# Center for Research on Economic and Social Theory
# CREST Working Paper

## When is Inducing Self-Selection Sub-optimal for a Monopolist?

Stephen W. Salant

February, 1987
Number 87-32

# When is Inducing Self-Selection Suboptimal for a Monopolist?

by

Stephen W. Salant

**Abstract.** Stokey (1979) showed in an intertemporal context that, under reasonable assumptions, price discrimination is never optimal if a monopolist can pre–commit to a price path. This note explores the implications of Stokey's result for the optimality of inducing self–selection in the static quantity and quality contexts of Spence (1980) and Mussa–Rosen (1978). It is shown that Stokey's result carries over to these other contexts under appropriate curvature assumptions. Moreover, even under traditional curvature assumptions, inducing self–selection may be suboptimal. Necessary and sufficient conditions for discrimination to be optimal are derived for the two–type case.

**Address.** Prof. Stephen W. Salant, Department of Economics, University of Michigan, Ann Arbor, MI. 48109, U.S.A.

# When is Inducing Self–Selection Suboptimal for a Monopolist?

Stephen W. Salant

## 1. Introduction

Stokey (1979) considers the case of a monopolist selling in continuous time to heterogeneous consumers with unit demands. The seller can precommit to any price path. By charging declining prices, the monopolist can induce customers with higher valuations to purchase sooner than those with lower valuations. Although price discrimination by inducing self–selection is, therefore, *feasible* in her model, Stokey shows that (under her base–case assumptions) it is never *optimal*. The monopolist is better off precommitting to a fixed price over time—thereby inducing everyone who would earn surplus at that price to purchase at the first opportunity.

This is a striking result—and, in light of the related papers of Spence (1980) and Mussa–Rosen (1978)—a puzzling one. Spence showed that it was optimal for a monopolist to induce self–selection among heterogeneous customers by offering a menu of quantities, each requiring a different outlay. Mussa–Rosen showed in the quality context that non–linear pricing to induce self–selection was likewise optimal. How can second–degree price discrimination be optimal in these latter contexts but inevitably suboptimal in the intertemporal one?

To facilitate a comparison of these models, we extract their essential features and treat them in a common framework. Stokey's result that price discrimination—although feasible—is never optimal is shown to have its counterpart in these other models if analogous curvature assumptions are made. While these are not the assumptions made by Spence or Mussa–Rosen, Stokey's result nonetheless has important and unnoticed implications for the contexts these authors considered.

Stokey's contributions were 1) to show that in the intertemporal case a corner solution inevitably arises and 2) to clarify its economic interpretation—that discrimination

is suboptimal. Under the curvature assumptions of Spence and Mussa–Rosen, a corner solution need not arise. But *whenever* it does, Stokey's point that discrimination is suboptimal remains valid. The note concludes by deriving necessary and sufficient conditions for inducing self–selection to be optimal.

## 2. Stokey's Corner and Its Implications

It is instructive to cast Stokey's problem in a framework which facilitates comparison with the work of Spence and Mussa–Rosen. This process of homogenization inevitably omits some aspects of each model, but the omitted aspects are inessential for our purposes.

We begin with Mussa–Rosen's problem of quality choice when customers have unit demands. We then define two transformations; using one, the static problem of Mussa–Rosen becomes Stokey's continuous–time problem; using the other, their problem is transformed into Spence's problem. For simplicity, we will assume that there are two types of buyers. Let $f_i$ denote the number of buyers of type $i$ for $i = 1, 2$. Assume each customer prefers lower prices and higher quality but that the two types differ in their relative valuations of these attributes. Let $s$ denote quality and assume that $U(s, i) = v_i s$ and $v_2 > v_1$. Hence, the second type cares more about quality. Assume that the cost per unit output to the monopolist of producing a good of quality $s$ is $c(s)$ and that $c'(s) > 0$—it is more expensive to produce a good of higher quality. Assume the monopolist's quality choice is nonnegative. Moreover, since we will be investigating corner cases and relating them to Stokey's corner, let us assume that there is also an *upper bound* to the monopolist's quality choice. Let $s \in [0, 1]$. Since the valuations and cost function can be re–scaled, normalizing the upper bound in this way entails no loss of generality. To make the problem nontrivial, assume $c'(0) < v_2$; to make it tractable, assume $c(0) = 0$.

