Who should practice price discrimination in an asymmetric duopoly?

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Abstract

Price discrimination is generally thought to improve firm profits by allowing firms to extract more consumer surplus. In competition, however, price discrimination may also be costly to the firm because restrictive incentive compatibility conditions may allow the competing firm to gain market share at the discriminating firm's expense. Therefore, with asymmetric competition, it may be the case that one firm would let the other firm assume the burden of price discrimination. We investigate optimal segmentation in a market with two asymmetric firms and two heterogeneous consumer segments that differ in the importance of price and product attributes. In particular, we investigate second-degree price discrimination under competition with explicit incentive compatibility constraints thus extending prior work in marketing and economics. Focusing on the managerial implications, we explore whether it would be profitable for either or both firms to pursue a segmentation strategy using rebates as a mechanism. We identify conditions under which one or both firms would want to pursue such segmentation. We find that segmentation lessens competition for the less price-sensitive consumer segment and that this results in higher profits to both firms. A key to understanding this result is the customer remixing that segmentation leads to. We establish the key result that if firms are asymmetric in their attractiveness to consumers, the disadvantaged firm in our model is more likely to pursue a segmentation strategy than its rival in equilibrium. We then ask whether this result obtains in practice. To this end, we explore competitive segmentation empirically and are able to verify that disadvantaged firms indeed pursue segmentation through rebates with greater likelihood.

Key Words: *Segmentation; Competition; Game Theory; Pricing, Rebates; Printers*

1. Introduction

In this paper we investigate the role of segmentation under competition. In particular we are interested in studying competition among asymmetric firms and exploring the consequences of segmentation for *customer remixing*-- changing the clientele mix that each firm serves-- for firms' profits and for equilibrium prices. To this end, we explore a market with two firms that compete for two types of consumers. The two types are heterogeneous in how they make a trade-off between their desired combination of attributes and price. We capture the competition between the firms in a Hotelling framework. We then ask if it would be profitable for both firms to segment the market. Specifically, we assume that segmentation can be implemented by the mechanism of rebates, although our model would apply equally to the case of coupons. We solve for equilibrium pricing and rebate strategies and then derive the profits with segmentation by one firm, both firms, or neither firm. In other words, either, both or neither firm can implement a segmentation mechanism based on rebates. We obtain an interesting insight into how segmentation affects competition. It softens competition for one segment of consumers that values attributes more than price, while simultaneously increasing the competition for the other segment. In sum, this allows firms to charge higher prices for the segment of consumers that values attributes more than price, as we might expect given the attendant price discrimination. What is more interesting is that the net effect of segmentation is higher profits for both firms, despite their asymmetry. In their pioneering work, Shaffer and Zhang (1995) demonstrated that targeting in competition can be a dominant strategy (if costs of targeting are low) but Pareto inferior in equilibrium—i.e., prisoner's dilemma. They discuss second degree price discrimination by *symmetric* firms in a *two-stage* setting and find that it is always profitable, for example through mass distribution of coupons. Nevertheless, their work mainly focuses on targeting and the difference between targeted and mass distributed coupons. We extend their work on second degree price discrimination by focusing on mass distribution of rebates by *asymmetric* firms in a *simultaneous-move* game with *explicitly* modeled incentive compatibility constraints. These three modeling differences allow us to identify when untargeted second degree price discrimination may or may not be profitable in an asymmetric duopoly. We find that under some

conditions, unrelated to costs but rather related to incentive compatibility constraints; second degree price discrimination may not be profitable for one or both firms. Finally, we also establish which of the two firms in an asymmetric duopoly has greater incentive to engage in second degree price discrimination through untargeted rebates. Segmentation, whether pursued by one or both firms, leads to customer remixing, resulting in each firm's clientele being different from what would have occurred without segmentation. Indeed, this remixing is central to firms' ability to garner higher profits, thus providing new insights into the strategic consequences of segmentation under competition. Given our focus on competition between asymmetric firms, we also find that the firm whose attractiveness is lower is more likely to pursue a segmentation strategy, through the use of rebates, than its rival.

We then investigate competitive rebate offerings empirically. There are few studies that have explored this in the context of product markets, an exception being Nevo and Wolfram (2002) who studied competitive segmentation based on coupons in the cereal market. Their main focus was to verify that coupons were indeed acting as price discrimination mechanisms. Similarly, other empirical works investigated the existence of second degree price discrimination in various markets with a specific emphasis on the effects of competition. These include Shepard (1991) (gasoline sales), Seim and Viard (2006) (cellular phone services), and Borzekowski, Thomadsen and Taragin (2006) (mailing lists). Lal and Rao (1997) studied competitive segmentation but in grocery retailing with the focus being on positioning and pricing. We are interested in examining the role of rebates in competitive segmentation with particular emphasis on which among asymmetric firms would find such a strategy more profitable. Several interesting questions come up in light of firm asymmetry. Do stronger brands employ segmentation mechanisms more often? What is the relationship between brand price and extent of segmentation? Even within a product category, are high-end or low-end sub-categories more likely to engage in segmentation? Our analytical model provides some guidance in examining these issues. For the empirical analysis, we use data from the printer market to see if the firm whose attractiveness is lower is more likely to pursue a segmentation strategy.

The rest of the paper is organized as follows. In section 2 we provide an extensive review of the literature that positions our work in context relative to the large body of prior work. We then describe our consumer model and how firms compete both on segmentation strategy and pricing strategy in section 3. In section 4 we analyze our model and derive the equilibrium segmentation and pricing strategies. We present our central result contained in propositions 2-4. In section 5 we analyze the printer market and present our empirical results. We end with concluding comments in section 6.

2. Review of Literature

Market segmentation has been studied extensively, with particular emphasis on making differentiated offerings to markets with heterogeneous consumers. For example, if consumers are heterogeneous in how they value quality, a firm would want to offer a product line with vertically differentiated products rather than a single product to cater to the different types of customers. This would allow the firm to charge differential prices and earn higher profits (see, for example, Moorthy, 1984). In general, segmentation involves price discrimination entailing different offers to different groups of consumers. For example, segmentation based on price sensitivity can be accomplished often by the use of coupons if price sensitive consumers also happen to have a low cost of time (Narasimhan, 1984). Thus, coupons serve as a mechanism to engage in price discrimination and segment the market based on price sensitivity. The usefulness of coupons in this context has been explored by numerous researchers: Gerstner and Hess (1991), Shaffer and Zhang (1995), Moraga-Gonzalez and Petrakis (1999), and Krishna and Zhang (1999). Of course, what makes price discrimination and the resulting segmentation profitable is consumer heterogeneity.

Turning to competition, the challenge of lessening it can be met in different ways. For example, Kim, Shi and Srinivasan (2001), following the arguments of Klemperer (1987), show that frequency based loyalty programs, effectively segmenting the market into heavy and light users, can soften price competition by introducing endogenous switching costs. In product markets in which some consumers are brand loyal and others willing to switch, competitive pricing often leads to price promotions (Narasimhan,

1988; Raju, Srinivasan and Lal, 1990; Rao, 1991). These temporary promotions, arising from mixed strategies, do not involve a constant low price targeted at switchers because that would leave money on the table for loyal consumers whose brand choice is unaffected by price promotions. In this way price promotions can be seen as a way to lessen price competition for a segment of consumers. Segmentation under competition has also been addressed by Coughlan and Soberman (2005) and Lal and Rao (1997) in the context of retailing.

In the absence of segmentation, consumer types that are willing to pay more for a product end up paying less, in essence receiving a subsidy. Segmentation can have the beneficial consequence of lessening or eliminating this subsidy. This has been shown to be the case by Kumar and Rao (2006) who find that complete retail segmentation by offering rewards eliminates subsidy to one segment of consumers when segments are heterogeneous in their shopping baskets. Economists have also studied third-degree price discrimination under competition and the obvious implications for segmentation. Borenstein (1985) studied price discrimination in monopolistic competition using simulations and found that such discrimination can increase profits for firms by lessening competition for one segment. Holmes (1989) provided a rigorous analytical framework for a similar setting in a duopoly and Corts (1998) extended the work of Holmes and relaxed some of the assumptions to provide additional insights. Corts (1998) modeled third degree price discrimination in order to determine when price discrimination[1] would have unambiguous price and welfare effects. One of his concerns was with preventing prisoner's dilemma, an equilibrium in which all firms are worse off relative to some other outcome but in which all firms choose their dominant strategies. This is an important feature since price discrimination is a dominant strategy in the third degree price discrimination as Corts modeled it. As we will show, second-degree price discrimination is not a dominant strategy given explicit incentive compatibility constraints. Corts (1998) remains focused on how firms can strategically commit to not price discriminate when price

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[1] Corts introduces the concept of 'best-response-asymmetry" that arises when firms rank their best market segments differently; i.e., firms have different "strong" and "weak" markets. The best-response-asymmetry is the necessary condition to have what Corts calls "all-out-competition" where prices fall for all consumer groups with third degree price discrimination.

discrimination leads to prisoner's dilemma, perhaps through the use of some mechanisms such as every day low price. We show that the decision not to price discriminate can occur in a natural way when incentive compatibility must be explicitly considered.

In contrast to Holmes (1989) and Corts (1998), we are able to make less ambiguous predictions and provide sharper insights into the effects of asymmetry. Prior work on price discrimination in competitive environments has made use of targeting, third-degree price discrimination or second-degree price discrimination with *implicit* incentive compatibility constraints that allows firms to target without fear of leakage[2]. Our investigation is a useful contribution because it is often difficult or impossible to offer different coupons and prices to different consumers (third-degree price discrimination).

Competitive second degree price discrimination between symmetric firms is often found in works concerned with product line design (Armstrong and Vickers, 2001; Locay and Rodriguez, 1992; Schmidt-Mohr and Villas Boaz, 2008; Desai, 2001; Champsaur and Rochet, 1989). The closest settings to ours were investigated in Desai (2001) and Schmidt-Mohr and Villas-Boas (2008) in the context of product line design. In these works, as in the present work, consumers are distributed on a continuum with both horizontal and vertical preferences. As in the present work, firms face two-dimensional decisions—price and a second variable. Incentive compatibility and individual rationality are the key considerations in these works as well. The difference between these works and ours is on the focus and generality. In Desai (2001) and Schmidt-Mohr and Villas-Boas (2008) symmetry is crucial for tractability, whereas in the present work asymmetry is the focus. In the above two works, product line design allows the firms to serve segments with different quality valuations, whereas here the segments differ in their price saliency—more appropriate for coupon and rebate problems. While this does not change the basic tradeoff between the desire for discrimination and the desire to lessen competition (see Champsaur and Rochet, 1989), it drastically changes the solution concept. Finally, whereas the focus of Schmidt-Mohr

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[2] For example, in Coughlan and Soberman (2005), consumers with a high cost of time, who also have lower price sensitivity and higher value for service than low cost of time consumers, are assumed to find the outlet mall too far for convenience. This allows for perfect segmentation since the incentive compatibility constraints are assumed to be met.

and Villas-Boas (2008) is to establish the sufficient conditions for the existence of a pure strategy Nash equilibrium outcome and in Desai (2001) to characterize the conditions under which the markets provide complete or incomplete coverage, the present work is concerned with characterizing the equilibria, primarily mixed strategy equilibria, in an asymmetric environment.

