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Author(s): Susanne Bobzien

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# *Wholly Hypothetical Syllogisms*

SUSANNE BOBZIEN

## ABSTRACT

In antiquity we encounter a distinction of two types of hypothetical syllogisms. One type are the ‘mixed hypothetical syllogisms’. The other type is the one to which the present paper is devoted. These arguments went by the name of ‘wholly hypothetical syllogisms’. They were thought to make up a self-contained system of valid arguments. Their paradigm case consists of two conditionals as premisses, and a third as conclusion. Their presentation, either schematically or by example, varies in different authors. For instance, we find ‘If (it is) A, (it is) B; if (it is) B, (it is) C; therefore, if (it is) A, (it is) C’. The main contentious point about these arguments is what the ancients thought their logical form was. Are A, B, C schematic letters for terms or propositions? Is ‘is’, where it occurs, predicative, existential, or veridical? That is, should ‘A ἐστὶ’ be translated as ‘it is an A’, ‘A exists’, ‘As exist’ or ‘It is true/the case that A’? If A, B, C are term letters, and ‘is’ is predicative, are the conditionals quantified propositions or do they contain designators? If one cannot answer these questions, one can hardly claim to know what sort of arguments the wholly hypothetical syllogisms were. In fact, all the above-mentioned possibilities have been taken to describe them correctly. In this paper I argue that it would be mistaken to assume that in antiquity there was one prevalent understanding of the logical form of these arguments – even if the ancients thought they were all talking about the same kind of argument. Rather, there was a complex development in their understanding, starting from a term-logical conception and leading to a propositional-logical one. I trace this development from Aristotle to Philoponus and set out the deductive system on which the logic of the wholly hypothetical syllogisms was grounded.

At the close of antiquity we encounter a distinction of two types of hypothetical syllogisms. One type, sometimes called ‘mixed hypothetical syllogisms’, contain a mixture of complex and simple premisses; they encompass e.g. arguments of the kind *modus ponens*.<sup>1</sup> The other type is the one to which the present paper is devoted. These arguments variously went by the names of ‘syllogisms by analogy’, ‘hypothetical syllogisms’, ‘wholly hypothetical syllogisms’, ‘wholly hypothetical arguments’, ‘“through three” arguments’, ‘“through three” syllogisms’. They were thought to make up a self-contained system of syllogisms or valid arguments.

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<sup>1</sup> I discuss this type of arguments and their system in Stoic philosophy in my ‘Stoic syllogistic’, *Oxford Studies in Ancient Philosophy* 1996, 133-92.

The paradigm case of these arguments consists of two conditionals as premisses, and a third one as conclusion, in which the antecedent clause of the first premiss is identical with that of the conclusion; the consequent clause of the first premiss corresponds to the antecedent clause of the second premiss; and the consequent clause of the second premiss is identical with that of the conclusion. The presentation of such arguments, either schematically or by example, varies slightly in different authors. For instance we find

si est A est B; at si est B, est C: si igitur est A, necesse est ut sit C. (Boeth. *HS* 2.9.5 Obertello)

εἰ τὸ Α, τὸ Β; εἰ τὸ Β, τὸ Γ; εἰ ἄρα τὸ Α, τὸ Γ (Alex. *APr.* 326.22-3)

εἰ ἄνθρωπος, καὶ ζῶον· εἰ ζῶον, ἔμψυχον· εἰ ἄνθρωπος ἄρα, ἔμψυχον ([Amm.] *APr.* 67.24-30)

εἰ ἄνθρωπός ἐστι, ζῶόν ἐστιν, εἰ ζῶόν ἐστιν, οὐσία ἐστίν, εἰ ἄρα ἄνθρωπός ἐστιν, οὐσία ἐστίν (Alex. *APr.* 326.24-5)

The main contentious point about these arguments is what the ancients thought their logical form was. Are A, B, C schematic letters for terms or for propositions? Do the arguments accordingly belong to term logic or propositional logic?<sup>2</sup> Is 'is' (ἔστι, *est*), where it occurs, predicative, existential, or veridical? That is, should for instance 'A ἔστι' be translated as 'it is an A', 'A exists', 'As exist' or 'It is true/the case that A'? If A, B, C are term letters, and 'is' is predicative, are the conditionals to be understood as some kind of quantified propositions (e.g. 'If anything is A, it is B') or as containing designators (e.g. 'If this thing here is A, it is B')?

Evidently, if one cannot answer these questions, one can hardly claim to know what sort of arguments the hypothetical arguments at issue were. In fact, all the above-mentioned possibilities have at some time or other been taken to give the correct description of these arguments. In this paper I argue that it would be mistaken to assume that in antiquity there was one prevalent understanding of the logical form of these hypothetical arguments – even if the ancients themselves seem to have thought they were all talking about the same kind of argument all along. Rather, there was a complex development in the understanding of these arguments, starting from a term-logical conception and leading to a propositional-logical one.

<sup>2</sup> Some authors prefer to talk about sentences and sentence logic; so J. Barnes, in his comprehensive study 'Terms and sentences: Theophrastus on hypothetical syllogisms', *Proceedings of the British Academy* 69, 1983, 279-326 (T&S). For my purposes here this difference is of minor importance.

In the following I refer to the arguments under discussion as ‘wholly hypotheticals’ (abbreviated as ‘WHs’), using this name as a neutral way of referring to whatever arguments at any particular stage were thought to belong to this particular class of arguments. Note that, following the ancient habit, I use the expression to denote formally valid arguments only.

### 1. *The deductive system of WHs*

Before I sketch the historical development of the WHs, let me present the deductive system the *basic* wholly hypotheticals (BWHs) were thought to form (i.e. the system of two-premiss WHs with simple component clauses). This system is neutral with respect to the different interpretations or developmental stages of the arguments – a fact that may have facilitated the development. The easiest way of conceiving of this system is perhaps as follows:<sup>3</sup>

- An argument has the syntactic structure of a BWH if it has this form: it has two *premisses* and a *conclusion*, each being a *conditional* of the general form ‘If  $\pm A$ ,  $\pm B$ ’, in which ‘ $\pm A$ ’ and ‘ $\pm B$ ’ are called the *components* of the conditional ( $\pm A$  is the *antecedent component*,  $\pm B$  the *consequent component*):

1st premiss	If $\pm A$ , $\pm B$
2nd premiss	If $\pm C$ , $\pm D$
conclusion	If $\pm E$ , $\pm F$

‘ $\pm A$ ’ is intended to leave the quality of the component indeterminate. It indicates that the component may be either a *positive component*, ( $A$ ), or a *negative component*, ( $\neg A$ ). Negative and positive components are structurally on a par. (Typically, a negative component would contain a negative particle such as ‘not’ at a designated position, whereas a positive component would not contain such a particle.)

- A *mode* is any schematic presentation of a WH which differs from an actual WH in that it has letters in place of ordinary language expressions such as nouns or clauses.

<sup>3</sup> In the presentation of this system I have shamelessly helped myself to similar presentations by Boethius (*De hypotheticis syllogismis* (HS) 2.9.1-3.6.4 Obertello) and Barnes, T&S; but note that Barnes, unlike myself, interprets the system as a system of sentence logic, not as neutral; that he attributes the whole system to Theophrastus; and that he gives structural rules that differ from mine.

- The system contains eight “axiomatic” modes, 1.1-8, namely

1.1 $\frac{\text{If } A, B}{\text{If } B, C}$ $\frac{\quad}{\text{If } A, C}$	1.2 $\frac{\text{If } -A, B}{\text{If } B, C}$ $\frac{\quad}{\text{If } -A, C}$	1.3 $\frac{\text{If } A, -B}{\text{If } -B, C}$ $\frac{\quad}{\text{If } A, C}$	1.4 $\frac{\text{If } -A, -B}{\text{If } -B, C}$ $\frac{\quad}{\text{If } -A, C}$
1.5 $\frac{\text{If } A, B}{\text{If } B, -C}$ $\frac{\quad}{\text{If } A, -C}$	1.6 $\frac{\text{If } -A, B}{\text{If } B, -C}$ $\frac{\quad}{\text{If } -A, -C}$	1.7 $\frac{\text{If } A, -B}{\text{If } -B, -C}$ $\frac{\quad}{\text{If } A, -C}$	1.8 $\frac{\text{If } -A, -B}{\text{If } -B, -C}$ $\frac{\quad}{\text{If } -A, -C}$

These eight modes are the result of producing all the permutations of 1.1 which one obtains by using a negative component instead of any one, two, or three different positive components.

- The system of BHWs contains one rule, *CR*, a conversion rule, which operates on the conditionals. When applied it switches round the components of the conditional and at the same time changes the quality (negative or positive) of each component. If we use ‘ $\pm A^*$ ’ to indicate the qualitative ‘opposite’ of  $\pm A$ , the rule can be presented as

$$CR \quad \frac{\text{If } \pm A, \pm B}{\text{If } \pm B^*, \pm A^*}$$

- By applying *CR* to the ‘conclusions’ of each of 1.1-8, one obtains another eight modes, 1.9-16.<sup>4</sup> By applying *CR* to the ‘first premisses’ of 1.1-16 one obtains another sixteen modes, 2.1-16. By applying *CR* to the ‘second premisses’ of 1.1-16 one obtains another sixteen modes, 3.1-16.<sup>5</sup> We thus have a system of 48 modes of BWHs.
- Let us call a component without any negation sign it may contain a *bare component* (e.g. the bare components of ‘If A, -B’ are ‘A’ and ‘B’); and let us say that a bare component is *in* a component, if the latter is identical with it or surpasses it by a negation sign. It is then a necessary (but not sufficient) condition for an argument of the syntactic structure of a BWH to be ‘provable’ in the system that the premisses share one bare component.<sup>6</sup> Call this the *shared component* of the argument.

<sup>4</sup> Strictly speaking, *CR* applies to arguments, not modes, and by applying *CR* to arguments in modes 1.1-8 one obtains arguments in another eight modes, namely 1.9-16, and so forth. I put ‘premiss’, ‘conclusion’, etc. in quotes, to indicate this fact.

<sup>5</sup> By applying *CR* to both the ‘first and second premisses’ of 1.1-16, one ends up with the same set of modes 1.1-16, with reversed ‘premiss’ order. Similarly, by applying *CR* to any ‘premiss’ or ‘conclusion’ of 2.1-16 or 3.1-16 one obtains no new modes. More modes than 1.1-3.16 cannot be had in the system.

<sup>6</sup> Axioms 1.1-1.8 each contain a shared component, and any application of *CR* on a BWH preserves the bare components in each conditional.

- Call those BWHs in which the shared component is in the antecedent component in one premiss, in the consequent component in the other, *first figure* BWHs; those in which the shared component is in the antecedent component in both premisses, *second figure* BWHs; and those in which it is in the consequent component in both premisses, *third figure* BWHs.
- A *reduction* is any use of *CR* on a premiss of a second or third figure argument which transforms the argument into a first figure argument; any use of *CR* on 1.9-16 arguments which transforms them into arguments of modes 1.1-8; and any two uses of *CR* that transforms an argument from 2.9-16 or 3.9-16 to an argument of 1.1-8.

All this may appear to be much ado about very little. However, if, like the proponents of this system of arguments, one has no concept of substituting negative for positive components (no concepts of substitution and instantiation at all, perhaps), this may not be so. Moreover, whereas most would regard arguments in modes 1.1-1.8 as evidently valid, few would consider all arguments in modes 1.9-3.16 as *evidently* valid, and the reduction by means of *CR* may thus be seen to provide sufficient reason for their validity.

The system sketched so far covers BWHs only. The ancients also allowed for complex wholly hypotheticals, i.e. those with more than two premisses, of the form 'If A, B<sub>1</sub>; if B<sub>1</sub>, B<sub>2</sub>; . . . if B<sub>n</sub>, C; therefore if A, C.' (and any arguments that can obtain this form by changing the order of the premisses or by using *CR* on any premisses and/or the conclusion.) These arguments can be reduced to BWHs by using an inference rule *IR* (with  $n \geq 1$ )

$$IR \frac{\text{If A, B}_1; \text{if B}_1, \text{B}_2 \dots \text{if B}_n, \text{C; therefore if A, C} \quad \text{If A, C; if C, D; therefore if A, D}}{\text{If A, B}_1; \text{if B}_1, \text{B}_2 \dots \text{if B}_n, \text{C; if C, D; therefore if A, D.}}$$

This is a rule of the cut type.<sup>7</sup> Such rules were common in antiquity. One can easily see that the number of modes of complex WHs with three premisses is very large, let alone those of modes of complex WHs with four or more premisses. The distinction of figures cannot be transferred to complex WHs, except that one may want to call arguments in the above given 'standard' form first figure complex WHs.

<sup>7</sup> In the lower line "therefore if A, C – If A, C;" is cut.

## 2. An outline of the development of the wholly hypothetical syllogisms

Our evidence for WHs is patchy and heterogeneous. Any interpretation should ideally be such that it is consistent with the entire surviving evidence. However, even if one takes all relevant passages into account (which has, I think, so far not been done), there remain several consistent interpretations. Decisions between them need to be made on the basis of philosophical and historical plausibility. It would be too space-consuming to present all consistent interpretations, and then weigh up their persuasiveness. Instead, I present a story of the development of the WHs which I believe consistent with the evidence and historically and philosophically highly plausible. Two basic ideas form the main connecting thread in this story. First, many apparent difficulties in the sources disappear once one realizes that there was not one understanding of the WHs throughout antiquity, but several, the later ones developing from the earlier. Second, I assume that in the wake of Aristotle's term-logical syllogistic this process starts out from a term-logical perspective of syllogisms tout court and, presumably under the influence of Stoic logic, develops into a propositional-logical understanding. That is – as one would naturally expect – the direction is from term-logic to propositional logic. In some more detail, the development, as I envisage it, was as follows:<sup>8</sup>

The origin of the theory of WHs lies in a passage from Aristotle's *Prior Analytics*. In *APr.* I 32 47a28-31 Aristotle gives an example of an argument in which the conclusion necessarily follows from the premisses (i.e. which is valid), but which is not a syllogism, since the premisses are not related in the required way. The argument is:

<p>(I<sub>ex</sub>) ἀνθρώπου ὄντος ἀνάγκη ζῶον εἶναι καὶ ζῴου οὐσίαν, ἀνθρώπου ὄντος ἀνάγκη οὐσίαν εἶναι</p>	<p>It being the case that being human it is necessarily an animal and being an animal, a substance, being human, it is necessarily a substance</p>
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Slightly tidied up, its grammatical form (and 'mode') would be

<p>(I<sub>mode</sub>) τοῦ Α ὄντος ἀνάγκη τὸ Β εἶναι τοῦ Β ὄντος ἀνάγκη τὸ Γ εἶναι τοῦ Α ὄντος ἀνάγκη τὸ Γ εἶναι</p>	<p>being A, it is necessarily B being B, it is necessarily C being A, it is necessarily C</p>
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<sup>8</sup> I restrict myself to WHs of the form I.1. Peculiarities concerning arguments in the other modes and questions of reduction will be discussed later in sections 4 to 10.

with A, B, C for terms. The phrase ἀνάγκη <εστι> indicates *necessitas consequentiae*. According to Aristotle's remarks in *APr.* I 32 47a10-40, the argument would become a proper (and that is categorical) syllogism if reformulated as

- (0<sub>ex</sub>) substance belongs to every animal  
 animal belongs to every human being  
 therefore substance belongs to every human being

which is in *modus Barbara*

- (0<sub>mode</sub>) C belongs to every B  
 B belongs to every A  
 therefore C belongs to every A

From (1) to (0) premisses and conclusion have each been transformed in the same way. On the assumption that the non-syllogistic valid argument (1) was regarded as reformulable as (0), it must be possible to understand it as – more or less – equivalent to (0). Thus premisses and conclusion in (1) have to be understood as something like

- (C<sub>1</sub>) If anything is T<sub>A</sub>, it is necessary that it is T<sub>B</sub>. (with T<sub>A</sub> and T<sub>B</sub> for terms)

i.e. as (expressing) a universally quantified conditional of sorts. This is a natural way of paraphrasing sentences of the kind 'T<sub>A</sub> ὄντος ἀνάγκη T<sub>B</sub> εἶναι'.<sup>9</sup> For they have exactly two terms (T<sub>A</sub>, T<sub>B</sub>), and there is no singular subject term explicitly mentioned in either the genitive absolute or the main clause.

So far what we find in Aristotle. From here a theory of WHs was developed in a first step as follows: the argument from *APr.* I 32 was taken as a paradigm for a specific kind of argument, with standard formulations of examples and modes:

- |                      |  |  |
|----------------------|--|--|
| (2 <sub>ex</sub> )   | εἰ ἄνθρωπος ἐστὶ, ζῷον ἐστὶ,<br>εἰ ζῷον ἐστὶν, οὐσία ἐστίν,<br>εἰ ἄρα ἄνθρωπος ἐστὶν, οὐσία ἐστίν. | If it is human, it is an animal<br>If it is an animal, it is a substance<br>Hence, if it is human, it is an animal |
| (2 <sub>mode</sub> ) | εἰ A ἐστὶ, B ἐστίν,<br>εἰ B ἐστὶ, Γ ἐστίν,<br>εἰ ἄρα A ἐστὶ, Γ ἐστίν.                              | If it is A, it is B<br>If it is B, it is C<br>Therefore, if it is A, it is C.                                      |

This would be the normal way of straightening out Aristotle's genitive absolute in Greek. The *necessitas consequentiae* is however no longer

<sup>9</sup> Cf. Arist. *Top.* 112a17-19, ἄνθρωπον εἶναι for 'something is a man'.



stated.<sup>10</sup> As long as (2) is also still regarded as reformulable as (0), it must be possible to paraphrase its conditionals as universally quantified, now as

(C<sub>1</sub>') If anything is T<sub>A</sub>, it is T<sub>B</sub>.

