mu'_+$ . Thus, it is optimal to consume  $c_1 < w'_1$ . By Assumption (A4), lifetime utility is strictly concave so that an interior optimum  $c_1 < w'_1$  exists.

If  $w_1 > w_2$ , the reference-independent optimum is to consume  $\bar{w}'$  in both periods. The first derivative of lifetime utility given this level of consumption is

$$U'(c_1; \bar{\mathbf{w}}', \mathbf{w}, \gamma) = \frac{1 + \mu'(x_1)}{\bar{w}'} - \frac{1 + \gamma \mu'(x_2)}{\bar{w}'} = \frac{\mu'(x_1) - \gamma \mu'(x_2)}{\bar{w}'}.$$

By (17), this first derivative is strictly negative if  $w'_1 = w_2$ , so that  $c^*_{RD,1}(\mathbf{w}', \mathbf{w}, \gamma) < c^*_{RI,1}(\mathbf{w}')$ . As  $w'_1$  becomes higher, the difference in gain-loss utility  $\mu'(x_1) - \gamma \mu'(x_2)$  will converge to zero by declining diminishing sensitivity. As a result, since  $\gamma < 1$ , some  $w^{**}$  exists such that

$$\frac{\mu'(\log(\bar{w}'/w_1)) - \gamma\mu'(\log(\bar{w}'/w_2))}{\bar{w}'} = 0 \text{ with } \bar{w}' = \frac{w^{**} + w_2}{2}$$

so that  $c^*_{RD,1}(\mathbf{w}', \mathbf{w}, \gamma) > \bar{w}'$  for all  $w'_1 > w^{**}$  and  $c^*_{RD,1}(\mathbf{w}', \mathbf{w}, \gamma) < \bar{w}'$  for all  $w'_1 < w^{**}$ .

### **PROOF PROPOSITION 3**

This proof first analyzes optimal savings for a naive consumer (RD,N) given the plan to consume  $\mathbf{c}_{RI}^*(\mathbf{w})$ , with  $c_{RI,1}^*(\mathbf{w}) = \min\{w_1, \bar{w}\}$ . At t = 1, a consumer with this plan solves

$$\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{c}_{RI}^*(\cdot), \gamma) = \max_{c_1 \le w_1, c_2} m(c_1) + N(c_1; c_{RI,1}^*(\cdot)) + \gamma N(c_2; c_{RI,2}^*(\cdot)) + m(c_2),$$

$$c_2 = w_1 + w_2 - c_1, \text{ and}$$

$$\mathbf{c}_{RD,N}^*(\mathbf{w}, \mathbf{c}_{RI}^*(\cdot), \gamma) = \arg\max_{c_1 \le w_1, c_2} U(\mathbf{c}; \mathbf{w}, \mathbf{c}_{RI}^*(\cdot), \gamma).$$

The marginal utility from  $c_1$  when planning to consume the reference-independent optimum is

$$m'(\bar{w}) + N'(\bar{w}; B_1(\cdot)) - m'(\bar{w}) - \gamma N'(\bar{w}; B_2(\cdot)) \text{ if } w_1 > \bar{w},$$
  
$$m'(w_1) + N'(w_1; B_1(\cdot)) - m'(w_2) - \gamma N'(w_2; B_2(\cdot)) \text{ if } w_1 \le \bar{w}.$$

To find an explicit expression for gain-loss utility  $N(\cdot)$ , define  $P_1(c_1) \equiv P(\min\{W_1, \bar{W}\} \leq c_1)$  as the probability that  $W_1 < c_1$  or, if  $W_1 \geq c_1$ , that  $\bar{W} < c_1$ , with  $\bar{W} = (W_1 + W_2)/2$ . Dropping time subscripts for income (because income is i.i.d.), we obtain

$$P_1(c_1) \equiv P(\min\{W_1, \bar{W}\} \le c_1) = P(W < c_1) + P(W \ge c_1)P(\bar{W} < c_1|W \ge c_1).$$

The derivative of gain-loss utility for consumption in the first period is

$$N'(c_1; B_1(\cdot)) = m'(c)\eta \left( P_1(c_1) + (1 - P_1(c_1))\lambda \right) = m'(c_1)\eta \left(\lambda - (\lambda - 1)P_1(c_1)\right).$$

Consumption is higher than planned for income realizations that occur with probability  $P_1(c_1)$ , yielding a gain for which utility is weighted by  $\eta$ . Consumption is lower than planned for income levels that occur with probability  $1 - P_1(c_1)$ , yielding a loss for which utility is weighted by  $\eta\lambda$ . The derivative of gain-loss utility for consumption in the second period is defined in a similar way:

$$N'(c_2; B_2(\cdot)) = m'(c_2)\eta \left(P_2(c_2) + (1 - P_2(c_2))\lambda\right) = m'(c_2)\eta \left(\lambda - (\lambda - 1)P_2(c_2)\right),$$

where  $P_2(c_2) \equiv P(\max\{W_2, \bar{W}\} \le c_2)$ , the probability that  $W_2 < c_2$  and that  $\bar{W} < c_2$ , so that the consumer gains relative to the plan given these income realizations:

$$P_2(c_2) \equiv P(W < c_2) P(\bar{W} < c_2 | W < c_2).$$

Using these expressions for gain-loss utility  $N(\cdot)$ , the first derivative of life-cycle utility with respect to  $c_1$  reduces to

$$m'(\bar{w})\eta \left(\lambda - (\lambda - 1)P_1(\bar{w}) - \gamma(\lambda - (\lambda - 1)P_2(\bar{w}))\right) \quad \forall w_1 \ge \bar{w},$$
  
$$m'(w_1) \left(1 + \eta \left(\lambda - (\lambda - 1)P_1(w_1)\right)\right) - m'(w_2) \left(1 + \eta \gamma(\lambda - (\lambda - 1)P_2(w_2))\right) \quad \forall w_1 < \bar{w}.$$

