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# When is the Design of a Manufacturing System Acceptable in the Presence of Uncertainty?

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### ABSTRACT

In the design of a manufacturing system, the design specification is often suggested by a design team. Managers are interested in verifying that this specification will satisfy the production requirements. Because the future production environment will likely differ from the one assumed, it is important to determine in which situations the suggested design becomes unacceptable. This paper suggests an approach that allows determining which uncertain parameters are important and which combinations of these parameters can lead to an unacceptable design. This approach combines several methods, namely, simulation, bootstrapping, and metamodeling. The methodology is explained and illustrated through a stochastic simulated manufacturing system, which includes uncertain parameters related to the arrival and the processing times of jobs. This example shows the conditions under which the system does not meet the requirements.

Keywords: Uncertainty; Design; Robustness and sensitivity analysis; Simulation; Manufacturing systems

### **1. INTRODUCTION**

Designing a manufacturing system is known to be complex. The type and quantity of resources have to be determined, the transportation system has to be efficient enough, the operator must be trained and assigned to the machines in a suitable way, parts routing has to be chosen, buffers location and sizes must be defined, the layout selected, etc. The system designers' aim is to suggest design specifications such that the future system will meet the expected performance requirements, while being as inexpensive as possible. Hence, an important question, which is addressed in this article, is: are these specifications acceptable? Verifying that the total cost is acceptable is possible on the basis of the investment and building costs; meanwhile, decision makers need also to be convinced that the performance of the future system will be good enough to meet the forecasted customer demand. Given the complex dynamic behavior of a manufacturing system, its stochastic features, and the unavailability of precise and certain data about the actual future operating conditions of the manufacturing system available at the time of the design study, this question is difficult to answer.

Simulation analysis is well known to play an important role in evaluating a 'to be' manufacturing system. In the relevant literature and in practical applications, two approaches are based on simulation. The first approach assumes that specific system parameters are unknown (e.g., buffer sizes, number of pallets, and conveyor speed). The aim of such research is either to determine these parameters through a search that optimizes one or more performance criteria using so-called simulation

estimated through the expected value of the simulation outputs, which is generally not relevant in practical cases [5]. The second pitfall is that at the time when the system is designed, the exact system parameters or environmental conditions, which are used in the simulation model, are not exactly known; obviously these parameters and conditions may have a great effect on system performance. An interesting method for performing simulation studies for design purposes with uncertain parameters is given in [6]. That publication explains how fuzzy theory can be combined with simulation; unfortunately, it places less emphasis on the acceptability of a given design and on the significance of the important uncertain factors.

allows designers to determine when a suggested design can be considered to be acceptable, regarding its performance when parameters are uncertain. This new approach is based on simulation, bootstrap estimation, and simulation metamodeling.

optimization approaches ([7] and [14]) or to evaluate the importance of these design parameters on the system

performance using so-called sensitivity analysis ([4] and

[11]). Such an approach, however, does not answer our

initial question since in our case the design team has

already specified the future system. In the second

approach (typically performed in practice) a simulation

model is built from the system specification and the current knowledge about its future operating conditions.

The model is used to predict the future performance.

Unfortunately, there are two important pitfalls in this

approach. The first is that the performance is generally

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The remainder of this paper is organized as follows. Section 2 explains how to formulate the problem, given our initial question. Section 3 presents the approach in more detail. Section 4 illustrates this approach through a manufacturing example, inspired by [5]. Section 5 summarizes our conclusions and gives topics for future research.

### 2. PROBLEM STATEMENT

In the literature, several research publications are interested in determining decision variables (e.g., number of operators, buffer sizes) to optimize a given perfomance criterion. In our study, however, the design has already been given; emphasize is placed on those parameters that are actually *uncertain*. A typical example is the customer demand for a given product. We will consider these uncertain parameters as input variables, and study the effects of their variations. Let  $\mathbf{x} = (x_1, ..., x_i, ..., x_k)$  be the *k*-dimensional vector of input variables. We assume that each  $x_i$  may have values within an interval  $I_i = [L_i; H_i]$ . Obviously, these uncertain variables  $x_i$  can have important effects on system performance, and hence on the acceptability of the future system.

