

WHO MARRIES WHOM AND WHY*

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Abstract

This paper proposes and estimates a static transferable utility model of the marriage market. The model generates a non-parametric marriage matching function with spillover effects. It rationalizes the standard interpretation of marriage rate regressions as well as pointing out its limitations. The model was used to estimate US marital behavior in 1971/72 and 1981/82. The marriage matching function estimates show that the gains to marriage for young adults fell substantially over the decade. Unlike contradictory marriage rate regression results, the marriage matching function estimates showed that the legalization of abortion had a significant quantitative impact on the fall in the gains to marriage for young males and females.

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1 Introduction

Thirty years ago, Gary Becker (1973, 1974; summarized in Becker 1991) expounded a static transferable utility model of the marriage market. While implications of his model have been tested and applied,¹ it has seldom been estimated.² There are two problems that have to be solved before a transferable utility model of the marriage market can be estimated. First, equilibrium transfers in modern marriages are seldom observed. Second, individuals may differ by age, religion, education, wealth, ethnicity, and so on. Different types of individuals may not agree on the rankings of individuals of the opposite gender as spouses. Thus an empirical model of the marriage market should not impose too much apriori structure on the nature of preferences for marriage partners. However without apriori structure, it is unclear what can be identified from the data.

To understand the identification problem, consider a society with I types of men and J types of women participating in the marriage market. A type is defined by an age range, ethnicity, education, geographic location and so on. Each individual chooses who to marry or to remain single. For each type of man (woman), there are potentially J (I) preference parameters to characterize his (her) choice of whether to marry and who to marry. In total, there are as many as $2 \times I \times J$ preference parameters. What is observable to a researcher? In principle, the researcher observes the quantity of each type of men in the marriage market,

¹ Angrist 2002, Botticini and Siow 2004, Chiappori, et. al. 2000, Edlund 2000, Grossbard-Shectman 1993, Hamilton and Siow 2000, Rao 1993, Seitz 1999, South and Lloyd 1992, South and Trent 1988. Bergstrom 1997 and Weiss 1997 provide surveys of the economics literature up to the mid-nineties. Casper and Bianchi 2002; and Waite, et. al. 2000 show his influence outside economics.

² Bergstrom and Lam 1994; Suen and Lui 1999 are exceptions.

m_i for type i men (I observations), the quantity of each type of women, f_j for type j women (J observations), and the quantity of type i men married to type j women, μ_{ij} ($I \times J$ observations). So the total number of observables are $I + J + I \times J$. For $I, J > 2$, the number of observables are less than the number of unknown preference parameters. Thus **any** behavioral empirical model will need to make identifying assumptions to reduce the number of unknown parameters.³

To finesse the identification problem, demographers use a reduced form approach in the form of *marriage matching functions*, to estimate the behavior of the entire marriage market.⁴

A marriage matching function is defined as follows. Let M be the vector of available men by types, $i = 1, \dots, I$ at that time. The i 'th element of the vector M is denoted by m_i . Let F be the vector of available women by types, $j = 1, \dots, J$, where the j 'th element of the vector is denoted by f_j . Let Π be a matrix of parameters. A marriage matching function is an $I \times J$ matrix $\mu(M, F; \Pi)$, whose i, j element is μ_{ij} . Denote the number of unmarried men of type i as μ_{i0} and the number of unmarried women of type j as μ_{0j} . The marriage matching function $\mu(M, F; \Pi)$ must satisfy the following accounting constraints:

$$\mu_{0j} + \sum_{i=1}^I \mu_{ij} = f_j \quad \forall j \tag{1}$$

$$\mu_{i0} + \sum_{j=1}^J \mu_{ij} = m_i \quad \forall i \tag{2}$$

$$\mu_{0j}, \mu_{i0}, \mu_{ij} \geq 0 \quad \forall i, j \tag{3}$$

Demographers have mostly estimated marriage matching functions without spillover ef-

³ The identification problem is not well known because economists calibrate or estimate models with strong apriori identifying assumptions (E.g. Aiyagari, et. al. 2000, Bergstrom and Lam, Fernandez, et. al. 2004, Hamilton and Siow, Seitz, Suen and Lui, Wong 2003a,b).

⁴ Our discussion borrows heavily from Pollak 1990a,b.

facts.⁵ This paper proposes and estimates a static transferable utility model of the marriage market. The model produces a simple non-parametric marriage matching function with spillover effects which will fit any cross-section marriage distribution.⁶

There are three conceptual benefits for considering transferable utility models of the marriage market. First, marriage market clearing equilibrium must satisfy all the accounting constraints, (1), (2) and (3). Second, the reduced form for equilibrium quantities of a market clearing model do not include prices, i.e. equilibrium transfers. Thus the absence of observable transfers to the researcher may not be a problem. Third, transferable utility models provide a solution to the identification problem discussed above. To see how the identification problem may be resolved, let the marital output of an i type male and a j type female only depends on i and j . Then there are $I \times J$ number of these marital outputs plus $I + J$ outputs of the types being single. If the behavior of the marriage market is characterized by these outputs alone, then we may be able to estimate all the parameters which are necessary to determine marital behavior. In particular, we do not have to estimate separate male and female preferences for spouses. A well known property of transferable utility models of the marriage market is that they maximize the sum of marital output in the society (For example, Roth and Sotomayor 1990; Chapter 8). Thus behavior in transferable utility models can be characterized by knowledge about marital output alone, and knowledge about male and female preferences separately is not necessary. The novelty of this paper is to exploit this property to specify a just identified econometric model of the marriage market, and minimize a priori restrictions on preferences for spousal types.

⁵ $\mu_{ij} = g(m_i, f_j, \pi_{ij})$ do not have spillover or substitution effects (E.g. Schoen 1981). Variations in $m_{\neq i}$ or $f_{\neq j}$ do not affect μ_{ij} . This deficiency is known (Pollak, Pollard 1993,1997).

⁶ It also satisfies the conditions in Pollak 1990a, sufficient to generate a well posed two-sex model of population growth.

An important theoretical antecedent to our work is Dagsvik (2000) who also derived a behavioral marriage matching function.⁷ We follow his lead in using McFadden's 1974 extreme value random utility functions. We use a transferable utility framework whereas he uses a non-transferable utilities model (For a fuller comparison, see our working paper, Choo Siow 2003).

The current paper has two limitations. While our model admits spillover or substitution effects, we do not know how restrictive our substitution patterns are on the marriage matching function. Another limitation of our static approach is that it ignores dynamic considerations. Choo Siow 2005 extends this model into a dynamic framework.

Using ages as the only types for males and females in the benchmark model, the second part of the paper estimates the model using data from the 1970 and 1980 *US Census*, and 1971/72 and 1981/82 *Vital Statistics*. The baby boom generation came into marriageable age between the two decades and thus there were substantial changes in the population vectors between the decades. Our marriage matching function can capture some changes in marital patterns in the US between 1971/72 and 1981/82 due to changes in population vectors between the two periods. However our benchmark model could not capture the drastic fall in the marriage rate among young adults over the decade.⁸

There were many social changes between 1970 and 1980 which could have affected the gains to marriage over the decade. A major change was the national legalization of abortion in 1973. Legal abortions were partially available in some states by 1970. If the partial legalization of abortions in a state reduced the gains to marriage in that state, we would

⁷ Also see Johansen and Dagsvik 1999; Dagsvik, et. al. 2001, Logan, et.al. 2001.

⁸ The drop is well known (E.g. Qian and Preston 1993, Qian 1998).

expect to see lower gains to marriage in the early legalizing states relative to later legalizing states in 1970 but not in 1980. Moreover this difference in difference in the gains to marriage should be concentrated among women of child bearing age. Using marriage rate regressions, Angrist and Evans 1999 showed that the marriage rates of young men and women were lower in early legalizing states relative to later legalization states in the early seventies. We show that the estimates of the number of marriages affected are extremely sensitive to whether we use male or female marriage rate regressions. We extend the benchmark model to include whether an individual resided in a state which allowed legal abortions or not as part of the definition of the type of an individual. Methodologically, we extend the standard difference in differences estimator to estimate the effect of a policy change on bivariate distributions. Estimating this extended model, we show that the partial legalization of abortion in some states can explain up to twenty percent of the drop in the gains to marriage among young adults in the seventies.