The first set of derivatives for  $w_1 \ge \bar{w}$  are strictly positive if  $\gamma = 0$  because  $\lambda > \lambda - 1 > (\lambda - 1)P_1(\bar{w})$ . These derivatives are strictly negative if  $\gamma = 1$ , because  $P_1(\bar{w}) > P_2(\bar{w})$ , that is,

$$P_1(\bar{w}) - P_2(\bar{w}) = P(W < \bar{w})(1 - P(\bar{W} < \bar{w}|W < \bar{w})) + P(W \ge \bar{w})P(\bar{W} < \bar{w}|W \ge \bar{w}) > 0.$$

So if  $w_1 \ge w_2$  and  $\gamma = 1$ , then a consumer has an incentive to reduce  $c_1$  relative to the plan to consume  $\bar{w}$ . If, in contrast,  $\gamma = 0$ , then the consumer will always have an incentive to increase  $c_1$  relative to the plan to consume  $\bar{w}$ . We can hence define a threshold

$$\gamma^*(\bar{w}) \equiv \frac{\lambda - (\lambda - 1)P_1(\bar{w})}{\lambda - (\lambda - 1)P_2(\bar{w})} < 1$$

such that  $c_{RD,N,1}^*(\mathbf{w}, \mathbf{c}_{RI}^*(\mathbf{w}), \gamma) < \bar{w}$  for all  $w_1 > \bar{w}$  and  $\gamma > \gamma^*(\bar{w})$ . By continuity, we can more generally define parameters  $\gamma^*, \varepsilon^* > 0$  so that also the derivatives of life-cycle utility for  $w_1 < \bar{w}$  are positive for all  $w_1 \ge \bar{w} - \varepsilon^*$  and  $\gamma \ge \gamma^*$ :

$$\frac{m'(\bar{w}-\varepsilon^*)}{m'(\bar{w}+\varepsilon^*)} = \frac{1+\eta\,\gamma^*\,(\lambda-(\lambda-1)P_2(\bar{w}+\varepsilon^*))}{1+\eta\,(\lambda-(\lambda-1)P_1(\bar{w}-\varepsilon^*))}.$$

As a result,  $c^*_{RD,N,1}(\mathbf{w}, \mathbf{c}^*_{RI}(\mathbf{w}), \gamma) < \min\{w_1, \bar{w}\}$  for all  $w_1 \ge \bar{w} - \varepsilon^*, \gamma > \gamma^*$ .

For sophisticated consumers, this difference is more pronounced. To see this, note that in a personal equilibrium, if  $\gamma \ge \gamma^*$ , the first set of derivates of life-cycle utility for a naive consumer satisfies

$$m'(c_{RD,N,1}^*) + N'(c_{RD,N,1}^*; c_{RI,1}^*(\cdot)) - m'(c_{RD,N,2}^*) - \gamma N'(c_{RD,N,2}^*; c_{RI,2}^*(\cdot)) = 0 \text{ if } w_1 > \bar{w} - \varepsilon^*.$$

A sophisticated consumer will anticipate that her consumption in the first period will be lower than  $\mathbf{c}_{RI}^*(\cdot)$ for all  $w_1 > \bar{w} - \varepsilon^*$ . For her, life-cycle utility in the naive optimum will satisfy

$$m'(c_1) + N'(c_1; c^*_{RD,N,1}(\cdot)) - m'(c_2) - \gamma N'(c_2; c^*_{RD,N,2}(\cdot)) < 0 \ \forall \ w_1 \ge \bar{w} - \varepsilon^* \text{ if } c_1 = c^*_{RD,N,1}(\mathbf{w}, \mathbf{c}^*_{RI}(\cdot), \gamma).$$

because  $c_{RD,N,1}^*(\mathbf{w}, \mathbf{c}_{RI}^*(\cdot), \gamma) < c_{RI,1}^*(\mathbf{w})$  for all  $w_1 \ge \bar{w} - \varepsilon^*$ , and  $\mathbf{c}_{RD,N}^*(\mathbf{w}) = \mathbf{c}_{RI}^*(\mathbf{w})$  for all  $w_1 < \bar{w}$ , so that  $N'(c_1; c_{RD,N,1}^*(\cdot)) < N'(c_1; c_{RI,1}^*(\cdot))$  and  $N'(c_2; c_{RD,N,2}^*(\cdot)) > N'(c_2; c_{RI,2}^*(\cdot))$ . This completes the proof.

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