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Where Is the Σ_b?

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The masses of s-wave bottom baryons are discussed in a semirelativistic quark model, on the basis of a quark-distance relation. We stress that the Σ_b is heavier than $\Xi_b'(b[su], b[sd])$ containing the antisymmetric su(or sd) subsystem. We conclude that the two candidates for Λ_b with very different masses are different states; Basile et al.'s result 5425^{+175}_{-75} MeV is $\Lambda_b(b[du])$, but Arenton et al.'s result $\sim$5750 MeV is $\Xi_b'^0(b[su])$.

Where is the Σ_b? There are many experimental data of bottom mesons,[1] but as for bottom baryons, there are only two candidates for Λ_b[2] with large experimental errors. If the mass of Λ_b is smaller than 5510 MeV, the decay Υ (11020)$\to\Lambda_b+\bar{\Lambda}_b$ is allowed. Higher excited states of Υ may be expected to decay to $\Lambda_b+\bar{\Lambda}_b$, $\Sigma_b+\bar{\Sigma}_b$ and so on. We expect that the masses of Λ_b, Σ_b and Ξ_b' will be established experimentally soon.

In the previous paper,[3] we discussed baryon spectroscopy in a semirelativistic quark model. Modifying the quark distances by forces due to the Fermi-Breit terms, we get all the well-known s-wave baryon masses within ± 9 MeV accuracy. The new data[1] of Σ_c^{++}, Σ_c^0, $\Xi_c'^+(c[su])$ and $\Xi_c'^0(c[sd])$ get closer to our results. As for Λ_c^+, Alvarez et al.[4] and Frabetti et al.[5] have obtained experimental values which just fit our results. The fit is so good that it is interesting to discuss bottom-baryon masses in our model.

Ξ_c baryons of the quarks c, s, u (or d) have two states. One contains the symmetric state of s and u (or d) subsystem, which is denoted by Ξ_c in our notation, and the other, the antisymmetric of them, by Ξ_c'. We note that the Σ_c is near Ξ_c'. There are two main reasons for it. One reason is that the difference of the spin-spin interaction term $(16\pi\alpha_s/9)\langle\Sigma\delta(\boldsymbol{r}_{ij})(\boldsymbol{s}_i\cdot\boldsymbol{s}_j)/(m_im_j)\rangle$ of the one gluon exchange potential partly cancels the mass difference between s and $u(d)$. The other is that the distances of the quark pairs in these baryons are not equal.

In this paper, we further discuss these circumstances about the bottom baryons. Using the values of the parameters obtained in the previous paper,[3] we estimate bottom-baryon masses. One of our main results is that Ξ_b' with the antisymmetric $[su]$ configuration has a smaller mass than Σ_b with the symmetric $\{du\}$ configuration.

Our Hamiltonian for the three-quark system consists of the relativistic kinetic terms $\Sigma(m_i^2+\boldsymbol{p}_i^2)^{1/2}$, the harmonic-oscillator potentials $\Sigma(1/2)Kr_{ij}^2$, the Fermi-Breit terms U and all the others H_0, where m_i, $\boldsymbol{p}_i$ and $\boldsymbol{r}_i$ are the mass, momentum and

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position of the ith quark, respectively and $\boldsymbol{r}_{ij}=\boldsymbol{r}_i-\boldsymbol{r}_j$. The solutions[3] of the Schrödinger equations with harmonic-oscillator potentials are the Gaussian wave functions. Distances between two quarks in the baryons depend on the masses of the quarks and get the modification by the forces due to the Fermi-Breit terms. We reduce the size of the Gaussian wave function according to the quark-distance relation[3]

$$r_{ij}=R_{ij}-\kappa\frac{\partial}{\partial r_{ij}}\langle U\rangle|_{r_{ij}=R_{ij}}\,, \tag{1}$$