This is again a natural reading, as there are again exactly two terms per conditional, and no explicit mention of a subject. From arguments of the form (2<sub>mode</sub>) a whole system of WHs was developed along the lines of the previous section. Since what was at issue is the relation *between terms*, for brevity, the arguments and their modes were also presented as

(2' <sub>ex</sub> )	εἰ ἄνθρωπος, ζῷον, εἰ ζῷον, οὐσία, εἰ ἄρα ἄνθρωπος, οὐσία.	If human being, animal If animal, substance Therefore, if human being, substance
(2' <sub>mode</sub> )	εἰ τὸ A, τὸ B, εἰ τὸ B, τὸ Γ, εἰ ἄρα τὸ A, τὸ Γ.	If A, B If B, C Therefore, if A, C

Again, understanding the premisses and conclusion as conditionals of type (C<sub>1</sub>') seems natural. However, once the standard formulations (2) and (2') are generally adopted, even though (C<sub>1</sub>) may be a natural reading of the conditionals taken in isolation (and is required for reformulation as (0)), there are several ways of understanding the whole arguments. They are bound up with different ways of relating the six – implicit – grammatical subjects in the argument to each other. In a complete WH the cross-referring can happen in three ways:

(A <sub>1</sub> )	(A <sub>1</sub> ')	(A <sub>1</sub> '')
If anything is T <sub>A</sub> , it is T <sub>B</sub>	If anything is T <sub>A</sub> , it is T <sub>B</sub>	If anything is T <sub>A</sub> , it is T <sub>B</sub>
If anything is T <sub>B</sub> , it is T <sub>C</sub>	If it is T <sub>B</sub> , it is T <sub>C</sub>	If it is T <sub>B</sub> , it is T <sub>C</sub>
Hence, if anything is T <sub>A</sub> , it is T <sub>C</sub>	Hence, if anything is it is T <sub>C</sub>	Hence, if it is T <sub>A</sub> , T <sub>A</sub> , it is T <sub>C</sub>

If one tried to construct three – very roughly – corresponding complex propositions in the style of modern predicate logic the following would perhaps come closest:

<sup>10</sup> If ἀνάγκη in *APr.* I 32 47a28-31 (= I<sub>ex</sub>) had the function of an Aristotelian necessity operator, the categorical propositions in the reformulation (= 0) were perhaps all necessary propositions. In that case the modal operators from (1) may have been deliberately dropped at some point, and the applicability of the system thus widened.

$$\begin{array}{ll}
 (T_1) & [\forall x [Fx \rightarrow Gx] \ \& \ \forall x [Gx \rightarrow Hx] ] \rightarrow \forall x [Fx \rightarrow Hx] \\
 (T_1') & [\forall x [ [Fx \rightarrow Gx] \ \& \ [Gx \rightarrow Hx] ] ] \rightarrow \forall x [Fx \rightarrow Hx] \\
 (T_1'') & \forall x [ [ [Fx \rightarrow Gx] \ \& \ [Gx \rightarrow Hx] ] \rightarrow [Fx \rightarrow Hx] ]
 \end{array}$$

But whereas the propositional formulae  $(T_1)$ ,  $(T_1')$ , and  $(T_1'')$  would be considered as tautologies and as equivalent to each other, there are some logically relevant differences between the three argument forms  $(A_1)$ ,  $(A_1')$ , and  $(A_1'')$ .  $(A_1)$  can be reformulated in *modus Barbara* (see above), transforming the conditionals one by one;  $(A_1')$ , and  $(A_1'')$  cannot.  $(A_1'')$  is in fact not an argument in a strict sense, since it does not have a detachable conclusion (premisses and conclusion being as it were in the scope of one universal quantifier) – although this fact would not be apparent in the Greek formulation, and hence would not necessarily have been recognized by the ancient logicians. And whereas  $(A_1)$  may have been used in order to establish a relation between terms  $(T_A, T_B)$  on the basis of other relations between terms  $(T_A, T_B$  and  $T_B, T_C)$ ,  $(A_1'')$  could also be understood as establishing some fact about a particular thing – only that it has been left indeterminate which thing it is. A change in the understanding of arguments of type (2) from  $(A_1)$  to  $(A_1'')$  may have led to a more significant change the understanding of the logical form of the WHs.

This change takes place at some point after the WHs are no longer regarded as needing reformulation as (0) in order to count as syllogisms, but are taken to be arguments that are valid because of their own specific form (and in the case of WHs of modes 1.1-8 as evidently so). It is perhaps triggered by bigger changes in the understanding of logic generally, its tasks and its justifications. Thus in Hellenistic philosophy it is no longer a fundamental task of syllogistic to provide necessary truths, derived from prior necessary truths, which involve generic terms only; this may be reflected in the fact that syllogisms which yield conclusions about individual things became the standard examples. Moreover, Platonists take Plato as highest authority in matters of logic, and seek to attest the use of all kinds of syllogisms by Plato in his dialogues. An interest in WH-type arguments that deal with individual things may have arisen either way. In any case, at some point the interpretation of arguments of the linguistic forms (2) and (2') changed, so that conditionals 'if (it is)  $T_A$ , (it is)  $T_B$ ' are now read as of the logical form:

$$(C_2) \quad \text{If } S \text{ is } T_A, S \text{ is } T_B. \quad (\text{with } S \text{ as singular term and logical subject})$$

and the whole arguments as of the logical form:



this is only a matter of switching perspective as to what for the validity of a WH are the logically relevant components of the conditionals. Take the above example (2/3<sub>ex</sub>). So far it has been interpreted as an argument of form (A<sub>2</sub>):

If the One is T<sub>A</sub>, it (i.e. the One) is T<sub>B</sub>.  
 If it (i.e. the One) is T<sub>B</sub>, it (i.e. the One) is T<sub>C</sub>.  
 Therefore, if the One is T<sub>A</sub>, it (i.e. the One) is T<sub>C</sub>.

Now consider first that the logicians of the time have available the concept of conditionals of the form (C<sub>3</sub>) 'If P, Q', and second that they accept as valid arguments of forms such as 'If P, Q; P; therefore Q'. Such logicians would presumably be tempted to read the form of the example (2/3<sub>ex</sub>), instead of as (A<sub>2</sub>), as (A<sub>3</sub>). But once arguments such as (2/3<sub>ex</sub>) have been accepted as valid *because* they have form (A<sub>3</sub>), of course also arguments which have *different* subject terms in antecedent and consequent of the conditionals can be allowed, e.g.

(3<sub>ex</sub>) If the sun is above the earth, it is day.  
 If it is day, it is light.  
 Therefore, if the sun is above the earth, it is light.

And at that point we have reached a full propositional-logical understanding of the WHs. This is the story of the historical development of WHs I suggest. There are from the point of view of logic two equally important steps in this development: one from an understanding of the conditionals in terms of universal quantification to an understanding of them in terms of designators (from (A<sub>1</sub>)-(A<sub>1</sub>'') to (A<sub>2</sub>), or *stage 1* to *stage 2*); the other from a term-logical perspective to a propositional-logical one (from (A<sub>2</sub>) to (A<sub>3</sub>), or *stage 2* to *stage 3*). In the following I hope to move from fiction to fact and to substantiate the story with our extant texts on WHs, at the same time providing more details about the respective theories at each stage.

### 3. *The evidence*

In order to keep the main part of the paper as uncluttered as possible with historical and philological interruptions, I here adduce separately a brief survey of the available evidence, together with some general remarks about the nature of the sources. Apart from one detailed work on hypothetical syllogisms (Boethius' *De hypotheticis syllogismis*, HS), our identifiable sources are all either commentaries on Aristotle's *Prior Analytics* (by Alexander, Philoponus, [Ammonius], and various scholia), or belong

to a tradition of very brief outlines of the entire Peripatetic-Platonist syllogistic (e.g. in Alcinous, *Didasc.* ch. 6). The two groups overlap. This origin of the passages on WHs already hints at the fact that the WHs and their development are a specifically Peripatetic-Platonist affair. There is no evidence that the Stoics ever distinguished such a kind of argument.<sup>11</sup> WHs are absent in all our main sources for Stoic logic, such as Sextus Empiricus, Diogenes Laertius, Gellius or even Galen.

Theophrastus is the first philosopher we know to have dealt systematically with WHs. We do not have any direct evidence for his theory, but are dependent on reports of much later authors. Alexander's Commentary on Aristotle's *Prior Analytics* is the earliest extant work in which WHs are mentioned. Alexander wrote almost 500 years after Theophrastus. Much happened in logic in the meantime, although we have little first-hand evidence. Stoic propositional logic, an enormous, elaborate, system, had been developed in the 3rd and 2nd centuries BC. It had been studied by Peripatetics at least from the first century BC onwards. As a result, post-Theophrastean interpretations of early Peripatetic logic and later developments within Peripatetic logic are coloured by their authors' knowledge of propositional logic. All this can be clearly witnessed in the development of *modus ponens* type arguments. However, the WHs are a special case, since there was no Stoic model for them, and hence the influence of Stoic propositional logic on the WHs is more subtle and less tangible than in the case of other types of arguments.

Using the Aristotle commentators as evidence for the logic of their time or for the history of logic up to any particular commentator is generally a difficult enterprise. Alexander, and presumably most of the commentators, were perfectly familiar with contemporary logic. They were all acquainted at least with the elements of Stoic logic. (Alexander has detailed first-hand knowledge of it.) They all report views of logicians that wrote long before their time (such as Theophrastus, Eudemus, Chrysippus, Galen, Alexander), and we may conjecture that the works of some 'intermediate Peripatetics', such as Boethus and Aristo, and Alexander's teacher Herminus (who wrote on the *Prior Analytics*), also left their traces. We can thus assume neither that everything we find in a commentator's text which is not marked as someone else's view is contemporary logic, or the commentator's view; nor that what is marked as some earlier philoso-

<sup>11</sup> Cf. my 'The Stoics on hypotheses and hypothetical arguments', *Phronesis* 1997, 299-312.

pher's view is uncontaminated by later developments. Commentators are intrinsically conservative, and seldom flag what is their own view, or their innovation. They tend to adopt, adapt, improve, expand, shorten, summarize, juxtapose, criticise, misinterpret, or misunderstand what they find in their predecessors. Thorough reading of the commentaries strongly suggests that overall consistency or even homogeneity of view in one book was not sought, or if sought, was certainly not achieved. Commentators are conservative also in the sense that their main goal is to present and explicate the views of Aristotle to their contemporaries. Hence Aristotelian terminology and approach may prevail even where modern terminology and later theories on the same topic exist and are known to the commentator. As a result of all this we frequently find in different parts of a commentary, or even side by side, bits of textual exegesis from different epochs, without any mention of the fact. This becomes obvious for example in the many inconsistencies, in the frequent use of parallel sets of technical terms, lists of alternative explanations of a passage or phrase, simply connected by 'or else . . .' or 'perhaps rather . . .', often without the commentator taking a stand himself. Occasionally we find two or more interpretations of an Aristotle passage, all plainly inadequate. Another peculiar feature in the commentaries (which is the upshot of the commentators' acquaintance with contemporary and historical views, and the fact that it is Aristotle whom they explain) is that we may find two or even more nomenclatures (Aristotelian, early Peripatetic, Stoic, later Peripatetic) blended together in one and the same passage but still describing conflicting or ill-matching bits of theories.

All this makes it extremely difficult to date individual comments or passages with any reasonable precision (except in some cases where parallel information from other datable authors is available). Temporal ranking of passages is sometimes possible, as well as separating out different strands and traditions of influences, and establishing dependencies of one commentator on another, or on a shared source. The fact remains that most commentators present some sort of historical mosaic, where the origin of many individual elements can be ascertained if at all only approximately. For the history of the WHs this means more specifically that different passages from the same commentator may belong to different developmental stages. We cannot simply take a passage from one part of the book in order to sort out one from another part, nor can we assume that a particular passage actually presents *the* view of the commentator on WHs; and generally, the historical claims of the commentators have to be viewed with a critical eye.

Here is a list of the surviving evidence on WHs, in what I consider their rough historical order. My account of their development is based on this classification. The ordering here can be no more than approximate, because one and the same passage often conflates material from different periods. In the list I indicate material of earlier origin in a passage as ‘a source of x’. (The developmental stages are added in brackets.)

- Arist. *APr.* I 32 47a10-40, in particular 28-30 (*stage 0*)
- a source of Alex. *APr.* 347.15-348.23 (together with Alex. *APr.* 22.25-30); a source of Alex. *APr.* 326.8-14, 20-2, 328.1-5; a source of Philop. *APr.* 302.14-15; a source of Anon. Latin Scholium in *APr.* I 29 (Minio-Paluello); a source of [Amm] *APr.* 67.24-30 (*stage 1*)
- Boethius *HS* 2.9.1-3.6.4; a source of Anon. Greek Scholium in *Org.* (Waitz); (*stage 1a*)<sup>12</sup>
- a source of Alex. *APr.* 326.20-328.6; Philop. *APr.* 302.6-23; [Amm] *APr.* 67.24-30, Anon. Latin Scholium in *APr.* I 29 (*stage 1b*)
- Alcin. *Didasc.* 158.23-7, 159.7-24 (*stage 2*)
- Alex. *APr.* 374.21-35 (*stages 1/2/3*)
- Alex. *APr.* 330.28-30, 265.13-19, 348.9-19; 390.16-19 (*stages 1/2/3*)
- Philop. *APr.* 413.8-24; Boethius *HS* 1.6.2-3, 1.8.6-9.1; Anon. Greek Scholium in *Org.* (Waitz) (*stages 1/2/3*)
- Philop. *APr.* 243.11-36; Scholium in Amm. *APr.* XI.1-6; (*stage 3*)<sup>13</sup>

I now begin with the story, telling it in what I consider the chronological order of events.

#### 4. Aristotle expanded: Theophrastus and the first generation of Peripatetics

There is a passage in Alexander which is evidence that *someone* in antiquity (presumably before Alexander) thought that Aristotle’s second example in *APr* I 32 was a WH.<sup>14</sup> Alexander reports three alternative

<sup>12</sup> Among the medieval sources, Garlandus Compotista *Dialectica* 6; Abelard *Dialectica*; Walter Burleigh *De Puritate Artis Logicae, Tractatus Longior*; Holobolos trl. of Boethius’ *HS* all follow Boethius, and thus preserve *stage 1* (see K. Ierodiakonou, ‘The Hypothetical Syllogisms in the Greek and Latin Medieval Traditions’, *Cahiers de L’institut du Moyen-age Grec et Latin* 66, Copenhagen 1996, 96-116).

<sup>13</sup> A number of Byzantine texts, in the tradition of Philop. *APr.* 243.11-36, are further evidence for *stage 3*: Anon. *Log.&Quadr.* 38 30.16-32.7 Heiberg; Psellus, Scholia in *APr.*, ff.120v-121r (Vat. gr. 209) = ff.158v-159r (Vat. gr. 243); Blemmydes Ἐπιτομή Λογικῆς 973-980; Holobolos’ Scholia on Boethius’ *HS* (cf. D.Z. Nikitas, 1982); John Pediasimus in *APr.* 43.31-45.8; two anonymi (see Ierodiakonou, ‘Medieval Traditions’).

<sup>14</sup> The passage implies that Alexander thought that Aristotle himself knew and dis-

explanations of this passage (Alex. *APr.* 347.15-348.23); the second connects Aristotle's example (quoted in section 2) with the wholly hypothetical arguments:

Evidently he means by these <sentences> also that the so-called 'through three' argument has its consequence <that it being the first, it is the third> from necessity, but not syllogistically, and that neither the 'through three' argument nor generally the so-called wholly hypothetical is a syllogism. This may perhaps rather be the reason why in the case of the example 'again, if it being a human being it is necessary for it to be an animal' he said 'but this has not yet been syllogized – for the premisses are not related in the way we have said', for they were taken neither probatively<sup>15</sup> nor universally. For there would be a syllogism if they were taken in this way: 'every human being is an animal, every animal a substance'. But taken in the above way, that which results is necessary but it does not result syllogistically, since every syllogism sets out to show that something belongs or does not belong.<sup>16</sup> (Alex. *APr.* 348.9-19)

Thus Aristotle's 'it being a human being it is necessary for it to be an animal' (ἀνθρώπου ὄντος ἀνάγκη ζῶον εἶναι) was considered as the premiss of a WH, and hence as an alternative way of saying 'if it is a human being, it is an animal' (εἰ ἄνθρωπος ἐστὶ, ζῶον ἐστὶ).<sup>17</sup> An argument with this conditional as first premiss and 'if it is an animal, it is a substance' as second premiss is regarded as becoming a proper syllogism if reformulated as 'every human being is an animal, every animal a substance, etc.' (πᾶς ἄνθρωπος ζῶον, etc.).

This is supported by a comment by Alexander on the τῶ ταῦτα εἶναι clause of Aristotle's definition of the syllogism (Arist. *APr.* 24b20), in

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cussed WHs as a particular type of arguments (and so do some other passages, e.g. Philop. *APr.* 413.8-24), but this may be little more than wishful thinking.

<sup>15</sup> The use of δεικτικῶς instead of a form of κατηγορικῶς, suggests an early origin for this passage.

<sup>16</sup> δηλὸς δὲ ἐστὶ διὰ τούτων καὶ τὸν διὰ τριῶν λεγόμενον λόγον ἐξ ἀνάγκης μὲν λέγων ἔχειν τὸ ἐπόμενον τὸ τοῦ πρώτου ὄντος τὸ τρίτον εἶναι, οὐ μὴν συλλογιστικῶς, οὐδ' εἶναι τὸν διὰ τριῶν λόγον συλλογισμὸν οὐδὲ ὅλως τὸν δι' ὅλων ὑποθετικὸν λεγόμενον. διὸ καὶ μᾶλλον δύναται ἐπὶ τοῦ παραδείγματος τοῦ "πάλιν εἰ ἄνθρωπος ὄντος ἀνάγκη ζῶον εἶναι" εἰρηκέναι τὸ "ἀλλ' οὕτω συλλελογίσται· οὐ γὰρ ἔχουσιν αἱ προτάσεις, ὡς εἴπομεν", ὅτι μὴ δεικτικῶς μηδὲ καθόλου ἐλήφθησαν. ἔσται γὰρ συλλογισμὸς, ἂν οὕτω ληφθῶσι "πᾶς ἄνθρωπος ζῶον, πᾶν ζῶον οὐσία". Ἐκείνως δὲ ληφθέντων ἀναγκαῖον μὲν τὸ συμβαῖνον οὐ μὴν συλλογιστικῶς, ἐπεὶ πᾶς συλλογισμὸς κείται ὑπάρχειν ἢ μὴ ὑπάρχειν δεικνύναι.