2 The model

We begin by describing a transferable utility model of marriage. There are I types of men and J types of women. For a type i man to marry a type j woman, he must transfer τ_{ij} amount of income to her. There are $I \times J$ sub-marriage markets for every combination of types of men and women. The marriage market clears when given equilibrium transfers, τ_{ij} , the demand by men of type i for type j spouses is equal to the supply of type j women for type i men for all i, j .

To implement the above framework empirically, we adopt the extreme value random utility model of McFadden 1974 to generate market demands for marriage partners. Each

individual considers matching with a member of the opposite gender. Let the utility of male g of type i who marries a female of type j be:

$$V_{ijg} = \tilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg}, \quad \text{where} \quad (4)$$

$\tilde{\alpha}_{ij}$: Systematic gross return to male of type i married to female of type j .

τ_{ij} : Equilibrium transfer made by male of type i to spouse of type j .

ε_{ijg} : i.i.d. random variable with type I extreme value distribution.⁹

Equation (4) says that the payoff to a male g from marrying a female of type j consists of two components, a systematic and an idiosyncratic component. The systematic component, $\tilde{\alpha}_{ij} - \tau_{ij}$, is common to all males of type i married to type j females. This systematic return is reduced when τ_{ij} , the equilibrium transfer, is increased.

The idiosyncratic component, ε_{ijg} , measures the departure of his individual specific match payoff, V_{ijg} , from the systematic component. We assume that the distribution of ε_{ijg} does not depend on the number of type j females, f_j . The payoff to g from remaining unmarried, denoted by $j = 0$, is:

$$V_{i0g} = \tilde{\alpha}_{i0} + \varepsilon_{i0g} \quad (5)$$

where ε_{i0g} is also an i.i.d. random variable with type I extreme value distribution.

Individual g will choose according to:

$$V_{ig} = \max_j \{V_{i0g}, \dots, V_{ijg}, \dots, V_{iJg}\} \quad (6)$$

We assume that the numbers of men and women of each type is large. Let μ_{ij}^d be the number of i, j marriages demanded by i type men and μ_{i0}^d be the number of unmarried i type men.

⁹ The random variable $\varepsilon_{ijg} \sim EV(0, 1)$, with the cumulative distribution given by $F(\varepsilon) = e^{-e^{-\varepsilon}}$.

Then McFadden showed that (Appendix A includes a proof for convenience):

$$\begin{aligned}\ln \mu_{ij}^d &= \ln \mu_{i0}^d + \tilde{\alpha}_{ij} - \tilde{\alpha}_{i0} - \tau_{ij} \\ &= \ln \mu_{i0}^d + \alpha_{ij} - \tau_{ij}\end{aligned}\tag{7}$$

The term $\alpha_{ij} = \tilde{\alpha}_{ij} - \tilde{\alpha}_{i0}$, is the systematic gross return to an i type male from an i, j marriage relative to being unmarried. The above equation is a quasi-demand equation by type i men for type j spouses.

Let Γ be Euler's constant. Appendix A shows another well known result:

$$\mathbb{E}V_{ig} = \Gamma + \tilde{\alpha}_{i0} + \ln\left(\frac{m_i}{\mu_{i0}^d}\right)\tag{8}$$

$\mathbb{E}V_{ig}$ is the expected utility of a male of type i before he sees his realizations of his ε_{ijg} for all j . Equation (8) shows that it is proportional to the log of the ratio of the number of available type i men relative to the number of type i men who choose to remain single. The expected payoff of being single is given by $\mathbb{E}V_{i0g} = \Gamma + \tilde{\alpha}_{i0}$. Let $q_i = \ln\left(\frac{m_i}{\mu_{i0}^d}\right)$. It measures the expected gains or benefit from being able to participate in the marriage market for a type i male. As shown in the appendix and section 3, the expected gains depends on preference parameters, $\tilde{\alpha}_{ij}$ and $\tilde{\alpha}_{i0}$, as well as transfers, τ_{ij} .

The random utility function for women has similar form except that in marriage with a type i men, a type j women receives a transfer, τ_{ij} . Let $\tilde{\gamma}_{ij}$ denote the systematic gross gain that j type women get from marrying i type men, and $\tilde{\gamma}_{0j}$ be the systematic payoff that j type women get from remaining single. The term $\gamma_{ij} = \tilde{\gamma}_{ij} - \tilde{\gamma}_{0j}$, is the systematic gross gain that j type women get from marrying i type men relative to not marrying.

Let μ_{ij}^s be the number of i, j marriages demanded by j type women and μ_{0j}^s the number of type j women who want to remain unmarried. The quasi-supply equation of type j women

who marry type i men is be given by:

$$\ln \mu_{ij}^s = \ln \mu_{0j}^s + \gamma_{ij} + \tau_{ij}. \quad (9)$$

Following (8), the expected gains to entering the marriage market for a type j female is

$$Q_j = \ln \left(\frac{f_j}{\mu_{0j}^s} \right).$$

The $I \times J$ sub-marriage markets clears when given equilibrium transfers, τ_{ij} , the demand by men of type i for type j spouses is equal to the supply of type j women for type i men for all i, j .¹⁰ That is, for all i, j pairs, $\mu_{ij} = \mu_{ij}^d = \mu_{ij}^s$. Substituting this into equations (7) and (9) and adding the two equations yields:

$$\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2} = \frac{\alpha_{ij} + \gamma_{ij}}{2} \quad (10)$$

If we let $\pi_{ij} = \ln \Pi_{ij} = \frac{\alpha_{ij} + \gamma_{ij}}{2}$, we can rewrite Equation (10) as:

$$\Pi_{ij} = \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \quad (11)$$

which is our marriage matching function.

Equation (11) has an intuitive interpretation. The right hand side of (11) is the ratio of the number of i, j marriages to the geometric average of those types who are unmarried. The log of the left hand side, $\ln \Pi_{ij} = \pi_{ij}$, has the interpretation as the total systematic gain to marriage per partner for **any** i, j pair relative to the total systematic gain per partner from remaining single. One expects the systematic gains to marriage to be large for i, j pairs if one observes many i, j marriages. However there are two other explanations for numerous i, j marriages. First, there are lots of i type men and j type women in the population.

¹⁰See Roth and Sotomayor (1990) and Gretsky, et. al. (1992) for existence of equilibria in transferable utility market assignment models.

Second, there are relatively more i type men and j type women in the population than other types of participants. Scaling the number of i, j marriages by the geometric average of the numbers of unmarrieds of those types control for these effects. Equation (11) is homogeneous of degree zero in population vectors and the number of marriages. Thus our marriage matching function has no scale effect in population vectors.

2.1 Identification

A point estimate for Π_{ij} is given by $\frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}}$. Equation (11) is non-parametric in the sense that it fits any observed marriage distribution. Observing Π_{ij} however, is not sufficient for us to identify the individual specific systematic returns, α_{ij} and γ_{ij} .¹¹

In addition to π_{ij} , equations (7) and (9) allows us to identify $\alpha_{ij} - \tau_{ij}$ and $\gamma_{ij} + \tau_{ij}$:

$$\begin{aligned}\ln\left(\frac{\mu_{ij}}{\mu_{i0}}\right) &= \alpha_{ij} - \tau_{ij} = n_{ij} \\ \ln\left(\frac{\mu_{ij}}{\mu_{0j}}\right) &= \gamma_{ij} + \tau_{ij} = N_{ij}\end{aligned}\tag{12}$$

We refer to n_{ij} as the systematic gain to marriage for a type i male in an i, j marriage relative to not marrying, and N_{ij} as the systematic gain to marriage for a type j female in an i, j marriage relative to not marrying.

2.2 Comparative statics and policy evaluations

Given the preference parameters of the system, Π_{ij} , we are often interested in how variations in the supply population vectors, M and F , affect the distribution of marriages as represented

¹¹It is also not sufficient to estimate $(\alpha_{ij} - \gamma_{ij})$, which is needed to identify the equilibrium transfers.

by μ . Our marriage matching function may be rewritten as

$$\mu_{ij} = \Pi_{ij} \sqrt{\mu_{i0} \times \mu_{0j}} \quad (13)$$

$$= \Pi_{ij} \sqrt{\left(m_i - \sum_{k=1}^J \mu_{ik}\right) \left(f_j - \sum_{g=1}^I \mu_{gj}\right)} \quad (14)$$

If we take Π_{ij} , M and F as exogenously given, equation (14) defines a $I \times J$ system of quadratic equations with the $I \times J$ elements of μ as unknowns. Given population quantities M , F , μ and Π as defined in equation (11), local uniqueness of μ^* for new values of $M^* \neq M$, $F^* \neq F$ and holding Π fixed is given by the following result.