Following [5], we assume that the production requirements are expressed as the probability of the future system satisfying given production objectives. Typically, the system must produce at east a given number of products (to meet forecasted demand). If the system produces (say) s = 1 types of products (or product families), then the acceptability of the system can be quantified as its capability of producing more than  $b_j$  products of type j (j = 1,..., s), where  $b_j$ represents the threshold for product j given by the decision makers. The stochastic number of manufactured products of type j (say)  $q_i$  may be expressed as a function of the uncertain variables  $\mathbf{x}$ , so  $q_i = f_i(\mathbf{x})$ . Because of the system's uncertainties, the requirement for each product will be satisfied with a probability lower than 100%. Because expected values are not relevant in this context, we use quantiles, as suggested in [5]; also see [1] and [2]. Let  $q_{j;p}$  denote the  $p^{th}$  quantile (e.g., the 5% quantile) of the  $j^{th}$  output (say)  $q_i$ :

$$P(q_j < q_{j;p}) = p_q \ (j = 1, ..., s); \tag{1}$$

i.e., there is only a  $p_q$  chance that output j is lower than

$$q_{j,p}$$
. Hence, the *managerial requirement* can be  
 $q_{1;p} = b_1, \dots, q_{s;p} = b_s$  (2)

where  $\mathbf{b} = (b_1,...,b_s)'$  is given by management. If all *s* constraints hold, then the system gives acceptable output. Consequently, determining the acceptability of a given design solution consists in determining which combinations of uncertain factors, i.e.  $x_i$ , lead to output quantiles  $q_{j;p}$  that meet the threshold values in (2). Assuming that these combinations form a closed set, we wish to find the frontier (say)  $G(\mathbf{x})$  that separates

"acceptable" and "unacceptable" solutions, where (1) implies that acceptabe solutions x satisfy the constraints (2).

#### **3. ANALYZING THE ACCEPTABILITY**

### 3.1 Building a simulation metamodel

To solve the problem formulated in the previous section, we have to estimate the performance of the future system in various environments; see [7] and [13]. A simulation model with as inputs the uncertain environmental factors  $x_i$  and as outputs the quantiles defined in (1) is needed. Unfortunately, this simulation model alone does not give a frontier  $G(\mathbf{x})$  between acceptable and unacceptable environments. We suggest to derive a mathematical approximation of the relationship between the model inputs and outputs, using a simulation metamodel (see [4], [7] and [12]). For this approximation we use a first-order polynomial regression that expresses the quantile as a function of the environmental variables:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \varepsilon$$
(3)

where *y* denotes the metamodel's predictor of the simulation output,  $\beta_0$  denotes the intercept,  $\beta_k$  denotes the first-order effect of input  $x_k$ , and e denotes the 'residual'. If `we standardize the inputs so that they vary between -1 and +1, then the effects  $\beta_j$  measure the *relative* importance of the (uncertain) input  $x_j$  [7]. To estimate the effects in (3), at least 1 + k combinations of the *k* factors must be simulated. To the resulting simulation data, we apply Least Squares (LS) to get the effect estimator

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}' \mathbf{X})^{-1} \mathbf{X}' \hat{\boldsymbol{q}}$$
(4)

where **X** denotes the  $n \times (k + 1)$  matrix of regression variables and  $\hat{\mathbf{q}}$  the quantiles estimated though simulation.