Even though our work focuses on untargeted distribution of rebates, it is useful to relate it to the prior work on "targeted-promotions" by Shaffer and Zhang (1995, 2000 and 2002) and Chen, Narasimhan and Zhang (2001). Shaffer and Zhang (1995) employ a two-stage coupon targeting game by two *symmetric* firms and report that a single-stage simultaneous-move game does not have pure strategy Nash equilibrium. Our work suggests that explicitly incorporating incentive compatibility constraints can provide equilibrium in a simultaneous-move promotion game. Shaffer and Zhang (1995) also conclude that coupon targeting leads to a prisoner's dilemma outcome for two *symmetric* firms. Like Shaffer and Zhang (1995) we feel that asymmetry is critical in settings with price discrimination. Often the insights obtained by explicitly considering asymmetry turn out to be important. Shaffer and Zhang (2000) solve a simultaneous-move game of third-degree price discrimination model to study when a firm would target its own or its competitor's customers. They find that when demand is asymmetric both firms can benefit from coupon targeting. Shaffer and Zhang (2000) further report that price discrimination favors the firm with the smaller market share. Our finding for untargeted promotions that the disadvantaged firm benefits more that the advantaged rival is similar to this. In Shaffer and Zhang's (2000) work, the firm that can price discriminate does so at the expense of its rival. In contrast, we show that when only one firm employs a segmentation strategy (the disadvantaged firm) it is not necessarily at the expense of its rival. Shaffer and Zhang (2002) find that when customers are individually addressable and customer preferences are known, one-to-one promotions would favor the firm with larger or more loyal customer base. They show that the asymmetry in customer loyalty and firm size could help firms avoid prisoner's dilemma outcome. Similarly, if the targeting was less than perfect, the prisoner's dilemma outcome could be avoided for low targeting accuracy levels even when the firms are symmetric (Chen, Narasimhan, and Zhang, 2001).

3. The Model

FIRMS: In our model there are two firms, A and D. The two firms offer products that are differentiated. Differentiation is based on two factors. The first is a combination of attributes, represented by a one-dimensional space. We posit a spatial model of differentiation in a one dimensional space a la Hotelling, following Shaffer and Zhang (1995) among others. The two firms are assumed to be located at the ends of a line segment DA of unit length, as shown in Figure 1. The second basis of differentiation is the brand itself. This might capture "inimitable" characteristics that consumers might value.[3] In particular, we assume that all consumers agree that one of the brands is superior on this second factor. The firm that is superior in this sense is called the *advantaged* firm (A) and the other firm is called the *disadvantaged* firm (D). Finally, firm i, $i = A, D$, is assumed to incur a constant marginal cost C_i on its product.
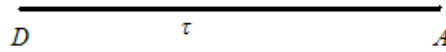

Figure 1: A spatial depiction of the competitive environment

CONSUMERS: In the spirit of the Hotelling model, we assume that consumers are located on the line segment DA. The location of a consumer defines the ideal combination of attributes desired by her. The utility of a product to a consumer τ then consists of three components: the utility for his or her ideal product, v; the disutility of a non-ideal product by virtue of its location far away from the consumer's ideal point at a distance $d_{\tau i}$, $i = A, D$; and a brand value b_i for firm i's product arising from the inimitable characteristics of firm i. Note that v and b_i are assumed to be the same for all consumers, and as a result consumer heterogeneity arises purely from considerations of location in the product space. We assume that the disutility due to a product's non-ideal location is linear in the distance of the product from the consumer's ideal point. Specifically, we model the disutility of a consumer τ, located at a distance τ from D and $(1-\tau)$ from A, as

$d_{\tau D} = \alpha \, \tau$ and $d_{\tau A} = \alpha \, (1-\tau)$.

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[3] As an example, vitamin tablets shaped like the *Flintstones* would possess an inimitable characteristic. Reputation, in the short run, would also be inimitable.

If α is high, consumers are less willing to sacrifice their desired attribute combination and we say that the product is more *salient* to consumers. On the other hand if α is low, price is more salient to consumers. Define the valuation of each brand as $V_i = v + b_i$. Then the utility of brand i to consumer τ is $(V_i - d_{\tau i})$. Finally, we assume that consumers are distributed uniformly over *DA*.

CONSUMER SEGMENTS: In our model, we assume that there are two segments of consumers, each distributed uniformly over *DA*. We denote the location at which consumers of type k are indifferent over the two firms by τ_k, $k = 1, 2$. The two segments differ in whether the product is more salient to them or price is more salient. Specifically, we assume that $\alpha_1 > \alpha_2$. This implies that for consumers in segment 1 the product is more salient while for those in segment 2, price is more salient. One way to interpret this is to say that consumers in segment 2 are more price oriented than consumers in segment 1, as in Corts (1998), Coughlan and Soberman (2005), Lal and Rao (1997) and Shaffer and Zhang (1995). This, in turn, would imply that consumers in segment 2 would also value rebates more than consumers in segment 1. To capture this, we assume that consumers in segment 1 incur a high cost of effort in redeeming rebates while consumers in segment 2 incur a lower cost of redemption. These redemption costs are exogenous to the firm[4]. Thus, denoting e_k to be the redemption cost for consumers in segment k, we assume $e_1 > e_2$. Given this inequality of redemption costs, a rebate would represent a higher value to consumers in segment 2. We refer to consumers in segment 1 as high type consumers and to consumers in segment 2 as low type consumers.[5] We assume that there is a discount associated with redeeming a rebate, denoted by

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[4] Redemption costs may be influenced by the firm's decision and as such could be treated as endogenous. However, the difference among consumers is still exogenous and we therefore treat e_1 and e_2 as exogenous.

[5] Note that in our model, redemption cost and price sensitivity are perfectly correlated. The assumption of perfect correlation is consistent with different consumer segments having different costs of time (Coughlan and Soberman, 2005; Winter, 1993). This assumption could be relaxed. As long as there is a positive correlation between low effort cost and price sensitivity, our results will hold.

δ.[6] The high type segment (segment 1) is of size γ and the low type segment (segment 2) is of size $(1-\gamma)$, where $0 < \gamma < 1$.

DYNAMICS: We assume that there is a time lag between purchase and rebate payment. This time lag is assumed to be specified exogenously, reflecting the technology of rebate processing. Thus, the time to redemption is not a strategic variable in our model. We denote time by t. Finally, we assume that both firms discount future cash flows by a discount factor δ_F. To keep the exposition simple, we assume that the firm's discount rate is the same as the consumer's discount rate, so $\delta_F = \delta$. This is not a crucial assumption and has no effect on the generality of our results.[7]

GAME: Firms decide simultaneously on their prices and rebate amounts that maximize their respective profits. The profit functions are:

$$\Pi_D = \gamma P_D \tau_1 + (1 - \gamma)(P_D - \delta^t R_D)\tau_2 \tag{1}$$

$$\Pi_A = \gamma P_A (1 - \tau_1) + (1 - \gamma)(P_A - \delta^t R_A)(1 - \tau_2) \tag{2}$$

The first term in (1) and (2) is the profit from high types and the second term is the profit from the low types. Note that the firm can offer any rebate amount, but not any rebate will effectively segment the population. If a firm decides to offer an effective rebate, it must satisfy incentive compatibility (IC) and individual rationality (IR) constraints such that consumers of each type pick the offering targeted at them. When a rebate does not meet the IC and IR conditions, the firm does not offer an effective rebate; in this case we say, without loss of generality, that the firm offers zero rebate.

Rebates are not targeted at high type consumers in segment 1, so we require that the value of the rebate be less than the cost of obtaining it for these consumers, which implies the following inequality:

$$\delta^t R_i \le e_1 \tag{3}$$

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[6] A higher discount factor is associated with a lower discount *rate*. We assume that the discounting is constant across consumers. One could alternatively let this discount factor vary in the consumer population. We chose to let the consumers' costs of effort vary and keep this discount constant, but we did work out the alternative formulation and the results are qualitatively unchanged.

[7] One could allow all three discount rates to differ. This would make the problem less tractable with little or no additional insight. Alternatively, one could have $\delta_F = \delta_1$ and leave type 2 as the "unusually patient" type. This would result in one or both firms targeting that type with rebates that mature at $t \to \infty$.

And the rebates must yield positive surplus to low type consumers in segment 2.

$$\delta^t R_i \geq e_2 \qquad \text{for all } R_i > 0 \qquad\qquad (4)$$

Consumers of each type must also obtain non-negative surplus if they decide to purchase the product. For the marginal consumer the two products must yield equal surplus. Therefore, for the marginal consumer who is indifferent to the two products, we have, for consumers in each segment the following conditions:

$$V_D - \alpha_1 \tau_1 - P_D = V_A - \alpha_1(1 - \tau_1) - P_A \geq 0 \qquad\qquad (5)$$

$$V_D - \alpha_2 \tau_2 - P_D + \delta^t R_D - e_2 = V_A - \alpha_2(1 - \tau_2) - P_A + \delta^t R_A - e_2 \geq 0 \qquad (6)$$

In equation (6) we assume that both firms offer effective rebates. If one firm, say firm i, does not offer an effective rebate, the corresponding value of $\delta^t R_i - e_2$ drops out of equation (6). Thus, the constraints change depending on whether one firm, both or neither firm offer effective rebates. To solve the problem, we characterize the best response functions for each firm and then search for the profit maximizing decisions over the full strategy space.

4. Equilibrium Outcomes and Analysis

Since the firms in our model make their rebate and price decisions simultaneously, there are discontinuities in the solution space. Each firm needs to decide whether or not to offer a rebate and determine the rebate amount and the price. There are four possible pure strategy outcomes: (1) Neither firm rebates, (2) only firm D rebates, (3) only firm A rebates and (4) both firms rebate. Under each outcome, the two firms simultaneously maximize profits subject to incentive compatibility and participation constraints. The solution procedure and the results for pure strategy equilibrium outcomes are provided in the appendix. We next examine the entire strategy space. We do this by noting that the four outcomes are more than a mere convenient solution approach. Instead, they are real discontinuities in the payoff functions for both firms. Below a certain rebate amount, the firm is essentially not offering a rebate at all. This discontinuity affects not only the firm's payoff function but its rival's payoff function as well. When the firm does not offer a rebate, the rival's incentives change in a discontinuous way.

Note that none of these four outcomes by itself constitutes an equilibrium. The procedure merely lists all feasible pure strategy solutions to the firms' maximization problems. To characterize the pure strategy equilibria for the firms, we verify whether a solution is an equilibrium by calculating the additional profit a firm can achieve by deviating from the given solution. Since the decisions are done simultaneously, when a firm deviates it is not constrained to move to another feasible solution but is free to change its decision variables, e.g. a firm that is offering a rebate in a given solution may deviate by ceasing its rebate offer and changing its price. Similarly, a firm who is not offering a rebate in a given solution may deviate to a positive rebate. We first find all feasible solutions. Then – in the appendix – characterize all pure strategy equilibria by deriving the cutoffs. The equilibrium cutoffs are reported in the appendix as conditions in terms of the valuation difference, $V_A - V_D$

Our main results are organized as follows. We start with a technical result to show that the disadvantaged firm D generally charges a lower price. This is Proposition 1. In section 4.2 we show in Proposition 2 that as the valuation difference increases, the advantaged firm is less attracted to pure strategy equilibria where it offers a rebate while the disadvantaged firm is more attracted to pure strategy equilibria where it offers a rebate. We use this result to develop the main intuition for the role that rebates play in our model. We then show that there exists no pure strategy equilibrium in which firm A offers a rebate but firm D does not. This is Proposition 3. Furthermore, in any mixed strategy equilibrium, firm D has a higher probability of offering a rebate. This is Proposition 4. Taken together, Proposition 3 and Proposition 4 suggest that in practice we are likely to see rebates offered by disadvantaged firms rather than advantaged ones. This will be the basis for our examination of data in section 5.

4.1. The pricing strategy

Note that in most equilibria, the advantaged firm's price is higher than its rival. The following proposition states this finding formally.