<sup>17</sup> Philop. *APr.* 413.8-24 (discussed in section 8), too, calls an argument with conditionals in genitive absolute formulation, as in Arist. *APr.* I 32, a syllogism 'through three'.



which Alexander connects the clause with *APr* I 32: ‘and <it is> also because of arguments which do not have canonical premisses and which must be transformed in order for there to be a syllogism,’ and after having adduced Aristotle’s first illustrative argument continues: ‘The conclusion does not come about “inasmuch as the premisses are the case”; rather, the premisses must be transformed.’ (Alex. *APr.* 22.25-30).

In Aristotle’s second example – we can assume – ‘it being a human being it is necessary for it to be an animal’ needs to be *transformed* to ‘every human being is an animal’ (cf. Alex. *APr.* 347.10-14). This transformation requires that the terms of the conditionals be made determinate. In the first premiss of the WH ‘human being’ is indeterminate as to its quantity. In the transformed version ‘every human being is an animal’ it is determinate, namely universal – and only when understood in this way is the original argument a syllogism. The WHs of mode 1.1 are thus not assumed to be proper syllogisms as they stand, since they do not have the right form (parallel to what Aristotle says in *APr.* I 32). They become proper syllogisms only when their premisses and conclusion are suitably transformed (μεταληφθῆναι). So far the commentary on Arist. *APr.* I 32.

I now turn to Theophrastus’ theory of WHs. I imagine Theophrastus’ logical activities as mainly systematizing and expanding on Aristotle’s logic, always remaining close to his teacher’s theory. He studied the *Prior Analytics* carefully, and elaborated on many of those types of arguments which Aristotle mentions or hints at, but which he does not discuss in the *Organon*: various kinds of syllogisms from a hypothesis (some of which became later the ‘mixed’ hypothetical syllogisms), prosleptic syllogisms and WHs.

We know very little about Theophrastus’ position on WHs. Our direct evidence comes from a couple of passages in Alexander (Alex. *APr.* 326.8-12, 20-2, 328.2-5), from Philoponus (Philop. *APr.* 302.6-19), and from a Latin *Scholium in APr.* (Arist. Lat. vol. 3.4 pp. 320.7-16 Minio-Paluello), henceforth the ‘Latin Scholium’.<sup>18</sup> In all these texts later views about the WHs are known to the authors and where exactly Theophrastus’ view ends and the more modern view begins is often not clear. Philoponus writes:

Theophrastus called ‘wholly hypothetical’ those <sylogisms> which take both the premisses and the conclusion from a hypothesis.<sup>19</sup> (Philop. *APr.* 302.9-10)

<sup>18</sup> This scholium belongs to the group of Latin Scholia in *APr.* which J. Shiel believes to be translations by Boethius of the (lost) commentary on *APr.* I by [Philoponus]; J. Shiel, *Vivarium* 1982, 188-141.

<sup>19</sup> δι’ ὅλου δὲ ὑποθετικούς ἐκάλει ὁ Θεόφραστος τοὺς καὶ τὰς προτάσεις καὶ τὸ

And Alexander reports:

The wholly hypotheticals, which Theophrastus calls '<sylogisms> by analogy', such as are those called 'through three'<sup>20</sup> (Alex. *APr.* 326.8-9)

and just before he presents his own view on the WHs:

But the wholly hypotheticals are reduced to the three above-mentioned figures <i.e. of categorical syllogisms> in a different manner, as Theophrastus has shown in the first book of his *Prior Analytics*.<sup>21</sup> (*APr.* 326.20-2)

and immediately after he has presented his own view on the figures of the WHs:

However Theophrastus in the first book of his *Analytics* says that in the case of the wholly hypotheticals the second figure is the one in which the premisses begin from the same <term> and end in different ones, whereas the third is the one in which the premisses begin from different <terms> but end in the same. But we set them out the other way around.<sup>22</sup> (Alex. *APr.* 328.2-5)

From these passages we can infer that Theophrastus acknowledged those arguments which were later called 'wholly hypothetical' as a special kind of syllogisms, and that he discussed them, their figures and their reduction, in his *Prior Analytics*. We have no examples for Theophrastean WHs or conditionals. But since Alexander in his report on WHs (below, section 6) mentions no differences between his and Theophrastus' theory of WHs besides the order of the figures, it appears that there were no obvious discrepancies in linguistic appearance between Theophrastus' WHs and those Alexander presents. Thus I presume that his (basic) WHs consist of two hypothetical premisses and a hypothetical conclusion,<sup>23</sup> that they have three terms in their premiss-pairings, of which exactly one is

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συμπέρασμα ἐξ ὑποθέσεως λαμβάνοντας, see also Philop. *APr.* 302.6-7 and cf. the Latin Scholium 'Dicit autem (i.e. Theophrastus) "per totum hypotheticos" qui et propositiones omnes et conclusionem habent hypotheticam . . .'

<sup>20</sup> οἱ δι' ὄλων ὑποθετικοί, οὓς Θεόφραστος κατὰ ἀναλογίαν λέγει, οἰοί εἰσιν οἱ διὰ τριῶν λεγόμενοι. That Theophrastus called them 'sylogisms' is clear from the context, e.g. Alex. *APr.* 326.12-14, quoted below.

<sup>21</sup> ἀνάγονται μέντοι καὶ οἱ δι' ὄλων ὑποθετικοί εἰς τὰ τρία τὰ προειρημένα σχήματα ἄλλω τρόπῳ, ὡς καὶ Θεόφραστος δέδειχεν ἐν τῷ πρώτῳ τῶν Προτέρων ἀναλυτικῶν.

<sup>22</sup> Θεόφραστος μέντοι ἐν τῷ προτέρῳ τῶν Ἀναλυτικῶν δεύτερον σχῆμα λέγει ἐν τοῖς δι' ὄλων ὑποθετικοῖς εἶναι, ἐν ᾧ ἀρχόμεναι ἀπὸ τοῦ αὐτοῦ αἱ προτάσεις λήγουσιν εἰς ἕτερα, τρίτον δέ, ἐν ᾧ ἀπὸ διαφόρων ἀρχόμεναι λήγουσιν εἰς ταῦτόν. ἀνάπαλιν δ' ἡμεῖς ἐξεθέμεθα.

<sup>23</sup> This is also suggested by Philop. *APr.* 302.9-10 and the Latin Scholium (quoted above).

shared by the premisses, and that the linguistic form e.g. of arguments in mode 1.1 was something like ‘If <it is>  $T_A$ , <it is>  $T_B$ ; if <it is>  $T_B$ , <it is>  $T_C$ ; therefore if <it is>  $T_A$ , <it is>  $T_C$ .’ I assume that Theophrastus considered his WHs as term-logical, since (i) nothing in our evidence speaks against this assumption, (ii) the theories that developed from his were term-logical (see below), and (iii) his claim of the reducibility of the WHs to categorical syllogisms makes a lot more sense that way (see below). It is possible that Theophrastus developed his theory from the above-quoted Aristotle passage, since the passages that include reports of Theophrastus’ views on WHs, and which evolved from his theory, use this very example, if in conditional formulation (Alex. *APr.* 326.24-5, the Latin Scholium).<sup>24</sup>

From Alexander’s third passage (Alex. *APr.* 328.2-5, see above) we can further infer that Theophrastus distinguished three figures of WHs, and what they were. Since Alexander mentions no difference between Theophrastus’ and his own view about the first figure, we can assume that (at least on the surface) there was none. The order of the terms in Theophrastus’ first figure premiss pairings should thus be **AB, BC**. His three figures then have the following order of terms in their premiss pairings: (1) **AB, BC**; (2) **BA, BC**; (3) **AB, CB**. The figures thus display a superficial structural similarity to those of Aristotle’s categorical syllogisms based on the position of the shared or middle term: Aristotle formulated his categorical propositions characteristically with the predicate term before the subject term, e.g. ‘A does not belong to some B’.<sup>25</sup> If we restrict ourselves to the order of the terms in Aristotle’s syllogisms, we can schematically present the premiss pairings of the three categorical figures as (1) **CB, BA**; (2) **BC, BA**; (3) **CB, AB**.<sup>26</sup> The analogy between the figures of the WHs and those of the categorical syllogisms is thus plain.

This explanation tallies with what Philoponus suggests about Theophrastus’ first figure:

For when we say ‘if A, also B, if B, also C’ and conclude ‘hence if A, also C’, then A is analogous to the minor term, i.e. the subject, B to the middle, which is predicated of A and subject of C, which is analogous to the major term; hence this will be the first figure.<sup>27</sup> (Philop. *APr.* 302.16-19)

<sup>24</sup> It seems that Theophrastus proceeded in a similar way with the so called prolepetic premisses and syllogisms, which are based on Aristotle *APr.* 49b14-33, esp. 49b27-30; 58a29-32, b8-10; 58b37-8, 59a28-9.

<sup>25</sup> e.g. Arist. *APr.* 25a17-18, 25b37-9, 26a23. This form of wording was adopted by Theophrastus, cf. Alex. *APr.* 31.4-10, Philop. *APr.* 48.11-18.

<sup>26</sup> Cf. e.g. Arist. *APr.* I 32 47b1-7.

<sup>27</sup> ὅταν γὰρ εἴπωμεν “εἰ τὸ Α, καὶ τὸ Β, εἰ τὸ Β, καὶ τὸ Γ” εἶτα συμπεράνωμεν “εἰ τὸ

Philoponus is slightly sloppy here. To bring out this analogy to the first figure categorical syllogisms ('C belongs to . . . B; B belongs to . . . A; C belongs to . . . A') more clearly, I add indices 'WH' and 'CS' to indicate to which type of syllogism Philoponus each time refers:

. . . then  $A_{WH}$  is analogous to the minor term $_{CS}$ , i.e. the subject $_{CS}$ ,  $B_{WH}$  to the middle $_{CS}$ , which is predicated of  $A_{CS}$  and subject of  $C_{CS}$ , <and  $C_{WH}$ > is analogous to the major term $_{CS}$ ; hence this will be the first figure.

Thus the WH consequent term is analogous to the major term, the WH antecedent term to the minor term in the (conclusions of the) categorical syllogisms. If we add this analogy to Alexander's statement about Theophrastus' 2nd and 3rd WH-figures, we receive the following extended picture of the analogy between the figures (with the indexes 'M' and 'm' to indicate the major and minor terms respectively, B for the middle term, and 'of' as short for 'is predicated of'):

First Figure		Second Figure		Third Figure	
$C_M$ of B	If A, B	B of $C_M$	If B, A	$C_M$ of B	If A, B
$B$ of $A_m$	If B, C	$B$ of $A_m$	If B, C	$A_m$ of B	If C, B
$C_M$ of $A_m$	If A, C	$C_M$ of $A_m$	If A, C	$C_M$ of $A_m$	If A, C

The most vexing question about Theophrastus' WHs is how he envisaged them to be reduced to categorical syllogisms. Alexander's report (quoted above), that Theophrastus thought that they 'are reduced to the three above-mentioned figures <i.e. of categorical syllogisms> in a different manner' is supported by several other passages. Philoponus writes:

Theophrastus says that these <i.e. the wholly hypotheticals, cf. Philop. *APr.* 302.13>, too, can be reduced to the three figures.<sup>28</sup> (Philop. *APr.* 302.14-15)

and the Latin Scholium has:

A ἄρα, καὶ τὸ Γ", ἀναλογεῖ τὸ μὲν Α ἐλάττωνι ὄρω καὶ ὑποκειμένω, τὸ δὲ Β μέσῳ κατηγορουμένω μὲν τοῦ Α ὑποκειμένω δὲ τῷ Γ, ὅπερ ἀναλογεῖ μείζονι ὄρω ὥστε οὕτως ἔσται τὸ πρῶτον σχῆμα. This analogy with minor and major term of the categorical syllogisms is absent in Alexander, from whom Philoponus seems to have drawn for some of what he says about WHs; it may thus be Theophrastus' own. (In any case we would expect *this* correspondence of terms for WH-mode 1.1 and *modus Barbara*.) Note however that immediately afterwards Philoponus wrongly attributes Alexander's order of the 2nd and 3rd figures to Theophrastus, and that the analogy may originate from a later time.

<sup>28</sup> ἔλεγε δὲ ὁ Θεόφραστος ὅτι δύνανται καὶ οὗτοι ὑπὸ τὰ τρία σχήματα ἀνάγεσθαι. 'Reduced to the three figures' means that each such syllogism reduces to at least one of these figures – as is clear from the context in Aristotle *APr.* I 29, i.e. the passage on which all the texts on reduction are comments (in particular *APr.* 45b38-41).

Here Theophrastus tries to argue otherwise, claiming that the wholly hypothetical syllogisms do not require this manner <of reduction>.<sup>29</sup> (T 113D)

There is moreover a parallel passage in [Amm.] *APr.* 67.24-31:

... not only the hypothetical syllogisms that are mixed from a categorical and a hypothetical syllogism are reduced to the three figures by means of the categorical <syllogism>, but also the wholly hypothetical ones (if human, animal; if animal, animate; hence if human, animate) and the prosleptic syllogisms are reduced to the three figures; ... but the wholly hypothetical syllogisms and the prosleptic syllogisms do not belong to the three figures by means of something else, as all the mixed hypotheticals, but because of their own structure.<sup>30</sup>

The context of the passages makes clear that the way in which the WHs are *not* reduced is that in which the ‘other syllogisms from a hypothesis’ are reduced (Arist. *APr.* I 42), i.e. those which have a categorical second premiss. The latter were traditionally reduced by selection (ἐκλογή), i.e. by proving the categorical premiss through a categorical syllogism by selecting a middle term for this premiss.<sup>31</sup> The syllogism from a hypothesis has then – indirectly – been proved by a categorical syllogism and is *thus* reduced to one of the categorical figures. This method does not work for the WHs, because they have no categorical premiss and a categorical syllogism can have no hypothetical conclusion.<sup>32</sup> It is less easy to say what the reduction of WHs to categorical syllogisms *did* consist in. The [Ammonius] passage suggests that it works not by the addition of further premisses, but by means of the structure of the WHs, presumably by transforming one or more of its premisses.

<sup>29</sup> ‘hic Theophrastus conatur redarguere, per totum hypotheticos syllogismos inquiens non indigere huiusmodi via.’ (T 113D)

<sup>30</sup> ... οὐ μόνον οἱ ὑποθετικοὶ οἱ μικτοὶ ἐκ κατηγορικοῦ συλλογισμοῦ καὶ ὑποθετικοῦ εἰς τὰ τρία σχήματα ἀνάγονται διὰ μέσου τοῦ κατηγορικοῦ ἀλλὰ καὶ οἱ δι’ ὅλου ὑποθετικοὶ· εἰ ἄνθρωπος, καὶ ζῶν· εἰ ζῶν, ἔμψυχον· εἰ ἄνθρωπος ἄρα ἔμψυχον· καὶ οἱ κατὰ πρόσληψιν δὲ συλλογισμοὶ εἰς τὰ τρία σχήματα ἀνάγονται... καὶ οὐδέ διὰ μέσου ἄλλου εἰς τὰ τρία σχήματα τελοῦσι οἱ δι’ ὅλου ὑποθετικοὶ καὶ οἱ κατὰ πρόσληψιν, καθάπερ πάντες οἱ μικτοὶ ὑποθετικοί, ἀλλὰ διὰ τὴν πλοκὴν τὴν οἰκείαν. (The same example occurs in Boethius *HS* 2.9.6.) I assume that this passage reports a thought that ultimately goes back to Theophrastus, since the prosleptic syllogisms were as far as we know Theophrastus’ invention (passages on prosleptic syllogisms are collected as T 110 A-D in Fortenbaugh), and Theophrastus is the only philosopher who is said to have held that the WHs reduce differently from the later so called mixed hypothetical syllogisms.

<sup>31</sup> Cf. Barnes, T&S, 386 n. 3, 387 n. 1.

<sup>32</sup> So pointed out rightly by the author of the Latin Scholium.

We cannot rule out entirely that Theophrastus' reduction of the WHs to categorical syllogisms consisted in nothing but pointing out the superficial structural similarity between them which I outlined above. I prefer to think that he thought that this analogy provided the basis for a reduction of a logically more significant kind. If Theophrastus considered Arist. *APr.* I 32 as presenting the core idea of such a reduction, which would be unfolded along the lines of the above-quoted commentary on the passage (Alex. *APr.* 348), then we can assume a closer relation at least between *modus Barbara* and WH-mode 1.1.<sup>33</sup>

A WH would reduce to a categorical syllogism in this stronger sense, if its validity was derived from the latter; that is, if one can logically transform it into a categorical syllogism. Or at least it must be possible to understand it in such a way that it logically follows from the categorical syllogism. This can be done in any straightforward sense only if the arguments are understood as term-logical and as involving some sort of universal quantification; e.g. a WH of mode 1.1 must be taken roughly along the lines of

- (A<sub>1</sub>) If anything is T<sub>A</sub>, it is T<sub>B</sub>; if anything is T<sub>B</sub>, it is T<sub>C</sub>; therefore if anything is T<sub>A</sub>, it is T<sub>C</sub>.

We should hence understand the Theophrastean conditionals accordingly, and as logically transformable to categorical propositions of the form 'Every T<sub>A</sub> is T<sub>B</sub>'.

In a related context it has been objected to such a logical transformation that a proposition 'If it (i.e. anything) is T<sub>A</sub>, it is T<sub>B</sub>' does not imply the existence of anything that is T<sub>A</sub>, whereas Aristotle's 'Every T<sub>A</sub> is T<sub>B</sub>' does.<sup>34</sup> Now, it is true that an Aristotelian universal premiss 'Every T<sub>A</sub> is T<sub>B</sub>' implies the existence of things that are T<sub>A</sub> and things that are T<sub>B</sub>, and also that a universally quantified conditional such as  $\forall x ((x \text{ is } T_A) \rightarrow (x \text{ is } T_B))$  does not

<sup>33</sup> A much later text (Anon. *Log.&Quadr.* 38 30.16-20) claims that WHs of mode 1.1 are distinguished from categorical syllogisms in *modus Barbara* only in that they conclude on the basis of hypotheses.