To determine if these estimated effects are actually important, we need their Confidence Intervals (CIs). LS assumes white noise. Unfortunately, this assumption often does not hold in practice, so CIs should not use the classic t statistic; instead we use bootstrapping. Bootstrapped observations are obtained by resampling with replacement the original (say) m replicated simulation observations, see [3]. This resampling is executed for each scenario i (i = 1,...,n). This gives the bootstrapped quantiles  $\hat{q}_b^*$ , where the superscript \* denotes bootstrapped (resampled) values. These bootstrapped quantiles give the bootstrapped estimated factor effects:

$$\hat{\boldsymbol{\beta}}_{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{q}}_{b}^{*} (b=1,...,B)$$
(5)

where B denotes the 'bootsrap sample size'; i.e., we repeat the bootstrap procedure B times. To estimate a

(say) 95% CI per factor effect *j*, we sort the *B* values of  $\hat{\beta}_{j;b}^*$  and find the order statistics (denoted by the (.) where  $\hat{\beta}_{j;b}^*$  and  $\hat{\beta}_{j;b}^$ 

subscript)  $\hat{\beta}^{*}_{(0.025B);j}$  and  $\hat{\beta}^{*}_{(0.975B);j}$ , which are the lower and upper bound for the CI.

## **3.2** Validating the metamodel and testing the importance of uncertain factors

Before using the CIs to test the importance of the individual estimated factor effects, we should *validate* the estimated metamodel as a whole. This model is based on (3) and (4):

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k.$$
(6)

Several methods may be used to validate a metamodel, for example, *Absolute Relative Error (ARE)*, *leave-one-out cross-validation* and *normalized prediction errors*; see [8] and [10]. Once the metamodel is validated, the individual factor effects should be tested, to determine which factors are most important. Statistically speaking, we reject the null-hypothesis  $H_0$ :  $\beta_j = 0$  (and call the factor nonsignificant), if the CI interval based on (5), i. e.,

 $[\hat{\beta}^{*}_{(0.025B);j}, \hat{\beta}^{*}_{(0.975B);j}]$  does not contain the value 0. Next we get the (say) h = k significant (uncertain) factors  $x_i$  (i = 1, ..., h).

### 3.3 What is acceptable?

Once the metamodel is validated (i.e.,  $\hat{y}$  correctly approximates  $\hat{q}$ ), substituting the fitted metamodels (6) into (2) yields:

$$\hat{\beta}_{0;j} + \hat{\beta}_{1;j} x_1 + \dots + \hat{\beta}_{h;j} x_j \ge b_j \text{ with } j = 1, \dots, s \quad (7)$$

which defines the frontier:

$$G(\mathbf{x}) = \hat{\mathbf{\beta}} \cdot \mathbf{x}' - \mathbf{b} \tag{8}$$

We can see that the values  $\mathbf{x}$  that make  $G(\mathbf{x})$  positive in (8) give an acceptable design.

### 4. EXAMPLE: A MANUFACTURING SYSTEM

### 4.1 System features

In our example, the factory consists of four workstations. Each workstation has identical machines. We assume that the number of machines per workstation is given by a design team (here, we select one of the solutions found in [5]). The factory produces two types of products (so s = 2), called prod1 and prod2. These product types have different routings. If we denote WorkStation *i* by  $WS_i$  with i = 1,..., 4, then prod1 has routing  $WS_1$ ,  $WS_3$ ,  $WS_4$  and prod 2 has  $WS_2$ ,  $WS_3$ ,  $WS_4$ ; also see Figure 2 in [5]. If we denote the exponential distribution with mean  $\mu$  and standard deviation *s* by

 $N(\mu, s)$ , then Table 1 gives the *base scenario*, see [13], the scenario with the input datas which enabled to obtain the design, which is assumed during the design, (reproduced from [5]).

| Product | Interarrival time a | Processing time p |            |             |            |
|---------|---------------------|-------------------|------------|-------------|------------|
|         |                     | <b>WS</b> 1       | WS 2       | <b>WS</b> 3 | WS 4       |
| prod1   | <i>E</i> (2)        | N(5,0.1)          | -          | N(3,0.05)   |            |
| prod2   | <i>E</i> (1.4)      | -                 | N(5.5,0.1) | N(3,0.05)   | N(2.5,0.1) |

Table 1. Base scenario (parameters in minutes)

The outputs of the simulated factory are the number of products of each type produced during a thirty-day period, after a warm-up period of 10 days; see [5].

These random responses  $q_1$  and  $q_2$  must satisfy given `threshold' values. Using the 0.05 quantiles  $q_{j;0.05}$ , the production requirements are

$$q_{1;0.05} = 15000 \text{ and } q_{2;0.05} = 17000$$
 (9)

where 15000 and 17000 are the threshold values given by the company (managers, decision makers); see bin (2). In practice, the parameters of the distributions are *uncertain*; e.g., the interarrival times of product 1 in Table 1 may have a parameter not equal to 2. We therefore focus on these uncertain factors: if the parameters in Table 1 change, will the design suggested in [5] still satisfy the requirements in (1)?

### 4.2 Experiment

We select a  $2^{14-10}$  design [7], which implies 16 scenarios that enable estimation of 14 effects  $\beta$  in (3); i.e., we study the 14 uncertain parameters in Table 1, which are the parameters of each distribution. In this experiment, each factor has 2 values; the factor changes by either +10% or +30%. We use m = 100 replications for each of the 16 scenarios. Simulation gives  $\hat{q}_1$ , the estimated 5% quantile for product 1; see column 2 in Table 2. We do not give the results for product 2, because product 2 gives only acceptable solution for all 16 scenarios; also see [9]. We analyze the results of Table 2 as follows.

First we compare  $\hat{q}_1$  (simulation estimate of quantile) with the threshold 15000 in (9). Nine of the sixteen scenarios give unacceptably low responses. We observe that the CIs are very short! The cause is the low variability in the simulation responses: the simulation runs of 30 days are very long compared with the processing time (maximally 5.5 minutes).

|    | $\widehat{q_1}$ | Confidence<br>interval | $var(\widehat{q_{(1)}^*})$ | $\widehat{\mathcal{Y}_1}$ |
|----|-----------------|------------------------|----------------------------|---------------------------|
| 1  | 13289           | (13287,13289)          | 1.919                      | 13314.31                  |
| 2  | 12721           | (12719,12722)          | 1.529                      | 12695.69                  |
| 3  | 15705           | (15702,15706)          | 3.465                      | 15679.69                  |
| 4  | 15425           | (15423,15425)          | 2.411                      | 15450.31                  |
| 5  | 13289           | (13288,13290)          | 1.779                      | 13263.69                  |
| 6  | 13289           | (13287,13290)          | 1.622                      | 13314.31                  |
| 7  | 15705           | (15704,15706)          | 1.868                      | 15730.31                  |
| 8  | 15704           | (15703,15706)          | 2.296                      | 15678.69                  |
| 9  | 13289           | (13288,13290)          | 3.242                      | 13263.69                  |
| 10 | 13290           | (13289,13290)          | 1.753                      | 13315.31                  |
| 11 | 15705           | (15703,15706)          | 5.108                      | 15730.31                  |
| 12 | 15023           | (15032,15034)          | 2.528                      | 15007.69                  |
| 13 | 13290           | (13288,13290)          | 1.594                      | 13315.31                  |
| 14 | 13288           | (13288,13290)          | 2.224                      | 13262.69                  |
| 15 | 15425           | (15424,15426)          | 1.958                      | 15399.69                  |
| 16 | 14056           | (14054,14056)          | 2.163                      | 14081.31                  |

Table 2. Simulated quantiles, CIs, bootstrapped variance, and predicted quantiles obtained from 16 scenarios Product 1

Next we examine whether the variability varies with the scenarios: we compute the ratio (say)  $r_j$  of the maximum and minimum CI lengths - which are 4 and 1 (see column 3 in Table 2) - for the 5% quantile of product 1:

$$r_{j} = \frac{\max_{i} [q_{(10);i;j} - q_{(1);i;j}]}{\min_{i} [q_{(10);i;j} - q_{(1);i;j}]} (j = 1, 2)$$
(10)

where the upper point of the CI is  $q_{(10)}$  and the lower point is  $q_{(1)}$ , which are computed from the m = 100original (not the bootstrapped) simulation responses; see [9].