Proposition 1 *Let $\Pi_{ij} = \mu_{ij} [(m_i - \sum_{k=1}^I \mu_{ik})(f_j - \sum_{g=1}^J \mu_{gj})]^{-\frac{1}{2}}$ and M and F be the vectors of m_i and f_j respectively. For M^* and F^* close to M and F , μ^* is uniquely determined.*

The proof using the implicit function theorem is given in Appendix B. ■

Our marriage matching function does not suffer from the zero spillover or substitution restriction that plague many marriage matching function in this literature. For example, holding preferences Π_{ij} constant, the marginal effect on μ_{ij} from an increase in m_r is given by

$$\frac{\partial \mu_{ij}}{\partial m_r} = \frac{1}{2} \Pi_{ij} \left[\left(\frac{\mu_{0j}}{\mu_{i0}}\right)^{1/2} \frac{\partial \mu_{i0}}{\partial m_r} + \left(\frac{\mu_{i0}}{\mu_{0j}}\right)^{1/2} \frac{\partial \mu_{0j}}{\partial m_r} \right].$$

The actual forms of $\partial \mu_{i0} / \partial m_r$ and $\partial \mu_{0j} / \partial m_r$ are given by equation 27 and 28 in Appendix B. These derivatives are not zero.

The expected gain to entering the marriage market for a type j female denoted by Q_j , is related to the marriage rate by:

$$Q_j = \ln \left(\frac{f_j}{\mu_{0j}} \right) = -\ln \left(1 - \frac{\sum_i \mu_{ij}}{f_j} \right) \approx \frac{\sum_i \mu_{ij}}{f_j} = \rho_j^f \quad (15)$$

This approximation is accurate for small marriage rates. The marriage rate for type j females is also related to the systematic net gains N_{ij} in (12) according to:

$$\rho_j^f \approx Q_j = \ln \left(1 + \sum_i \frac{\mu_{ij}}{\mu_{0j}} \right) = \ln \left(1 + \sum_i \exp(\gamma_{ij} + \tau_{ij}) \right) = \ln \left(1 + \sum_i \exp(N_{ij}) \right). \quad (16)$$

Equation (16) says that the marriage rate of type j women depends positively on the systematic gross gains to marriage, γ_{ij} , and equilibrium transfers, τ_{ij} . Thus (16) provides a formal justification for the standard interpretation of marriage rate regressions, where the marriage rate of type j females is assumed to vary positively with factors which increase the gains to marriage for these women.

We can also do policy evaluations with π_{ij} . Consider the following regression model for the total systematic gains to an i, j marriage:

$$\pi_{ij} = X_{ij}'\beta + u_{ij}, \quad (17)$$

where X_{ij} denote the vector of variables (including policy variables) that affect the total systematic gains to an i, j marriage. u_{ij} is an error term with mean zero and uncorrelated with X_{ij} . Since we can construct π_{ij} from equation (11), we can estimate β in equation (17). Policy changes will induce changes in π_{ij} as captured by (17). Changes in π_{ij} will affect marital behavior via the marriage matching function described in equation (11). So given estimates of β , one can predict the effect of changes in X_{ij} on marriage behavior including marriage rates.

3 Changes in the gains to marriage in the seventies

To estimate the marriage distributions by ages in 1971/72 and 1981/82, we use data from the 1970 and 1980 *US Census* to construct population vectors. Marriage records from the

1971/72 and 1981/82 *Vital Statistics* were used to construct the bivariate distributions of marriages. A state has to report the number of marriages to *Vital Statistics* to be in the sample. This requirement eliminated 10 states in 71/72 and 9 states in 81/82.¹²

For each period, we investigate a two year rather than one year marriage distribution because the two year distribution has thicker cells. For each period, we examine the marital behavior of individuals between the ages of 16 and 75 implied by the population vectors and preference parameters estimated from our model. Details of the construction of the data used are left to Appendix A.

In our sample (Table 1 below), there were 16.0 million and 19.6 million available (unmarried) men and women respectively between the ages of 16 to 75 in 1970. There were 3.24 million marriages in 1971/2. There were 23.4 million and 27.2 million available men and women respectively in 1980. Although the available population had increased by more than 39% over the decade, there were only 3.45 million marriages in 1981/2, an increase of 6.5%.

Figures 1a and 1b show the bivariate age distributions of the marrieds in 1971/2 and 1981/2 respectively. In both years, most marriages occurred between young adults and there was strong positive assortative matching by age.

In Figure 2, we graph the 1970 and 1980 age distributions of the population vectors.¹³ For both decades, there are more available men than women in the early ages and the reverse is true in the later ages. These gender differences are due to the fact that there are relatively more widows and lower remarriage rates of divorced women. The higher remarriage

¹²Arizona, Arkansas, Colorado, Nevada, New Mexico, New York, North Dakota, Oklahoma, Texas, and Washington were excluded in 71/72 and 81/82. Colorado was added in 81/82.

¹³The average age of available men and women in 1970 were 30.4 and 39.1 respectively. This gender difference reflected the larger fraction of available older women. The average age of the married men and women in 1971/2 were 27.1 and 24.5 respectively, reflecting the usual gender difference in ages of marriage. The statistics for 1980 are similar, as shown in Table 1.

TABLE 1: Data Summary

<i>US Census Data</i>	1970	1980	Δ
Number of Available Males (M^t)	16.0 mil	23.4 mil	46%
Number of Available Females (F^t)	19.6 mil	27.2 mil	39%
Average age of Available Males	30.4	29.6	
Average age of Available Females	39.1	37.1	
<i>Vital Statistics Data</i>	1971/72	1981/82	Δ
Number of Marrieds (μ^t)	3.24mil	3.45 mil	6.5%
Average age Married Males	27.1	29.1	
Average age Married Females	24.5	26.4	

rate of divorced men reduced the availability of younger women. The arrival of the baby boomers to the marriage market in 1980 is readily visible from the increase in the population of availables. This arrival should have had a substantial impact on the marriage market. However, as noted in Table 1, the number of marrieds in 1980 marginally increased.

Estimating the net gains to marriage by gender:

Our model allows us to estimate the systematic net gain relative to not marrying, for each party in any i, j marriage. The 1971/72 estimates for type i males, given by $n_{ij}^{71} = \ln(\frac{\mu_{ij}^{71}}{\mu_{i0}^{71}})$, and j type females, given by $N_{ij}^{71} = \ln(\frac{\mu_{ij}^{71}}{\mu_{0j}^{71}})$ are compared in Figure 3.¹⁴

Figure 3 plots \widehat{n}_{ij}^{71} and \widehat{N}_{ij}^{71} for 20 and 40 year old males and females by the ages of their spouses. The distributions of \widehat{N}_{i20}^{71} and \widehat{n}_{20j}^{71} are right skewed, with 20 year old females

¹⁴In the 1971/2 and 1981/2 marital records, there were many age pairs which had no marriage. This is a common problem in empirical discrete choice applications and is encountered throughout the empirical section of this paper. We employ kernel smoothers to deal with this thin cell problem. For details, see Choo Siow 2003. Yatchew 2003 provides an excellent overview of these techniques.

receiving the largest systematic net gain when marrying a slightly older male while the 20 year old males receiving the largest systematic net gain when marrying a slightly younger female. Comparing the distribution of systematic net gain for a 40 year old female, \widehat{N}_{i40}^{71} , with her 20 year old counterpart, we find the distribution for a 40 year old female is more dispersed. Again she receives the largest net gain when she marries someone slightly older. If we consider the distribution for 40 year old males, \widehat{n}_{40i}^{71} , we also find the distribution to be more dispersed than for 20 year old males. Again his largest net gain is to marrying someone slightly younger.