<sup>34</sup> Barnes, T&S pp. 316-17. Barnes points out that Boethius in *HS* 1.2.2 makes a remark to the effect of non-equivalence of these kinds of sentences. Boethius clearly wants to argue that there is a difference between 'man is an animal' and 'If it is a man, it is an animal', but I am unsure about the exact point Boethius intends to make; and even if the point is the one Barnes assumes, this does not say anything about *Theophrastus'* understanding of WHs some 1000 years earlier. (Boethius does not use the Aristotelian way of expressing universal sentences (*Every A is B*), but the Platonist way (*A is B*), which becomes common only in the 2nd century AD).

imply the existence of either  $T_{AS}$  or  $T_{BS}$ , or of any thing at all. However, I doubt that this affects Theophrastus' conditionals in the WHs. For they are not defined as the universal closure of an open conditional. They were most probably phrased as

εἰ ἄνθρωπός <έστι>, ζῷόν <έστι> (if <it is a> human being, then <it is an> animal)

and this kind of sentence could have been understood in all manner of ways; in particular, I suggest, as:

If it (i.e. anything) is one of *the* human beings, it is also one of *the* animals.<sup>35</sup>

Thus understood, a hypothetical proposition 'If it is  $T_A$ , it is  $T_B$ ' is logically equivalent to a categorical proposition 'Every  $T_A$  is  $T_B$ '. Both can be paraphrased as 'If anything is  $T_A$ , it is  $T_B$ ;' and there are things that are  $T_A$  and things that are  $T_B$ .' I conjecture then – on the grounds that it is compatible with all the evidence and gives a plausible story – that Theophrastus understood the conditionals in his WHs in this way. A WH of mode 1.1 can then indeed be reduced to a *modus Barbara* syllogism.

It still needs to be determined how the remaining kinds of WHs reduce to categorical syllogisms. We do not know how many kinds Theophrastus distinguished. But since his theory includes three *figures* of WHs, we can assume that it also provides for several *modes* in each figure. There are two very different ways in which one can imagine the reduction of the arguments in the remaining modes. One is very neat, the other very muddled. Neither is wholly satisfactory, and the following remarks are conjectural only.

The neat way works on the assumption that all WH modes can either be 'reduced' to *modus Barbara* or to other WH modes that can be thus 'reduced'. The reduction of WHs to WHs can be conceived of as set out in section 1 above. With a conversion rule in place, arguments in modes 1.9-3.16 are reduced by conversion to arguments in modes 1.1-8. Arguments in modes 1.1-8 are reduced to *modus Barbara*. For such reductions to be possible, both conversion and the negative components of the conditionals need to be understood in a particular way.

<sup>35</sup> No examples with terms that denote the empty class have been transmitted; hence we have no convenient test case. But the suggested reading finds some support in the fact that Philoponus, when giving the conclusion in the scheme for the third figure (in Philoponus' counting) writes εἰ μὴ τὸ A ἄρα, οὐδὲ τὶ τῶν Γ, 'hence if not A, one of the Cs' (Philop. *APr.* 302.22-23). τὸ A is here understood as equivalent in form to τὶ τῶν Γ.

The negation particles in the negative components of the conditionals have to negate *terms*, not whole propositions (as Stoic negation does) or whole predicates (i.e. term plus copula). Historically this poses no problems. Aristotle discussed the negation of terms, and so did Theophrastus.<sup>36</sup> For example, 'Εἰ οὐκ ἄνθρωπος <ἐστι>, οὐ ζῷον <ἐστι>' needs to be understood as 'if it is a not-human being, it is a not-animal'. Thus the WH of type 1.8 'if not-animal, not-human; if not-substance, not-animal; therefore if not-substance, not-human' reduces to the *modus Barbara* syllogism 'all not-substances are not-animals; all not-animals are not-humans; therefore all not-substances are not-humans.'

Similarly, the conversion (ἀντιστροφή) of the conditionals (which is required to get WHs of modes 1.9-3.16 down to WHs of modes 1.1-8) cannot have been contraposition of whole component propositions, since there are no independent component propositions. Conversion cannot be a change from 'If P, Q' to 'If not Q, not P', since the conditionals do not have the logical form 'If P, Q'. The best way to describe the change from 'if anything is T<sub>A</sub>, it is T<sub>B</sub>' to 'if anything is not T<sub>B</sub>, it is not T<sub>A</sub>' would presumably be by saying that the *terms* have been exchanged (ἀναστρέφειν, ἀντιστρέφειν), and then their oppositions (ἀντίθεσις) have been taken.<sup>37</sup> The understanding of the negations in the conversions is determined by that of the negatives in the WHs. For the negation in the conversion must be of the same kind as the negation by which arguments, e.g. in modes 1.1 to 1.8, are distinguished from each other, i.e. term negations. Thus 'if it is a cat it is an animal' converts into 'if it is a not-animal it is a not-cat'.<sup>38</sup>

<sup>36</sup> Cf. Arist. *APr.* I 46, Int. 20a20-3, *Top.* 113b15-26. See also Barnes T&S, 314 and 314 n.2.

<sup>37</sup> This assumption is in harmony with Galen, *Inst.Log.* 6.4, who, when introducing the conversion (ἀντιστροφή) of *complex propositions*, says that an exchange (ἀναστρόφη) of the *terms* is required, together with an opposition (ἀντίθεσις) of the *terms*. His example is εἰ ἡμέρα ἐστί, φῶς ἐστίν, which, with exchanged terms, becomes εἰ φῶς ἐστίν, ἡμέρα ἐστίν; if in addition the oppositions were taken one would get εἰ οὐ (μὴ) φῶς ἐστίν, οὐχ ἡμέρα ἐστίν (the text is corrupt and no example survived here). From Galen's definition to contraposition it is only one step: note the chameleon example he uses: it is of Stoic origin, and by them interpreted to be of the form 'if P, Q', but in its linguistic structure it is identical with εἰ ἄνθρωπος ἐστίν, ζῷον ἐστίν, and thus invites term-logical interpretation (cf. also below section 8). It is the Stoics who have an ἀντιστροφή of propositions that equals contraposition, cf. Diog.Laert. 7.76, and my *Die stoische Modallogik*, Würzburg 1986, 113.

<sup>38</sup> All this is in line with what Aristotle says in the *Topics*, which Theophrastus knew well. There we have the *topos* that if one term follows another, then the



The muddled way of reduction works on the assumption that some WH modes reduce directly to categorical modes other than *Barbara*. For instance mode 2.5 could be reduced to *modus Cesares*. This goes as follows: conditionals of the form ‘If <it is> A, <it is> B’ are understood as ‘If it is one of the As, it is one of the Bs’ and interpreted as equivalent to categorical propositions ‘Every A is B’. Conditionals of the form ‘If <it is> A, <it is> not B’ are understood as ‘If it is one of the As, it is not one of the Bs’ and interpreted as equivalent to categorical propositions ‘No A is B’.<sup>39</sup> A WH of mode 2.5 (If A, B; if C, not B; therefore if A, not C) is then ‘reduced’, conditional by conditional, to *modus Cesares* (Every A is B; no C is B; therefore no A is C), based on the assumed equivalences. This kind of ‘reduction’, however, has a very limited application. Conditionals of the form ‘If <it is> not one of the As, <it is> not one of the Bs’ would presumably also be equivalent to ‘Every A is B’; however, it is hard to say to what sort of categorical proposition conditionals of the form ‘If <it is> not one of the As, <it is> one of the Bs’ should be equivalent. Still this muddled kind of reduction has the advantage of being based on the structure (πλοκή) of the arguments themselves, as Theophrastus seems to have claimed they would (see above).

It remains to ask why Theophrastus described the WHs as being ‘syllogisms by analogy’. Alexander suggests: ‘Theophrastus calls them “by analogy”, since the premisses are analogous, and the conclusion is analogous to the premisses; for in all of them there is similarity’<sup>40</sup> (Alex. *APr.* 326.10-12). This reason as it stands can hardly have been Theophrastus’.<sup>41</sup> It holds e.g. equally of *modus Barbara*. Perhaps we have a distorting

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negation (ἀπόφασις, a type of ἀντίθεσις) of the second follows the negation of the first, illustrated by the example that if ‘animal’ follows ‘human’, then ‘not-human’ follows ‘not-animal’. The conversion rule here plainly concerns terms; and the negations (ἀποφάσεις) are negations of *terms*. (Arist. *Top.* 113b15-27 αἱ ἀντιθέσεις τέτταρες . . . ἐπὶ . . . τῶν ἀντιφάσεων ἀνάπαλιν ἐκ τῆς ἀκολουθήσεως . . . οἷον εἰ ὁ ἄνθρωπος ζῶον, τὸ μὴ ζῶον οὐκ ἄνθρωπος . . . τῷ μὲν γὰρ ἀνθρώπῳ τὸ ζῶον ἔπεται . . . τῷ μὴ ζῶῳ τὸ οὐκ ἄνθρωπος . . . δῆλον οὖν ὅτι πρὸς ἄμφω ἀντιστρέφει . . .).

<sup>39</sup> Cf. Boethius, *In Cic. Top.*, 356 (Orelli/Baiter), who considers a proposition of the form ‘If <it is> T<sub>A</sub>, not <it is> T<sub>B</sub>’ as the negation of ‘If <it is> T<sub>A</sub>, <it is> T<sub>B</sub>’. A passage in Epictetus (*Diss.* 2.20.2-3) implies that some Stoics transformed a negative universal statement of the form ‘No T<sub>A</sub> is T<sub>B</sub>’ into a conditional of the form ‘If something is T<sub>A</sub>, not: that thing is T<sub>B</sub>.’

<sup>40</sup> Λέγει δὲ αὐτοὺς ὁ Θεόφραστος κατὰ ἀναλογίαν, ἐπειδὴ αἱ τε προτάσεις ἀνάλογον καὶ τὸ συμπέρασμα ταῖς προτάσεσιν· ἐν πᾶσι γὰρ αὐτοῖς ὁμοίότης ἐστίν.

<sup>41</sup> Barnes, T&S, 288 n. 2, is tempted to guess that Theophrastus wrote λέγω δὲ αὐτοὺς συλλογισμοὺς κατὰ ἀναλογίαν, meaning ‘It is by analogy (i.e. with categorical syllogisms) that I call them “syllogism”’ and that Alexander wrongly took the sentence to mean ‘I call them “syllogisms by analogy”’. I, too, find this tempting.

abbreviation of ‘... since the premisses and the conclusion are analogous to the premisses <and conclusion of probative syllogisms>; for in all of them there is similarity <to the premisses and conclusion of probative syllogisms>.’<sup>42</sup> We have seen that such a similarity exists at least for the cases of WH mode 1.1 and *modus Barbara*. (Alternatively, the syllogisms could be called ‘by analogy’, since in them, analogous to the case of the categorical syllogisms, all propositions are of the same kind.)

### 5. An early theory of WHs? Boethius HS 2.9.1-3.6.4 and an anonymous scholium

Boethius’s *De hypotheticis syllogismis* (*HS*) is the only ancient text that provides a detailed presentation of the entire system of BWHs, setting out and proving (by reduction) all modes of all figures, and disproving (by example) the majority of the invalid premiss pairings which share one component. The work is thus invaluable as evidence for the ancient systematization of WHs. (My section 1 is largely based on it.) The problem is the dating of the theory presented. It is generally and correctly assumed that in *De hypotheticis syllogismis* Boethius drew from more than one source, and that some parts of the work show some influence of Stoic or later ancient logic. But there are very good reasons for an early, pre-Stoic dating of much of the theory of WHs. For example, the main passage shows no traces of Stoic propositional logic and the theory seems to predate the exposition of WHs in Alexander. In the present section I restrict myself to Boethius’ systematic presentation of the WHs in *HS* 2.9.1-3.6.4.<sup>43</sup> I list those elements in the passage that suggest an early source, and show that the passage has a term-logical understanding of WHs. In this way I hope at the same time to further substantiate the thesis that the early Peripatetics had a term-logical understanding of WHs.<sup>44</sup>

<sup>42</sup> ... αἴ τε προτάσεις ἀνάλογον καὶ τὸ συμπέρασμα ταῖς προτάσεσιν (καὶ τῷ συμπεράσματι τῶν δεικτικῶν συλλογισμῶν)· ἐν πᾶσι γὰρ αὐτοῖς <i.e. ταῖς προτάσεσιν καὶ τῷ συμπεράσματι τῶν συλλογισμῶν κατ’ ἀναλογίαν> ὁμοίότης ἐστὶν (ταῖς προτάσεσιν καὶ τῷ συμπεράσματι τῶν δεικτικῶν συλλογισμῶν).

<sup>43</sup> In *HS* Boethius mentions only Aristotle, Theophrastus, Eudemus and Cicero as authors whose views he reports. Cicero can be ruled out with reasonable certainty as the source for the parts on WHs. Boethius’ focus on all sorts of funny term-logical syllogisms that develop from *one propositio*, and which show some similarity to Theophrastus’ prosleptic syllogisms supports the view that much of the *HS* preserves pre-Stoic material.

<sup>44</sup> For the two brief passages *HS* 1.6.2-3 and 1.8.6-1.9.1, which also mention WHs, see below section 8.

Boethius does not call the WHs by any specific, technical, name. In particular, unlike most other sources, he does not call them ‘wholly hypothetical syllogisms’. This suggests that Boethius’ ultimate source stems from a time before ‘wholly hypothetical syllogism’ became a generally accepted standard name for WHs. Boethius simply treats the WHs as one of a handful of different types of hypothetical syllogisms. Like Theophrastus, but unlike Alexander, he never doubts that they are syllogisms. Another peculiar feature which is absent in other later sources links up with Theophrastus’ proleptic syllogisms: Boethius often, instead of talking about syllogisms, talks about the premiss-pairing, referring to it as *propositio*, i.e. in the singular.<sup>45</sup> Boethius’ usual way of describing the WHs is by saying that its premiss-pairing is constructed or woven together (*constant, componuntur, textitur*) from three terms (*termini*).<sup>46</sup>

Boethius tends to present the premiss pairings schematically thus: ‘si A non est, B est, si B est, C esse necesse est’ (*HS* 2.11.5); and with example: ‘si est animal, non est inanimatum, si non est animal, est sensibile’ (*HS* 3.2.1); ‘si est homo, lapis non est; si lapis non est, non est inanimatum’ (*HS* 2.10.2). By the ‘three *termini*’ of such premiss-pairings Boethius means terms such as animal, inanimate, stone;<sup>47</sup> A, B, C are term-letters which stand for the terms from which the syllogisms are thought to be composed. The forms of the arguments and of their conditionals are thus clearly envisaged to be term-logical. In principle, the conditionals can be taken either as ‘designator conditionals’ (C<sub>2</sub>), or as ‘quantified conditionals’ (C<sub>1</sub>) (see section 2 above). Since the numerous examples and schemata without exception contain exactly two terms, and never a term for the subject of the sentence, I suspect that these conditionals were at least originally seen as (implicitly) quantified.

Boethius’ positioning of the negative particles in the conditionals of the WHs, too, suggests a term-logical understanding of the WHs. He uses formulations like ‘A non est’ and ‘non est A’ interchangeably. In the first of these it is plainly the predicate that is negated; the second is neutral with respect to sentence negation and predicate negation.<sup>48</sup> Furthermore, the

<sup>45</sup> E.g. *HS* 3.1.1, 3.2.1, 3.2.3, 3.2.4; generally in the passage *propositio* means premiss, and not proposition, cf. *HS* 3.1.1.

<sup>46</sup> *HS* 2.10.6, *HS* 3.1.1 twice, *HS* 3.6.4 twice. (This recalls *πλοκή* from [Amm] *Apr.* 67.28-30.)

<sup>47</sup> Cf. e.g. ‘animal, quod est A’ (*HS* 3.1.1), ‘si enim sit A animal, B inanimatum, C insensitive’ (*HS* 3.2.1), ‘si B terminum negat assumptio’ (*HS* 3.3.3 and 5), ‘quomodo B atque C termini varientur’ (*HS* 3.3.6). For *HS* 1.6.3 see below, section 8.

<sup>48</sup> However, as the standard positioning of *non* before *est* rather than directly before

order of figures of WHs in Boethius is the same as Theophrastus' (*HS* 2.9.2; 3.1.1, 3.4.2; cf. 1.6.2), and differs from that later adopted by Alexander, Alcinous, and Philoponus. Boethius' source is thus closer to Theophrastus than to Alexander (see also below). Finally, there are a number of characteristics in the Boethius passage which show a close connection to Aristotle's *Prior Analytics*, and hence support an early dating, and which are absent in the other later sources: the proofs of invalidity of certain premiss-pairings by example is modelled on *APr.* I 4-6; so are the choice of terms in the examples and the frequent indication of *necessitas consequentiae* in the syllogisms by phrases like 'by necessity' (e.g. *necesse est, necessario, ex necessitate, HS* 2.9.1-3.6.4 *passim*).

Taking all these points together, I conclude that in Boethius *HS* 2.9.1-3.6.4 we find a tradition of WH syllogistic in which a considerable number of features of an early – perhaps first generation Peripatetic – theory of WHs are preserved, which are absent in the tradition(s) exemplified in authors such as Alexander, Alcinous and Philoponus (for which see sections 6-10).

There is one further text, viz. the anonymous Greek Scholium in Aristotle's *Organon*, which provides a witness to the stage of the theory of WHs as we find it in Boethius:<sup>49</sup>

Among the hypothetical syllogisms there are first those that come to be from two terms that have been connected and those from two terms that have been separated; then the syllogisms that come to be from three terms by means of two conditionals. . . . From three terms that have been connected there <come to be> eight syllogisms through the repeating of the first term and eight through the repeating of the last . . . such as in the first mode 'if A, B; if B, C; hence if A, C.' The

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the term indicates, Boethius' negations are not term-negations. The arguments of modes 1.2-8 thus do not reduce to *modus Barbara* syllogisms. If we disregard the possibility that Boethius mistranslated his Greek source, and if we assume that Boethius' source drew on Theophrastus directly, then Theophrastus cannot have reduced the WHs to categorical syllogisms in the 'neat' way, since this method of reduction – unlike the 'muddled' way – required term-negations (see above section 4). Since Boethius does not mention reduction to categorical syllogisms at all, I prefer to think that an intermediate source (e.g. Porphyry, see below) ditched the requirement of reducing the WHs to categorical syllogisms, and thereby any possible need for understanding the negations as term-negations.

<sup>49</sup> This scholium has to my knowledge rarely been discussed as a source on WHs and mixed hypothetical syllogisms; also the fact that it is the closest surviving parallel to the Boethius passage on WHs has, I believe, so far not been pointed out. The terminology of this scholium is amazingly idiosyncratic within the corpus of Greek passages on hypothetical syllogisms.

figure is the first; for the term which conjoins (συνάγωγος), which is middle, is taken twice, and follows in the first conditional but leads in the second. . . . The second figure is the one in which the term that conjoins has the same position with respect to each of the terms that are conjoined (συναγομένων), insofar as it leads in either conditional, except that it is taken affirmatively in one, and negatively in the other . . . if A, B; if not A, C; hence if not B, C. . . . The third figure is the one in which the term that conjoins has the same position with respect to each of the terms that are conjoined, following in both conditionals, affirmatively in the one, negatively in the other, such as in the first mode again, which is put together from two affirmative terms that are conjoined: if A, B; if C not B; hence if A, not C.<sup>50</sup> (Schol. Anon. Waitz, Aristotle's *Organon* vol.1, p. 9-10)

Here is a brief list of the similarities to Boethius *HS* 2.9.1-3.6.4 which are absent in other texts on hypothetical syllogistic: the short outline of the 'mixed' hypothetical syllogism<sup>51</sup> provides the most striking correspondence. It is a close parallel to Boethius *HS* 2.1.7-2.4.3, 3.10.3-3.11.7, and is in fact the only such parallel. Regarding the WHs,

1. the name 'wholly hypothetical' is absent in spite of juxtaposition of WHs and 'mixed' hypothetical syllogisms. Instead of a name we are given a description, in terms of the number of terms involved in the two types of hypothetical syllogisms, and their relation, and (perhaps) the kind of proposition, in the case of WHs.
2. the order of the second and third figure is the same as in Boethius, and Theophrastus.
3. the general description of the second figure arguments states that the middle term needs to be taken once affirmatively, once negatively (cf. *HS* 3.1.2).
4. the classification of modes is by means of the permutations one obtains by taking the terms (ὄροι, *termini*) affirmatively or negatively.

<sup>50</sup> Ἐν τοῖς ὑποθετικοῖς συλλογισμοῖς πρῶτοί εἰσιν οἱ ἐκ δύο ὄρων συνημμένων ἢ διαλελυμένων, εἴτα οἱ ἐκ τριῶν δύο συνημμένοις ὄρων γίνονται συλλογισμοί. . . . ἐκ δὲ τριῶν ὄρων συνημμένων ὁκτώ μὲν συλλογισμοὶ διὰ τῆς ἐπὶ τὸν πρῶτον ἐπανόδου, ὁκτώ δὲ διὰ τῆς ἐπὶ ὑστερον . . . οἷον ὡς ἐπὶ ἐνὸς τρόπου· εἰ τὸ Α, τὸ Β· εἰ τὸ Β, τὸ Γ· εἰ τὸ δὲ Α ἄρα, τὸ Γ. τὸ σχῆμα πρῶτον· ὁ γὰρ συνάγων ὄρος μέσος ἐστὶ δις λαμβανόμενος, λήγων μὲν ἐν τῷ πρωτέρῳ συνημμένῳ ἡγούμενος δὲ ἐν τῷ δευτέρῳ. . . . δευτερον δὲ σχῆμα ἐν ᾧ ὁ συνάγων ὄρος τὴν αὐτὴν ἔχει σχέσιν πρὸς ἑκάτερον τῶν συναγομένων καθὸ ἡγεῖται ἐν ἑκατέρῳ, συνημμένῳ, πλὴν ἐν μὲν τῷ ἐτέρῳ καταφατικῶς ἐν δὲ τῷ ἐτέρῳ ἀποφατικῶς . . . εἰ τὸ Α, τὸ Β· εἰ οὐ τὸ Α, τὸ Γ· εἰ οὐ τὸ Β ἄρα, τὸ Γ. . . . τρίτον σχῆμα ἐν ᾧ ὁ συνάγων ὄρος τὴν μὲν αὐτὴν ἔχει σχέσιν πρὸς ἑκάτερα τῶν συναγομένων, λήγων δὲ ἐν ἑκατέρῳ τῷ συνημμένῳ καταφατικῶς τε καὶ ἀποφατικῶς, οἷον ὡς ἐπὶ ἐνὸς τρόπου πάλιν ἐκ καταφατικῶν τῶν συναγομένων ὄρων συνεστῆτος· εἰ τὸ Α, τὸ Β· εἰ τὸ Γ, οὐ τὸ Β· εἰ τὸ Α ἄρα, οὐ τὸ Γ.

Reading ὄρων for ὄροις (ms) in line two; alternatively perhaps ἐκ τριῶν [δύο] συνημμένων ὄρων.

<sup>51</sup> I have not quoted this part of the scholium.

This scholium thus belongs to the same tradition of WH syllogistic as Boethius.<sup>52</sup> The pre-Stoic theory of ‘mixed’ hypothetical syllogisms, the Theophrastean order of the figures, and the absence of the names ‘wholly hypothetical syllogisms’ and “‘through three” syllogisms’ seem, again, to be remnants of an ultimate source that is early Peripatetic. (However, as we will see in section 8, the author of the scholium gives the WHs a propositional-logical interpretation.)

As to this ultimate source and the intermediate transmission we can only speculate. One possibility would be this: Boethius draws on Porphyry (possibly via a later commentator) either on parts of a commentary on the *Prior Analytics* or on a work on hypothetical syllogisms. Porphyry in turn draws from and elaborates on a work by Theophrastus, presumably his *Prior Analytics*. At least this hypothesis tallies with our scanty historical information:

1. Porphyry wrote commentaries on the *Categories* and the *De Interpretatione* (and often disagrees with what Alexander says in his commentaries).
2. he read Theophrastus’ *On affirmation and negation* and wrote a commentary on it.
3. he follows Theophrastus’ view on a number of issues in categorical syllogistic.
4. Theophrastus wrote about WHs in his *Prior Analytics*.
5. Boethius draws a lot on Porphyry.

We can infer with some plausibility that Porphyry read and knew Theophrastus’ *Prior Analytics* (2, 3); hence that he knew Theophrastus’ view on WHs (3, 4); that Porphyry studied and wrote about syllogistic (3); that Porphyry may have written on Theophrastus on WHs (1, 2, 3, 4); and that Boethius may have drawn from Porphyry’s writings on Theophrastus on WHs (5). The anonymous Greek Scholium (Waitz) would then present a late systematic summary of Porphyry’s writings on WHs, perhaps via intermediate sources.

#### 6. Ambiguity: Alexander *APr.* 326.22-328.2

The next part of the story is found in Alexander *APr.* 326.22-328.2, a passage which provides a self-contained outline of a theory of WHs, with

<sup>52</sup> M. Maroth, ‘Die hypothetischen Syllogismen, *Acta Antiqua* 27, 1979, 407-36, suggests a Greek source for Boethius’ *HS* on the basis of some Arabic parallel passages which are taken from the Greek.

continuous emphasis on its similarity to categorical syllogistic. This outline presents, as it were, an ‘updated’ version of Theophrastus’ theory.<sup>53</sup> It features a couple of changes: terminologically, there are distinctions between ‘categorical’ and ‘hypothetical’ arguments and between ‘categorical’ and ‘hypothetical’ premisses; systematically, the order of the second and third figures of WHs has been reversed. I assume that this theory was not developed by Alexander himself, but that he draws – with approval – from an earlier source,<sup>54</sup> which considered the conditionals in the WHs as universally quantified propositions.

The WHs are called ‘wholly hypothetical’, since in them <besides all the premisses> also the conclusion is hypothetical (Alex. *APr.* 326.22-4). A basic wholly hypothetical (BHW) consists of two hypothetical premisses and a hypothetical conclusion,<sup>55</sup> which are conditionals presented in the general, standardized formulation ‘if <it is>  $\pm T_A$ , <it is>  $\pm T_B$ ’, where terms (ὄροι) will take the places of  $T_A$  and  $T_B$ , and the addition of ‘it is’ (ἔστι) is optional.<sup>56</sup> Examples are ‘If it is an animal, it is a substance’, ‘if stone, not animal’, ‘if not rational, non-rational’. In addition to examples, we are given schematic presentations, which instead of terms have term-letters A, B, C,<sup>57</sup> e.g. ‘If A, B’, ‘if not A, C’. I assume that the conditionals can be parsed as ‘If anything is/is not  $T_A$ , it is/is not  $T_B$ ’.

The conditionals of these BHWs thus do not have logically independent component sentences as antecedent- and consequent-clauses. The logical properties and relations which characterize this system of WHs can therefore not be expressed in terms of component sentences. Instead, they are expressed by means of the properties of and relations between the terms of the conditionals. This is done by appropriating terminology from categorical syllogistic.  $T_A$  and  $T_B$  are called terms (ὄροι). In a conditional

<sup>53</sup> Theophrastus’ view on WHs is mentioned both just before and just after this passage: Alex. *APr.* 326.20-2 and 328.2-5 (also 326.8-10).

<sup>54</sup> My main reason for this assumption is that in Alexander we also find elements of later views on WHs (see below sections 8 and 9). The assumption that the theory in Alex. *APr.* 326.22-328.2 is still relatively early finds support in the facts that the terminology is purely Peripatetic; that there are no traces of Stoic logic in the passage; that πρότασις is used to refer to premisses, not propositions; and that the WHs are defined in terms of their premisses and conclusion (Alex. *APr.* 326.22-4), not their constituent propositions (as e.g. Alcinous has it, see section 7).

<sup>55</sup> ὑποθετικάι πρότασις Alex. *APr.* 327.2-3, ὑποθετικὸν συμπέρασμα Alex. *APr.* 326.23-4.

<sup>56</sup> Alex. *APr.* 326.24-5, 327.12-13; 19-20.

<sup>57</sup> Alex. *APr.* 326.22-3, 36-7, 327.11-12, 17-18.

'If  $\pm T_A$ ,  $\pm T_B$ ' term  $T_A$  is said to be at the beginning (ἀρχεσθαι) or to precede (ἡγείσθαι), or to be the preceding term (ἡγούμενος ὄρος, or simply τὸ ἡγούμενον) – viz. relative to  $T_B$ ;<sup>58</sup> term  $T_B$  is said to be at the end (λῆγειν) or to follow (ἔπεσθαι), or to be the following term (ἐπόμενος ὄρος, or τὸ ἐπόμενον), viz. relative to  $T_A$ .<sup>59</sup> Given that there are exactly two terms in each conditional, these expressions are unambiguous.<sup>60</sup>

The term which two conditionals need to share in order to make up a premiss pairing of a BWH is called the middle term (μέσος ὄρος, Alex. *APr.* 326.25-7). The position of the middle term in the premisses determines the figure to which the BWH belongs (Alex. *APr.* 326.27-9). In the first figure, the middle term follows in the first premiss and precedes in the second; in the second figure, the same term follows in both premisses, and in the third figure the same term precedes in both premisses. In this account of the figures the negation sign is not considered as part of the term. For instance, 'if  $T_A$ ,  $T_B$ ' and 'if  $T_C$ , not  $T_B$ ' are regarded as having the *same* term as following term.<sup>61</sup> In this case the middle term ( $T_B$ ) is said to be taken as following the two preceding terms ( $T_A$ ,  $T_C$ ) in *opposite ways* (ἀντικειμένως); i.e. once affirmatively ( $T_B$ ) and once negatively (not  $T_B$ ).<sup>62</sup> For the *reduction* of WHs of modes 1.9-3.16 to modes 1.1-8 and the *construction* of WHs of modes 1.9-3.16, their premisses or conclusion need to be converted (ἀντιστρέφεσθαι). A premiss or conclusion is converted if (i) its two terms change place and (ii) each term obtains the quality opposite to the one it had (σὺν ἀντιθέσει).<sup>63</sup>

<sup>58</sup> E.g. Alex. *APr.* 326.29, 31, 36, 327.8-10.

<sup>59</sup> E.g. Alex. *APr.* 326.29, 32, 35, 327.8-10.

<sup>60</sup> ἡγούμενον and ἐπόμενον, etc., are also used in ancient propositional logic to refer to the component propositions of conditionals. But note that Aristotle and Alexander use these expressions for terms elsewhere. For ἡγείσθαι see e.g. Alex. *Top.* 193.21. Aristotle uses ἔπεσθαι for 'is predicated of' e.g. in *APr.* 43b3, 44a13, 56a20. Alexander uses ἔπεσθαι and ἐπόμενον for relations between terms passim in Alex. *APr.* 294-328, see below.

<sup>61</sup> Ἐπεὶ γὰρ ἐν τοῖς ὑποθετικοῖς τὸ ἐπόμενον κατηγορουμένου χώραν ἔχει, ὅταν ἐν ταῖς δύο προτάσεσι ταῦτὸν ἐπόμενον λαμβάνηται, τὸ δεύτερον ἔσται σχῆμα (Alex. *APr.* 327.5-7; cf. 13-14). This shows that ἡγούμενον and ἐπόμενον, and the corresponding expressions here cannot mean 'antecedent' and 'consequent' in the propositional-logical sense. For, both in Peripatetic and Stoic tradition (as in modern logic), when two conditionals were said to have the same antecedent or consequent, this always *includes* the quality of the proposition. (Cf. also Alex. *APr.* 327.30-2.)

<sup>62</sup> Alex. *APr.* 327.7-10; cf. 16-17. Both passages are quoted below.

<sup>63</sup> Alex. *APr.* 326.27-327.2; 327.23-35. Cf. also Alex. *APr.* 29.15-18: ἔστι δὲ καὶ ἐν



For the second and third figure we are given a clear distinction between 2nd or 3rd figure premiss pairings in general, and syllogistic<sup>64</sup> premiss pairings, i.e. those that lead to a BWH.<sup>65</sup> Second figure pairings are conclusive

whenever <the following term> is taken as following each of the preceding terms in opposite ways, for instance “if A, C; if B not C”. For here the middle term, C, is taken as following the preceding terms, A and B, in opposite ways. For this reason, if they are taken in this way, it will be concluded “if the one of the leading terms, not the other”<sup>66</sup> (Alex. *APr.* 327.7-11).

In the third figure,

the following <term>, which has the place of the subject, is the same in both propositions. If this is taken in opposite ways, the figure will be conclusive, such as “if A, B; if not A, C”. For it will be concluded “if not the one of the ending <terms>, then the other”.<sup>67</sup> (Alex. *APr.* 327.15-18).

These accounts of valid second and third figure WHs may have been formulated in such a way as to cover – in one sentence – all the modes of the figure:<sup>68</sup> (i) nothing is said about the qualities of the extreme terms; (ii) it is left open in which of the two premisses the middle term is negative; (iii) it is left open whether or not the conclusion is converted

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προτάσεσιν ἀντιστροφή σὺν ἀντιθέσει· ἀντιστρέφει γὰρ τῇ “ἄνθρωπος ζῷον ἐστίν” ἢ λέγουσα “τὸ μὴ ζῷον οὐδὲ ἄνθρωπός ἐστιν”. Cf. Galen *IL* 6.4, discussed above section 4 n. 37. Alex. *APr.* 328.24 uses ἀντιστρέφειν for terms.

<sup>64</sup> The use of συλλογιστικός in Alex. *APr.* 327.7 is revealing. It suggests that Alexander copies the passage from someone who unlike himself (see below, section 9) has no qualms about referring to the WHs as syllogisms.

<sup>65</sup> The distinction is not made for the first figure. It must have been something like this: a conclusive premiss pairing in the first figure is one in which the middle term is taken in the same quality in both premisses (i.e. either twice positively, or twice negatively).

<sup>66</sup> συλλογιστικὴ δὲ ἡ συζυγία, ἂν ἀντικειμένως ἐπόμενον ἐκατέρω τῶν ἡγουμένων λαμβάνηται, οἷον εἰ τὸ Α, τὸ Γ, εἰ τὸ Β, οὐ τὸ Γ; τὸ γὰρ Γ μέσος ὢν ὅρος ἀντικειμένως εἴληπται ἐπόμενος τοῖς ἡγουμένοις, τῷ τε Α καὶ τῷ Β. διὸ καὶ συναχθήσεται οὕτως ληφθέντων τὸ ‘εἰ θάτερον τῶν ἀρχομένων, οὐ θάτερον’.

<sup>67</sup> τὸ γὰρ ἡγουμένον ὑποκειμένου χώραν ἔχον ἐν ἀμφοτέραις ταῖς προτάσεσι ταυτόν ἐστιν. ὅταν δὲ ἀντικειμένως τοῦτο ληφθῇ, συνακτικὸν ἔσται, οἷον εἰ τὸ Α, τὸ Β, εἰ τὸ Α, τὸ Γ. συναχθήσεται γὰρ, εἰ μὴ θάτερον τῶν ληγόντων, θάτερον.

<sup>68</sup> In fact, for full generality, the conclusions would have to be described for the 2nd figure as ‘if the one of the leading terms, the contradictory of the other’ and for the 3rd figure as ‘if the contradictory of one of the ending terms, then the other ending term’; or else a double negation rule would be needed. (Alexander appears to accept such a rule at Alex. *APr.* 18.6-7.)

(relative to the first figure). The discussion of WHs thus comes on three levels of generality: the individual arguments, such as the examples given; the schematic presentations in form of the modes (sorted according to figures); and the general accounts, in ordinary language, perhaps designed to cover all arguments in all modes of one figure. The modes of a figure could then all be independently produced by using the general account as a sort of ‘generation rule’, and there would be no need for a substitution rule.

The coming into being of second and third figure WHs is explained as the result of premiss conversion in first figure WHs.<sup>69</sup> However, the paradigm modes chosen for the 2nd and 3rd figures are not obtained by simply converting the respective first or second ‘premiss’ of the first figure mode 1.1:<sup>70</sup> Rather, the modes selected as paradigmatic (2.5 and 3.4) furnish the valid arguments in those figures with independent plausibility: The second figure embodies the principle that *if* all things that are  $T_A$  are also  $T_B$  and all things that are  $T_C$  are not  $T_B$ , *then* if something is one of  $T_A$ ,  $T_C$ , it is not the other – since otherwise it would be both  $T_B$  and not  $T_B$  (i.e. a version of the Principle of non-contradiction would be violated). The third figure embodies the principle that *if* all things that are  $T_B$ , are also  $T_A$ , and all things that are not  $T_B$ , are  $T_C$ , *then* if a thing is not one of  $T_A$ ,  $T_C$ , it is the other – since otherwise it would be neither  $T_B$  nor not  $T_B$  (i.e. a version of the *tertium non datur* would be violated).<sup>71</sup>

The main emphasis of the presentation of the figures lies on their correspondence to Aristotle’s categorical figures, based on an analogy between categorical and hypothetical propositions. Alexander puts it thus: “ending” and “following” are analogous to “being predicated”, and “leading” to “being subject” (Alex. *APr.* 326.31-2).<sup>72</sup> I interpret this sentence

<sup>69</sup> Ἡ γένεσις ὡςπερ ἐν τοῖς κατηγορικοῖς τῷ δευτέρῳ καὶ τρίτῳ σχήματι ἀπὸ τῶν ἀντιστροφῶν τῶν ἐν τῷ πρώτῳ προτάσεων, οὕτως δὲ καὶ ἐν τούτοις <i.e. the WHs>. (Alex. *APr.* 327.21-3)

<sup>70</sup> In that case, we would expect ‘if A, B, if not C, not B, therefore if A, C (therefore if not C, not A)’ for the second figure and ‘if not B, not A, if B, C, therefore if A, C (therefore if not C, not A)’ for the third. Philoponus *APr.* 302.20-3, in a brief passage otherwise close to the one under discussion, simply converts the respective first figure premisses in this way – but then gets the conclusions wrong!

<sup>71</sup> Relative to their premisses the conclusions of Alexander’s basic cases (modes 1.1, 2.5, 3.4) exemplify consequence (1), exclusion (2), and complementation (3).

<sup>72</sup> ἀνάλογον γὰρ τὸ μὲν λήγειν καὶ ἔπεισθαι τῷ κατηγορεῖσθαι, τὸ δὲ ἀρχεσθαι τῷ ὑποκείσθαι.

as: 'having  $T_x$  as 'following term' in a hypothetical proposition is analogous to having  $T_x$  as predicate term in a categorical premiss; and having  $T_x$  as 'preceding term' in a hypothetical proposition is analogous to having  $T_x$  as subject term in a categorical premiss.' Thus 'If (it is) A, (it is) B' is analogous to 'Every A is B' in the sense that in both cases we have the underlying sequence of term letters A, B.

This analogy in turn leads to the analogy of the three hypothetical figures with the categorical ones.<sup>73</sup> In Theophrastus, this analogy was based on the position of the middle terms in the premiss-pairings. In Alexander, the order of the 2nd and 3rd hypothetical figures has been reversed. Still, his analogy, too, is based on the position of the middle terms. How can this be? The answer is this:<sup>74</sup> Some time after Theophrastus and before Alexander the canonical way of formulating categorical propositions changed. For example, in Aristotle a universal affirmative would have the linguistic form 'A belongs to every B', whereas at Alexander's time it would display the form 'Every B is A'.<sup>75</sup> In categorical *sylogisms*, in addition to such reformulation of the premisses and conclusion, the order of the premisses is turned round. As a consequence the position of the middle terms in the premisses in the 2nd and 3rd categorical figures has changed, and thus the Theophrastean analogy between the categorical and hypothetical figures no longer holds. The parallel between the position of the middle terms in the categorical and hypothetical figures is however easily reinstalled by simply reversing the ordering of the 2nd and 3rd hypothetical figures. And this, I take it, is what as a matter of fact motivated the reversal. The resulting new correspondence of the categorical and hypothetical figures is this:

First Figure

A is B     If (it is) A, (it is) B  
 B is C     If (it is) B, (it is) C

Second Figure

A is B     If (it is) A, (it is) B  
 C is B     If (it is) C, (it is) B

<sup>73</sup> Cf. Alexander's use of οὕτως . . . ὡς (Alex. *APr.* 326.30) and of ἀνάλογον (Alex. *APr.* 327.3 and 14).

<sup>74</sup> The answer is given in more detail in my 'Why the order of the figures of the hypothetical syllogisms was changed', *Classical Quarterly* 50.1, 2000, 247-51.

<sup>75</sup> Cf. e.g. Alex. *APr.* 348, quoted above, section 4.

## Third Figure

**B** is **A**      If (it is) **B**, (it is) **A**  
**B** is **C**      If (it is) **B**, (it is) **C**

Alexander then connects his analogy between the figures with the reduction of the WHs to categorical syllogisms:

Thus, as the premiss pairings in these figures are similar in this way to those in the categorical figures, they should reasonably reduce to them (i.e. to those categorical premiss pairings).<sup>76</sup> (Alex. *APr.* 327.20-1)

This sentence is vague, and it allows for the same general range of interpretation as was possible for Theophrastus (section 4). (i) The sentence could mean that first figure BWHs reduce to first figure categorical syllogisms, second to second, and third to third. There is no way of doing this if reduction implies the derivability of BWHs from categorical syllogisms. Hence *either* ‘reduction’ here is used in a very loose sense, e.g. as denoting the fact that the WHs share a superficial structural similarity with the categorical syllogisms; *or* we have a careless logician at work, who perhaps just reduced arguments of mode 1.1 to *modus Barbara*, arguments of mode 2.5 to *modus Cesares*,<sup>77</sup> and then – wrongly – assumed the rest would work out in a similar way. (ii) The sentence could mean that because of the structural similarity the second and third figure WHs can be reduced to first figure WHs, as in the case of categorical syllogisms, and the first figure WHs then reduce to *modus Barbara*, e.g. along the lines given in section 4 for Theophrastus. This is more satisfactory from a logical perspective, but I am not too confident about it.<sup>78</sup>

Leaving aside the uncertainty connected with the reduction to categorical syllogisms, we have in the passage Alex. *APr.* 326-8 a consistent, intelligent, presentation of wholly hypothetical syllogistic, with the conditionals taken as universally quantified propositions.

What is remarkable about this passage is how easily it can be read as dealing with a piece of propositional logic which discusses arguments of the basic form ‘If P, Q; if Q, R; therefore if P, R’. The passage thus beautifully illustrates how a change from a term-logical understanding to a propositional-logical understanding of the WHs could have happened. It

<sup>76</sup> Ταύτη τε οὖν ὅμοιαι αἱ ἐν τούτοις συμπλοκαὶ ταῖς ἐν τοῖς κατηγορικοῖς σχήμασιν οὔσαι εἰκότως ἂν εἰς ἐκείνας ἀνάγοντο.

<sup>77</sup> As described in section 4.

<sup>78</sup> The phrase ‘and furthermore’ (καὶ ἔτι), which introduces the reduction of second and third figure WHs to first figure WHs in the next sentence, is not very encouraging; one would expect ‘for’ (γάρ).

seems that all one has to do is to take the expression ὅρος ('term') as meaning 'component proposition'. Thus in Alexander's schematic presentation of the abbreviated conditionals the term-letters A, B, C can be seen as standing for component propositions such as 'it is a human being' (ἄνθρωπος ἐστὶ) which now are seen as containing two terms each – one implicit subject term (the designator 'it' which is implicit in the verb) and one explicit predicate term (ἄνθρωπος).

Jonathan Barnes has demonstrated the possibility of such a propositional-logical reading by providing an exemplary interpretation of the entire passage Alex. *APr.* 326.22-328.2 in that vein.<sup>79</sup> Perhaps I should add briefly why, notwithstanding this interpretation, I prefer to think that the passage deals with quantified conditionals.<sup>80</sup> The main reasons are: the

<sup>79</sup> In his T&S, see above note 2.

<sup>80</sup> Barnes has in the main adduced three reasons in support of a propositional-logical interpretation:

*Reason (i)*: only the replacement of the schematic letters A and B by complete sentences makes grammatical sense of Alexander's schema 'if A, then B' (Alex. *APr.* 326.22-3); and Alexander himself replaces 'A' and 'B' by complete sentences, 'it is a human being', 'it is an animal' (T&S 290, cf. 293 n. 2). *Reply*: the schemata of the ancients were not strictly syntactical schemata. What mattered seems to be the parallel structure between actual examples and schematic presentation of modes. If the examples are abbreviated sentences, the schemata may become abbreviated sentences when the schematic letters are appropriately replaced; so for instance in the correlation of the schemata and examples which Alexander gives for the 2nd and 3rd figures: εἰ γὰρ τὸ A, τὸ Γ . . . οἷον εἰ ἄνθρωπος, ζῷον . . . (Alex. *APr.* 327.11-13, cf. 327.18-20). See also Galen, *Inst. Log.* 14.3.

*Reason (ii)*: Given that the Peripatetics did not restrict themselves to specific forms in the case of 'mixed' hypothetical syllogisms, but aspired to generality ('if P, Q; P; therefore Q' rather than 'if Fa, Ga; Fa; therefore Ga'), why should they have restricted themselves in the case of WHs (T&S 293-4)? *Reply*: This is a plausible argument. However, if – as I assume – the conditionals were understood as universally quantified, this point does not apply: WHs with quantified conditionals are not a subclass of arguments of the kind 'If P, Q; if Q, R; therefore if P, R', and the Peripatetics therefore could not restrict themselves to these.

*Reason (iii)*: There are some texts which provide examples for WHs which do not fit a term-logical mould, but do fit sentential (or propositional) interpretations. Hence a term-logical interpretation of the WHs is less plausible than a sentential one (T&S 294). *Reply*: This is a compelling reason for assuming that the authors of the texts which present such examples have a propositional-logical understanding of the WHs. (These texts are all from the 6th AD or later – see below, section 10.) But it does not provide sufficient reason for assuming that in our Alexander passage, too, the understanding was propositional. Rather, the change in the kinds of examples may reflect a change in the understanding of the WHs, from a term-logical to a propositional-logical understanding (see below).

absence of any exclusively propositional-logical vocabulary and of any noticeable Stoic influence; the prominence of term-logical vocabulary; the fact that the examples without exception fit a quantified conditional reading; that they are in fact of exactly the same kind as those in Boethius – which are unambiguously term-logical (see section 5);<sup>81</sup> and that Alexander's first example is taken from Aristotle *APr.* I 32, where only a quantified conditional interpretation makes sense; and last but not least the point made above in note 61.

A different kind of support for the assumption that Alexander (at Alex. *APr.* 326-8) and the early Peripatetics understood their conditionals 'if <it is>  $T_A$ , <it is>  $T_B$ ' as universally quantified propositions, and the WHs as term-logical arguments, would be forthcoming if it were attested that Aristotle and the Peripatetics had an independent interest in relations between *term* that could be expressed as 'if  $T_A$ ,  $T_B$ ', and which may have triggered or fuelled the development of a logic of WHs. There is indeed such evidence.

In *APr.* I 27, in the context of his instructions for producing syllogisms, Aristotle says we must determine, about a subject, what terms follow it, and what terms the subject follows (ἔπεται, ἀκολουθεῖ, Arist. *APr.* 43b1-4). Aristotle specifies this relation of 'following': one must select (ἐκλέγειν) those terms which follow the subject as a whole, and those which as wholes the subject follows (Arist. *APr.* 43b11-17); i.e.  $T_A$  follows  $T_B$  (in this specific sense) precisely if whatever is  $T_A$  is also  $T_B$ . In Alex. *APr.* 295-328 Alexander uses ' $T_A$  follows  $T_B$ ' generally in this specific sense. Aristotle and the Peripatetics thus had a logical interest in the relation of following *between terms*.<sup>82</sup> If one knows which terms follow which, this enables one to construct categorical syllogisms. How this works Aristotle shows in *APr.* I 27-8, and the commentators extensively elaborate on these sections.

Alexander regards the relation between ' $T_A$  follows  $T_B$ ' and 'every  $T_B$  is  $T_A$ ' as one of equivalence.<sup>83</sup> He also seems to endorse the view that 'if . . . , –' means the same as '– follows . . .' (Alex. *APr.* 373.28-35). It is

<sup>81</sup> Boethius has as schemata for WHs throughout 'si A est, B est', etc., parallel to his own examples, and to Alexander's example for the first figure; and Boethius' examples are very close to Alexander's.

<sup>82</sup> In *APr.* I 27-8 Aristotle uses ἔπεσθαι / ἐπόμενον for relations between terms over 30 times, and ἀκολουθεῖν 4 times. Alexander uses ἔπεσθαι / ἐπόμενον in the specific sense (of a term following a term as a whole) over 100 times in Alex. *APr.* 295-328.

<sup>83</sup> Thus he writes τὸ Β παντὶ τῷ Α ὑπάρχον· ἦν γὰρ ἐπόμενον αὐτῷ (Alex. *APr.* 319.4-5, cf. 319.13 ἀλλὰ μὴν τὸ Ε παντὶ τῷ Η ὑπάρχει· ἔπεται γὰρ αὐτῷ) and Alex. *APr.* 325.20 τὸ μὲν ἀγαθὸν ἔπεσθαι τῷ αἰρετῷ (πᾶν γὰρ αἰρετὸν ἀγαθόν).

thus possible that the WHs, as presented in Alexander (and the early Peripatetics) were to capture this relation of following between terms; i.e. that 'T<sub>A</sub> follows T<sub>B</sub>' was regarded as equivalent to 'If T<sub>B</sub>, T<sub>A</sub>'.<sup>84</sup> (The transitivity relation of 'following' of terms is discussed by Aristotle at *APr.* 43b22-32 and Alexander uses in part the same examples in both cases.)<sup>85</sup> Even if there was no direct connection between Arist. *APr.* I 27-8 and the Peripatetic theory of WHs, the extended discussion of the Aristotle passage shows that the Peripatetic logicians were *used* to thinking of terms as standing in a relation of following. This helps to explain the predominance of formulations without a verb ('if T<sub>A</sub>, T<sub>B</sub>' rather than 'if it is T<sub>A</sub>, it is T<sub>B</sub>') and the use of term-logical vocabulary in the theory of WHs. The equivalence between 'T<sub>B</sub> follows T<sub>A</sub>' and 'every T<sub>A</sub> is T<sub>B</sub>' also supports the above suggestion that the conditionals 'if <it is> T<sub>A</sub>, <it is> T<sub>B</sub>' can be paraphrased as 'if anything is *one of the* T<sub>A</sub>s, it is *one of the* T<sub>B</sub>s'.

### 7. From quantified conditionals to designators: Alcinous *Didasc.* ch. 6

Historically, the closest parallel to the theory from Alex. *APr.* 326-8 is a short passage in Alcinous *Didasc.* ch. 6, where Alcinous introduces a three-fold classification of syllogisms and presents the WHs and their three figures with an illustrative argument for each figure.<sup>86</sup> In fact, Alcinous reports a further updated, and it seems Platonist, variety of it. First the classification of syllogisms:

Of syllogisms some are categorical, some hypothetical, and some mixed from these. Categorical are those in which both premisses and conclusion are simple propositions, hypothetical those in which they are hypothetical propositions, and mixed those which combine the two <kinds of propositions>.<sup>87</sup> (Alc. *Didasc.* 158.23-7)

<sup>84</sup> Cf. e.g. Alex. *APr.* 347.25-9 ἔστι δὲ ἡ καθόλου πρότασις "πάν τὸ ἐπόμενόν τι ἐπεται καὶ ᾧ ἐκεῖνο ἔπεται". ἐν δὲ τῷ "εἰ ἀνθρώπου ὄντος ζῴον ἐστὶ καὶ ζῴου ὄντος οὐσία" τῷ μὲν ζῴω ἔπεται ἡ οὐσία, τῷ δ' ἀνθρώπῳ ἔπεται τὸ ζῴον· καὶ τῷ ἀνθρώπῳ ἄρα ἔπεται ἡ οὐσία etc. I take this passage to discuss relations of consequence between terms. The quoted premiss pairing from Arist. *APr.* I 32 is shortly after interpreted as belonging to a WH (see section 4).

<sup>85</sup> 'Animal' follows 'human being' (as Aristotle himself observed, *APr.* 43b25-31); 'rational' follows 'human being' (e.g. Alex. *APr.* 295.31); 'ensouled' follows 'animal' and 'animal' follows 'human being' (Alex. *APr.* 295.13), which is the same triad as used for WH 1.1 in [Amm.] *APr.* 67.24-30, cf. section 4.

<sup>86</sup> Alcinous' dates are uncertain (cf. T. Göransson, *Albinus, Alcinous, Arius Didymus*, Göteborg 1995, chs. 6-9). A *floruit* sometime between the middle of the 2nd AD and the early 3rd AD is likely.

<sup>87</sup> Τῶν δὲ συλλογισμῶν οἱ μὲν εἰσι κατηγορικοί, οἱ δὲ ὑποθετικοί, οἱ δὲ μικτοὶ ἐκ

This is presumably the earliest surviving passage which provides this threefold distinction of syllogisms. The classificatory criterion is based on the kinds of *propositions* the syllogisms are composed of: the WHs are defined as those composed entirely of hypothetical propositions – not of hypothetical premisses and hypothetical conclusions, as in Alex. *APr.* 326-8. This recognition of hypothetical *propositions* as the components of WHs is one important step in the direction of propositional logic. So is the fact that they are discussed together with the ‘mixed’ hypothetical syllogisms, since Alcinous presents the latter as unambiguously propositional-logical. He also classifies the WHs straightforwardly as syllogisms. (I surmise that this has something to do with the fact that Alcinous, like the Platonists in general, took Plato as authority in logic, and that generally a lot of Plato exegesis consisted in finding syllogisms employed by Plato in his works. Hence the readiness to accept as syllogisms arguments or argument forms which Plato himself used.) Here is Alcinous’ presentation of the WHs, which follows upon a similar one of categorical syllogisms:

We will find Plato arguing by employing the hypothetical < syllogisms > in many of his books, but mostly in the *Parmenides* we find arguments like these:

If the One has no parts, it has neither a beginning, nor a middle, nor an end.

If it has neither a beginning, nor a middle, nor an end, it does not have a limit.

If it does not have a limit, it does not partake in shape.

Therefore, if the One has no parts, neither does it partake in shape.

And in accordance with the second hypothetical figure (which most people call the third), in which the common term follows the extremes in either premiss, he argues thus:

If the One does not have parts, it is neither straight nor curved.

If it partakes in shape, it is either straight or curved.

Therefore, if it does not have parts, it does not partake in shape.

And in accordance with the third figure (which some take to be the second), in which the common term precedes the extreme terms, in the *Phaedo* he argues in effect thus:

If we acquired knowledge of the equal, and have not forgotten it, we know it.

If we have forgotten it, we recollect it.<sup>88</sup> (Alc. *Didasc.* 159.7-24 Whittaker)

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τούτων· κατηγορικοί μὲν, ὧν καὶ τὰ λήμματα καὶ τὰ συμπεράσματα ἀπλάι προτάσεις ὑπάρχουσιν, ὑποθετικοὶ δὲ οἱ ἐξ ὑποθετικῶν προτάσεων, μικτοὶ δὲ οἱ τὰ δύο συνειληφότες.