The observed value of 4 for  $r_j$  suggests that the variances are not constant.

The  $2^{14-10}$  design gives an orthogonal **X**, so (5) simplifies to

$$\hat{\beta}_{h; j} = \frac{\sum_{i=1}^{16} x_{i; h} \hat{q}_{i; j}}{16} (h = 0, ..., 14) (j = 1, 2)$$
(11)

This gives the LS estimates displayed in Table 3.

| $\hat{\boldsymbol{\beta}}$ 0; j | $\hat{oldsymbol{eta}}$ 1; j | $\hat{\beta}_{2; j}$ | β̂3; <i>j</i>                 | $\hat{\beta}_{4; j}$ |
|---------------------------------|-----------------------------|----------------------|-------------------------------|----------------------|
| 14281.438                       | 109.438                     | 25.688               | -1063.313                     | 180.688              |
| $\hat{\beta}$ 5; j              | $\hat{f eta}$ 6; $j$        | $\hat{f eta}$ 7; j   | $\hat{oldsymbol{eta}}$ 8; $j$ | $\hat{\beta}$ 9; j   |
| -131.563                        | -108.563                    | -74.563              | -96.563                       | 9.188                |
| $\hat{\beta}$ 10; j             | $\hat{f eta}$ 11; j         | $\hat{\beta}$ 12; j  | β̂13; <i>j</i>                | $\hat{\beta}$ 14; j  |
| -109.563                        | 60.438                      | 96.688               | 145.438                       | 61.438               |
|                                 |                             | 0.01                 |                               |                      |

Table 3. LS estimates of first-order polynomial for product 1

After bootstrapping as described in Subsection 3.1, we get  $var(\widehat{g_{11}})$  displayed in Table 2. We see that these variances are small, which agrees with our comment on

the lengths of the CI in Table 2. For the validation of the metamodels, we apply measures that are also used in [5]. First, we compute the classic *coefficient of determination*  $R^2$  and  $R^2_{adjusted}$ : these two measures turn out to be 0.9995 and 0.993. These values are excellent compared with the maximum value, which is equal to 1. Moreover, we compute the *Absolute Relative Error* (*ARE*):

$$ARE_{i} = \frac{\left|\widehat{y_{i}} - q_{i}\right|}{q_{i}}$$
 (*i* = 1,...,*n*). (12)

Our experiment gives an average ARE of 0.0018 and a maximum of 0.0020, so no ARE exceeds the given threshold 0.10, see [6]. Altogether we find the AREs to be excellent. We also make a *scatterplot* with the 16 simulated quantiles versus the quantiles predicted through the fitted first-order polynomial (6); see Figure 1. These points in this figure lie close to a  $45^{\circ}$  line, which means the simulated quantiles have approximately the same value than the quantiles Predicted using the metamodel.

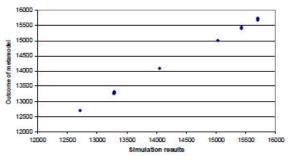


Figure 1. Quantiles simulated versus quantiles predicted through first-order polynomial for product 1

In addition, we perform *leave-one-out cross-validation*: We delete I/O combination *i* from the complete set of 16 combinations, which gives the I/O data set  $(\mathbf{X}_i, \mathbf{q}_i)$ , see [7]. We recompute the original LS estimator defined in (4):

$$\hat{\boldsymbol{\beta}}_{-i} = (\mathbf{X}'_{-i}\mathbf{X}_{-i})^{-1}\mathbf{X}'_{-i}\hat{\mathbf{q}}_{-i} \ (i=1,...,n)$$
(13)

We use this recomputed estimator  $\hat{\beta}_{-i}$  to compute the regression predictor for the deleted combination:  $\hat{q}_{-i} = \mathbf{x}'_i \hat{\beta}_{-i}$ . This gives *n* predictions  $\hat{q}_{-i}$  with i = 1,...,16. Then we calculate the ARE between the predicted and simulated quantiles.

| -i | $\hat{q}_{-i;1}$ | ARE    |
|----|------------------|--------|
| 1  | 16107            | 0.0255 |
| 2  | 14322            | 0.0273 |
| 3  | 18792            | 0.0209 |
| 4  | 18960            | 0.0216 |
| 5  | 15304            | 0.0255 |
| 6  | 16107            | 0.0256 |
| 7  | 19598            | 0.0209 |
| 8  | 18793            | 0.0209 |

| 9  | 15302 | 0.0256 |
|----|-------|--------|
| 10 | 16107 | 0.0255 |
| 11 | 19595 | 0.0209 |
| 12 | 17593 | 0.0223 |
| 13 | 16108 | 0.0255 |
| 14 | 15303 | 0.0255 |
| 15 | 18156 | 0.0216 |
| 16 | 17013 | 0.0242 |

Table 4. Predicted quantiles and ARE for<br/>cross-validation

In Table 4, the AREs are acceptable so we accept our first-order polynomial metamodel.

Next we find the bootstrapped factor effects, which define the CI, using (5). This gives Table 5, where all effects are significant. We conclude that first-order polynomials give adequate approximations and that all fourteen factors are important.

|    | $\hat{\beta}^{*}_{(2.5);1}$ | $\hat{\beta}^{*}_{(97.5);1}$ |
|----|-----------------------------|------------------------------|
| 0  | 14281.438                   | 14286.813                    |
| 1  | 108.188                     | 110.844                      |
| 2  | 24.250                      | 26.938                       |
| 3  | -1064.813                   | -1062.719                    |
| 4  | 179.625                     | 181.438                      |
| 5  | -133.563                    | -130.250                     |
| 6  | -181.719                    | -179.719                     |
| 7  | -74.938                     | -74.094                      |
| 8  | -97.344                     | -96.094                      |
| 9  | 8.563                       | 10.313                       |
| 10 | -110.094                    | -109.188                     |
| 11 | 60.125                      | 61.406                       |
| 12 | 96.188                      | 97.031                       |
| 13 | 145.000                     | 145.875                      |
| 14 | 61.125                      | 62.000                       |

Table 5. CIs for factor effects computed from 100boots trapped quantiles of product 1

Now we return to Table 3 and interpret this table as follows:

- Factor 1 (?1) has a plus sign, whereas we expect a minus sign because higher mean interarrival time implies fewer products arriving into the system so the output decreases. However, workstations 1 and 3 may reach saturation; i.e., the number of products arriving exceeds the workstations' capacities.
- 2) Factor 3 ( $\mu_{1;1}$ ) is the most important factor ( $\hat{\beta}_{8;1} = -1063.313$ ). Its minus sign implies that a higher mean processing time at work station 1 decreases the output of product 1.
- 3) Factor 9  $(s_{1;1})$ 's plus sign means that a higher standard deviation increases the output, which is hard to explain.
- 4) Factors 5  $(\mu_{1;3})$ , 7  $(\mu_{1;4})$ , 11  $(s_{1;3})$  and 13  $(s_{1;4})$  have the same signs as factors 3 and 9, because their influences are similar.
- 5) Factors 4  $(\mu_{2,2})$  and 10  $(s_{2,2})$  do not have direct

influences on the output of product 1, but a higher processing time at workstation 2 for product 2 obviously gives fewer arrivals of product 2 before workstation 3. Because workstation 3 serves FIFO, fewer products 2 in its queue implies smaller waiting time for product 1.