According to equations (15) and (16), the area below the transformed net gains, $\exp(n_{ij})$ and $\exp(N_{ij})$ is proportional to the type specific marriage rates. From Figure 3, we also observe that the estimated net gains are negative which reflects the fact that the systematic net gains to marriage are smaller than not marrying. This is not surprising since at any age, most individuals do not marry.¹⁵ The model predicts a match occurs only when the match specific idiosyncratic utility is large. Most of the features of the empirical distributions in Figure 3 are expected; What is new is that our model provides a normative interpretation of these empirical distributions.

Estimating the total gains to marriage:

Using age as the only type differentiating individuals, Figure 4 shows the smoothed non-parametric plot of $\widehat{\pi}_{ij}^{71}$ for the period 1971/72. Compared with Figure 1a, the distribution of the estimated total gains are less peaked and less concentrated. In particular, the total gains are larger off the age diagonal and for older individuals than would be predicted from bivariate marriage distribution of Figure 1a. Like the estimates of the net gains from marriage

¹⁵ $n_{ij} > 0$ implies $\mu_{ij} > \mu_{i0}$ which is counterfactual for all i, j .

in the previous section, we observe that the estimated total gains relative to remaining single is also negative. This reflects the empirical fact that most available individuals do not marry. Figure 4 also shows the standard result that there is strong positive assortative matching by age.¹⁶

3.1 Drop in the gains to marriage

Figure 5 plots of the change in the gains to marriage over the decade, $\Delta \hat{\pi}_{ij} = \hat{\pi}_{ij}^{81} - \hat{\pi}_{ij}^{71}$ for spouses that are close in age (where the most of the data lie). The striking feature of the data is the sharp drop in the estimated total gains to marriage to young adults between the age of 16 and 30 in 81/2.¹⁷ Technological innovations and social changes like the invention of the pill and the legalization of abortion in the seventies affected the gains to marriage by changing the opportunities available to women. In this section, we explore the role of differential access to legal abortions across states in the seventies in affecting the gains to marriage.¹⁸ Before 1967, legal abortion was generally unavailable. Between 1967 and 1973, legal abortion became easier to obtain in several states (reform states).¹⁹ The reform states included in our analysis are: Alaska, California, Delaware, Florida, Georgia, Hawaii, Kansas, Maryland, North Carolina, Oregon, South Carolina and Virginia.

In January 22, 1973, due to the United States Supreme Court ruling in *Roe v. Wade*, legal abortions became available in the entire country. This ruling was less restrictive on access to abortion than what was available previously in the reform states.

¹⁶Choo and Siow (2005) provide an explanation based on dynamic considerations.

¹⁷Refer to earlier working papers for illustrations.

¹⁸Akerlof, et. al. 1996 argued that the legalization of abortion may substantially reduce the gains to marriage. Also see Goldin and Katz 2002 and Siow 2002.

¹⁹Thirteen states passed “Model Penal Code” legislation. Alaska, Florida, Hawaii, New York and Washington enacted even more liberal laws. California’s restrictive abortion laws were struck down by the state courts. See Merz, Jackson and Klerman 1995 for details.

If partial availability of legal abortions in a state reduced the gains to marriage in that state, we would expect to see lower gains to marriage in reform states relative to non-reform states in 1971/2 but not in 1981/2. Moreover this difference in difference in the gains to marriage should be concentrated among women of child bearing age and the men who marry them.

In order to empirically study the impact of the partial legalization of abortions on the gains to marriage, consider an expansion of the type space of individuals. A type of an individual is now defined by his or her age, whether the individual lives in a reform state (r for male and R for female), or non-reform state (n for male and N for female), and time, t . We will use the convention s and S to denote the states of residences for a male and female respectively, where $s \in \{r, n\}$ and $S \in \{R, N\}$. We assume that all individuals at time t , living in a reform state or otherwise, are available to other individuals at the same time t in one national marriage market. So an individual living in a reform state at time t may marry someone living at the same time in either a reform or non reform state, and vice versa. Let $t = 71$ refer to the marriage market in the years 1971 and 1972, and $t = 81$ refer to 1981 and 1982. The number of i, j marriages between male and female individuals from states (s, S) respectively at time t is denoted by μ_{ijt}^{sS} .

To provide a benchmark for our analysis, consider the marriage rate regression, where ρ_{jt}^S is the marriage rate of age j females living in state S at time t :

$$\rho_{jt}^S = h(j) + h_t(j) \cdot (1 - D_{jt}) + h^R(j) \cdot D_j^R + h_t^R(j) \cdot D_j^R \cdot D_{jt} + v_{jt}^S \quad (18)$$

We use D to denote dummy variables and $h(x)$ to denote some general nonparametric function which has x as its argument. The variable D_{jt} takes a value of 1 for $t = 1971/72$ and

zero otherwise; D_j^R takes a value of 1 if the female individual is from a reform state, and zero otherwise; v_{jt}^S is an error term with mean zero.

The terms $h_t(j)$ and $h^R(j)$ allow for age specific time trend and age specific state effect respectively. Then $h_t^R(j)$ measures the impact of living in a reform state at $t = 71$ on the marriage rate of type j females. The function $h_t^R(j)$ can be estimated non-parametrically by appropriately smoothing the difference in differences (DD) estimator:

$$\Delta^2 \rho_j^f = (\rho_{j71}^R - \rho_{j81}^R) - (\rho_{j71}^N - \rho_{j81}^N). \quad (19)$$

The total gains to marriage can be parameterized in a similar manner. Let the marriage gains to an age i male living in state s with an age j female living in state S at time t , π_{ijt}^{sS} , be given by:

$$\begin{aligned} \pi_{ijt}^{sS} = & g(i, j) + g_t(i, j) (1 - D_{ijt}) + g^{rR}(i, j) D_{ij}^{rR} + g^{nR}(i, j) D_{ij}^{nR} + g^{rN}(i, j) D_{ij}^{rN} \quad (20) \\ & + g_t^{rR}(i, j) D_{ij}^{rR} D_{ijt} + g_t^{nR}(i, j) D_{ij}^{nR} D_{ijt} + g_t^{rN}(i, j) D_{ij}^{rN} D_{ijt} + \varepsilon_{ijt}^{sS}. \end{aligned}$$

The notational convention adopted in equation (18) applies. The dummy variable D_{ijt} takes a value of 1 for age combinations in years $t = 1971/72$ and zero otherwise; the variable D_{ij}^{rN} takes a value of 1 for couples where the male resides in the reform states, r , and the females in the non-reform state, N and zeros otherwise, and so on. The function $g(i, j)$ captures the systematic gain to marriage for an age i male in a non-reform state with an age j female in a non-reform state in 1971/72. It forms the base gains to marriage that varies according to the ages of the couples, (i, j) . The functions $g^{rR}(i, j)$, $g^{nR}(i, j)$ and $g^{rN}(i, j)$, captures the remaining fixed effects arising from the state of residence of the couple. For example, $g^{rR}(i, j)$ is the increment in systematic gains added to the base $g(i, j)$ if the couples are both from the reform states.

The increment to the gains to marriage in years 1981/82 for an (i, j) pair is captured by the function $g_t(i, j)$. This time effect is assumed to be independent of the state of residence. The function $g_t^{sS}(i, j)$ is the increment to the gains to marriage in $t = 1971/72$ between a male in state s and female in state S for state combinations $sS \neq nN$. If we expect the legalization of abortion in the reform states to have lowered the gains to marriages among young adults who both reside in those states, then $g_t^{rR}(i, j) < 0$ for young couples. The mean zero error term is denoted by ε_{ijt}^{sS} .

Our model for the systematic gains to marriage in equation (20) has some advantages over the marriage rate formulation in equation (18). First, the formulation using the systematic gains satisfies all the restrictions of a marriage matching function while the marriage rate models of the form in equation (18) do not impose any restriction between different marriage rates. Second, our model can distinguish between the effect of the legalization of abortion on the systematic gains to marriage for age i males with different types of females. For example, $g_t^{rR}(i, j)$ need not be the same as $g_t^{rR}(i, j')$.