The monopolist can price–discriminate by offering a menu of goods of two qualities, $s_1$ and $s_2$, priced appropriately. For any quality pair such that $s_2 \geq s_1$, the monopolist can obtain maximum profits by charging a price for $s_1$ high enough to extract all the surplus of the first type. This unavoidably leaves the second type the opportunity to get some positive surplus by purchasing the good intended for the first type and puts an upper limit

2

on how much can be charged for the higher quality ($s_2$) good. Having priced each quality good to extract maximum profits, the monopolist need merely select the two qualities optimally:

$$\max_{s_1, s_2} \Pi = f_1 v_1 s_1 + f_2 [v_2 s_2 - (v_2 - v_1)s_1] - f_1 c(s_1) - f_2 c(s_2).$$

To transform Mussa–Rosen's problem into Stokey's problem, let $s_i = e^{-rt_i}$. Since $s_i \in [0, 1]$, $t_i \in [0, \infty)$. Re–write the foregoing problem in terms of the new decision variable, $t_i$.

$$\max_{t_1, t_2} \Pi = f_1 v_1 e^{-rt_1} + f_2 [v_2 e^{-rt_2} - (v_2 - v_1)e^{-rt_1}] - f_1 c(e^{-rt_1}) - f_2 c(e^{-rt_2}).$$

Since $t_i$ is a monotonic function of $s_i$, the problem is unchanged. But notice that if we interpret $t_i$ as the date when the good is delivered and $r$ as the force of interest we have the objective function of a monopolist who can precommit to delivery dates $i$, ($i = 1, 2$), prices in continuous time, and sells to two types of consumers: Stokey's problem (with two types rather than a continuum).

If the monopolist charges a discounted price of $v_1 e^{-rt_1}$ for delivery at $t_1$ and a discounted price of $v_2 e^{-rt_2} - (v_2 - v_1)e^{-rt_1}$ for delivery at $t_2 \le t_1$, then the type–one individual will be indifferent between not participating and purchasing at $t_1$ while the type–two individual will be indifferent between the two offerings. The undiscounted cost to the monopolist of producing one unit of the good of quality $i$ is $e^{rt_i} c(e^{-rt_i})$. For future reference, denote this undiscounted cost as $k(t)$.

To transform Mussa–Rosen's problem into Spence's problem, let $x_i = c(s_i)$. Then $s_i = c^{-1}(x_i)$. Denote this inverse function as $U(x_i)$.[1] Since $s_i \in [0, 1]$, $x_i \in [0, c(1)]$. Since there is a monotonic relationship between $s_i$ and $x_i$, we can again re–write the original problem in terms of the new decision variable, $x_i$:

$$\max_{x_1, x_2} \Pi = f_1 v_1 U(x_1) + f_2 [v_2 U(x_2) - (v_2 - v_1)U(x_1)] - (f_1 x_1 + f_2 x_2).$$

---

[1] This second transformation is discussed in Tirole (1986), p.37 of chapter 3.

In this version, $x_i$ is interpreted as the quantity of the $i^{th}$ offerring. This is Spence's problem of a monopolist with constant marginal costs (scaled to unity) facing two types of consumers. Their utility functions have the form $u(x_i, i) = v_i U(x_i)$ for $i = 1, 2$.

Stokey assumed that the undiscounted cost of producing a good and delivering it at $t_i$ is a *constant*: $k(t_i) = c$. Hence, the problem she considered is linear in $e^{-rt_i}$ and must achieve its maximum at a corner. The equivalent assumption in Mussa–Rosen would be $c(s_i) = cs_i$ and in Spence $U(x_i) = Ux_i$.