<sup>88</sup> Τοὺς δὲ ὑποθετικούς ἐν πολλοῖς βιβλίοις εὐρήσομεν ἐρωρωμένους ὑπ’ αὐτοῦ, μάλιστα δ’ ἐν τῷ Παρμενίδῃ τοιοῦτους εὐροίμεν ἂν λόγους· εἰ μὴ ἔχει μέρη τὸ ἓν, οὔτε ἀρχὴν οὔτε μέσον οὔτε τελευτὴν ἔχει· εἰ μῆτε ἀρχὴν μῆτε μέσον μῆτε τελευτὴν ἔχει, οὐδὲ πέρασ ἔχει· εἰ μὴ ἔχει πέρασ, οὐδὲ σχήματος μετέχει· εἰ ἄρα μὴ ἔχει μέρη τὸ ἓν, οὐδὲ σχήματος μετέχει. Κατὰ δὲ τὸ δεύτερον ὑποθετικὸν σχῆμα, ὃ οἱ πλείστοι τρίτον φασί, καθ’ ὃ ὁ κοινὸς ὅρος ἀμφοτέροις τοῖς ἄκροις ἔπεται, οὕτως ἐρωτᾷ· εἰ μὴ ἔχει μέρη τὸ ἓν, οὔτε



Alcinous reports the same change in the order of figures as Alexander (Alex. *APr.* 326-8), and adopts the same order as the latter. He also uses the terminology of preceding and following terms in the conditionals. Hence both passages appear to belong to the same tradition.<sup>89</sup> But, as in the case of the classification of syllogisms, there are a number of logically relevant differences. Like the Platonizing authors Galen and Apuleius,<sup>90</sup> Alcinous uses the expression 'shared term' (κοινὸς ὅρος), instead of Alexander's 'middle term' (μέσος ὅρος); this has the advantage of providing a nomenclature equally suitable for all three figures.

More importantly, the assumed logical form of the conditionals seems to have changed: since Alcinous gives no schematic presentation of modes, we have to go by his examples on this point. Alcinous has replaced the examples modelled on Aristotle's *Prior Analytics* by examples from Plato's dialogues, in order to show that Plato made use of WHs. All the conditionals in his examples share the following general form (C<sub>2</sub>):

If S is/is not T<sub>A</sub>, it (S) is/is not T<sub>B</sub>.<sup>91</sup>

Unlike in Alexander, we here have (explicitly) *three* terms per conditional; in each component clause a generic term as predicate term, and a singular term as subject term shared in antecedent and consequent clause. This is clearly no longer a quantified understanding of the conditionals. The subject term is a designator ('we', 'the one') in its first occurrence, i.e. in the antecedent of each first premiss, and it is indicated by a cross-reference to that designator in all further occurrences in the same argument. Thus the arguments are of form (A<sub>2</sub>) (cf. section 2), and their three figures in their simplest forms would be:<sup>92</sup>

εὐθύ ἐστιν οὔτε στρογγύλον· εἰ μετέχει σχήματος, ἢ εὐθύ ἐστιν ἢ στρογγύλον· εἰ ἄρα μὴ ἔχει μέρος, οὐ μετέχει σχήματος. Καὶ μὴν καὶ κατὰ τὸ τρίτον σχῆμα, πρὸς τινῶν δὲ δεύτερον, καθ' ὃ ὁ κοινὸς ὅρος ἀμφοτέρων ἡγεῖται, ἐν τῷ Φαίδωνι οὕτως ἐρωτᾷ δυνάμει· εἰ λαβόντες τὴν τοῦ ἴσου ἐπιστήμην μὴ ἐπιλελήσμεθα, ἐπιστάμεθα, εἰ δὲ ἐπιλελήσμεθα, ἀναμνησκόμεθα.

<sup>89</sup> That both authors draw from the same theory is also suggested by the same classification of 'mixed' hypothetical syllogisms (see Alc. *Didasc.* 158.16-7 & 159.24-9, Alex. *Top.* 165.6ff & 174.5ff), which, to my knowledge, is nowhere else extant.

<sup>90</sup> Gal. *Inst.Log.* 7.6 and 7, Apul. *Int.* 7, pp. 198-9 (Moreschini).

<sup>91</sup> Some of the terms T<sub>A</sub>, T<sub>B</sub> are complex, e.g. 'either straight or curved'.

<sup>92</sup> I assume that Alcinous abbreviated his source, and that at the beginning of the short passage on hypothetical syllogisms originally we had something like: 'there are three figures in the case of the hypothetical syllogisms: the first, in which the common term follows the extreme term in one premiss, and precedes it in the other.' (τῶν δὲ ὑποθετικῶν σχημάτων ὄντων τριῶν· τὸ πρῶτον, καθ' ὃ ὁ κοινὸς ὅρος τῷ μὲν τῷ ἄκρῳ

## First Figure

If S is/is not  $T_A$ , S is/is not  $T_B$   
 If S is/is not  $T_B$ , S is/is not  $T_C$   
 If S is/is not  $T_A$ , S is/is not  $T_C$

## Second Figure

If S is/is not  $T_A$ , S is/is not  $T_B$   
 If S is/is not  $T_C$ , S is/is not  $T_B$   
 If S is/is not  $T_A$ , S is/is not  $T_C$ .

## Third Figure

If S is/is not  $T_B$ , S is/is not  $T_A$   
 If S is/is not  $T_B$ , S is/is not  $T_C$   
 If S is/is not  $T_A$ , S is/is not  $T_C$

In section 2 I have outlined two different ways in which the transition from quantified conditionals to designator conditionals could have occurred. Either way, the resulting syllogisms of form (A<sub>2</sub>) can no longer be reduced or reformulated as categorical syllogisms, as those of form (A<sub>1</sub>) could.

A syllogism of the form (A<sub>2</sub>) necessarily also has the form (A<sub>3</sub>) 'If P, Q; if Q, R; therefore if P, R'. Whether Alcinous (or his source) understood the logical form of the WHs to be (A<sub>2</sub>) or (A<sub>3</sub>) depends on how he understood the expression 'term' (ὄρος). Since (i) he has just defined ὄρος in the context of categorical syllogisms as parts of propositions, such as 'man' and 'animal' in 'man is an animal' (*Didasc.* 158.36-7), and (ii) all examples have in fact the form (A<sub>2</sub>), and (iii) he uses term-logical vocabulary as he does not in the subsequent description of the (mixed) hypothetical syllogisms,<sup>93</sup> and (iv) he is plainly in the tradition of Alex. *APr.* 326-8, ὄρος presumably denotes the (predicate) terms of the conditionals. (I suspect that instead of seeing the inferences as dealing with whole propositions, the perspective was still that the inferences really deal with the relations between predicate terms.)

We can however see that it would be only a minor step to get from (A<sub>2</sub>) to (A<sub>3</sub>). It is simply the change of perspective on what counts as logically relevant components of the conditionals, from the (predicate) terms,

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ἔπεται, τοῦ δὲ ἡγεῖται, cf. Alex. *APr.* 326.28-9.) Alcinous may have omitted the general description of the first figure deliberately, since it did not fit the *three*-premiss example he takes from Plato.

<sup>93</sup> Ὁ κοινὸς ὄρος, οἱ ἄκροι ὄροι, *Didasc.* 159.15-16 versus τὸ ἡγούμενον, τὸ λήγον, used of whole, simple, component propositions, *Didasc.* 159.27-8.

to whole component propositions. At the point at which this happens, at the latest, a decision about terminology will have to be made. 'Opoc will either have to be endowed with a new meaning (Alex. *APr.* 374), or replaced by another expression (Philop. *APr.* 347), in order to make clear that the logically relevant components of the conditionals are now component propositions.

#### 8. *Fusion of Stoic and Peripatetic logic: Alexander APr. 374.21-35*

The first of these options seems to have been adopted – if in a confused way – by the author of another passage from Alexander's commentary on the *Prior Analytics*:

In this way one must proceed also in the case of the argument which has been put forward in order to discredit the 'through three' <syllogism>, i.e. the argument 'if nothing is, neither is night; if night is not, day is; hence if nothing is, day is.' For since on 'nothing is' 'night is not' does not follow simply, but 'neither is night', one must take the same term as leading term of the second conditional, viz. 'if neither is night'. Taken in this way, 'day is' will no longer follow. Moreover, taken by itself 'if night is not, day is' is true, but with the previous conditional, i.e. 'if nothing is, neither is night', it is no longer true, since the middle, which follows in the first conditional and precedes in the second, is not taken in the same way in both conditionals. For in the first conditional 'neither night' was taken as the same as 'in addition to the other things, neither is night' upon which 'day is' no longer follows.<sup>94</sup> (Alex. *APr.* 374.21-35)

This passage is a comment on *APr.* I 40, 49b10, where Aristotle distinguishes between the use of terms with or without the definite article: 'pleasure is (a) good' differs from 'pleasure is *the* good'. Alexander's point is that a similar distinction helps to rebut a piece of fallacious reasoning which attempts to invalidate the 'through three' arguments. This attempt proceeds by proposing that there is an argument which satisfies the formal requirements for being 'through three', but which is evidently invalid,

<sup>94</sup> Οὕτως καὶ ἐν τῷ εἰς διαβολὴν τοῦ διὰ τριῶν φερομένῳ λόγῳ τῷ "εἰ μηδὲν ἐστίν, οὐδὲ νύξ ἐστίν, εἰ μὴ νύξ ἐστίν, ἡμέρα ἐστίν, εἰ μηδὲν ἄρα ἐστίν, ἡμέρα ἐστίν" χρῆ ποιεῖν. Ἐπεὶ γὰρ ἔπεται τῷ μηδὲν εἶναι οὐχ ἀπλῶς τὸ μὴ εἶναι νύκτα ἀλλὰ μηδὲ νύκτα, τοῦτον χρῆ τῆς δευτέρας συνεχείας ὄρον ἡγούμενον λαμβάνειν τὸν "εἰ μηδὲ νύξ ἐστίν". ὦ οὕτως ληφθέντι οὐκέτι ἀκολουθήσει τὸ ἡμέραν εἶναι. Ἔτι καθ' αὐτὸ μὲν λαμβανόμενον τὸ "εἰ μὴ νύξ ἐστίν, ἡμέρα ἐστίν" ἀληθές, ἂν δὲ ἐπὶ προκειμένῳ συνημμένῳ τῷ "εἰ μηδὲν ἐστίν, οὐδὲ νύξ ἐστίν" οὐκέτι ἀληθές τῷ τὸ μέσον τὸ ἐπόμενον μὲν ἐν τῷ πρώτῳ συνημμένῳ ἡγούμενον δὲ ἐν τῷ δευτέρῳ μὴ ὁμοίως ἐν ἀμφοτέροις λαμβάνεσθαι. ἔληφθη γὰρ ἐν τῷ πρώτῳ συνημμένῳ τὸ "οὐδὲ νύξ" ὡς ἴσον τῷ πρὸς τοῖς ἄλλοις μηδὲ νύκτα ἔσεσθαι, ὦ οὐκέτι ἔπεται τὸ ἡμέραν εἶναι.

and thus undermines the assumption that ‘through three’ arguments are formally valid. The argument is ‘if nothing is, neither is night; if night is not, day is; therefore if nothing is, day is.’ (The sophistic argumentation works better in Greek than in English, ‘day is’ (ἡμέρα ἐστίν) and ‘night is’ (νύξ ἐστίν) being the normal way of saying ‘it is day / night’. Thus these sentences are grammatically parallel to existence statements such as ‘nothing is (i.e. exists)’. Of course they differ logically from the latter in two essential respects: unlike ‘day’ and ‘night’, ‘nothing’ is not a designator, and ‘is’ in ‘it is day’ is not existential.) Alexander objects – in brief – that the consequent of the first premiss differs from the antecedent of the second (since ‘neither is night’ means ‘night, *like everything else*, is not’), and that the argument is hence not a ‘through three’ argument.

The component sentences ‘it is night’ and ‘it is day’, and the composite propositions formed from them, are standard examples from Stoic, and later ancient, propositional logic. Another element of Stoic origin in the passage is the term for conditional, *συνημμένον*, which is used in parallel with the Peripatetic one, *συνέχεια*. (The fact that premisses and conclusion are expressly called conditionals also shows that they are recognized as composite propositions.) Thus, as elsewhere in Alexander,<sup>95</sup> we find in the same passage the use of two different sets of terminology, which indicates a fusion of different theories. We can infer that the logicians who adduced the argument were acquainted with Stoic or contemporary propositional logic. They may have exploited the ambiguities that result from the clash of Peripatetic and Stoic logic. (Unlike other standard examples, such as ‘if Dion is walking, Dion is moving’, ‘if it is not night, it is day’ sports the peculiarity that its grammatical structure is identical with that of early ‘Peripatetic’ conditionals in WHs such as ‘If it is not rational, it is non-rational.’)

The essential point for us in this passage is that the form of the would-be ‘through three’ argument is treated as propositional-logical, and that the expression *ὅρος* is endowed with a new meaning. In the lines before Alexander turns to the ‘through three’ arguments (Alex. *APr.* 374.21-4), he uses *ὅρος* to denote terms – just as Aristotle did in the passage commented upon. Then, however, Alexander uses *ὅρος*, or more precisely, ‘preceding term (*ὄρος*) of the second conditional’, in order to refer to a whole clause, ‘if neither is night’, i.e. to the antecedent of a conditional (Alex. *APr.* 374.27-8).<sup>96</sup> Similarly, the expression ‘follows’ (*ἐπεται*) is

<sup>95</sup> E.g. Alex. *APr.* 262-5, *Top.* 165-6.

<sup>96</sup> *Ὅρος* may have taken on the general meaning ‘component’, referring to terms

twice used for component propositions, not for terms (Alex. *APr.* 374.26 and 34). Thus the form of the argument is envisaged as (A<sub>3</sub>) ‘If P, Q; if Q, R; therefore if P, R’.

What caused this change of focus we can only guess. First, note the term-logical vocabulary (μέσον, etc.), and that the example, like all previous ones, at least superficially, still fits a term-logical mould; the author of the counter-example may have thought that conditionals in ‘through three’ arguments must have the grammatical form ‘if it is T<sub>A</sub>, it is T<sub>B</sub>’. The passage (unlike Alex. *APr.* 326-8) shows clear signs of Stoic (or later propositional-logical) influence, and this may well explain the altered perspective. If the authors of the counter-example were Stoic, or obtained their logical education in Stoic logic, they may have (perhaps deliberately) misunderstood the ‘through three’ arguments as propositional-logical. In any event, the passage is the earliest we have which – indubitably – treats WHs as propositional-logical arguments.

There are two further passages which occupy the twilight zone between term-logical and propositional-logical WHs: first, a misguided comment on Aristotle’s *Prior Analytics* II 4 reported in Philop. *APr.* 413.8-24, which – unusually – *contrasts* the ‘through three’ syllogisms with the hypothetical syllogisms, reserving the title ‘hypothetical syllogism’ for ‘mixed’ hypothetical arguments. The ‘through three’ syllogisms are said to be so called since they take three terms (ὅρος, Philop. *APr.* 413.21-4), but it remains unclear what counts as a ὅρος. Equally Philoponus’ example for a ‘through three’ syllogism (a slight adaptation of Aristotle’s own) leaves one guessing what the ὅροι in it are: εἰ τοῦ Α λευκοῦ ὄντος ἀνάγκη τὸ Β μέγα εἶναι, τοῦ δὲ Β μεγάλου <ὄντος> ἀνάγκη τὸ Γ μὴ εἶναι λευκόν, ὥστε τοῦ Α ὄντος λευκοῦ τὸ Γ μὴ εἶναι λευκόν. Aristotle used Α, Β, and Γ for subject terms, but this would here produce ‘through *three*’ syllogisms with *six* terms. Thus I translate Philoponus (not Aristotle!) as ‘if being Α (white), necessarily it is Β (big), and being Β (big), necessarily it is not C (white), then being Α (white), it is not C (white)’.<sup>97</sup> If this gets Philoponus right, we can infer that the ὅροι were either whole predicates

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in categorical propositions, component clauses in hypothetical propositions This would tally with Alexander’s general definition of ὅρος, Alex. *APr.* 14.29-15.1.

<sup>97</sup> This interpretation not only has the advantage of producing a standard ‘through three’ syllogism, there are also parallels to such kind of formulation. Compare e.g. Boethius *HS* 2.8.7 ‘si cum sit a animatum, est b homo, est c animal, . . . si cum sit a animatum, est b homo, non est c equus’ with *HS* 2.7.1 ‘si cum sit a, est b, est c: si cum sit a, est b, non est c’.

or – more probably – component sentences, here in genitive absolute and accusative with infinitive formulations.

The second passage is *HS* 1.6.2-3, where Boethius, in an overview of what follows in the work, introduces the WHs. When explaining their systematic place between two other kinds of hypothetical syllogisms, he states that in the first figure the premisses are expressed as follows:

(1) “si est A, est B, et si est B, est C”, (2) igitur B in utrisque numeratur, (3) et sunt tres quidem termini hi, “est A”, “est B”, “est C”; (4) duae vero conditionales hoc modo: “si est A, est B, si est B, est C”, (5) namque B utrisque communis est.

Here, in (3) whole component clauses (“est A”, etc.) are regarded as *termini*, whereas in (2) and (5) the predicated terms (A, B, etc.) seem to count as *termini*. In the later passages which Boethius here anticipates, *termini* are undoubtedly terms (see above, section 4). Thus I believe that what has happened here in Book One is this: a later philosopher, familiar with Stoic-type propositional logic (perhaps Boethius himself), carelessly reinterpreted the term-logical WHs of forms (A<sub>1</sub>) of the source of *HS* 2.9.1-3.6.4 as being of propositional-logical form (A<sub>3</sub>), and hence took ‘terminus’ to refer to whole, independent, component propositions, or at least wavered between the two possibilities.<sup>98</sup>

A passage in which the author may have interpreted ‘term’ (ὄρος) throughout as ‘component sentence’ is the anonymous Greek Scholium (Waitz), of which I have shown above (section 5) that its ultimate source is an early theory of WHs. In the scholium the letters A, B, C stand for ‘terms’ which can be used affirmatively or negatively, e.g. if C, not B (εἰ τὸ Γ, οὐ τὸ Β).<sup>99</sup> At the end of the scholium the author attaches examples for all three figures (to which, in the manner of the *later* commentators, he refers as ‘matter’ (ὑλη)). These are all unambiguously propositional-logical, e.g. ‘If it is day, it is light; if it is light, the visible things are seen; if it is day, the visible things are therefore seen’. Again, I assume that what happened is that – at some point in the history of transmission of this piece of theory – an initially term-logical theory was re-interpreted in the light of the author’s acquaintance with propositional logic, and that corresponding examples were then attached.

<sup>98</sup> In the only other passage in Book One of *HS* in which Boethius mentions WHs (*HS* 1.8.6-1.9.1) his *termini* are terms, exemplified by letters A, B, C (*HS* 1.8.6).

<sup>99</sup> See the quotation of the passage in section 5.

9. *Interlude: A brief note on the inferential status of WHs in Alexander*

Alexander is the first of whom we know with certainty that he used the names ‘wholly hypotheticals’ (δι’ ὅλου ὑποθετικοί) and ‘through three’ (διὰ τριῶν) for the WHs. (The name ‘through three’ developed from an abbreviation of the general description of the BWHs as arguments or syllogisms composed from three terms.)<sup>100</sup> He treats the ‘through three’ arguments as a subclass of the wholly hypotheticals (e.g. Alex. *APr.* 326.8-9), and as valid. We do not know whether Alexander contributed any original ideas to the theory of WHs; but he seems to have had strong views about their inferential status. In all other sources which talk about arguments that are ‘wholly hypothetical’ or ‘through three’ they are marked out as syllogisms: they are wholly hypothetical *syllogisms*. This is not Alexander’s view. For him, the WHs are arguments (λόγοι), and valid,<sup>101</sup> but not syllogisms, or at least not syllogisms proper (ἀπλῶς).<sup>102</sup> He provides in his commentary on the *Prior Analytics* three different necessary conditions for syllogismhood, which may reflect a development of the Peripatetic concept of a syllogism; and he claims, or at least implies, that none of them is satisfied by the WHs.

The strictest (and perhaps earliest) condition requires that the *premisses* of a syllogism have the right form. The right form includes that they are probative (δεικτικῶς).<sup>103</sup> The context of the passage suggests that this criterion is satisfied precisely by the categorical syllogisms.<sup>104</sup> The WHs fail, because they do not have probative premisses. A second necessary condition requires either that a syllogism is a categorical syllogism or that at least one of its premisses is backed up by a categorical syllogism.<sup>105</sup> If this criterion is taken to be sufficient for syllogismhood, ‘mixed’ hypotheticals come out as syllogisms because of their assumed dependency on categorical syllogisms. WHs do not, since they are neither categorical

<sup>100</sup> Cf. e.g. Boethius, *HS* 3.1.1, Philop. *APr.* 413.21-4 and the anonymous Greek Scholium (Waitz).

<sup>101</sup> Thus he says that the ‘through threes’ are sound (ὀρθός, Alex. *APr.* 265.16) or conclusive (περαίνοντες, Alex. *APr.* 390.14), or that the conclusion follows from necessity (ἐξ ἀνάγκης, Alex. *APr.* 348.9-11, 17-18). Alexander produces no reason for their validity except that he implies that Aristotle says so in *APr.* I 32 (Alex. *APr.* 348.9-11).

<sup>102</sup> Alex. *APr.* 265.17, 19-20; 330.28-30; 348.12-13; 390.10-13.

<sup>103</sup> Alex. *APr.* 348.16, in part discussed above, section 4.

<sup>104</sup> Similarly suggested at Alex. *APr.* 265.13-16, where the ὑπαρξίς condition (see below) seems to be a necessary but not sufficient condition.

<sup>105</sup> Alex. *APr.* 390.14-19.

syllogisms nor do they have any categorical premisses that could be backed up by a categorical syllogism. A third necessary condition requires that a syllogism demonstrate that something belongs (ὕπαρχει) or does not belong (to something).<sup>106</sup> This criterion concerns the *conclusion* of a syllogism. The WHs fall short, since they do not – unconditionally – state that something does or does not belong. We encounter this criterion also in a more technical, perhaps later, formulation: the WHs are probative not of a belonging (ὕπαρξις) but only of a consequence (ἀκολουθία).<sup>107</sup> Here we have a positive description of what *is* validly inferred in the WHs, and thus implicitly the acknowledgement *that* something of a specific kind is inferred. With this criterion, the ‘mixed’ hypotheticals become proper syllogisms since they demonstrate a belonging (ὕπαρξις); they seem at this point recognized as valid because of their own particular form. This focus on the form of the ‘mixed’ hypotheticals, together with their independence from the categorical syllogisms, opens the door for the WHs to become accepted as syllogisms as well: they, too, have their validity because they have particular valid forms which make them the kind of arguments they are. But Alexander does not consider their form as sufficient for syllogismhood. He tries to be faithful to Aristotle’s view (or what he takes it to be). We can infer that he did not yet regard the WHs as being of the same general sort of arguments as the ‘mixed’ hypothetical syllogisms.

10. *Propositional logic at last: Philoponus* *APr.* 243.11-36; 244.13-26

At some point in antiquity the WHs received an unambiguous propositional interpretation. It is hard to say when and where exactly this happened. The best evidence for the fact *that* it happened are examples for WHs which do not allow a term-logical interpretation. This criterion provides two passages from late antiquity (Philop. *APr.* 243.11-36, 6th AD; Scholium in *Amm. APr.* XI.1-6, 6th AD or later) and some passages from Byzantine logic.<sup>108</sup> These passages seem all to belong in the same general tradition. The longest, presumably earliest, and most informative one is Philoponus. It forms part of a self-contained digression of several pages

<sup>106</sup> Alex. *APr.* 265, 347-8. The criterion is based on Arist. *APr.* I 23 40b23-4.

<sup>107</sup> Alex. *APr.* 330; cf. the Latin Scholium (Minio-Paluello) and Philop. *APr.* 302.

<sup>108</sup> The earliest of these is Anon. *Log.&Quadr.* 38 30.16ff, dated 1007. On the Byzantine sources for wholly hypothetical syllogisms cf. K. Ierodiakonou, ‘Medieval Traditions’.



on hypothetical syllogistic.<sup>109</sup> This digression starts with a juxtaposition of Stoic and Peripatetic logical terminology (Philop. *APr.* 242.18-243.10). Thus the author is familiar with Stoic logic. The theory given and terminology used thereafter are however purely Peripatetic. Philoponus' discussion of WHs displays many similarities to the tradition of WHs we found in Alex. *APr.* 326-8, Alcin. *Didasc.* ch. 6, and in Philop. *APr.* 302.6-23.<sup>110</sup> It is evidently a later development of this theory. Apart from the above-mentioned fact that some of the examples do not fit a term-logical interpretation any more, there are a number of further noteworthy modifications.

Neither Philoponus nor any of the later sources voices any doubt that the WHs are proper syllogisms. From Philoponus we can infer that this is a conscious change from Alexander. Philoponus defines syllogisms disjunctively as those which (1) either demonstrate that something is or is not, or (2a) that if something is, something is or something is not, or (2b) that if something is not, something is or something is not.<sup>111</sup> Syllogisms of type (1) demonstrate a belonging or being the case (ὑπαρξίς), those of type (2) a consequence (ἀκολουθία).<sup>112</sup> Thus whereas Alexander used the ὑπαρξίς / ἀκολουθία distinction in order to exclude WHs from syllogismhood, Philoponus explicitly includes syllogisms with a 'consequence' as conclusion.

The 'wholly hypotheticals' and 'through three' are then defined as the syllogisms of type (2).<sup>113</sup> The 'wholly hypotheticals' are so called, because all the premisses (or propositions) introduced in them are hypothetical.<sup>114</sup> There are two changes in the account of the name 'through three': the WHs are called 'through three' because they conclude through at least

<sup>109</sup> The Anon. *Log.&Quadr.* and the other Byzantine texts provide a comparable overview of syllogistic, and the Scholium, in a different way, too.

<sup>110</sup> E.g. the two names 'wholly hypothetical' and 'through three, the explanation of the name 'wholly hypothetical', the explicit requirement of a shared component in the two premisses.

<sup>111</sup> Λάβωμεν δὲ λοιπὸν ἐκ διαίρεσεως τὰ εἶδη τῶν ὑποθετικῶν συλλογισμῶν. καθόλου πᾶς συλλογισμὸς ἢ τὸ ἔστιν ἢ τὸ οὐκ ἔστι δεικνυσιν, ἢ τίνος ὄντος τί ἔστιν ἢ τί οὐκ ἔστιν, ἢ τίνος μὴ ὄντος τί ἔστιν ἢ τί οὐκ ἔστιν.

<sup>112</sup> The terminological distinction ὑπαρξίς / ἀκολουθία is not made in the definition, but later at Philop. *APr.* 244.13-26, where Philoponus distinguishes 'mixed' hypothetical syllogisms from the WHs.

<sup>113</sup> οἱ μὲν οὖν τίνος ὄντος ἢ μὴ ὄντος τί ἔστιν ἢ τί οὐκ ἔστι δεικνύντες, οὗτοι καλοῦνται διὰ τριῶν καὶ δι' ὅλων ὑποθετικοί, Philop. *APr.* 243.13-15.

<sup>114</sup> Δι' ὅλων μὲν, ὅτι πᾶσαι αἱ παραλαμβανόμεναι προτάσεις ὑποθετικάί, Philop. *APr.* 243.15-16. Shortly after (Philop. *APr.* 243.32-5) the syllogisms are said to be wholly hypothetical because their conclusion is a consequence.

three hypotheses.<sup>115</sup> Thus first, the ‘through three’ arguments are no longer regarded as a subclass of the wholly hypotheticals, but as co-extensive with them.<sup>116</sup> Consequently, as they must then include ‘complex’ wholly hypothetical syllogisms as well, we have the addition of ‘at least’: they conclude from *at least* three, perhaps more, components.<sup>117</sup>

The second modification is more significant: instead of ‘three terms (ὅροι)’ Philoponus says ‘three hypotheses (ὑποθέσεις)’. This is another indication of the progression from term logic to propositional logic: the essential elements of the WHs are no longer seen to be terms, but hypotheses. In line with this alteration in the account of the ‘through three’ arguments, when presenting his wholly hypothetical syllogistic, Philoponus uses ‘hypothesis’ where Alexander and Alcinous used ‘term’. A hypothesis in this context is the component sentence of a conditional.<sup>118</sup> We have noticed above that, at the point when the change to a propositional-logical understanding occurs, ideally an expression would be needed to denote the component sentences of the conditionals; but the use of the word ‘hypothesis’ in this function, i.e. for both antecedent and consequent, is peculiar and seems unparalleled in ancient texts.<sup>119</sup> Whoever introduced it must have been looking for a suitable replacement for ‘term’, since they felt that ‘term’ was an inadequate expression for component sentences. Hence ὅρος can hardly have been an established term for ‘component sentence’.

The theory of WHs is then as follows: The definition of WHs as syllogisms of type (2) (see above) serves at the same time as a new classificatory scheme. It identifies four types of WHs by way of the forms of their conclusion: (i) If P, Q; (ii) if P, not Q; (iii) if not Q, P, (iv) if not Q, not P.

<sup>115</sup> διὰ τριῶν δέ, ὅτι τοῦλάχιστον οὗτοι οἱ συλλογισμοὶ διὰ τριῶν ὑποθέσεων περαίνονται, Philop. *APr.* 243.16-17.

<sup>116</sup> Cf. the Scholium in Amm. *APr.* XI.1-2, ὁ λεγόμενος διὰ τριῶν καὶ δι’ ὅλου ὑποθετικὸς <i.e. συλλογισμὸς>.

<sup>117</sup> I assume that this is the result of a misunderstanding of the extensional relation between ‘through three’ arguments and wholly hypotheticals, and a subsequent adaptation of the explanation of the name, to make it fit.

<sup>118</sup> Cf. e.g. Philop. *APr.* 243.17-19, text quoted below.

<sup>119</sup> One might have expected the use of ‘hypothesis’ in lieu of ‘hypothetical propositions’ (cf. Arist. *APr.* 50a16-28); but – as K. Ierodiakonou (in her ‘Rediscovering some Stoic arguments’, in P. Nicolacopoulos, ed., *Greek Studies in the Philosophy and History of Science*, Dordrecht 1990, 137-48) at 138 rightly argues – this is not borne out by the text. The wholly hypothetical syllogisms are said to conclude (περαίνειν) from at least three hypotheses, and if the hypotheses were whole conditionals, this would not be true. For the same reason, the other plausible understanding of ‘hypothesis’, as denoting the antecedents of the conditionals only can be ruled out. Both points are also clear from *APr.* 243.17-19, quoted below.

In accordance with this fourfold distinction, Philoponus presents four examples, perhaps based on Plato's *Phaedrus*,<sup>120</sup> in the order introduced by the definition. They can be schematically represented as:

If P, Q	If P, Q	If not P, Q	If not P, Q
If Q, R	If Q, R	If Q, R	If Q, not R
If P, R	If R, not S	If not P, R	If not P, not R <sup>121</sup>
	If P, not S		

This fourfold distinction is the only classification given. There is no mention of figures of WHs. The examples are all in the first figure.<sup>122</sup> This is a characteristic of the sources which present a propositional-logical understanding of the WHs.<sup>123</sup> The philosophically most interesting bit of the passage is the following, which provides some information about the construction of WHs:

For instance, I want to show that the hypothesis which says 'god is good' is followed by 'the all is eternal'. Since I want to show that these two hypotheses follow each other,<sup>124</sup> it is wholly necessary to establish this through something else. For if we assumed just it itself, we would postulate the initial point again. We then need several hypotheses or at least one other hypothesis through which, as intermediate, we will connect these with each other; so that we may, for instance, say: 'if god is good, he brings about good things. If he brings about good things, he brings about eternal things. Therefore, if god is good, he brings about eternal things.'<sup>125</sup> (Philop. *APr.* 243.17-24)

This suggests the following general method for proving a conditional 'If  $\pm P$ ,  $\pm Q$ ': one picks a hypothetical proposition that one wants to prove, i.e.

<sup>120</sup> Cf. *Phdr.* 249a. *Log.&Quadr.* 38 p. 30.20-23 provides a very similar example of a WH in mode 1.1.

<sup>121</sup> The examples for the third and fourth type are given in one go, and thus run into each other; but their separate reconstruction is easy.

<sup>122</sup> One could easily integrate 2nd and 3rd figure syllogisms into this new scheme: e.g. the syllogistic form 'If P, Q; if not R, not Q; therefore if P, R' would belong to the first of Philoponus' categories.

<sup>123</sup> With the examples in the anonymous Greek Scholium being the exception.

<sup>124</sup> 'Each other' is strange. Perhaps Philoponus gives only part of a longer argumentation. But cf. Galen *Inst.Log.* ch. 14 for biconditionals with the linguistic form 'If P, Q'.

<sup>125</sup> οἷον βούλομαι δεῖξαι ὅτι τῇ ὑποθέσει τῇ λεγούσῃ ἀγαθὸν εἶναι τὸν θεὸν ἀκολουθεῖ τὸ αἰδιδίον εἶναι τὸ πᾶν. Ἐπειδὴ οὖν δύο ταύτας ὑποθέσεις βούλομαι δεῖξαι ἐπομένως ἀλλήλαις, πᾶσα ἀνάγκη δι' ἑτέρου τοῦτο κατασκευάσαι· ἐπεὶ εἰ αὐτόθεν τοῦτο λάβωμεν, τὸ ἐν ἀρχῇ πάλιν αἰτησόμεθα. Οὐκοῦν χρεῖα ἡμῖν πλείονων ἢ τοῦλάχιστον ἄλλης μιᾶς ὑποθέσεως, δι' ἧς μέσης ταύτας ἀλλήλαις συνάψομεν· οἷον ἴνα εἰπωμεν οὕτως, "εἰ ὁ θεὸς ἀγαθός, ἀγαθὰ ποιεῖ· εἰ ἀγαθὰ ποιεῖ, αἰδιδία ποιεῖ· εἰ ὁ θεὸς ἄρα ἀγαθός, αἰδιδία ποιεῖ".

the *demonstrandum*. One then needs to find intermediate simple propositions ('hypotheses')  $\pm R_1 \dots \pm R_n$  such that

$$\begin{array}{l} \text{If } \pm P, \pm R_1 \\ \text{If } \pm R_1, \dots \\ \text{If } \dots, \pm R_n \\ \hline \text{If } \pm R_n, \pm Q \\ \hline \text{If } \pm P, \pm Q \end{array}$$

This approach, although not present in earlier sources on WHs, is thoroughly Peripatetic, modelled on how one finds a syllogism for a 'problem' (πρόβλημα) in categorical syllogistic (cf. Arist. *APr.* I 26-30).<sup>126</sup> The examples are Platonic or Christian (but certainly not Stoic) and the terminology is entirely in the Peripatetic-Platonist tradition (πρότασις, κατασκευάζειν), and thus not Stoic either.

There is no longer any talk about reducing the WHs to categorical syllogisms, and indeed this would be surprising, given that arguments of the form 'If P, Q; if Q, R; therefore if P, R', etc. cannot in any straightforward manner be reduced to categorical syllogisms.<sup>127</sup> At this point the story I intended to tell comes to an end. We have reached a stage where the WHs are evidently understood as having a propositional-logical form.<sup>128</sup>

*The Queen's College  
Oxford*

<sup>126</sup> Anon. *Log.&Quadr.* 38 p. 30.16ff talks explicitly about the πρόβλημα.

<sup>127</sup> In this context it is odd that Anon. *Log.&Quadr.* 38 p. 30.23-6 (similarly p. 30.17-19) claims that a WH (in the first mode of the first figure) differs from a categorical syllogism in the first mode of the first figure only in its hypothetical form (οὗτος μὲν οὖν ἐν τῷ πρώτῳ τρόπῳ τοῦ πρώτου σχήματος τῶν κατηγορικῶν συλλογισμῶν ἀναφαίνεται μόνῳ τῷ ὑποθετικῷ διαλλάττων). For the example given belongs to propositional logic and such arguments can only with considerable logical contortions be reduced to *modus Barbara*; cf. Barnes, 'T&S', 313-318 for an heroic attempt, based on Brentano.

<sup>128</sup> I would like to thank the editors for their meticulous editing of the piece; Paolo Crivelli for helpful notes on parts of an early draft; and especially Jonathan Barnes for incisive and insightful comments on the penultimate draft.