For any age combination (i, j) with observed marriages, the systematic gains, π_{ijt}^{sS} is estimated by $\widehat{\pi}_{ijt}^{sS} = \ln(\mu_{ijt}^{sS} / \sqrt{\mu_{i0t}^{sS} \mu_{0jt}^{sS}})$. The increment in the gain to marriage for an i, j pair in 1971/72 who lived in reform states, $g_t^{rR}(i, j)$ can be estimated by the DD estimator:

$$\Delta^2 \pi_{ij}^{rR} = (\widehat{\pi}_{ij71}^{rR} - \widehat{\pi}_{ij81}^{rR}) - (\widehat{\pi}_{ij71}^{nN} - \widehat{\pi}_{ij81}^{nN}) \quad (21)$$

Note the similarity between $\Delta^2 \pi_{ij}^{sS}$ and the standard DD marriage rate estimator, $\Delta^2 \rho_t^f$, for $l = i, j$. Although $\Delta^2 \pi_{ij}^{sS}$ is defined for an age pair (i, j) , and state pair (s, S) rather than for male or female ages alone, it is as easy to estimate as equation (19).

Data:

Using information on the place of residence from the *US Census*, and the marriage records from the *Vital Statistics*, we classify the data described in Section 3 according to whether the place of residence of an individual is a reform or non-reform state. Table 2 provides a summary of the data used.

The sample of available males and females on the marriage market from the non-reform states is considerably larger than that of the reform states. The increase in the population observed over the decade in the two groups of states also differ in magnitude. In the reform states, the population of available males and females increased by 50.4 % and 54.6 % respectively, compared to a more modest increase of 38.2 % and 30.7 % for available males and females respectively in the non-reform states. The average age of males and females in the two groups of states are comparable to that of the entire sample reported in Table 1.

As expected, marriages between individuals in the same state of residence are more likely relative to marriage between individuals living in different states. There are 2.1 million marriages between couples in the non-reform states, (μ^{nN}) , compared to 1.05 million between couples in the reform states, (μ^{rR}) , in 1971/72. The number of cross-marriages in 1971/72, (μ^{rN}, μ^{nR}) , is around 40,000.

The changes in the total number of marriages across the four groups over the decade differ in magnitude and sign. Marriages between reform state males and non-reform state females decreased by 14.8 % while marriages between males from non-reform states and females from the reform states decreased by almost 30 %. In the reform states where there was little change in access to legalized abortion over the decade, we find total marriages increased by 17 % while in the non-reform states where legalized abortion became more accessible, total

TABLE 2: Data summary based on place of residence

<i>US Census Data</i>	1970	1980	Δ
No. of available males in reform states (M^r)	5.76 mil	9.24 mil	60.41%
No. of available females in reform states (F^r)	6.70 mil	10.36 mil	54.63%
No. of available males in non-reform states (M^n)	10.25 mil	14.17 mil	38.24%
No. of available females in non-reform states (F^n)	12.90 mil	16.86 mil	30.70%
Aver. age of available males in reform states	30.00	29.62	
Aver. age of available females in reform states	38.93	36.93	
Aver. age of available males in non-reform states	30.64	29.53	
Aver. age of available females in non-reform states	39.22	37.24	
<i>Vital Statistics</i>	1971/72	1981/82	Δ
No. of marriages in rR states (μ^{rR})	1.05 mil	1.26 mil	17.17%
No. of marriages in rN states (μ^{rN})	45,456	38,730	-14.80%
No. of marriages in nR states (μ^{nR})	39,367	30,358	-29.68%
No. of marriages in nN states (μ^{nN})	2.10 mil	2.11 mil	0.56%
Aver. age married males in reform states	27.5	29.6	
Aver. age married males in non-reform states	26.9	28.9	
Aver. age married females in reform states	24.8	26.8	
Aver. age married females in non-reform states	24.4	26.2	

marriages only increased by 0.56 %. It is this differential change in marriage patterns in the four groups and the changes in the population of marriage market participants that provide identification of the fall in marriage gains due to legalizing abortion.

Results:

Figure 6(a) shows estimates of the decrease in marriage rates in the reform states from the DD marriage rate estimators, $\Delta^2 \rho_l^k$ ($k = m, f$). Consistent with the findings in Angrist and Evans 1999, $\Delta^2 \rho_l^k$ are negative for both young males and young females. There is evidence of a small increase in the marriage rate of males, and a smaller increase in the marriage rates of females, between the ages of 30 to 40. As explained later, it is problematic that the estimated effects for males are significantly larger than that for females.

Figure 6(b) shows estimates of $\Delta^2 \pi_{ij}^{rR}$ for same aged spouses where $i = j$. This slice of the distribution is informative because there are many same aged spouses. The drop in the gains to marriage for same age spouses, between the ages of 19 to 26, living in reform states in 1971/72 relative to those living in non-reformed states, is substantial. We also see a small increase in the gains to marriage for same age spouses, between the ages of 27 to 40. An explanation of these gains is that these are young individuals who would have gotten married young had abortion not been legalized. This social change allow these individuals to delay marriage to an older age.

We interpret the effects displayed in Figures 6 as due to the partial legalization of abortion on marriage rates and the gains to marriage. The standard DD argument for identification is based on the claim that the policy intervention of interest generates year and location specific interaction effects that would otherwise not be there. In addition to the standard argument, we also expect partial legalization to affect young adults more than older adults which is consistent with the evidence in Figures 6.

In order to quantify the effect of the partial legalization of abortion on marriage rates, we

use the two estimators, $\Delta^2 \pi_{ij}^{sS}$ and $\Delta^2 \rho_l^k$ ($k = m, f$), to do a counterfactual experiment. Consider an experiment where the non-reform states also partially legalize abortion in 1971/72 like the reform states. The estimates from the DD marriage rate equation (19), allow us to construct a counterfactual marriage rate for male and female in the non-reform states.²⁰ Using the counterfactual marriage rates in the non-reform states and the observed rates in the reform states, we construct an aggregate male and female marriage rate in the scenario where there was no differential access to abortion in 1971/72.

A comparable counterfactual marriage rate can be constructed using the DD marriage gains estimator. Using the estimates from equation (21), we first construct gains to marriage in the non-reform states in the counterfactual scenario that abortion was partially legalized in these states in 1971/72.²¹ We then compute the number of marriages (and the implied male and female marriage rates) that would have been observed using these counterfactual marriage gains and the observed marriage gains for the reform states.

Let the counterfactual aggregate marriage rates in 1971/72 constructed using the DD marriage rate and DD marriage gains estimator be denoted by C_{j71}^ρ and C_{j71}^π respectively. The graphs in Figure 7 (a) and (b) compares the total observed change in marriage rates for age k over the decade, $\Delta \rho_k^l = \rho_{k81}^l - \rho_{k71}^l$ (where $l = m, f$ is an index for gender), with the change in marriage rate in 71/72 in the above counterfactual scenario as suggested by the DD marriage rate estimator, $\Delta^\rho \rho_k^l = C_{k71}^l - \rho_{k71}^l$, and by the marriage gain estimator,

²⁰Using our estimate of $\widehat{h_{71}^s(k)}$, let the counterfactual marriage rates for males and females be denoted by $\widetilde{\rho_{i71}^n}$ and $\widetilde{\rho_{j71}^N}$ respectively, where $\widetilde{\rho_{k71}^s} = \rho_{k71}^s - \widehat{h_{71}^s(k)}$ where $(s, k) \in \{(n, i), (N, j)\}$.

²¹We first estimate $\widehat{g_{71}^{rR}(i, j)}$, $\widehat{g_{71}^{rN}(i, j)}$, and $\widehat{g_{71}^{nR}(i, j)}$ according to equation (21). These counterfactual marriage gains, $\widehat{\pi_{ij71}^{nN}}$, $\widehat{\pi_{ij71}^{rN}}$, and $\widehat{\pi_{ij71}^{nR}}$ are estimated according to this equation, $\widehat{\pi_{ij71}^{sS}} = \widehat{\pi_{ij71}^{sS}} - \widehat{g_{71}^{sS}(i, j)}$ where $sS \in \{nN, rN, Rn\}$, $\forall i, j$.