Had these functions been assumed linear, the respective maximization problems would have been linear and the optimum would inevitably have occurred at a corner as in the intertemporal case. Consider, for example, Mussa–Rosen's version of the problem. Differentiating the objective function, we obtain:

$$\frac{d\pi}{ds_2} = f_2(v_2 - c'(s_2))$$
$$\frac{d\pi}{ds_1} = f_1 \left[ v_1 - \frac{f_2}{f_1}(v_2 - v_1) - c'(s_1) \right].$$

If

$$v_1 - \frac{f_2}{f_1}(v_2 - v_1) - c'(1) \geq 0,$$

then $s_1 = 1$; otherwise $s_1 = 0$. In either event, $s_2 = 1$. In short, the monopolist either provides the highest quality output to both types (and hence must charge the identical price for the two offerings) or he provides that quality only to the second type and does not serve the first type (the plausible interpretation of $s = 0$). Price discrimination is never optimal if the cost function is linear.

This proposition clearly generalizes for *any* finite number of types. Facing n–types, the *full* problem of the monopolist is to select $n$ price–quality pairs to:

$$\max_{p_i, s_i} \sum_{i=1}^{n} f_i(p_i - c(s_i)) = \max_{s_i} \max_{p_i} \sum_{i=1}^{n} f_i(p_i - c(s_i))$$

4

$$\text{subject to } s_i \geq 0,$$

$$p_i \geq 0,$$

$$1 - s_i \geq 0,$$

$$v_i s_i - p_i \geq 0,$$

$$v_i s_i - p_i - (v_i s_j - p_j) \geq 0, \text{ for } i = 1, \ldots, n \text{ and } j \neq i.$$

Most authors "max out" the prices as a preliminary step since this inner maximization problem is always linear and easy to solve.[2] What remains in the two–type case is the familiar problem analyzed above.

In principle, one might examine the full problem directly but this is rarely instructive. The one exception is when $c(\cdot)$ is linear. In that case, the foregoing is a simple linear–programming problem and achieves its maximum at $s_i = 0$ or $s_i = 1$ for $i = 1, \ldots, n$. Suppose $s_i = s_j = 1$ for $i \neq j$. Then the monopolist *must* charge the same prices $(p_i = p_j)$ or the customer–type charged the higher price would prefer the other's offering—and the final constraint would be violated. Hence, price discrimination is never optimal in the n–type case if $c(\cdot)$ is linear. Given the transformations defined above, the analogous proposition holds for the quantity and temporal interpretations.

## 3. Extension to the Non–linear Case

Even when the monopolist's cost function (resp. consumers' utility function) is strictly convex (resp. strictly concave), a corner solution might still result. To conclude the analysis, therefore, we re–examine the quality–choice model above and derive necessary and sufficient conditions for price discrimination to be optimal. These conditions are then transformed for use in the intertemporal and quantity interpretations of the model. Although the approach generalizes, we confine ourselves in this section to an analysis of the two–type case; the general case is tedious and uninstructive.

Assume in the quality model that the cost function is at least weakly convex. Then any solution to the Kuhn–Tucker conditions is optimal. There are *in principle* five classes

---

[2]  It is always optimal to set $s_{i+1} \geq s_i$ and $p_i = v_i s_i - s_{i-1}(v_i - v_{i-1}) - \cdots - s_1(v_2 - v_1)$. For details, see Spence,[1980, p.823].

5

of programs in which no price discrimination occurs: when both types of customers are provided the identical quality goods at an interior point or at one of the two boundaries and when one of the two types is served but not the other. Hence if the optimal program $(s_1, s_2)$ has any of the following five characteristics, then price discrimination is suboptimal: (1) $s_1 = s_2 = s \in (0,1)$; (2) $s_1 = s_2 = 0$; (3) $s_1 > 0$ and $s_2 = 0$; (4) $s_1 = s_2 = 1$; and (5) $s_1 = 0$ and $s_2 > 0$. Assumptions which insure that none of these cases arises insure that price discrimination is optimal.

For (1) to occur, $c'(s) = v_2 = v_1 - [f_2/f_1](v_2 - v_1)$, contradicting the assumption that $v_2 - v_1 > 0$. For (2) or (3) to occur, $s_2 = 0$ and $v_2 - c'(0) \le 0$, contradicting $v_2 - c'(0) > 0$. Hence if price discrimination is suboptimal then either $s_1 = s_2 = 1$ or $s_1 = 0$ and $s_2 > 0$. The following condition is sufficient to rule out both possibilities: $c'(0) < v_1 - [f_2/f_1](v_2 - v_1) < c'(1)$. Moreover, given the concavity of the problem if price discrimination is optimal this condition *must* hold.