Proposition 1: In any pure strategy equilibrium where firms are asymmetric in consumer valuations, the disadvantaged firm's price is always lower than its rival's unless only the disadvantaged firm offers a

rebate. When only the disadvantaged firm offers a rebate, its price is lower only if the difference between the firms' consumer valuations is higher than a threshold value (u*), given by

$$u* = \frac{\alpha_1(1-\gamma)[3(\alpha_1 - \alpha_2) - e_2]}{4[(1-\gamma)\alpha_1 + \gamma\alpha_2]}.$$

Let (q, w) be a mixed strategy equilibrium profile, where q (w) is the probability that the disadvantaged (advantaged) firm will offer a rebate in a given mixed strategy equilibrium. In any mixed strategy equilibrium, the disadvantaged firm's price is lower than its rival's only if the difference between the firms' consumer valuations is higher than a threshold value (u**), given by

$$u** = \frac{\alpha_1(1-\gamma)(q-w)[6(\alpha_1 - \alpha_2) - (2+qw)e_2]}{2(4-qw)[(1-\gamma)\alpha_1 + \gamma\alpha_2]}.$$

Proof: All the proofs are relegated to the appendix.

4.2. Rebating and Segmentation in Asymmetric Duopoly

It is clear that the equilibrium depends on the model parameters . To compare all possible equilibria,, we find it useful to examine "equilibrium attraction". Attraction, or basins of attraction, is a concept often used in game theory in analysis of games with multiple equilibria (Ellison, 1993; Fudenberg and Levin, 1998). It allows the us to gauge the strength of one equilibrium relative to another. The attraction of an equilibrium can be described as the best response region of the simplex that corresponds to that particular equilibrium.

We specifically want to see how asymmetry in firm valuations affects the attraction of rebates. We find that the attraction of offering a rebate increases for the disadvantaged firm as it becomes more disadvantaged vis-à-vis its rival. Therefore, offering a rebate becomes more attractive for the disadvantaged firm. At the same time, the exact opposite result is observed for the advantaged firm. The advantaged firm benefits less from offering rebates if its advantage is higher. As the difference between

consumer valuations for the two firms widens we can expect rebates to be provided by the disadvantaged firm rather than the advantaged one. The above result seems to suggest that the basin of attraction of a rebating equilibrium is asymmetric for the two firms in a predictable manner. Using the equilibrium cutoffs, we can state the attraction to a rebating equilibrium by each firm.

Proposition 2: As the relative advantage of the advantaged firm increases, the attraction of the advantaged firm to a pure strategy equilibrium where it offers a rebate decreases while the attraction of the disadvantaged firm to a pure strategy equilibrium where it offers a rebate increases.

What is Proposition 2 telling us? When only the disadvantaged firm D offers a rebate (to the low type consumers), in effect its pricing is determined by its desire to compete for the high type consumers. Since the high types are not as price oriented as the low types, moving competition away from the low types leads to higher prices, benefiting both firms even if only one firm offers a rebate. Clearly, when only D offers a rebate, some low type consumers would still consider (the unrebated) product A, depending on the location of their ideal product. In this case, the non-zero rebate by D does not completely separate the two types of consumers, but changes the customer mix more towards high types so that "average" price salience is lower. This separation of the two types of consumers is a segmentation strategy. Under competition, firms would like to compete separately for the two segments. In particular, price is more salient for the low types, and so competing for them lowers prices for the high types. This can be avoided if the low types could be addressed using a tool other than price – rebates in this case. When only one firm offers a rebate, segmentation is partial and prices are higher relative to the no rebate case. When both firms offer rebates, segmentation is complete, and prices are highest. Thus we can see that segmentation, whether partial or full, benefits both firms by enabling them to extract higher surplus from the high type consumers.

Many interesting insights emerge from the pure strategy equilibrium cutoffs. First note that the cutoffs depend critically on the value of the redemption cost for the low type consumers (e_2). We find that, given the asymmetry level, the attractiveness of a pure strategy equilibrium where one or both firms

offer rebate is higher as the redemption cost asymptotically approaches to zero. However, whether or not a given solution constitutes equilibrium depends on both the redemption cost and the asymmetry level. For instance, if the redemption cost is high enough even at low valuation difference levels the advantaged firm may not want to offer a rebate. This is because the firm that offers a rebate incurs a cost to make the second-degree price discrimination through consumer self-selection work. The redemption cost along with the asymmetry between the two firms determines the pure strategy equilibria. Clearly, this feature of our model does not exist in third-degree and other price discrimination models where consumer segments are assumed to be perfectly separable.

We can get a clear picture of how the equilibria change with the extent of asymmetry between the firms by investigating the equilibria cutoffs. Figure 2 depicts a numerical example of how the pure strategy equilibria change. The results shown in this figure can be generalized using the equilibrium cutoffs given in the appendix. We present one such generalization in Proposition 3 and discuss other interesting ones later in this section. With this in mind, we computed the equilibria by varying $u = V_A - V_D$ over the interval [0, 20]; holding all other parameters constant[8]. Even though the figure shows that pure strategy equilibrium exists for all the valuation differences, this is not true for other parameter values. It is possible that we may not have pure strategy equilibrium for a range of valuation difference levels.

For low values of asymmetry (region I) we observe multiple equilibria. In region I, both firms offering rebates is equilibrium as also neither firm offering rebates. There is also a mixed strategy equilibrium in this range and we derive it in the next section. The equilibrium in which both firms offer rebates results in higher profits than the other two equilibria for low values of asymmetry. So, it is preferable for both firms, and so firms face a coordination problem. For high values of asymmetry (region III), on the other hand, both firms improve their profits if only firm D offers a rebate as Proposition 2 suggests. Thus, in this example, it is equilibrium for both firms to offer rebates for small asymmetries, and only firm D offering a rebate for larger asymmetries. In the intermediate range of asymmetry (region II), neither firm offering a rebate emerges as the unique equilibrium. This is because firm A prefers only

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[8] We set $V_A = 100$, $\alpha_I = 10$, $\alpha_2 = 7.5$, $\delta = 0.9$, $\gamma = 0.4$, $e_2 = 0.2$, $e_I = 5$ $t = 8$.

firm *D* to offer a rebate in this intermediate range; however, firm *D* prefers both firms to offer rebates. Even though both firms rebating is a better outcome for both firms than neither firm rebating, it is not in equilibrium.
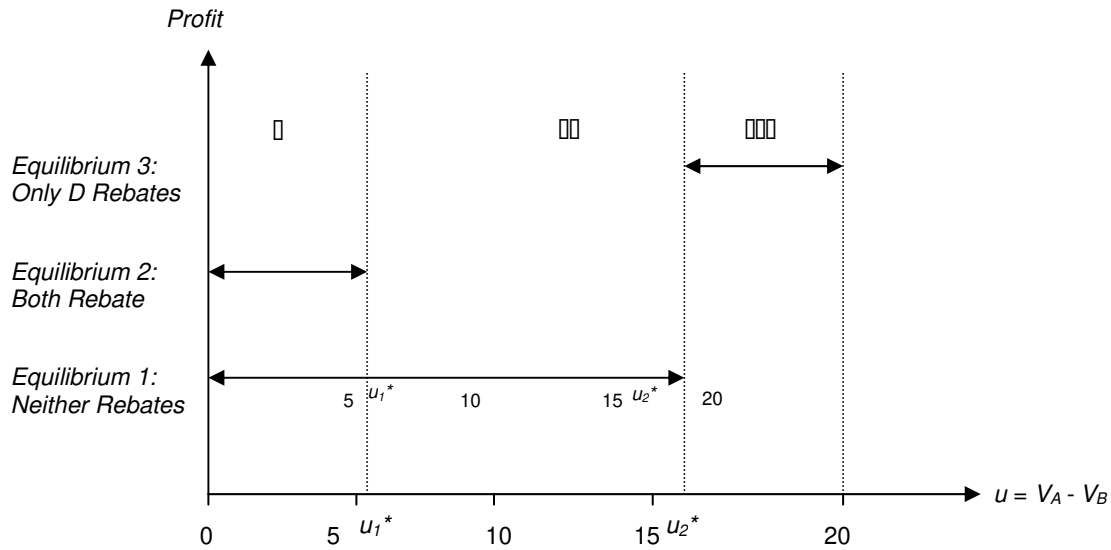

Figure 2: Equilibria with changing asymmetry in consumer valuations (*u*)

In the above numerical example, we see no equilibrium in which only firm *A* offers a rebate. Is this result generalizable? The answer is yes and presented as Proposition 3. The result makes intuitive sense; with its higher valuation the advantaged firm (*A*) prefers to focus on the high types.

Proposition 3: There is no pure strategy Nash equilibrium outcome where the advantaged firm rebates and the disadvantaged firm does not rebate.

The figure also exhibits other results that can be generalized. Note that the asymmetric equilibrium wherein the disadvantaged firm rebates alone does not coincide with a symmetric equilibrium where neither firm rebates, so the two types of equilibria cannot co-exist. The proof is provided in the appendix under generalization 1. We next examine how the sizes of the consumer segments affect the uniqueness of equilibrium in region III. We set the value of asymmetry equal to 17.5 (to see all possible equilibria) and vary the size of the high type segment over the interval [0, 1]. Figure 3 shows the

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equilibria. We see that only one firm offering a rebate emerges as the unique equilibrium when the size of the high type segment is intermediate (region II). When the proportion of high type consumers is moderate, both firms will have much to gain by separating the segments. However, partial segmentation entails one firm taking the less lucrative low type segment. Though both firms benefit from segmentation relative to no segmentation, each firm would like to be the one with the high type segment. That is, each firm would like the other firm to offer the rebate. Even when it is profitable for a firm to offer a rebate, it is more attractive if the other firm offers a rebate as well.[9] In equilibrium, the advantaged firm, who would win out in the competition for the high type segment, has less to gain from offering a rebate. Hence, whenever only one firm offers a rebate in equilibrium (only one firm finds the benefit from segmentation to exceed the cost of foregoing the lucrative high type segment), it will be the disadvantaged firm.

When the size of the high type segment is small (region I), i.e. the size of the low type segment is large, the firms once again face a Prisoners' Dilemma. Each firm prefers the other to offer rebates. Even though both firms would profit if they both offered rebates, it is not in equilibrium. Finally, when the size of the high type segment is large (regions III and IV), both firms offering rebates is in equilibrium. We find that for a given value of asymmetry in the interval $[u_2^*, 20]$, the sizes of segments impact the nature of the equilibrium.

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[9]A similar strategic complementarity is shown in Kumar and Rao (2006), where a firm benefits more from a data-analytics program that allows it to segment the market if the competing firm also obtains the data-analytics program.
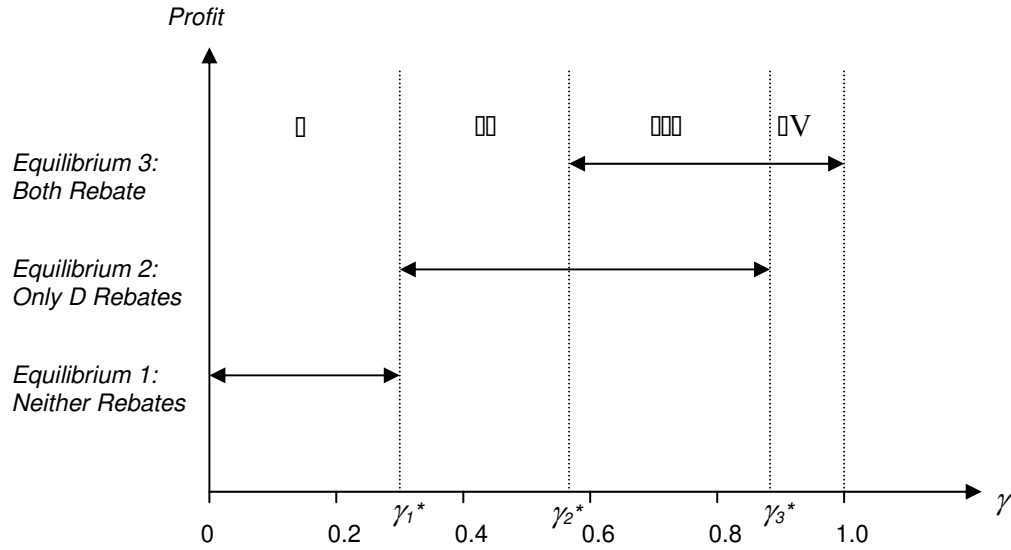
Profit

Ⅰ Ⅱ Ⅲ ⅠV

Equilibrium 3:
Both Rebate

Equilibrium 2:
Only D Rebates

Equilibrium 1:
Neither Rebates

γ_1^* γ_2^* γ_3^* γ

0 0.2 0.4 0.6 0.8 1.0

Figure 3: Equilibria with changing proportion of high type consumers (γ)

4.3 The Mixed Strategy Equilibrium

Under certain conditions, the equilibrium for the full space is in mixed strategies over rebating and not rebating (recall that not rebating includes all rebate amounts that do not satisfy the IC and IR conditions), in addition to equilibria in pure strategies. Given our interest in examining data on rebates, it is useful to characterize the mixed strategy equilibrium also, since in practice many firms in the same industry offer rebates.[10] Since rebates are temporary price reductions, they are properly viewed as promotions. And many kinds of promotions have been identified by mixed strategies in prior analytical work by Varian (1980) and Narasimhan (1984), and in empirical work by Rao, Arjunji and Murthi (1995) and Villas-Boas (1995). Indeed, Rao, Arjunji and Murthi offered the generalization that competitive promotions are mixed strategies based on empirical findings in many product categories. The solution procedure for mixed strategy equilibrium outcomes is provided in the appendix. In our model, we find that that the disadvantaged firm would rebate more frequently than its advantaged rival in any mixed

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[10] In fact we will see in the next section that in the printer market we observe rebate promotions by virtually all manufacturers and these promotions appear to be uncorrelated, with the exception of seasonal promotion such as end of year sales. This seems to indicate that the mixed strategy equilibrium would be a good candidate to describe empirical observation.

strategy Nash equilibrium. This is consistent with proposition 3 that shows that the disadvantaged firm will never find itself in a pure strategy Nash equilibrium outcome not offering a rebate in response to a competitor's rebate. The mixed strategy finding is formalized in the following proposition.

Proposition 4: In any mixed strategy Nash equilibrium, the disadvantaged firm offers a rebate with a probability as high, or higher, than the advantaged firm.

5. Analysis of Rebates in the Printer Category

We next examine some data on rebates in the printer category. The printer category is a dynamic category, with firms continually devising pricing, promotion and new product initiatives as part of their competitive strategy (Shankar, 2005). There are three major subcategories in the printer category: Inkjet, Laser, and Multi-function (printer with other functions such as photo printing, copying, and fax functions) with a handful of firms dominating each sub-category. While *Hewlett-Packard* has long established itself as the leader in all printer categories, there is fierce competition for second place. Given our interest in rebates, we want to ensure that rebates occur with sufficient frequency in the product category we choose to examine. In the printer category, looking at all brands, in any given week 30% of SKU's carry a rebate on average. For all these reasons, we chose to study the printer category.

5.1 Description of Data

We collected data on printers from the website of a major retailer specializing in office equipment. The time period for the data is November 2004 to August 2005. We gathered daily price and rebate information for each SKU that the store carries. We can identify the brand name for each SKU. For purposes of analysis we decided to aggregate daily data over a week. The reason for this is that most rebates run for a week. Table 1 gives a frequency distribution of the duration of rebates. We can see that over 75% of the rebates are offered for one week. The price variable was the average price in the week. In addition we gathered data on 2004 market share for each brand in the laser sub-category from International Data Corporation (IDC).

Rebate length (weeks)	1	2	3	4	5	6	8	10	11
Frequency	199	28	8	5	14	6	1	1	1
%	75.67	10.65	3.04	1.90	5.32	2.28	0.38	0.38	0.38

Table 1: Rebate length distribution

5.2 Hypotheses

We want to test our main result that the disadvantaged firm is more likely to offer rebates. One thing to note in our data is that all brands and most SKU's offer a rebate. This might suggest a mixed Nash strategy equilibrium as in our model. As a first step, we wanted to see if the retailer might act in such a way as to coordinate the rebates across brands (Lal, 1990). We tested for independence of rebate offers across pairs of brands using a Chi-squared test. This test is conservative for our purpose. [11] Table 2 contains the test statistics for pair-wise independence of rebates. We find that in all but four cases, the hypothesis of independence cannot be rejected at the 5% level. Given the conservative nature of the test, we would be justified in rejecting any coordination of rebate offers by competitors. We will therefore assume that rebates result from mixed strategies. Now, invoking Proposition 4 of our analysis, we should expect that the disadvantaged firm would offer rebates with greater probability than the advantaged firm. This is what we wish to test next. To render this into a specific hypothesis it is necessary to map a firm's advantage to observable variables. This is challenging because in practice it is not easy to determine who exactly is an advantaged (or disadvantaged) firm. We met this challenge by conducting our analysis using several proxies.

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[11] See Rao (1996), for example.

Firm	Brother	HP	Konica	Lexmark	Okidata	Xerox	**Number of Rebates[12]**
Brother	0						**26**
HP	0.01	0					**22**
Konica	0.68	0.20	0				**14**
Lexmark	0.003	0.02	0.88	0			**13**
Okidata	0.12	0.09	0.39	0.01	0		**13**
Xerox	0.49	0.39	0.18	0.46	0.46	0	**1**

Table 2: Test statistics (p-values) from tests for independence

We analyzed the laser printer market at the brand level using two proxies: market share and price. A firm with a higher market share in equilibrium can reasonably be argued to have greater brand equity, and so would be the advantaged firm. Turning to price, a brand with a higher consumer valuation would charge a higher price (by Proposition 1). As such, a brand that is priced higher in equilibrium would be the advantaged firm. In addition to the brand level analysis, we examined the relationship between price and the likelihood of a rebate at the SKU level in each of the three sub-categories. Finally, we also conducted an analysis across sub-categories relating price to rebate frequency. The inkjet category is the most disadvantaged and the laser category the most advantaged. As such the laser category should carry rebates least often and inkjet the most[13]. In this way, we want to see how robust our findings are at the brand level, SKU level and sub-category level. We now formally state our hypothesis, corresponding to each proxy for the advantaged firm:

Hypothesis 1a: Brands with higher market share will offer rebate less frequently.

Hypothesis 1b: Brands and/or SKUs with higher average price will offer rebate less frequently.

Hypothesis 1c: Subcategories with higher price will rebate less frequently.

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[12] To be precise, this is how often *at least* one SKU in the brand carries a rebate in a given week.
[13]In theory, we could use features to assess quality. The problem there is that the number of features is large relative to the number of SKU's and these features are highly correlated with each other as well as with brand names.

5.3 Results and Discussion

Our first analysis is at the brand level in the laser sub-category. The brands are ordered, from highest to lowest, by market share as follows: HP, Xerox, Konica, Brother, Lexmark. The ordering, by price, is: Xerox, HP, Konica, Lexmark, and Brother. We can see in the histogram in Figure 4 that displays the frequency of rebates along with market share and price. Rebate frequency is operationalized as the proportion of SKU's promoted each period by the brand given that our analysis is at the brand level.
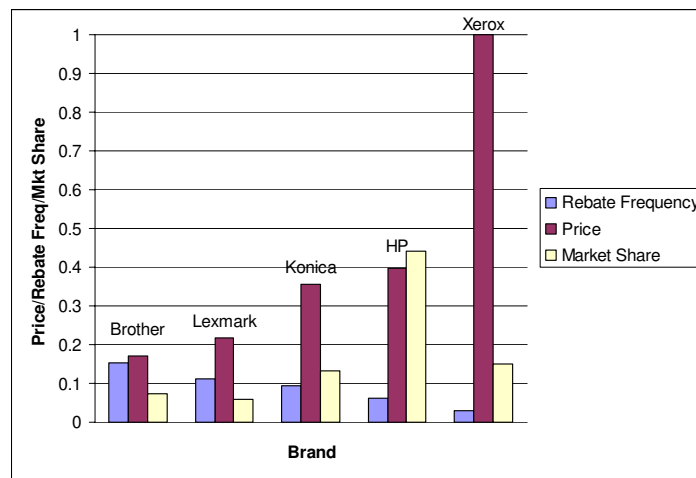

Figure 4: Price, Rebate Frequency and Market Share Data for Laser Printer Category

As we can see there is a negative relationship between price and rebate frequency. There is also a negative relationship between market share and rebate frequency, with the possible exception of Xerox. The brand with the highest market share and the second highest price, HP, has the lowest rebate frequency, while the brand with the lowest price and the second lowest market share, Brother, has the highest rebate frequency.

To get a better feel for this data we conducted paired comparisons for these brands. The *t*-test results for these comparisons are shown in Table 3. In the first column, each pair is ordered with the first brand being the advantaged brand relative to the second with two exceptions: HP-Xerox and Brother-Lexmark. Our proxies have different predictions for these two pairs. The dependent variable is the difference in the

rebate frequencies of the first and the second brands in any given week. Therefore, according to our hypothesis this difference should be negative for all but two aforementioned cases. In those two cases, our prediction is positive for one of the proxies whereas it is negative for the other. The number of observations for each hypothesis is the number of weeks in our sample. In the two right-most columns, we provide the predictions of hypothesis 1a and 1b. In the last row, we display a count of the number of predictions which are inconsistent with our hypothesis.

Comparison	DF	t Value	Pr > |t|	1a Prediction	1b Prediction
HP-Brother	37	-6.93	<.0001	Negative	Negative
HP-Lexmark	37	-3.02	0.0045	Negative	Negative
HP-Konica	37	-3.25	0.0025	Negative	Negative
HP-Xerox	37	7.01	<.0001	Negative	Positive
Konica-Brother	37	0.65	0.5175	Negative	Negative
Konica-Lexmark	37	0.77	0.4481	Negative	Negative
Xerox-Konica	37	-4.68	<.0001	Negative	Negative
Xerox-Brother	37	-9.41	<.0001	Negative	Negative
Xerox-Lexmark	37	-4.50	<.0001	Negative	Negative
Brother-Lexmark	37	0.41	0.6820	Negative	Positive
Total Number of Significant Differences Consistent with Hypothesis				6/7	7/7

Table 3: Paired comparisons of the proportion of SKU's promoted in the laser printer category.

From Table 3 we see that 13/14 significant cases support our hypothesis 1a and 1b that the advantaged brand offers rebates less frequently. The exceptions involve Xerox which has low market share and a high price and offers few SKUs that are high volume printers. As such, the comparisons in that case may be less valid. The results in Table 3 confirm the general picture that Figure 4 depicts.

We think it would be worthwhile to examine the data at the SKU level since brands vary in the number of SKUs they offer. Therefore, we next look at SKUs within each of the three sub-categories,

separately. To this end, we formulated a simple model for the probability of an SKU carrying a rebate. Define W_{jt} to be value of a rebate to an SKU j in week t. We model W_{jt} as

$$W_{jt} = \alpha_{Bj} + \beta P_{jt} + \mu_t + \varepsilon_{jt} \tag{7}$$

where Bj is the brand of SKU j, P_{jt} is the price of SKU j at time t, μ_t is a common shock in week t and ε_{jt} is the idiosyncratic shock for SKU j in week t. The common shock should take care of any effects in a given week that affects all printers, such as the introduction of new products, the state of the economy and possible seasonality. We model it as normally distributed, with mean 0 and standard deviation σ. The idiosyncratic shock is assumed to be i.i.d , following an extreme value distribution, yielding a binary logit model for estimation. Our interest is in the sign of the price parameter. To correct for endogeneity of price since price and rebates are simultaneously decided, we used a two-step procedure. In the first step, we performed a regression of price on the brand and product attributes -- printer speed in BW and Color, resolution, Sheet Capacity, Business Use, Weight, copier speed, fax. We verified that the instruments are not correlated with the error term. In the second step we substitute the predicted price from the first step for the price. While we report the two-step results we also checked the regression results for a one-step procedure without correction for endogeneity and the results are remarkably close. The estimation results are given in Table 4.

From Table 4 we see that the price coefficient is significant for all three sub-categories, and has the right, negative, sign. The negative sign means that higher priced SKUs have a lower probability of carrying a rebate. This supports our hypothesis 1b. We also see that the brand effects vary by sub-category, because some brands are strong in one sub-category but not necessarily in another.

Subcategory	Inkjet		Multifunction		Laser	
# of obs.	724		1708		1947	
# of weeks	38		38		38	
# of sku's	25		60		62	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Intercept	-2.58	-5.32	-2.48	-7.23	-3.09	-7.12
Brother	NA	NA	-0.79	-2.79	1.28	3.44
HP	1.47	3.22	-0.15	-0.60	0.43	1.26
Canon	-13.36	-25.76	0.59	1.85	NA	
Konica					0.26	0.57
Lexmark	1.16	2.17	-0.18	-0.57	0.73	1.79
Sharp	NA	NA	-1.77	-2.46	NA	NA
Okidata	NA	NA			2.18	5.34
Epson	Benchmark		Benchmark		NA	NA
Xerox	NA	NA	NA	NA	Benchmark	
Price	-0.0030	-5.67	-0.0008	-1.83	-0.0003	-3.22
σ	4.39	0.24	2.55	1.29	3.59	0.92
Log-likelihood	-232.63		-488.09		-488.38	

Table 4: Rebate Frequency at SKU level

For the category level data we chose not to perform any statistical analysis. Instead we present a visual depiction of the data in Figure 5. We can see from Figure 5 that laser printer sub-category is the most expensive and yet has the lowest rebate frequency. This could be interpreted as supporting our hypothesis 1c.
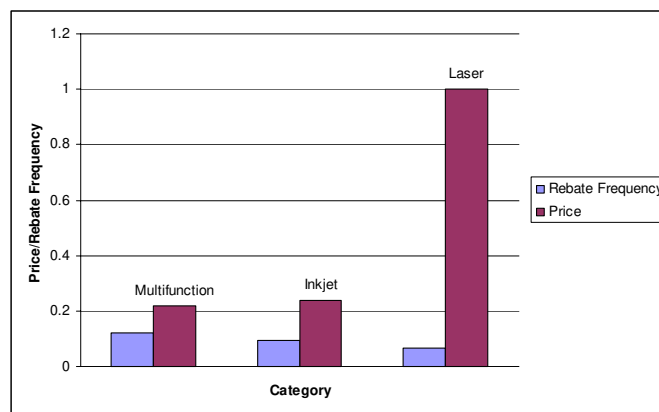

Figure 5: Price vs. Rebate Frequency for Printer Categories

Summarizing our results, we see that the advantaged competitor in our data offers rebates less often than the disadvantaged one. In this way we find support for our Proposition 4 which says that in any mixed strategy equilibrium the advantaged firm has a lower probability of offering a rebate.

6. Conclusions

Our main result is that in an asymmetric duopoly, when second degree price discrimination is an option, it is the disadvantaged firm that is more likely to engage in it. Moreover, the result of such price discrimination is unambiguous in its effect on firms' profits. Both firms realize higher profits under price discrimination regardless of whether only one firm or both firms discriminate. One mechanism for segmentation is a rebate offer, which is common in the promotion of big ticket items. We model the rebate decision in a competitive duopoly setting, with price and rebate as decision variables and heterogeneous consumers of different price orientations. Our analysis provides important insights in formulating a firm's rebate strategy. The main insight is that rebates, when used to segment a market, lessen competition that would result if firms were competing on prices separately over each segment. A second important insight is that rebates would more often be offered by the disadvantaged firm[14]. Indeed, the greater the disadvantage of the firm relative to its rival, the more beneficial rebates become to the firm's profits. Third, if consumer effort involved in obtaining rebates is not too high, the disadvantaged firm will typically offer a rebate, but the advantaged competitor would not. Finally, we identify a mixed strategy Nash equilibrium of rebate offers. In such an equilibrium the disadvantaged firm always offers rebates more frequently than the advantaged firm.

A further interesting insight we obtain is on the reason for the higher profits. Specifically, price discrimination in our model causes customer remixing that softens the competition for the more profitable consumers, thus resulting in higher profits. Without segmentation the equilibrium price tends to be too

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[14] A similar result has been noted in a monopolistic setting by Bruce et al. (2006) who find that under certain conditions, a monopoly manufacturer with a lower durability product is more likely to give cash rebates than a higher durability manufacturer. However, in their work rebates are not modeled as a mechanism.

low for the more profitable consumers and too high for the less profitable consumers relative to the optimal price for a homogenous population consisting of that segment. This in turn means that without segmentation some money is left on the table for the more profitable consumers, who in effect receive a "subsidy". In general, the cost of this subsidy due to lower prices for the profitable segment is not offset by the higher prices for the less profitable segment in the absence of segmentation. What segmentation does is reduce or eliminate the subsidy, simultaneously lowering prices for the less profitable segment. The loss of profits on the less profitable segment is more than compensated for by the reduced subsidy to the more profitable segment. Thus, our analysis provides an important insight into marketing strategy through segmentation by identifying for segmentation the role of customer remixing.

Our theoretical framework and insights are not limited to rebates and could be extended to coupons without major modifications. The main theoretical findings would still hold that coupons could allow managers to lessen competition and that disadvantaged firms would be more prone to use coupons. We believe that distribution costs are a major determinant in the rebate-coupon decision. Specifically, if coupons are also available as a segmentation device, the decision between rebates and coupons would depend on costs of administering coupon or rebate programs. Future research should also examine the role of the channel, specifically, who in the channel is most likely to offer a rebate and who stands to benefit the most.

Recent works (Chen, Moorthy and Zhang, 2005; Lu and Moorthy, 2007; Soman, 1998) propose that firms could benefit from "slippage" because consumers often make purchases intending to redeem rebates but end up not redeeming them. In Chen, Moorthy and Zhang (2005) and Lu Moorthy (2007), slippage is rational in that consumers fully anticipate that they might find themselves in a future date in a state of the world where they are unable to redeem. In Soman (1998) and Lynch and Zauberman (2006), slippage is irrational in that a fraction of consumers buy the product erroneously, due to time inconsistency, expecting to redeem the rebate. Prospect Theory in particular, suggests that the chronological separation of gains from losses is salient as a basis for understanding how consumers respond to rebates. Models of irrational slippage appear inconsistent with our data. Since the marginal

revenue from rebating in such models is higher for the advantaged firm (the advantaged product is priced higher and so the benefit from irrational slippage is higher), the propensity to offer the rebate should be higher for the advantaged firm as well. Rational slippage models, on the other hand, could be combined with the present framework and this is a direction for future research.

One limitation of the model is that if price sensitivity and redemption costs are not negatively correlated, rebates will not be an effective price discrimination tool. Another limitation is that the consumer redemption costs are assumed to be exogenous. An extension that allows firms to endogenously determine consumers' redemption effort may yield interesting insights in asymmetric duopoly. A third limitation pertains to the nature of the asymmetry. In our model, the asymmetry was captured by asymmetric valuations. Asymmetries in the cost structure are an avenue for future research.

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Appendix

Solution procedure for pure strategy equilibrium outcomes

Firms simultaneously maximize the profits in equations 1 and 2 subject to inequalities 3-6. A critical feature of our model is that firms need to set rebate amounts to separate the segments of the consumer population (low types from high types). First, the optimal rebate amount should be high enough so that the low type consumers use it. Second, the optimal rebate amount should not be too high that even the high type consumers use it. The latter type of incentive compatibility constraints are relatively easy to handle since the violation of the second (high) type constraints does not alter the firms' profit function. The violation of the high type incentive compatibility constraints implies that a firm offers too large of a rebate that does not prevent the high type consumers from using it, i.e. all consumers use the rebate. It is straightforward to show that a rebate offer of this kind is suboptimal since it is equivalent to a price cut with added cost (e_1 or e_2) to all consumers. However, the violation of the first type of incentive compatibility constraints alters the profit function as well as the participation constraints. If a rebate offer is too small we should make sure it is dropped out of the firms' profit function and the low type consumers' participation constraint. To handle this properly we define characteristic function $\chi\{x\}$ which takes on the value 1 if $x > 0$ and 0 otherwise. Using the characteristic function we can re-write the participation constraints and the firms' profit functions as follows:

$$V_D - \alpha_1\tau_1 - P_D = V_A - \alpha_1(1-\tau_1) - P_A \geq 0 \qquad \text{High type}$$

$$V_D - \alpha_2\tau_2 - P_D + \chi\{\delta^t R_D - e_2\}(\delta^t R_D - e_2) = V_A - \alpha_2(1-\tau_2) - P_A + \chi\{\delta^t R_A - e_2\}(\delta^t R_A - e_2) \geq 0 \quad \text{Low type}$$

We let χ_A and χ_D denote $\chi\{\delta^t R_A - e_2\}$ and $\chi\{\delta^t R_D - e_2\}$, respectively.

We can derive the market share figures from the left-hand side of inequalities given above as

$$\tau_1 = \frac{V_D - V_A + P_A - P_D + \alpha_1}{2\alpha_1} \qquad \text{(A.1)}$$

$$\tau_2 = \frac{V_D - V_A + P_A - P_D + \chi_D(\delta^t R_D - e_2) - \chi_A(\delta^t R_A - e_2) + \alpha_2}{2\alpha_2} \qquad \text{(A.2)}$$

The maximization problem for the disadvantaged (D) firm:

$$\text{Max } \Pi_D = \gamma P_D\left(\frac{V_D - V_A + P_A - P_D + \alpha_1}{2\alpha_1}\right)$$

$$+(1-\gamma)(P_D - \chi_D\delta^t R_D)\left(\frac{V_D - V_A + P_A - P_D + \chi_D(\delta^t R_D - e_2) - \chi_A(\delta^t R_A - e_2) + \alpha_2}{2\alpha_2}\right)$$

The maximization problem for the Advantaged (A) firm:

$$\text{Max } \Pi_A = \gamma P_A\left(\frac{\alpha_1 - V_D + V_A - P_A + P_D}{2\alpha_1}\right)$$

$$+(1-\gamma)(P_A - \chi_A\delta^t R_A)\left(\frac{\alpha_2 - V_D + V_A - P_A + P_D - \chi_D(\delta^t R_D - e_2) + \chi_A(\delta^t R_A - e_2)}{2\alpha_2}\right)$$

Using the first order conditions we have the following reaction functions:

$$P_D = \frac{1}{2}[P_A + V_D - V_A] + \frac{(1-\gamma)\alpha_1}{2((1-\gamma)\alpha_1 + \gamma\alpha_2)}[\chi_D(2\delta^t R_D - e_2) - \chi_A(\delta^t R_A - e_2)] + \frac{\alpha_1\alpha_2}{2((1-\gamma)\alpha_1 + \gamma\alpha_2)}$$

$$P_A = \frac{1}{2}[P_D + V_A - V_D] + \frac{(1-\gamma)\alpha_1}{2((1-\gamma)\alpha_1 + \gamma\alpha_2)}[\chi_A(2\delta^t R_A - e_2) - \chi_D(\delta^t R_D - e_2)] + \frac{\alpha_1\alpha_2}{2((1-\gamma)\alpha_1 + \gamma\alpha_2)}$$

$$(\chi_D)^2 R_D = \frac{\chi_D[2P_D - P_A + \chi_D e_2 + \chi_A(\delta^t R_A - e_2) + V_A - V_D + \alpha_2]}{2\delta^t}$$

$$(\chi_A)^2 R_A = \frac{\chi_A[2P_A - P_D + \chi_A e_2) + \chi_D(\delta^t R_D - e_2) + V_D - V_A + \alpha_2]}{2\delta^t}$$

Depending on the values of χ_A and χ_D there are four possible outcomes. Note that none of these four possibilities necessarily constitutes equilibrium. The solution procedure merely lists all feasible pure strategy solutions to the firms' maximization problems. Since firms make their decisions simultaneously we cannot directly compare these solutions against each other either. To characterize the pure strategy equilibria for the firms, we examine each solution in turn to see if the solution is equilibrium. We do this by calculating the additional profit a firm can achieve, if any, by deviating from the given solution. Since the decisions are done simultaneously, when a firm deviates it does not necessarily move to another feasible solution but it is allowed to change its decision variables freely, e.g. a firm that is offering a rebate in a given solution may deviate by ceasing its rebate offer and changing its price. Similarly a firm who is not offering a rebate in a given solution may deviate by starting a rebate program. We first find all feasible solutions. Then, in the next section we characterize all pure strategy equilibria.

When $\chi_A = 0$ and $\chi_D = 0$, note that the last two reaction functions disappear and we have two reaction functions as $P_D = \frac{1}{2}[P_A + V_D - V_A] + \frac{\alpha_1\alpha_2}{2((1-\gamma)\alpha_1 + \gamma\alpha_2)}$ and $P_A = \frac{1}{2}[P_D + V_A - V_D] + \frac{\alpha_1\alpha_2}{2((1-\gamma)\alpha_1 + \gamma\alpha_2)}$. Solving the two reaction functions simultaneously for P_D and P_A we find

$$P_D^1 = \frac{1}{3}(V_D - V_A) + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2} \tag{A.3}$$

$$P_A^1 = \frac{1}{3}(V_A - V_D) + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2} \tag{A.4}$$

When $\chi_A = 0$ and $\chi_D = 1$, note that the last reaction function disappears and we have three reaction functions. Solving the three reaction functions simultaneously for P_D, P_A and R_D we find

$$P_D^2 = \frac{1}{3}(V_D - V_A) + \frac{\alpha_1(1-\gamma)e_2}{6[(1-\gamma)\alpha_1 + \gamma\alpha_2]} + \frac{\alpha_1((1-\gamma)\alpha_1 + (1+\gamma)\alpha_2)}{2[(1-\gamma)\alpha_1 + \gamma\alpha_2]} \tag{A.5}$$

$$P_A^2 = \frac{1}{3}(V_A - V_D) + \frac{\alpha_1(1-\gamma)e_2}{3[(1-\gamma)\alpha_1 + \gamma\alpha_2]} + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2} \tag{A.6}$$

$$R_D^2 = \frac{\alpha_1 - \alpha_2 + e_2}{2\delta^t} \tag{A.7}$$

When $\chi_A = 1$ and $\chi_D = 0$, note that the third reaction function disappears and we have three reaction functions. Solving the three reaction functions simultaneously for P_D, P_A and R_A we find

$$P_D^3 = \frac{1}{3}(V_D - V_A) + \frac{\alpha_1(1-\gamma)e_2}{3[(1-\gamma)\alpha_1 + \gamma\alpha_2]} + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2} \tag{A.8}$$

$$P_A^3 = \frac{1}{3}(V_A - V_D) + \frac{\alpha_1(1-\gamma)e_2}{6[(1-\gamma)\alpha_1 + \gamma\alpha_2]} + \frac{\alpha_1((1-\gamma)\alpha_1 + (1+\gamma)\alpha_2)}{2[(1-\gamma)\alpha_1 + \gamma\alpha_2]} \tag{A.9}$$

$$R_A^3 = \frac{\alpha_1 - \alpha_2 + e_2}{2\delta^t} \tag{A.10}$$

When $\chi_A = 1$ and $\chi_D = 1$, Solving the reaction functions simultaneously for P_D, P_A, R_D and R_A we get

$$P_D^4 = \frac{1}{3}(V_D - V_A) + \alpha_1 \tag{A.11}$$

$$P_A^4 = \frac{1}{3}(V_A - V_D) + \alpha_1 \tag{A.12}$$

$$R_D^4 = \frac{\alpha_1 - \alpha_2}{\delta^t} \tag{A.13}$$

$$R_A^4 = \frac{\alpha_1 - \alpha_2}{\delta^t} \tag{A.14}$$

Note that checking to see if $\delta^t R_i^j \geq e_2$ for each of the above solutions leads to the same requirement: $\alpha_1 - \alpha_2 \geq e_2$. As long as this condition holds, all the solutions above are feasible. When the condition is violated, i.e. $\alpha_1 - \alpha_2 < e_2$, each firms has to set its rebate amount equal to e_2 indicating a corner solution. We checked to make sure that our qualitative results are valid when $\alpha_1 - \alpha_2 < e_2$. The details are available from the authors upon request. In the interest of brevity, we report the results of the interior case when $\alpha_1 - \alpha_2 \geq e_2$ and the proofs given are for this case.

Equilibrium Conditions
As mentioned before, in this section we establish the equilibrium conditions for each of the four feasible solutions. In order for the first solution - where $\chi_A = 0$ and $\chi_D = 0$ - to be pure strategy equilibrium neither firm should be able to offer a rebate and change its price given the other firm's optimal price (reported in equation A.3 or A.4). Let Π_i' denote firm i's highest deviation profit, i.e.

$$\Pi_D' = \max_{P_D, R_D} \Pi_D \Big|_{P_A = \frac{1}{3}(V_A - V_D) + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2}} \qquad \Pi_A' = \max_{P_A, R_A} \Pi_A \Big|_{P_D = \frac{1}{3}(V_D - V_A) + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2}}$$

We first formulate and solve these two maximization problems. Then, we compare these deviation profits to the profits of the first solution. This leads us to the conditions under which the first solution is equilibrium. The equilibrium conditions are represented as cutoff values in terms of the difference between the two firms' valuations, i.e. $V_A - V_D$. We find that the disadvantaged firm will not deviate, i.e. $\Pi_D^1 - \Pi_D' \geq 0$, if

$$V_A - V_D \leq \frac{3[-\overline{\alpha}e_2^2 + 2\alpha_2(\overline{\alpha} + \alpha_1)e_2 - \gamma\alpha_2(\alpha_1 - \alpha_2)^2]}{4\overline{\alpha}e_2}, \text{ where } \overline{\alpha} = (1-\gamma)\alpha_1 + \gamma\alpha_2. \text{ Similarly, the first solution is}$$

equilibrium for the advantaged firm , i.e. $\Pi_A^1 - \Pi_A' \geq 0$ if $V_A - V_D \geq \dfrac{3[\overline{\alpha}e_2^2 - 2\alpha_2(\overline{\alpha} + \alpha_1)e_2 + \gamma\alpha_2(\alpha_1 - \alpha_2)^2]}{4\overline{\alpha}e_2}$.

Let u_i^j denote the cutoff value of $V_A - V_D$ for firm i in order for solution j to be equilibrium. The cutoffs for the first solution are as follows:

$$u_D^1 = \frac{3[-\overline{\alpha}e_2^2 + 2\alpha_2(\overline{\alpha}+\alpha_1)e_2 - \gamma\alpha_2(\alpha_1-\alpha_2)^2]}{4\overline{\alpha}e_2} \geq V_A - V_D \qquad (A.15)$$

$$u_A^1 = \frac{3[\overline{\alpha}e_2^2 - 2\alpha_2(\overline{\alpha}+\alpha_1)e_2 + \gamma\alpha_2(\alpha_1-\alpha_2)^2]}{4\overline{\alpha}e_2} \leq V_A - V_D \qquad (A.16)$$

For the second solution - where $\chi_A = 0$ and $\chi_D = 1$ - we find the highest deviation profits similarly. Note that for the second case to be pure strategy equilibrium the disadvantaged firm should not have incentive to cease offering a rebate and change its price given the advantaged firm's price (A.6). Furthermore, the advantaged firm should not offer a rebate and change its price given the disadvantaged firm's price and rebate combination reported in equations A.5 and A.7. We calculate the firms' deviation profit as follows

$$\Pi_D^{'} = \max_{P_D} \Pi_D \Big|_{P_A = \frac{1}{3}(V_A-V_D) + \frac{\alpha_1(1-\gamma)e_2}{3[(1-\gamma)\alpha_1 + \gamma\alpha_2]} + \frac{\alpha_1\alpha_2}{(1-\gamma)\alpha_1 + \gamma\alpha_2}}$$

$$\Pi_A^{'} = \max_{P_A, R_A} \Pi_A \Big|_{P_D = \frac{1}{3}(V_D-V_A) + \frac{\alpha_1(1-\gamma)e_2}{6[(1-\gamma)\alpha_1 + \gamma\alpha_2]} + \frac{\alpha_1((1-\gamma)\alpha_1 + (1+\gamma)\alpha_2)}{2[(1-\gamma)\alpha_1 + \gamma\alpha_2]}, R_D = \frac{\alpha_1 - \alpha_2 + e_2}{2\delta^t}}$$

Formulating and solving these two maximization problems and comparing the profits to the second solution profits leads to the following two conditions:

$\Pi_D^2 - \Pi_D^{'} \geq 0$ if

$$u_D^2 = \frac{-(\overline{\alpha}+2\gamma\alpha_2)e_2^2 + 6\alpha_2(\overline{\alpha}+\alpha_1)e_2 - 3\gamma\alpha_2(\alpha_1-\alpha_2)^2}{4\overline{\alpha}e_2} \leq V_A - V_D. \qquad (A.17)$$

$\Pi_A^2 - \Pi_A^{'} \geq 0$ if

$$u_A^2 = \frac{-(4\overline{\alpha}-7\gamma\alpha_2)e_2^2 - 6\alpha_2(3\overline{\alpha}+5\alpha_1)e_2 + 27\gamma\alpha_2(\alpha_1-\alpha_2)^2}{16\overline{\alpha}e_2} \leq V_A - V_D. \qquad (A.18)$$

For the third solution - where $\chi_A = 1$ and $\chi_D = 0$ - we find the highest deviation profits similarly. Note that in the third solution the roles of the two firms are exactly opposite to those of the second solution, i.e. for the third case to be pure strategy equilibrium the advantaged firm should not have incentive to cease offering a rebate while the disadvantaged should not deviate and offer a rebate and change its price. We omit the formulation details. The deviation profit comparison leads to the following two conditions:

$\Pi_D^3 - \Pi_D^{'} \geq 0$ if

$$u_D^3 = \frac{(4\overline{\alpha}-7\gamma\alpha_2)e_2^2 + 6\alpha_2(3\overline{\alpha}+5\alpha_1)e_2 - 27\gamma\alpha_2(\alpha_1-\alpha_2)^2}{16\overline{\alpha}e_2} \geq V_A - V_D. \qquad (A.19)$$

$\Pi_A^3 - \Pi_A^{'} \geq 0$ if

$$u_A^3 = \frac{(\overline{\alpha}+2\gamma\alpha_2)e_2^2 - 6\alpha_2(\overline{\alpha}+\alpha_1)e_2 + 3\gamma\alpha_2(\alpha_1-\alpha_2)^2}{4\overline{\alpha}e_2} \geq V_A - V_D. \qquad (A.20)$$

Finally, similar analysis for the fourth solution requires the following two conditions in order for the fourth solution to constitute pure strategy equilibrium:

$\Pi_D^4 - \Pi_D^{'} \geq 0$ if

$$u_D^4 = \frac{3[(1-\gamma)\alpha_1 e_2^2 + 4\alpha_1\alpha_2 e_2 - 4\gamma\alpha_2(\alpha_1 - \alpha_2)^2]}{4\bar{\alpha}e_2} \leq V_A - V_D.$$ (A.21)

$\Pi_A^4 - \Pi_A^{'} \geq 0$ if

$$u_A^4 = \frac{3[-(1-\gamma)\alpha_1 e_2^2 - 4\alpha_1\alpha_2 e_2 + 4\gamma\alpha_2(\alpha_1 - \alpha_2)^2]}{4\bar{\alpha}e_2} \geq V_A - V_D.$$ (A.22)

Solution procedure for mixed strategy equilibrium outcomes

Let (q, w) be a mixed strategy equilibrium profile, where q (w) is the probability that the disadvantaged (advantaged) firm will offer a rebate in a given mixed strategy equilibrium. We denote firm i's price charged at non-rebating periods with P_{inr}. While for rebating periods firm i chooses price P_{ir} and rebate R_i.

The expected profit of rebating for the disadvantaged firm (D) when the advantaged firm (A) mixes with probability w has two components: the profit the disadvantaged firm gets when both firms end up offering a rebate ($\Pi_{Dr,Ar-mix}$) and the profit when the disadvantaged firm is the only firm offering a rebate ($\Pi_{Dr,Anr-mix}$). We formulate each profit function using the market shares described in equations A.1 and A.2.

The advantaged firm executes its pure strategy with mixing probability w. We can formulate the disadvantaged firm's expected profit from rebating as $E(\Pi_{Dr-mix}) = w\Pi_{Dr,Ar-mix} + (1-w)\Pi_{Dr,Anr-mix}$ and its expected profit from not rebating is $E(\Pi_{Dnr-mix}) = w\Pi_{Dnr,Ar-mix} + (1-w)\Pi_{Dnr,Anr-mix}$. For the advantaged firm, the expected profit from rebating when the disadvantaged firm mixes with probability q is $E(\Pi_{Ar-mix}) = q\Pi_{Ar,Dr-mix} + (1-q)\Pi_{Ar,Dnr-mix}$ and the expected profit from not rebating is $E(\Pi_{Anr-mix}) = q\Pi_{Anr,Dr-mix} + (1-q)\Pi_{Anr,Dnr-mix}$.

Each firm will set prices for rebate and no rebate periods (P_{ir} and P_{inr}, respectively) and the rebate amount (R_i) for the rebating period given the other firm's mixed equilibrium profile. Therefore, there are four maximization problems which jointly determine firm actions using a similar set of participation and incentive compatibility constraints as before. Of the four profit functions above, two of them are for rebating firms and therefore each includes two decision variables, P and R. The other two are for non-rebating firms and therefore each includes one decision variable, P. This results in six decision variables: P_{Anr}, P_{Ar}, R_A, P_{Dnr}, P_{Dr} and R_D. The first order conditions of each profit function with respect to the decision variables it contains are taken.

$$\frac{\partial E(\Pi_{Dnr-mix})}{\partial P_{Dnr}} = 0, \ \frac{\partial E(\Pi_{Dr-mix})}{\partial P_{Dr}} = 0, \ \frac{\partial E(\Pi_{Dr-mix})}{\partial R_{Dr}} = 0$$

$$\frac{\partial E(\Pi_{Anr-mix})}{\partial P_{Anr}} = 0, \ \frac{\partial E(\Pi_{Ar-mix})}{\partial P_{Ar}} = 0, \ \frac{\partial E(\Pi_{Ar-mix})}{\partial R_{Ar}} = 0$$

We now have six equations with six unknowns. Solving them simultaneously, we get the solutions:

$$R_A = \frac{(2+q)(\alpha_1 - \alpha_2) + e_2(2-q-wq)}{(4-wq)\delta^t} \ , \ R_D = \frac{(2+w)(\alpha_1 - \alpha_2) + e_2(2-w-wq)}{(4-wq)\delta^t}$$

$$P_{Anr} = \frac{V_A - V_D}{3} + \frac{6\alpha_1\alpha_2 + e_2\alpha_1(1-\gamma)(2q+w)}{6((1-\gamma)\alpha_1 + \gamma\alpha_2)} \ , \ P_{Dnr} = \frac{V_D - V_A}{3} + \frac{6\alpha_1\alpha_2 + e_2\alpha_1(1-\gamma)(2w+q)}{6((1-\gamma)\alpha_1 + \gamma\alpha_2)}$$

$$P_{Ar} = \frac{V_A - V_D}{3} + \frac{\alpha_1[e_2(1-\gamma)(w^2q + 2wq^2 + 3wq - 2q - 4w) - 6\alpha_1(1-\gamma)(2+q) - 6\alpha_2(\gamma(2+q)+2-q-wq)]}{6(4-wq)((1-\gamma)\alpha_1 + \gamma\alpha_2)}$$

$$P_{Dr} = \frac{V_D - V_A}{3} + \frac{\alpha_1[e_2(1-\gamma)(q^2w + 2w^2q + 3wq - 2w - 4q) - 6\alpha_1(1-\gamma)(2+w) - 6\alpha_2(\gamma(2+w)+2-w-wq)]}{6(4-wq)((1-\gamma)\alpha_1 + \gamma\alpha_2)}$$

Since in any mixed strategy equilibrium each firm will rebate with probability so that its rival is indifferent between its two pure strategies (rebating or no rebating), the following two equations must be satisfied:

$$w\Pi_{Dr,Ar-mix} + (1-w)\Pi_{Dr,Anr-mix} = w\Pi_{Dnr,Ar-mix} + (1-w)\Pi_{Dnr,Anr-mix} \quad (A.23)$$

$$q\Pi_{Ar,Dr-mix} + (1-q)\Pi_{Ar,Dnr-mix} = q\Pi_{Anr,Dr-mix} + (1-q)\Pi_{Anr,Dnr-mix} \quad (A.24)$$

Plugging the values of six price and rebate figures reported above into each of the profit functions in preceding two equations, we can solve for firms' mixing probabilities (q, w).

To obtain numerical results, parameter values would have to be plugged into these equations and the solutions obtained using numerical methods. We tried the Maple software package and were able to obtain numerical solutions in this way.

Proof of Proposition 1

We start with firm D rebating alone equilibrium. Combining equations A.5 and A.6 gives

$$P_A - P_D = \frac{2}{3}(V_A - V_D) - \frac{\alpha_1(1-\gamma)[3(\alpha_1 - \alpha_2) - e_2]}{6[(1-\gamma)\alpha_1 + \gamma\alpha_2]}.$$ Hence, $P_A > P_D$ if

$$V_A - V_D > \frac{\alpha_1(1-\gamma)[3(\alpha_1 - \alpha_2) - e_2]}{4[(1-\gamma)\alpha_1 + \gamma\alpha_2]} = u*$$

Next prove the proposition for other pure strategy equilibria. In neither rebating equilibrium, using equations A.3 and A.4, we find $P_A - P_D = \frac{2}{3}(V_A - V_D) > 0$. When the advantaged firm rebating alone is equilibrium, we have

$$P_A - P_D = \frac{2}{3}(V_A - V_D) + \frac{\alpha_1(1-\gamma)[3(\alpha_1 - \alpha_2) - e_2]}{6[(1-\gamma)\alpha_1 + \gamma\alpha_2]}.$$ In this case $P_A - P_D > 0$ since $\gamma < 1$ and $\alpha_1 - \alpha_2 > e_2$.

In both firms rebating equilibrium, we get $P_A - P_D = \frac{2}{3}(V_A - V_D) > 0$.

In mixed strategy equilibrium, we compare the average prices charged by both firms.

$$E(P_A) = wP_{Ar} + (1-w)P_{Anr}$$
$$E(P_D) = qP_{Dr} + (1-q)P_{Dnr}$$
$$E(P_A) - E(P_D) = \frac{2}{3}(V_A - V_D) - \frac{\alpha_1(1-\gamma)(q-w)[6(\alpha_1 - \alpha_2) - (2+qw)e_2]}{3(4-qw)[(1-\gamma)\alpha_1 + \gamma\alpha_2]}$$

Hence, $E(P_A) - E(P_D) > 0$ if

$$V_A - V_D > \frac{\alpha_1(1-\gamma)(q-w)[6(\alpha_1 - \alpha_2) - (2+qw)e_2]}{2(4-qw)[(1-\gamma)\alpha_1 + \gamma\alpha_2]} = u**. \text{ QED}$$

Proof of Proposition 2

The proof directly follows from the pure strategy equilibria cut-offs. Recall that the advantaged firm offers rebates in two pure strategy equilibria given by solutions 3 and 4. Using the cutoffs u_A^3 and u_A^4 described earlier we see that the advantaged firm does not deviate from solutions 3 and 4, if $V_A - V_D \le u_A^3$ and $V_A - V_D \le u_A^4$, respectively.

Clearly, as the relative advantage $(V_A - V_D)$ increases, the preceding conditions are less likely to be satisfied. Therefore the advantaged firm is more likely to deviate for larger values of $V_A - V_D$.

Similarly there are two pure strategy equilibria where the disadvantaged firm offers a rebate. These are given by solutions 2 and 4. Using the cutoffs u_D^2 and u_D^4 described earlier we see that the disadvantaged firm does not deviate from solutions 2 and 4, if $V_A - V_D \geq u_D^2$ and $V_A - V_D \geq u_D^4$, respectively. It is clear that as the relative advantage $(V_A - V_D)$ increases, the preceding conditions are more likely to be satisfied. Therefore the disadvantaged firm is less likely to deviate for larger values of $V_A - V_D$. *QED*

Proof of Proposition 3

In order for firm A rebating alone in equilibrium $u_D^3 \geq u$ and $u_A^3 \geq u$ as given in A.19 and A.20, where $u = V_A - V_D$. Reorganizing firm D's equilibrium cutoffs we get:
$$(4\bar{\alpha} - 7\gamma\alpha_2)e_2^2 + 6\alpha_2(3\bar{\alpha} + 5\alpha_1)e_2 - 16\bar{\alpha}ue_2 \geq 27\gamma\alpha_2(\alpha_1 - \alpha_2)^2.$$
Similarly, reorganizing firm A's equilibrium cutoffs and multiplying both sides of the inequality by 9 leads to
$$27\gamma\alpha_2(\alpha_1 - \alpha_2)^2 \geq -9(\bar{\alpha} + 2\gamma\alpha_2)e_2^2 + 54\alpha_2(\bar{\alpha} + \alpha_1)e_2 + 4\bar{\alpha}ue_2.$$

Note that the right hand side of the first and the left hand side of the second inequality are the same. Therefore, combining the inequalities we get $e_2 \geq \dfrac{\alpha_2(36\bar{\alpha} + 24\alpha_1) + 20\bar{\alpha}u}{13\bar{\alpha} + 11\gamma\alpha_2}$. When firm A rebates alone, plugging in the optimal price and rebate values (A.8-A.10) into equation A.2 and noting the restriction on market share $\tau_2 \leq 1$ and rewriting it in terms of e_2 leads to $e_2 \leq \dfrac{2u\bar{\alpha} + 3\alpha_2(\alpha_1 + \bar{\alpha})}{2\bar{\alpha} + \gamma\alpha_2}$. Combining the two conditions on e_2 leads to

$-[u\bar{\alpha}(12\bar{\alpha} + 2(1-\gamma)\alpha_1) + \alpha_2[(33\bar{\alpha}^2 + 3\bar{\alpha}\gamma\alpha_2 + 9(1-\gamma)\alpha_1^2] \geq 0$. Since the left hand side of the preceding inequality is negative, firm A rebating alone cannot be Nash equilibrium. *QED*

Proof of Generalization 1

Using the equilibrium cutoffs we find $u_D^2 - u_D^1 = \dfrac{(1-\gamma)\alpha_1 e_2}{2\bar{\alpha}} > 0$, i.e. $u_D^2 > u_D^1$. Clearly the equilibrium conditions for firm D rebating alone outcome $(u_D^1 \geq u)$ and neither firm rebating outcome $(u_D^2 \leq u)$ cannot hold simultaneously. *QED*

Proof of Proposition 4
Reorganizing equations A.23 and A.24 gives

$$w(\Pi_{Dr,Ar-mix} - \Pi_{Dnr,Ar-mix}) - (1-w)(\Pi_{Dnr,Anr-mix} - \Pi_{Dr,Anr-mix}) = 0 \tag{A.25}$$
$$q(\Pi_{Ar,Dr-mix} - \Pi_{Anr,Dr-mix}) - (1-q)(\Pi_{Anr,Dnr-mix} - \Pi_{Ar,Dnr-mix}) = 0 \tag{A.26}$$

The profit differences are:

$$\Pi_{Dr,Ar-mix} - \Pi_{Dnr,Ar-mix} = \frac{(1-\gamma)e_2u}{6\alpha_2} - \frac{(1-\gamma)}{24\alpha_2(4-wq)^2\overline{\alpha}}[(12\gamma\alpha_2(2-w(1+q))(q-w)) + (1-\gamma)(4-wq)^2(2q+w)\alpha_1)e_2^2$$

$$+12\alpha_2(\alpha_1(4-wq)^2 - (\alpha_1-\alpha_2)\gamma(qw((2+q)w-8-q) + 2(2-w)^2))e_2 - 12\gamma\alpha_2(\alpha_1-\alpha_2)^2(2+w)(4+q-(1+q)w)]$$

$$\Pi_{Dnr,Anr-mix} - \Pi_{Dr,Anr-mix} = -\frac{(1-\gamma)e_2u}{6\alpha_2} - \frac{(1-\gamma)}{24\alpha_2(4-wq)^2\overline{\alpha}}[(12\gamma\alpha_2(2-w(1+q))(2+w) + (1-\gamma)(4-wq)^2(3-2q-w)\alpha_1)e_2^2$$

$$-12\alpha_2(\alpha_1(4-wq)^2 - (\alpha_1-\alpha_2)\gamma(qw((2+q)w-4) + 8 + 2w^2))e_2 + 12\gamma\alpha_2(\alpha_1-\alpha_2)^2(2+w)(2-(1+q)w)]$$

$$\Pi_{Ar,Dr-mix} - \Pi_{Anr,Dr-mix} = -\frac{(1-\gamma)e_2u}{6\alpha_2} - \frac{(1-\gamma)}{24\alpha_2(4-wq)^2\overline{\alpha}}[(12\gamma\alpha_2(2-q(1+w))(q-w) + (1-\gamma)(4-wq)^2(2w+q)\alpha_1)e_2^2$$

$$+12\alpha_2(\alpha_1(4-wq)^2 - (\alpha_1-\alpha_2)\gamma(qw((2+w)q-8-w) + 2(2-q)^2))e_2 - 12\gamma\alpha_2(\alpha_1-\alpha_2)^2(2+q)(4-w+(1+w)q)]$$

$$\Pi_{Anr,Dnr-mix} - \Pi_{Ar,Dnr-mix} = \frac{(1-\gamma)e_2u}{6\alpha_2} - \frac{(1-\gamma)}{24\alpha_2(4-wq)^2\overline{\alpha}}[(12\gamma\alpha_2(2-q(1+w))(2+q) + (1-\gamma)(4-wq)^2(3-2w-q)\alpha_1)e_2^2$$

$$-12\alpha_2(\alpha_1(4-wq)^2 - (\alpha_1-\alpha_2)\gamma(qw((2+w)q-4) + 8 + 2q^2))e_2 + 12\gamma\alpha_2(\alpha_1-\alpha_2)^2(2+q)(2-(1+w)q)]$$

Note that even though the above differences are functions of both (u, q, w), u does not interact with (q, w). Note that even though the above differences are functions of both (u, q, w), u does not interact with (q, w). Plugging in the expressions into equations A.25 and A.26 give the following equalities:

$$\frac{(1-\gamma)e_2u}{6\alpha_2} + \frac{(1-\gamma)}{24\alpha_2(4-wq)^2\overline{\alpha}}[(12\gamma(2-w-wq)^2\alpha_2 + (1-\gamma)(4-wq)^2(3-2q-4w)\alpha_1)e_2^2$$
$$-12\alpha_2(\alpha_1(4-wq)^2 - 2\gamma(\alpha_1-\alpha_2)(2+w)(2-w-wq))e_2 + 12\gamma(2+w)^2\alpha_2(\alpha_1-\alpha_2)^2] = 0 \qquad (A.27)$$

$$-\frac{(1-\gamma)e_2u}{6\alpha_2} + \frac{(1-\gamma)}{24\alpha_2(4-wq)^2\overline{\alpha}}[(12\gamma(2-q-wq)^2\alpha_2 + (1-\gamma)(4-wq)^2(3-4q-2w)\alpha_1)e_2^2$$
$$-12\alpha_2(\alpha_1(4-wq)^2 - 2\gamma(\alpha_1-\alpha_2)(2+q)(2-q-wq))e_2 + 12\gamma(2+q)^2\alpha_2(\alpha_1-\alpha_2)^2] = 0 \qquad (A.28)$$

For $u > 0$, the first term of equation A.27 is positive while the first term of equation A.28 is negative. Therefore, the second term of equation A.27 (in the square brackets) must be negative and the parallel term in equation A.28 must be positive in order to have A.27 and A.28 hold simultaneously. This can be stated as two inequalities:

$$12\gamma\alpha_2(\alpha_1-\alpha_2)^2 < \frac{\left[\begin{array}{c} -(12\gamma(2-w-wq)^2\alpha_2 + (1-\gamma)(4-wq)^2(3-2q-4w)\alpha_1)e_2^2 \\ +12\alpha_2(\alpha_1(4-wq)^2 - 2\gamma(\alpha_1-\alpha_2)(2+w)(2-w-wq))e_2 \end{array}\right]}{(2+w)^2} \quad \text{and}$$

$$12\gamma\alpha_2(\alpha_1-\alpha_2)^2 > \frac{\left[\begin{array}{c} -(12\gamma(2-q-wq)^2\alpha_2 + (1-\gamma)(4-wq)^2(3-4q-2w)\alpha_1)e_2^2 \\ +12\alpha_2(\alpha_1(4-wq)^2 - 2\gamma(\alpha_1-\alpha_2)(2+q)(2-q-wq))e_2 \end{array}\right]}{(2+q)^2}.$$

Since the left hand side terms are the same for both the inequalities, we can combine the two inequalities into a single inequality. This leads to the following condition that is true for any mixed strategy equilibrium for $u > 0$:

$$\frac{(q-w)(4-wq)}{(2+w)^2(2+q)^2}\left[\begin{array}{c} -[(1-\gamma)(4-wq)(20-2q^2-2w^2-5q-5w-6wq)\alpha_1 + 12\gamma(8-w^2q-wq^2-6wq)\alpha_2]e_2 \\ +12\alpha_2(\alpha_1(4-wq)(4+w+q) - 2\gamma(2+w)(2+q)(\alpha_1-\alpha_2)) \end{array}\right] > 0 \qquad (A.29)$$

In order for the term in the square brackets to be positive we should have

$$e_2 < \frac{12\alpha_2(\alpha_1(4-wq)(4+w+q) - 2\gamma(2+w)(2+q)(\alpha_1-\alpha_2))}{(1-\gamma)(4-wq)(20-2q^2-2w^2-5q-5w-6wq)\alpha_1 + 12\gamma(8-w^2q-wq^2-6wq)\alpha_2} \qquad (A.30)$$

Let $X_1 = 12\alpha_2(\alpha_1(4-wq)(4+w+q) - 2\gamma(2+w)(2+q)(\alpha_1-\alpha_2))$ and
$Y_1 = (1-\gamma)(4-wq)(20-2q^2-2w^2-5q-5w-6wq)\alpha_1 + 12\gamma(8-w^2q-wq^2-6wq)\alpha_2$.

We next show that inequality A.30 holds, i.e. $e_2 < \dfrac{X_1}{Y_1}$. In a mixed strategy equilibrium when firm D offers a rebate, plugging in the optimal price and rebate values into equation A.2 we get

$$\tau_2 = -\frac{u}{6\alpha_2} + \frac{1}{12\alpha_2(4-wq)\overline{\alpha}}\left[\begin{array}{l} -[(1-\gamma)(4-wq)(3+w-q)\alpha_1 + 6\gamma(2+w)\alpha_2]e_2 \\ +6\alpha_2(-\alpha_1(4-wq)+\gamma(2-w-wq)(\alpha_1-\alpha_2)) \end{array}\right].$$ Note that the first term of τ_2 is

negative when $u > 0$. Thus for any $\tau_2 > 0$, the second term should be positive, i.e., solving for e_2 we should have

$$e_2 < \frac{6\alpha_2(-(4-wq)\alpha_1 + \gamma(2-w-wq)(\alpha_1-\alpha_2))}{(1-\gamma)(4-wq)(3+w-q)\alpha_1 + 6\gamma(2+w)\alpha_2} \tag{A.31}$$

Let $X_2 = 6\alpha_2(-(4-wq)\alpha_1 + \gamma(2-w-wq)(\alpha_1-\alpha_2))$ and $Y_2 = (1-\gamma)(4-wq)(3+w-q)\alpha_1 + 6\gamma(2+w)\alpha_2$.
Using this cutoff on e_2 we will show that $\dfrac{X_1}{Y_1} < \dfrac{X_2}{Y_2}$, i.e. – since Y_1 and Y_2 are positive – $\dfrac{X_1Y_2 - X_2Y_1}{Y_1Y_2} > 0$.
Computing $X_1Y_2 - X_2Y_1$ we get

$$6\alpha_2(4-wq)((1-\gamma)\left[\begin{array}{l} ((2w^3 - 39wq + 4q^2 - 70w - 8 + 6w^2q^2 + 2wq^3 - 7w^2 - 18q + 7qw^2 + 11wq^2 + 2w^3q)\gamma \\ +16 - 4w^3q + 16w^2 - 3wq^2 - 19qw^2 + 76w - 6w^2q^2)\alpha_1^2 \\ +(42q - 2w^3q + 5qw^2 - 4q^2 + wq^2 - 2w^3 + 123wq + 142w + 19w^2 - 2wq^3 + 8 - 6w^2q^2)(1-\gamma)\gamma\alpha_1\alpha_2 \\ +(qw^2 + 6w + wq^2 + 7wq + 2q + w^2)12\gamma^2\alpha_2^2 \end{array}\right]$$

The above term is positive for all values of $q \in (0,1)$ and $w \in (0,1)$. Thus the right hand side of inequality A.31 is larger than the right hand side of inequality A.30 indicating that inequality A.30 is satisfied for any feasible mixed strategy. Therefore, the value in the square brackets in inequality A.29 is positive. Since all other terms are also positive, we need to have $q - w$ be strictly positive for A.29 to hold. Thus $q > w$ if $u > 0$. i.e., firm D offers a rebate with higher probability than firm A. *QED*