$\Delta^\pi \rho_k^l = C_{k71}^\pi - \rho_{k71}^l$.²² As discussed earlier, marriage rates for males and females fell over the decade. Both $\Delta^\rho \rho_k^l$ and $\Delta^\pi \rho_k^l$ suggest that a quantitatively significant part of the fall in aggregate marriage rates for young adults over the decade is attributable to the lack of partial legalization in the non-reform states in 1970. $\Delta^\pi \rho_{22}^m$ suggests that 20% of the observed fall in the 22 year old male marriage rates can be attributed to partial legalization compared to the 31% estimate from $\Delta^\rho \rho_{22}^m$. The estimates of female marriage rate decrease attributable to the partial legalization of abortion is more modest.

While the both estimators provide qualitatively similar results, the quantitative predictions of the two estimators are very different. The estimate from the female DD marriage rates estimator suggest that legalizing abortion in the non-reform states would have resulted in 7080 less marriages in 1971/72 while the estimate using the male marriage rates is 196,270.²³ The latter estimate is larger by a factor of 27 times!²⁴ This kind of discrepancy from male and female marriage rate regressions is not unusual. So while marriage regressions are easy to use and interpret, the biases in these estimators can be substantial. The estimate from the DD marriage gains estimator is around 45,440 less marriages among individuals aged 16 to 75 years of age. In other words, a partial legalization of abortion in the non-reform states in 1971/72 would have resulted in 1.4 % less total US marriages in that period. Among young individuals the decrease is more pronounced. For males aged 16 to 25 years old, partial abortion legalization in the non-reform states would have lowered the

²²Non-reform states went from no legalization to full legalization between 1970 and 1980. This change can be conceptually decomposed into (1) no legalization to partial legalization, and (2) partial legalization to full legalization. We are asking how much of the change in marriage rates over the decade can be attributed to the conceptual change from no legalization to partial legalization.

²³The total number of recorded marriages in 1971/72 is 3,235,806.

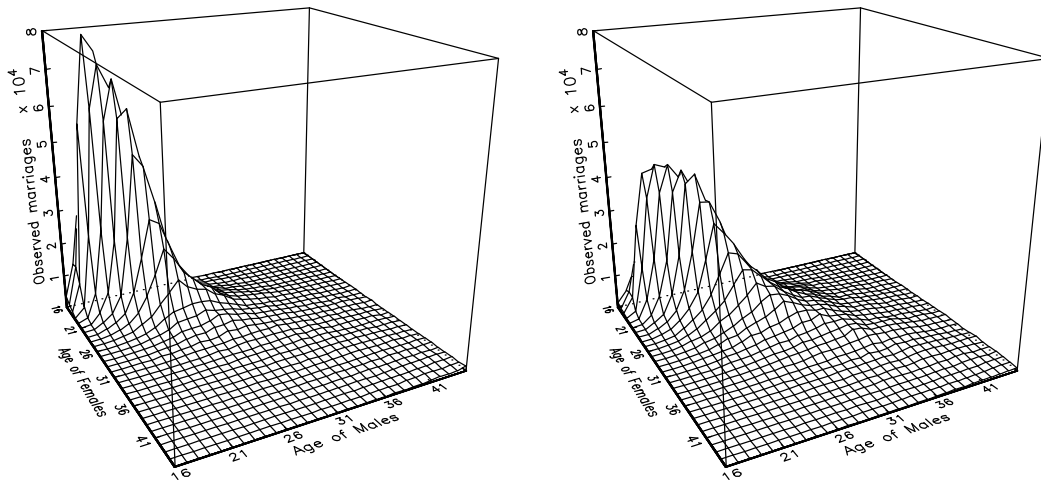
²⁴While smaller, significant disparity remains if we limit ourselves to marriages for individuals less than age thirty.

number of total marriages in this age group by 4.2 % while among 16 to 25 years old females, the decrease is around 3.6 %. For males older than 26 years of age, this social change would have increased the total number of marriages in this age group by 3.8 % and for females older than 26 years of age, the increase is around 5.2 %.

4 Conclusion

We provide brief suggestions for further research. Empirically, due to space constraints, we have only briefly investigated the effect of legalized abortions on the gains to marriage (Granberg and Granberg 1980). Other social changes also needs to be examined. Theoretically, dynamic considerations are needed (see Choo and Siow (2005)). This model of marriage matching should be intergrated with models of intrahousehold allocations such as Chiappori, et. al., Lundberg and Pollak 1993. Finally, substitution patterns of the current model need to be better understood.

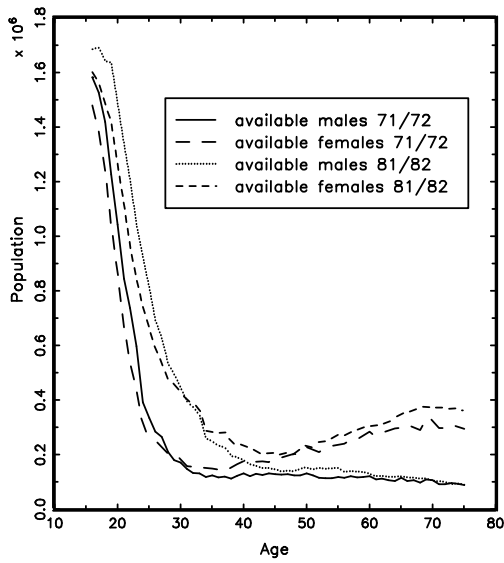
FIGURE 1



a) Surface of observed μ_{ij} for 1971/72

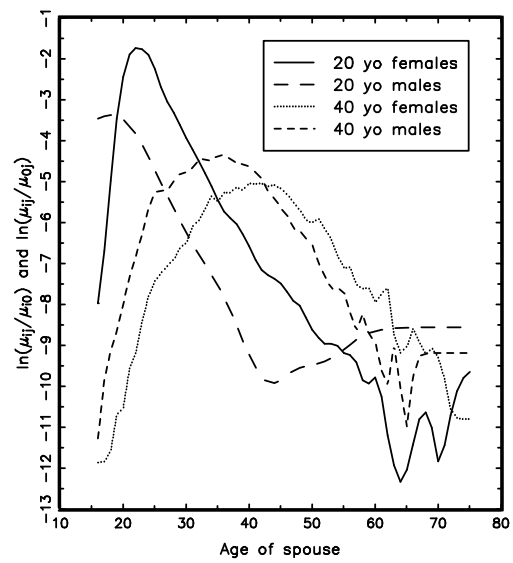
b) Surface of observed μ_{ij} for 1981/82

FIGURE 2



Availables in 71/72 and 81/82

FIGURE 3



Systematic net returns for 20 and 40 year old

FIGURE 4

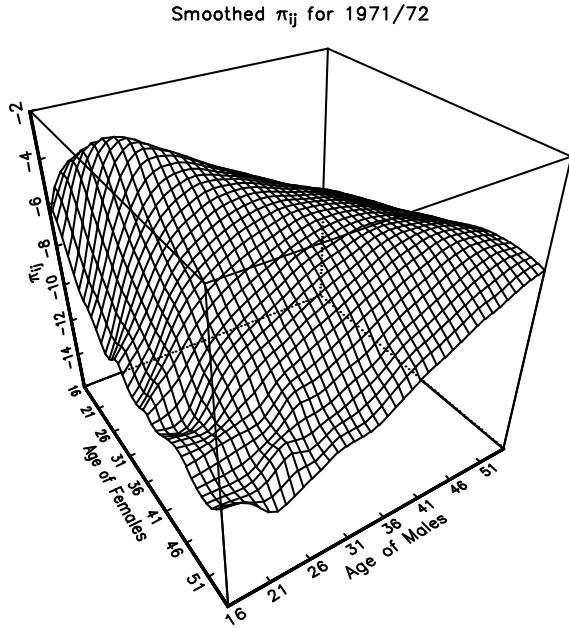


FIGURE 5

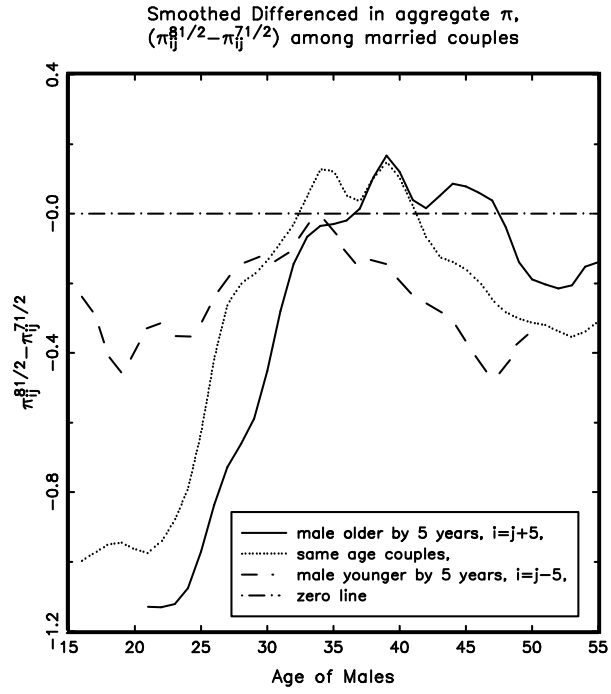


FIGURE 6

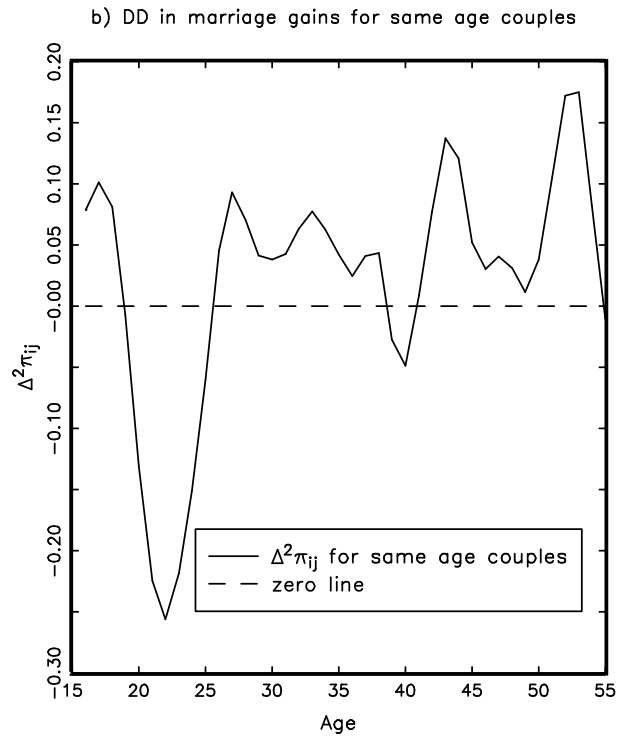
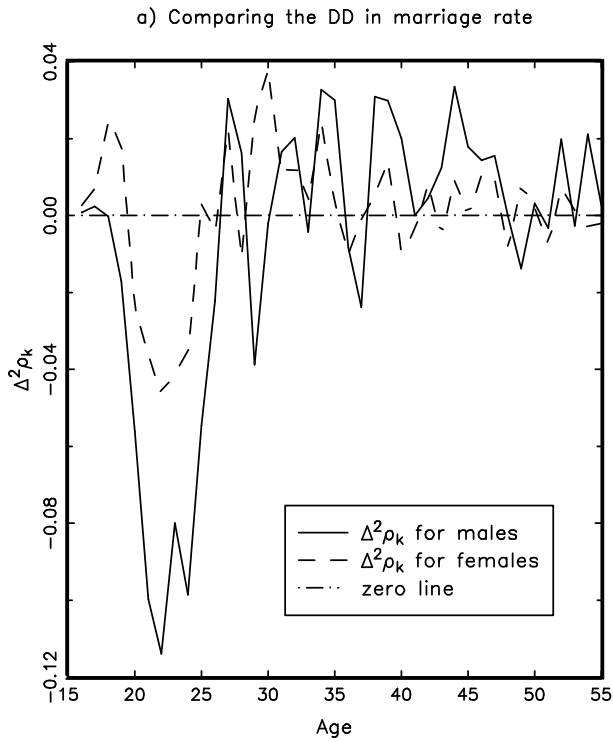
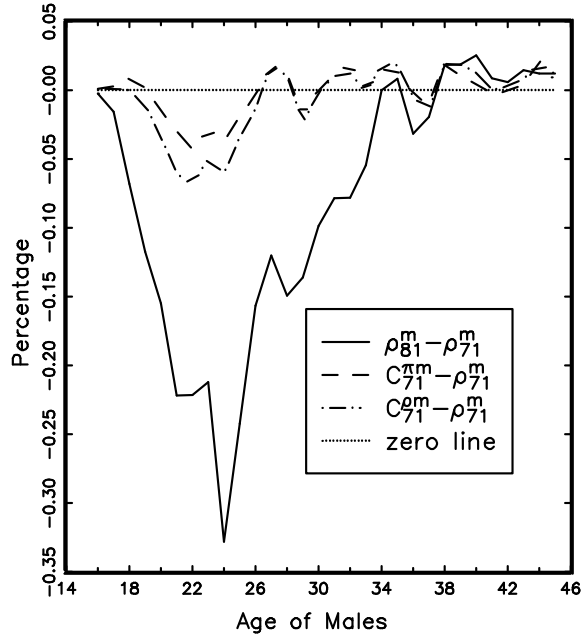
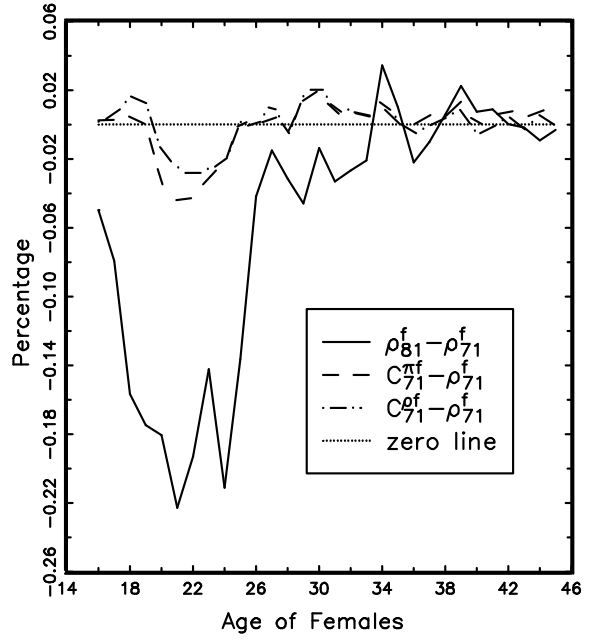


FIGURE 7

a) Comparing observed total change in male mr with change attributed to legalizing abortion from the two estimators



b) Comparing observed total change in female mr with change attributed to legalizing abortion from the two estimators



5 Appendix A

The derivations of (7) and (8) are known and included here for completeness.

5.1 Derivation of (7)

(4) may be rewritten as

$$V_{ijg} = \tilde{\alpha}_{ij} - \tau_{ij} + \epsilon_{ijg} = \eta_{ij} + \epsilon_{ijg}$$

where ϵ_{ijg} has an extreme value distribution with cdf $F(\epsilon) = e^{-e^{-\epsilon}}$. As specified by (6), individual g solves $V_{ig} = \max_j \{V_{i0g}, \dots, V_{ijg}, \dots, V_{iJg}\}$. The probability that a option j is chosen:

$$\begin{aligned} & \mathbb{P}\{j = \arg \max_{k=0, \dots, J} V_{ikg}\} = \mathbb{P}\{\epsilon_{ikg} < \eta_{ij} - \eta_{ik} + \epsilon_{ijg} \ \forall k \neq j\} \\ &= \int_{-\infty}^{\infty} \prod_{k \neq j} F(\eta_{ij} - \eta_{ik} + \epsilon_{ijg}) f(\epsilon_{ijg}) d\epsilon_{ijg} \\ &= \int_{-\infty}^{\infty} \prod_{k \neq j} \exp\left\{-\exp\left[-(\eta_{ij} - \eta_{ik} + \epsilon_{ijg})\right]\right\} = \exp\left\{-\epsilon_{ijg} - \exp(-\epsilon_{ijg})\right\} d\epsilon_{ijg} \end{aligned}$$

Let $z_k = \exp\{-(\eta_{ij} - \eta_{ik})\}$, and apply a change of variable $\psi = \exp(-\epsilon_{ijg})$, we get

$$\mathbb{P}(j) = \int_0^{\infty} \exp\left\{-\psi\left(1 + \sum_{k \neq j} z_k\right)\right\} d\psi = \frac{1}{1 + \sum_{k \neq j} z_k} = \frac{\exp \eta_{ij}}{\sum_{k=0}^J \exp \eta_{ik}}$$

When there are many men of each type, we may approximate $\mathbb{P}\{j = \arg \max_{k=0, \dots, J} V_{ikg}\}$ with $\frac{\mu_{ij}}{m_i}$. Then (7) follows.

5.2 Derivation of (8)

Conditional on male individual g of type i choosing to match with a type j woman, the expected utility of that individual is:

$$\mathbb{E}(V_{ijg} \mid j = \arg \max_{k=0, \dots, J} V_{ikg}) = \eta_{ij} + \mathbb{E}(\epsilon_{ijg} \mid \epsilon_{ijg} + \eta_{ij} > \eta_{ik} + \epsilon_{ikg} \ \forall k \neq j)$$

$$\begin{aligned}
& E(\varepsilon_{ijg} | \varepsilon_{ijg} + \eta_{ij} > \eta_{ik} + \varepsilon_{ikg} \forall k \neq j) \\
&= \frac{\int_{-\infty}^{\infty} \varepsilon \exp\{-\sum_{k \neq j} e^{-\varepsilon - \eta_{ij} + \eta_{ik}}\} e^{-\varepsilon - e^{-\varepsilon}} d\varepsilon}{\Pr\{V_{ig} = V_{ijg} | \eta\}}
\end{aligned} \tag{22}$$

Using the change of variable described above and the fact that $\int_{-\infty}^{\infty} x e^x \exp(-\phi e^x) dx = -(\Gamma + \ln \phi)/\phi$ where Γ is Euler's constant, $\simeq 0.577215$, thus

$$\mathbb{E}(V_{ijg} | j = \arg \max_{k=0, \dots, J} V_{ikg}) = \Gamma + \ln\left(\sum_k \exp \eta_{ik}\right) \tag{23}$$

which is independent of j . Since knowing the optimal choice of the individual is not informative about his expected payoff, $EV_{ig} = EV_{ijg}$. Then (23) and (7) imply:

$$EV_{ig} = \Gamma + \ln\left(\sum_k \exp(\tilde{\alpha}_{ik} - \tau_{ik})\right) = \Gamma + \tilde{\alpha}_{i0} + \ln m_i - \ln \mu_{i0} \tag{24}$$

which is (8).

6 Appendix B

The system (13) can be reduced to an $I + J$ system with $I + J$ number of unmarrieds of each type, μ_{i0} and μ_{0j} , as unknowns. This reduced system defined by equations (25) and (26), is derived by summing equation (13) over all i 's and j 's respectively.

$$f_j - \mu_{0j} = \sum_{i=1}^I \Pi_{ij} \sqrt{\mu_{i0} \times \mu_{0j}} \tag{25}$$

$$m_i - \mu_{i0} = \sum_{j=1}^J \Pi_{ij} \sqrt{\mu_{i0} \times \mu_{0j}} \tag{26}$$

If we can solve for μ_{i0} and μ_{0j} , then the μ_{ij} 's are fully determined by equation (13). We apply the implicit function theorem to the system (25) and (26). Consider taking derivatives

with respect to m_r , we get the linear system

$$\begin{bmatrix} D_J & B \\ C & D_I \end{bmatrix} \cdot \begin{bmatrix} \Delta_r f \\ \Delta_r m \end{bmatrix} = \begin{bmatrix} 0 \\ e_r \end{bmatrix},$$

- where D_J is a $J \times J$ diagonal matrix where the jj element is $-1 - \sum_{i=1}^I \frac{\mu_{ij}}{2\mu_{0j}}$,
- D_I is an $I \times I$ diagonal matrix where the ii element is $-1 - \sum_{j=1}^J \frac{\mu_{ij}}{2\mu_{i0}}$,
- B is a $J \times I$ matrix whose ji element is $-\frac{\mu_{ij}}{2\mu_{i0}}$.
- C is an $I \times J$ matrix whose ij element is $-\frac{\mu_{ij}}{2\mu_{0j}}$.
- $\Delta_r m$ is an $I \times 1$ vector where the i 'th element is $\partial\mu_{i0}/\partial m_r$,
- $\Delta_r f$ is a $J \times 1$ vector where the j 'th element is $\partial\mu_{0j}/\partial m_r$,
- and e_r is an $I \times 1$ zero vector with -1 at the r 'th element. ²⁵

We need to show that the Jacobian of the system is non-singular. As long as $\mu_{i0} \neq 0$ and $\mu_{0j} \neq 0$, we know D_I^{-1} and D_J^{-1} exist. Using the formula for a partition inverse, we get

$$\Delta_r f = D_J^{-1} \left[I_J - BD_I^{-1}CD_J^{-1} \right]^{-1} e_r, \quad (27)$$

$$\Delta_r m = -D_I^{-1}C \cdot \Delta_r f. \quad (28)$$

The Jacobian is non-singular as long as $\left[I_J - BD_I^{-1}CD_J^{-1} \right]^{-1}$ exists. Let $A = BD_I^{-1}CD_J^{-1}$, then $(I_J - A)$ is invertible if there is a matrix norm $\|\bullet\|$ such that $\|A\| < 1$. Consider the *maximum column sum matrix norm* defined by, $\|A\| = \max_j \sum_{i=1}^n |a_{ij}|$. Then:

$$\|CD_J^{-1}\| = \max_j \frac{\sum_i \mu_{ij}}{2\mu_{0j} + \sum_i \mu_{ij}} < 1$$

$$\|BD_I^{-1}\| = \max_i \frac{\sum_j \mu_{ij}}{2\mu_{i0} + \sum_j \mu_{ij}} < 1.$$

²⁵If we take a derivative with respect to f_r , the system of first derivatives will have a similar form except the position of 0 and e_r is reversed.

By definition of a matrix norm, $\|BD_I^{-1}CD_J^{-1}\| \leq \|BD_I^{-1}\| \cdot \|CD_J^{-1}\| < 1$, and hence $(I_J - A)^{-1}$ exists. \square

7 Appendix C: Data

Data used were extracted from the Integrated Public-Use Microdata (IPUMS henceforth) Files of the US Census. The samples used were the 5% state samples for 1980, and the 1% Form 1 and Form 2 samples for 1970. The 1970 datasets were appropriately scaled to be comparable with the 1980 files.²⁶ To maintain consistency between states reporting marriages to the *Vital Statistics* and the data collected from the respective *US Census*, some states to be excluded. This result in the data from the following states being used: Alabama; Alaska; California Connecticut; Delaware; District of Columbia; Florida; Georgia; Hawaii; Idaho; Illinois; Indiana; Kansas; Kentucky; Louisiana; Maine; Maryland; Massachusetts; Michigan; Mississippi; Missouri; Montana; Nebraska; New Hampshire; New Jersey; New York State; North Carolina; Ohio; Oregon; Pennsylvania; Rhode Island; South Dakota; Tennessee; Utah; Vermont; Virginia; West Virginia; Wisconsin and Wyoming.²⁷ The age range studied was 16 to 75 years of age. Education level was identified using the “**higradeg**” variable in the 1970 and 1980 samples. This variable allowed us to assign each person one of the following schooling types: less than highschool, highschool graduate, and college degree or more. We use the “**marst**” variable in the census to identify a person as either: never married, currently married (spouse present), or previously married (divorced or widowed).

²⁶State of residence in the 1970 census files can only be identified in the state samples (Form 1 and Form 2 samples, both of which are 1% samples). This is the reason that the other samples were not used for 1970 calculations. Further, the age of marriage variable is only available in Form 1 samples in 1970 which meant that only one sample, the Form 1 state sample, was used for calculations involving married couples in the 1970 census.

²⁷In other words the excluded states (cities) are: Arizona, Arkansas, Colorado, Iowa, Minnesota, Nevada, New Mexico, New York City, North Dakota, Oklahoma, South Carolina, Texas and Washington.

Further, the “marrno” variable (in 1970 and 1980 datasets) allows us to distinguish between married individuals in their first marriage and individuals in their second or later marriage. To calculate the number of unmarried individuals of each type, we simply collapse the census data into counts by type.

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