6) Factors 6 (µ<sub>2;3</sub>) and 8 (µ<sub>2;4</sub>) have the same influence on product 1; their minus signs are explained by the fact that products 1 and 2 share workstations 3 and 4. If the processing times for product 2 increase at workstations 3 or 4, then the output of product 1 decreases.

Replacing the quantiles in (9) by their regression estimates  $\hat{y}_j$  based on (3) using Table 3 enables us to estimate whether the thresholds are satisfied for specific scenarios. Because we found that the first-order polynomial (3) is a valid metamodel, a hyperplane in the k-dimensional input space is the frontier of the region of acceptable scenarios:

$$G(\mathbf{x}) = \hat{\beta}_{0;1} + \hat{\beta}_{1;1}x_1 + \dots + \hat{\beta}_{14;1}x_{14} - 15000 .$$
(14)

We illustrate this 14-dimensional frontier as follows. We assume that two factors (say)  $x_h$  and  $x_h$  deviate from their base values, while all other factors (say)  $\mathbf{x}_{(h;h')}$ remain at their base values, hence  $\mathbf{x}_{(h;h')} = 0$  using standardized code; we denote this scenario by  $x_h, x_h, \mathbf{x}_{(h;h')} = 0$ . Then (14) implies:

$$E\left\langle \hat{y}_{1} \middle| x_{h}, x_{h'}, \mathbf{x}_{-(h;h')} = \mathbf{0} \right\rangle = \hat{\beta}_{0;1} + \hat{\beta}_{h;1} x_{h} + \hat{\beta}_{h;1} x_{h'} \quad (15)$$

To illustrate (15), we plot the effects of the two most important factors:  $x_1$  corresponding with the original factor  $\mu_{1;1}$  and  $x_2$  or  $\mu_{2;2}$  (see (3) and  $\hat{\mathbf{B}}$  in Table 3) - and use the threshold 15000:

 $\hat{y} = 15000 - (14281.44 - 1063.31x_1 + 180.6875x_2)$  (16)

This equation gives Figure 2, axis X presents the processing time of product 1 at work station 1, axis Y presents the interarrival time of product 1, and axis Z presents the production volume of product 1. This figure shows that low values for  $x_1$  give acceptable production volumes.

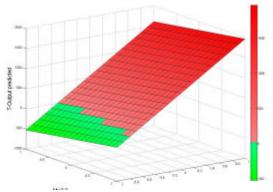


Figure 2. Acceptability frontier for product 1, when factors 1 and 2 change

We conclude that the simulated system is sensitive to the changes in the environmental factors. For example, a 16% change of the production time at workstation 1 for product 1, which means only 48 seconds more, makes the system performance unacceptable. The acceptability depends only on product 1 because the output volume for product 2 remains greater than the threshold, for every simulated scenario. The most important factor for product 1 is the mean processing time of product 1 at workstation 1; i.e., if we keep the mean processing time at its assumed value while all other factors change by 20%, then the design remains acceptable. As a consequence, these two factors should receive much attention from the decision makers.

### 7. CONCLUSIONS AND FUTURE RESEARCH

The approach presented in this paper allows decision makers to determine to what extend a design specification can be considered to be acceptable. Given the stochastic behavior of factories, the future system should be capable of producing the forecasted (monthly or annual) number of parts to satisfy a given customer demand, with a given probability that has to be high enough. This can be verified using the methodology that we have presented. The uncertainty about the data used in the simulation model can be taken into account using simulation metamodeling principles, which may highlight the conditions on the acceptability of a given possible design. Our example has shown that, depending on processing time of product 1, the system may become unacceptable so that this factor requires particular attention during the design study. In this respect, the approach presented in this paper should be able to provide useful assistance to decision makers.

First-order polynomial regression metamodels have been used in our method. However, other types of metamodels can also be studied to improve the identification of the acceptable frontier. This is contemplated in our future work.

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