where r_{ij} stands for the modified quark distance, R_{ij} denotes the quark distance in the harmonic-oscillator potentials, $1/R_{ij}=\langle 1/|\boldsymbol{r}_{ij}|\rangle$ and κ is an adjustable parameter. Baryon masses are calculated as the expectation values of the Hamiltonian. The details of the calculation are the same as given in Ref. 3) and the fixed parameters are

$$m_u=432\text{ MeV}\,,\quad m_d=437\text{ MeV}\,,\quad m_s=680\text{ MeV}\,,$$
$$m_c=1944\text{ MeV}\,,\quad \alpha_s=0.566\,,\quad K=3.87\times10^7\text{ MeV}^3\,,$$
$$\kappa=2.46\times10^{-9}\text{ MeV}^{-3}\,,\quad E_0=\langle H_0\rangle=-1021\text{ MeV}\,. \tag{2}$$

Figure 1 shows our results of the Ξ_b', Λ_b masses and the mass difference $\Xi_b'-\Sigma_b$ as the functions of the b quark mass m_b. The mass difference $\Xi_b'-\Sigma_b$ changes the sign at $m_b=2820$ MeV. The expected b quark mass ~5000 MeV is well above this crossing point. So we can expect Σ_b to be heavier than Ξ_b'. For example, for $m_b=5300$ MeV, we obtain $\Xi_b'^0-\Sigma_b{}^0=-25$ MeV, in contrast with $\Xi_c'^0-\Sigma_c{}^0=25$ MeV (expt. $=20$ MeV) obtained in Ref. 3). From Table I, we can see that this comes mainly from the spin-spin interactions, the harmonic-oscillator potentials and the Coulomb potentials, where the differences of quark distances in Ξ_b' and Σ_b play an essential role. Thus we see that the internal interactions overcome the quark mass difference.

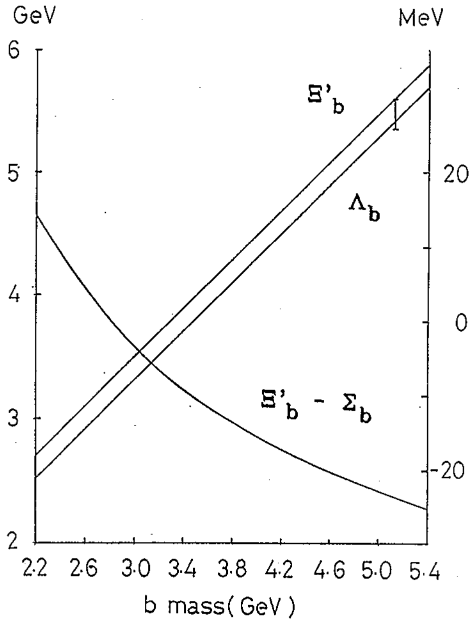

Fig. 1. The dependence of Ξ_b', Λ_b(the left-hand scale in GeV) and $\Xi_b'-\Sigma_b^0$ (the right-hand scale in MeV) on m_b (in GeV). The experimental data are for Λ_b observed by Basile et al.[2]

There are two candidates[2] for Λ_b with very different masses. One is 5425 $^{+175}_{-75}$ MeV by Basile et al. and the other is ~5750 MeV by Arenton et al. Though the experimental errors of them are very large, the mass difference between them is clearly larger than the experimental errors. This evidence is sufficient to suspect that these two experiments are of the same particle. Possible candidates for them are Λ_b and $\Xi_b'^0$, where $\Xi_b'^0$ contains $b[su]$ quarks of a flavor antisymmetric and spin antisymmetric

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Table I. The expectation values (in MeV) of the kinetic terms, the harmonic-oscillator potentials and the Fermi-Breit terms of $\Xi_b'^0$, $\Sigma_b{}^0$ and $\Xi_b'^0-\Sigma_b{}^0$. We put $m_b=5300$ MeV as an example.

	$\Xi_b'^0$	$\Sigma_b{}^0$	$\Xi_b'^0-\Sigma_b{}^0$
$\Sigma\langle(m_i{}^2+\boldsymbol{p}_i{}^2)^{1/2}\rangle$	6953	6672	281
$\Sigma\left\langle\frac{1}{2}Kr_{ij}^2\right\rangle$	475	581	-106
$\Sigma\left\langle\left(\alpha Q_iQ_j-\frac{2}{3}\alpha_s\right)\frac{1}{\lvert\boldsymbol{r}_{ij}\rvert}\right\rangle$	-551	-499	-52
$\Sigma\left\langle\left(\alpha Q_iQ_j-\frac{2}{3}\alpha_s\right)\left(-\frac{1}{2m_im_j}\right)\left(\frac{\boldsymbol{p}_i\boldsymbol{p}_j}{\lvert\boldsymbol{r}_{ij}\rvert}+\frac{\boldsymbol{r}_{ij}\cdot(\boldsymbol{r}_{ij}\cdot\boldsymbol{p}_i)\boldsymbol{p}_j}{\lvert\boldsymbol{r}_{ij}\rvert^3}\right)\right\rangle$	-88	-72	-16
$\Sigma\left\langle\left(\alpha Q_iQ_j-\frac{2}{3}\alpha_s\right)\left(-\frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\right)\left(\frac{1}{m_i{}^2}+\frac{1}{m_j{}^2}\right)\right\rangle$	122	128	-6
$\Sigma\left\langle\left(\alpha Q_iQ_j-\frac{2}{3}\alpha_s\right)\left(-\frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\right)\frac{16\boldsymbol{s}_i\cdot\boldsymbol{s}_j}{3m_im_j}\right\rangle$	-108	18	-126
E_0	-1021	-1021	0
total	5782	5807	-25

su state. The decay $\Xi_b'^0\to\Lambda K^0 2\pi^+2\pi^-$, which is observed by Arenton et al., is Cabibbo allowed. From Fig. 1, we can see that the mass difference $\Xi_b'^0-\Lambda_b$ is almost constant for a wide range of m_b; $\Xi_b'-\Lambda_b=181.9$ MeV for $m_b=5000$ MeV and $\Xi_b'-\Lambda_b=181.5$ MeV for $m_b=5300$ MeV. Our prediction $\Xi_b'^0-\Lambda_b=182$ MeV is consistent with these assignments within the experimental errors.

It is crucial to find Σ_b which may decay to $\Lambda_b\pi$. Thus, we will first experience a reversal $\Sigma_b>\Xi_b'$ among different generations with the same spin in spite of the quark mass relation $m_d<m_s$. On the other hand, there are no such drastic differences between the particles with a flavor symmetric quark pair:

$$\Xi^0-\Sigma^0=135\text{ MeV}\quad(\text{expt.}=122.35\text{ MeV}),$$

$$\Xi_c{}^0-\Sigma_c{}^0=127\text{ MeV},$$

$$\Xi_b{}^0-\Sigma_b{}^0=114\text{ MeV}\quad(\text{for } m_b=5300\text{ MeV}). \tag{3}$$

The former two values are the results given in Ref. 3).

Finally, we would like to add some comments. Our calculation is semirelativistic. We use nonrelativistic Gaussian wave functions. The numerical calculation of the kinetic term in the relativistic form $\sqrt{m_i{}^2+\boldsymbol{p}_i{}^2}$ leads[6)] to a more accurate fit and more reasonable parameters than those in the nonrelativistic form $m_i+\boldsymbol{p}_i{}^2/(2m_i)$. By this we include the relativistic effect of the kinetic term up to the first order. So we can expect a better fit with relativistic wave functions. A possible wave function is given in Ref. 7).

Equation (1) gives an excellent fit. Similar relation to Eq. (1) is used for the meson spectroscopy.[6),8)] We can show that our reduced quark distance Eq. (1) and the wave function are the same as those of the perturbation theory up to the $2s$ state of the harmonic oscillator for the mesons.[8)] This discussion can also be applied to the Fermi-Breit terms. As for the baryons, we would like to discuss this point elsewhere.

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