It is straightforward to translate this condition to the other contexts. In Stokey's version of the problem, recall that $k(t) = e^{rt}c(e^{-rt})$. This implies that $c'(1) = k(0) - k'(0)/r$ and $c'(0) = \lim_{t \to \infty} \{k(t) - k'(t)/r\}$. These terms can be substituted into the condition to restate it in terms of $k(\cdot)$.

Similarly, in Spence's version recall that $c^{-1}(x) = s = U(x)$. In this version, $x$ is interpreted as quantity. When $s = 0$, $x = c(0) = 0$; when $s = 1$, $x = c(1)$. Since $c^{-1}(x) = U(x)$, $c'(0) = 1/c'^{-1}(0) = 1/U'(0)$ and $c'(1) = 1/c'^{-1}((1)) = 1/U'((1))$. These terms can be substituted into the condition to restate it in terms of $U(\cdot)$. The condition not only implies that price discrimination will be suboptimal if $c(\cdot)$, $k(\cdot)$ or $U(\cdot)$ is linear but also characterizes when discrimination is optimal if any of these functions is strictly convex (resp. strictly concave).

Analogous conditions can be derived for the $n$–type although they are more cumbersome. If price discrimination is suboptimal, those consumers who are served must purchase the same offering (and hence will pay the same price). Assumptions insuring that none of the complementary slackness conditions for such corner solutions holds insure that price discrimination is optimal. If the monopolist's optimization problem is quasi–concave, then these conditions are also necessary for price discrimination to be optimal.

6

## 4. Conclusion

This note has pointed out the implications of Stokey's (1979) article on intertemporal price discrimination for the optimality in other contexts of inducing self–selection. If I were to speculate on why these implications have not been drawn previously, two reasons suggest themselves: 1) Stokey did not emphasize the relation of her problem to the literature on static self–selection problems; and 2) her assumption that the monopolist could precommit to a price path has lessened readership in this era of subgame–perfection.

Although a principal contribution of Stokey's (1979) analysis is its far–reaching implications for the optimality of second–degree price–discrimination, it should be pointed out that her analysis is relevant to at least some real–world situations of interest. Many businesses (delivery services such as Federal Express or United Parcel Service) charge different prices for delivery at different subsequent dates. By charging customers when the contract is *signed* rather than when delivery occurs, such sellers routinely induce self-selection of the kind which Stokey's monopolist considers. Of course, such services need to make credible their promises to deliver the package *on time*. They often accomplish this by contractually obligating themselves to pay a refund if delivery is late.

## 5. References

M. Mussa and S.Rosen. (1978) "Monopoly and Product Quality," *Journal of Economic Theory*, **18**, 301–317.

Spence, M. (1980) "Multi–Product Quantity–Dependent Prices and Profitability Constraints," *Review of Economic Studies*, **47**, 821-41.

Stokey, N. (1979) "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, **93**, 355-371.

Tirole, J. (1986) "The Theory of Industrial Organization," MIT, unpublished.

# Recent CREST Working Papers

87-1: Jeffrey K. MacKie-Mason, "Nonlinear Taxation of Risky Assets and Investment, With Application to Mining" September, 1984.

87-2: Jeffrey K. MacKie-Mason, "Sequential Decision Problems and Asymmetric Information" September, 1985.

87-3: Michelle J. White, "Contract Breach and Contract Discharge due to Impossibility: A Unified Theory" July 29, 1987.

87-4: Ted Bergstrom, "Systems of Benevolent Utility Interdependence" May 19, 1987.

87-5: Ted Bergstrom, "A Fresh Look at the Rotten Kid Theorem—and Other Household Mysteries" November, 1986.

87-6: Michelle J. White, "The Corporate Bankruptcy Decision" July, 1987.

87-7: Michelle J. White, "Location Choice and Commuting Behavior in Cities with Decentralized Employment" July, 1987.

87-8: Lawrence E. Blume and David Easley, "Implementation of Walrasian Expectations Equilibria" December, 1985.

87-9: Lawrence E. Blume, "Lexiocographic Refinements of Nash Equilibrium" April, 1986.

87-10: David Lam, "Lorenz Curves, Inequality, and Social Welfare Under Changing Population Composition" June 16, 1987.

87-11: Mark Bagnoli and Naveen Khanna, "Equilibrium with Debt and Equity Financing of New Projects: Why More Equity Financing Occurs When Stock Prices are High" June, 1987.

87-12: Mark Bagnoli and Barton L. Lipman, "Provision of Public Goods: Fully Implementing the Core through Private Contributions" March, 1987.

87-13: Mark Bagnoli and Barton L. Lipman, "Successful Takeovers without Exclusion" August, 1987.

87-14: Mark Bagnoli and Michael McKee, "Controlling the Game: Political Sponsors and Bureaus" May, 1987.

87-15: Mark Bagnoli and Michael McKee, "Can the Private Provision of Public Goods be Efficient?—Some Experimental Evidence" March, 1987.

87-16: Mark Bagnoli, "Non-Market Clearing Prices in a Dynamic Oligopoly with Incomplete Information" January, 1986.

87-17: John Laitner, "Bequests, Gifts, and Social Security" February 28, 1986.

87-18: John Laitner, "Dynamic Determinacy and the Existence of Sunspot Equilibria" May 12, 1986.

87-19: David Lam, "Does a Uniform Age Distribution Minimize Lifetime Wages?" August 12, 1987.

87-20: David Lam, "Assortative Mating with Household Public Goods" April, 1987.

87-21: Jeffrey A. Miron and Stephen P. Zeldes, "Production, Sales, and the Change in Inventories: An Identity that Doesn't Add Up" June 1987.

87-22: Jeffrey A. Miron and Stephen P. Zeldes, "Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories" December, 1986.

87-23: Hal R. Varian, "Differences of Opinion in Financial Markets" March, 1985.

87-24: Roger H. Gordon and Hal R. Varian, "Taxation of Asset Income in the Presence of a World Securities Market" August, 1986.

87–25: Hal R. Varian, "Measuring the Deadweight Costs of DUP and Rent Seeking Activities" November, 1982.

87–26: Hal R. Varian, "Price Discrimination" January, 1987.

87–27: Roger H. Gordon and Hal R. Varian, "Intergenerational Risk Sharing" October, 1985.

87–28: Hal R. Varian, "Three Papers on Revealed Preference" August, 1987.

87–29: Hal R. Varian, "Optimal Tariffs and Financial Assets" April, 1987.

87–30: Jonathan Cave and Stephen W. Salant, "Cartels That Vote: Agricultural Marketing Boards and Induced Voting Behavior" August, 1987.

87–31: Stephen W. Salant and Donald H. Negri, "Pastures of Plenty: When is the Standard Analysis of Common Property Extraction Under Free Access Incorrect?" July 10, 1987.

87–32: Stephen W. Salant, "When is Inducing Self-Selection Sub- optimal for a Monopolist?" February, 1987.

87–33: Stephen W. Salant, "Treble Damage Awards in Private Lawsuits for Price-Fixing" August, 1987.

87–34: Stephen W. Salant and Roy Danchick, "Air Force Academy Attrition: A New Perspective on the College Dropout Problem" August, 1987.

87–35: Stephen W. Salant and Eban Goodstein, "Committee Voting Under Alternative Procedures and Preferences: An Experimental Analysis" April 20, 1987.

87–36: Robert B. Barsky and Jeffrey A. Miron, "The Seasonal Cycle and the Business Cycle" June, 1987.

87–37: Robert B. Barsky, N. Gregory Mankiw, Jeffrey A. Miron and David N. Weil, "The Worldwide Change in the Behavior of Interest Rates and Prices in 1914" July, 1987.

87–38: Jeffrey K. MacKie-Mason, "Taxes, Information and Corporate Financing Choices" April 1986.

DATE DUE | BORROWER'S NAME

**DATE DUE**

| | | | |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |