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Why Defeating Insurgencies Is Hard: The Effect of Intelligence in Counterinsurgency Operations—A Best-Case Scenario

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In insurgency situations, the government-organized force is confronted by a small guerrilla group that is dispersed in the general population with no or a very small signature. Effective counterinsurgency operations require good intelligence. Absent intelligence, not only might the insurgents escape unharmed and continue their violent actions, but collateral damage caused to the general population from poor targeting may generate adverse response against the government and create popular support for the insurgents, which may result in higher recruitment to the insurgency. We model the dynamic relations among intelligence, collateral casualties in the population, attrition, recruitment to the insurgency, and reinforcement to the government force. Even under best-case assumptions, we show that the government cannot totally eradicate the insurgency by force. The best it can do is contain it at a certain fixed level.

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1. Introduction

In many recent military conflicts (e.g., Northern Ireland, Tuck 2007; Colombia, Phillips 2003; Afghanistan, Barno 2007; and Iraq, Hoffman 2004), government forces have been confronted by relatively small insurgency groups diffused in the general population. In terms of physical net assessment, insurgents are no match to a government force that is typically an order of magnitude larger than the insurgents (as of 2007 there are more than 500,000 Coalition and Iraqi Security Forces in Iraq, whereas the estimate for the number of insurgents ranges between 15,000 and 70,000; O'Hanlon and Cambell 2007). Also, the government force is usually better equipped and trained than the insurgents. The key advantage of the insurgents is their elusiveness and invisibility; the government troops have the military means and capabilities to effectively engage the insurgency targets, but they have difficulties finding them. Thus, although intelligence is a key component in any conflict situation, it is critical in counterinsurgency operations. Absent intelligence, not only might the guerrillas be able to continue their insurgency actions, but collateral damage caused to the general population from poor targeting by the government forces may generate an adverse response against the government, thus creating popular support for the insurgents. This popular support translates into new cadres of recruits to the insurgency (Lynn 2005, Hammes 2006).

Eliminating the insurgency is the main goal of the regime. The July 12, 2007 White House report to Congress entitled Initial Benchmark Assessment Report (U.S. Congress 2007, p. 4) states:

> We presently assess that degrading al Qaida in Iraq networks in these critical areas—together with efforts to degrade Iranian-backed Shi'a extremist networks—is a core U.S. national security interest and essential for Iraq's longer-term stability.

We develop a dynamic model that describes the effect of key parameters on the outcome of an insurgency: Force sizes (government and insurgency), attrition rates, recruitment (to the insurgency) rate, reinforcement (to the government) rate, and most of all—intelligence. We consider a scenario where the government confronts a single homogeneous insurgency. Our model and analysis represent a *best-case* situation from the government perspective because (a) the government force is steadily reinforced by new units, (b) it has unlimited endurance—it surrenders to the insurgents only when it is totally annihilated, and (c) the only recruitment to the insurgency is due to collateral casualties in the general population that generate resentment to the government, and therefore more recruits to the insurgency. In our model there is no ongoing "ideological" recruitment to the insurgency. Although the assumptions above lead to a best-case situation for the government, one could argue that it is not *the* best case; if the insurgents have limited endurance and they deliberately attack the population, and thus undermine their popular support, the situation would be even better for the government. However, one would expect that the endurance of insurgents, who are motivated by ideology and zeal, would be at least as high as that of the government. Moreover, the insurgents, who grow from the population, would know if coercive actions enhance recruitment or hinder it; they will not act deliberately against their own interests. Either violent coercive actions by the insurgents against the population increase their support and recruitment, in which case our assumptions are still best case because in our model we do not account for those extra insurgency cadres, or such attacks are mostly a result of interinsurgency feuds (e.g., Shiite against Sunni in Iraq.), which is not exactly the scenario studied in this paper.

As far as we know, this is the first attempt to model the effect of intelligence in a dynamic combat setting in general, and counterinsurgency in particular. The closest relevant work is the dynamic guerrilla warfare model introduced by Deitchman (1962) and followed by Schaffer (1968). Both papers are based on classical Lanchester equations (Lanchester 1916). Another body of relevant research studies dynamic models that describe the proliferation of fanatic ideas and terrorism (Castillo-Chavez and Song 2003, Udwadia et al. 2006). The relationship between inaccurate government fire and insurgency recruiting is discussed in Jacobson and Kaplan (2007), Kaplan et al. (2005), and Caulkins et al. (2006). Our model, which combines intelligence, attrition, and popular support, manifested in recruits to the insurgency, sheds new light on the dynamics of counterinsurgency operations, and it shows why it is almost impossible to eradicate insurgency by force only—"soft" actions such as civil support and psychological operations, that affect the attitude of the population, may be needed too. This conclusion is quite general and robust; it does not depend on the specific parameters of a particular insurgency situation, but on general assumptions regarding their characteristics. Many of these parameters, like attrition rates to government forces, intelligence levels and recruitment rates to the insurgency, are typically classified or unavailable.

2. Model

There are two explicit players in our model—the insurgents and the government force that fight each other—and one implicit player—the general population that sustains collateral casualties by the government's actions and provides new recruits to the insurgency. The combat situation is asymmetric; although the insurgents have perfect situational awareness regarding the government forces, the insurgents are mixed in the general population, and thus their signature as targets is inversely related to the size of the population in which they are imbedded. It follows that the effectiveness of the government not only depends on its force size and its effectiveness, but also on the insurgents' signature. Moreover, for a given level of combat intensity exerted by the government forces, smaller signature of the insurgency results in higher collateral damage—killing innocent bystanders—with an adverse effect to the government and favorable effect to the insurgency.

Let G, I, and P denote the sizes of the government forces, the insurgency, and the general population, respectively. Although G and I may vary over time, we assume that the size of the general population remains constant throughout. Absent any intelligence, the signature of the insurgency is measured by the ratio I/P, which may be interpreted as the probability that a randomly selected target is an insurgent. Following Deitchman (1962), the Lanchester model (Lanchester 1916) that describes this combat situation is

$$\dot{G} = -\alpha I \tag{1}$$

and

$$\dot{I} = -\gamma G \frac{I}{P}, \tag{2}$$

where α and γ are the attrition coefficients, which should be interpreted as the general intensity and effectiveness of insurgency and counterinsurgency operations, respectively.

Let $\mu \in [0, 1]$ denote the level of intelligence, which may be interpreted as the fraction of intelligence reports that correctly identify the location of insurgents. A fraction $1-\mu$ of these reports are erroneous to the point that the intelligence provided is completely useless; in these instances the government practically "shoots in the dark." However, the government force does not know a priori which report is which, and therefore it engages all targets with uniform vigor. In that case, the insurgency attrition becomes

$$\dot{I} = -\gamma G\left(\mu + (1-\mu)\frac{I}{P}\right). \tag{3}$$

With perfect intelligence ($\mu = 1$), we obtain the classical Lanchester's Square Law of aimed fire (Lanchester 1916), and absent intelligence ($\mu = 0$), we obtain Deitchman's guerrilla model (Deitchman 1962) in Equation (2). The initial conditions are G_0 and I_0, and the terminating conditions are the force endurance thresholds $\bar{G}$ and $\bar{I}$ at which the government and the insurgents declare defeat, respectively. We assume that both the government and the insurgency have unlimited endurance, that is, $\bar{G} = \bar{I} = 0$. Although for a determined insurgency this may be a reasonable assumption, for the government it clearly represents a best case. The engagement (insurgency) ends when either threshold is reached.

It is postulated that the main driver for population behavior in insurgency situations is security (Lynn 2005, Hammes 2006, U.S. Army 2006); the population will align with the side that is perceived as better protecting it, or is at least less threatening. We assume no sectarian or coercive violence, and therefore the insurgents do not deliberately attack the population; their attack is focused on the government force. Because the insurgents have perfect situational awareness, they do not harm the general population. On the other hand, absent perfect intelligence, the government forces may cause casualties in the population when missing their insurgency targets. This collateral damage triggers support to the insurgency, which is manifested in new recruits to their ranks. Let $\theta(C)$ denote the insurgency recruitment rate, where C represents the rate at which collateral casualties are generated. From Equation (3), it follows that $C = \gamma G(1-\mu)(1-I/P)$. We assume that θ is monotone increasing and, absent ideological recruitment to the insurgency, $\theta(0)=0$. Equation (3) becomes

$$\dot{I}=-\gamma G\left(\mu+(1-\mu)\frac{I}{P}\right)+\theta\left(\gamma G(1-\mu)\left(1-\frac{I}{P}\right)\right). \quad (4)$$

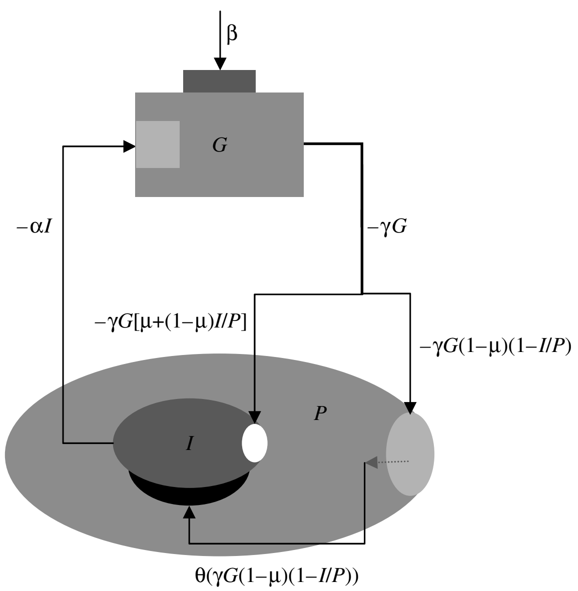
Finally, we assume that there is a steady stream of reinforcement to the government force. Thus, (1) is replaced by

$$\dot{G}=-\alpha I+\beta. \quad (5)$$

The dynamical system under consideration is the set of Equations in (4) and (5). The dynamics of G and I are illustrated in Figure 1.

If the level of intelligence μ is constant throughout and the insurgency recruitment rate $\theta(C)=\theta C$ is a linear function, a simple analysis (additional details are provided in the appendix) shows that if $\mu \leqslant \theta/(1+\theta)$, then the insurgency cannot be eradicated, regardless of the initial force sizes and attrition rates. If $\beta/\alpha < P(1-((1-\mu)\cdot(1+\theta))^{-1})$, then the government loses, and if the opposite is true, then the government can only control or *contain* the insurgency at a constant level $P(1-((1-\mu)(1+\theta))^{-1})$. If $\mu > \theta/(1+\theta)$, then the insurgency wins if $I_0 > \beta/\alpha + G_0(\gamma(\mu-\theta(1-\mu))/\alpha+\gamma\beta(1-\mu)(1+\theta)/P\alpha^2)^{1/2}$, and the government wins (eradicates the insurgency) otherwise.

Figure 1. The dynamics of the insurgency-counterinsurgency process.



Notes. The insurgency (I, dark grey) causes attrition (grey) to the government (G, medium grey) at a rate αI. The rate of reinforcement to the government is β (dark grey). The government operates with intensity γG where a fraction $\mu+(1-\mu)I/P$ of this intensity hits the insurgents (white), and a fraction $(1-\mu)(1-I/P)$ inadvertently hits the general population (light grey). The collateral casualties generate recruits to the insurgency (black) at rate $\theta(\gamma G(1-\mu)(1-I/P))$. The number of collateral casualties is very small compared to the size of the population, and therefore we assume that P remains constant throughout.

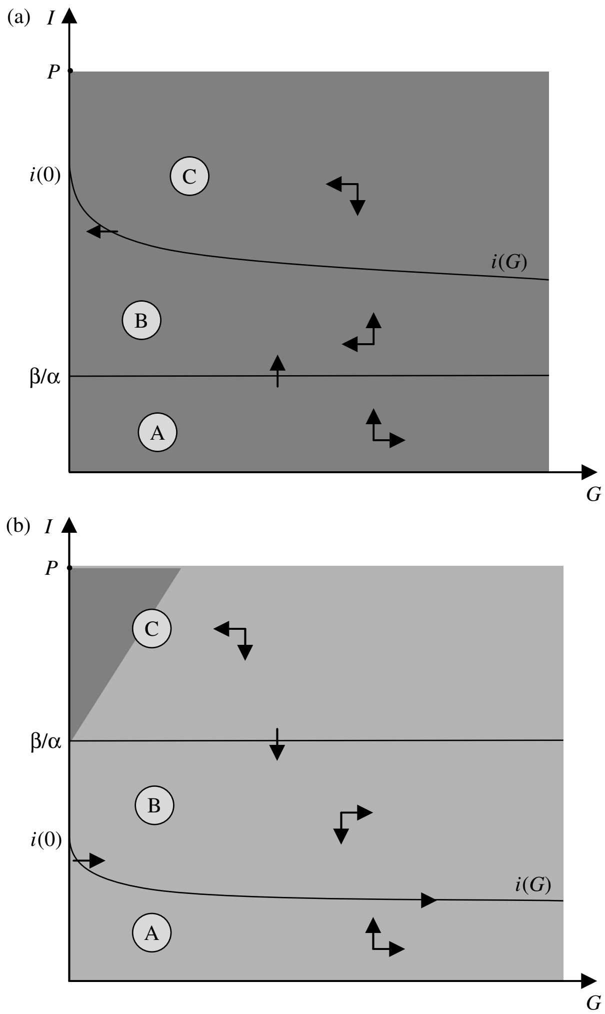
Although a constant intelligence level is a reasonable approximation to reality when the counterinsurgency operation is in some stable state, μ may become negligible if the number of insurgents is very small (either when the insurgency gets started, or when it has been weakened and fragmented into a small number of cells), or when the government force is weakened to the point that it loses its intelligence-gathering capabilities. Thus, it is natural to assume that $\mu=\mu(G,I)$ is monotone increasing in I and nondecreasing in G, and that $\mu(G,I)\to 0$ as either G or I go to zero. As we discuss in the next section, these assumptions imply that the government can, at best, *contain* the insurgency—control it at a certain fixed level. The government can never totally eradicate the insurgency by force. However, in some favorable situations, discussed below, regarding the characteristics of the intelligence and insurgency recruitment functions, the containment level can be very small, leading effectively to insurgency neutralization.

In Equation (4), $\dot{I}=0$ if $\gamma G(\mu+(1-\mu)I/P)=\theta(\gamma G(1-\mu)(1-I/P))$. If the functions μ and θ are continuously differentiable, based on our previous assumptions and the implicit function theorem, there is a continuously differentiable function $i(G)$ satisfying

$$-\gamma G\left(\mu(G,i(G))+(1-\mu(G,i(G)))\frac{i(G)}{P}\right)+\theta\left(\gamma G(1-\mu(G,i(G)))\left(1-\frac{i(G)}{P}\right)\right)=0, \quad (6)$$

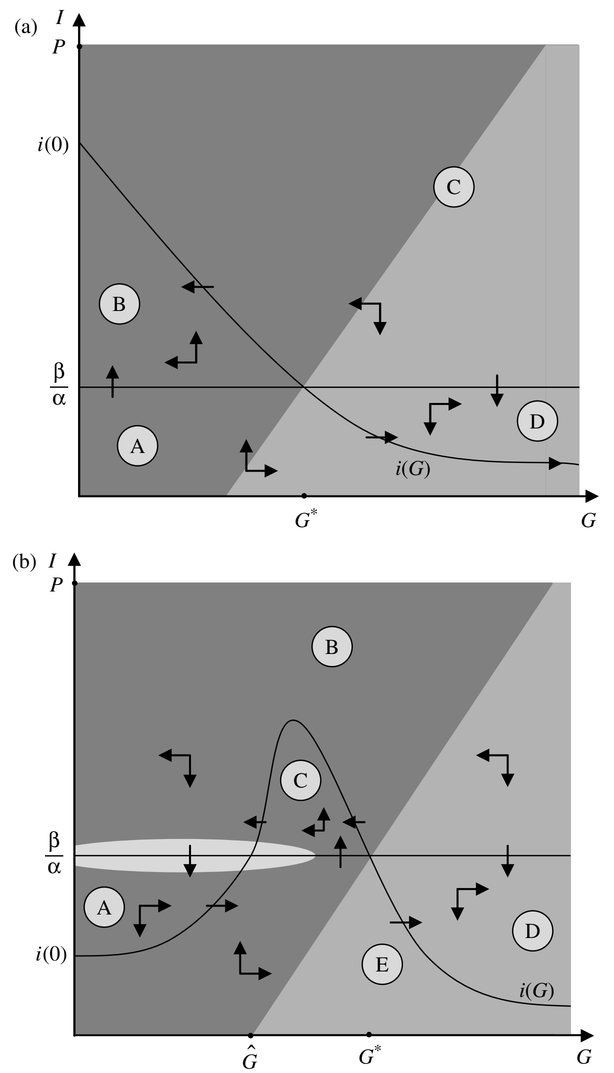
where we define $i(0)=\lim_{G\downarrow 0} i(G)$. The function $i(G)$ separates the region where the insurgency grows ($\dot{I}>0$) from the region where the insurgency gets smaller; see Figures 2 and 3. The shape of $i(G)$ depends on the functions μ and θ, but in all cases $i(G)$ is bounded away from zero for all G. This property follows from Equation (6) and the boundary conditions of μ, for if $i(G)=0$ for some $G>0$, then $\theta(\gamma G)=0$, in contradiction to the monotonicity of θ. Thus, the insurgency can never be totally eradicated physically; there is always a range of values of I where the insurgency grows. The operational explanation for this phenomenon is as follows: When the insurgency is small, the intelligence available to the government is poor, and therefore, when attempting to attack the insurgents, the government force inadvertently causes collateral innocent casualties in the population. These casualties generate popular resentment toward the government, and eventually new cadres to the insurgents. The attrition to the insurgency generated by higher values of G is offset by increased recruitment. If for some range of C the recruitment to the insurgency accelerates with the number of per-unit-time collateral casualties, and the growth more than makes up the increase in attrition of the insurgents, then $i(G)$ may actually increase, as shown in Figure 3(b). However, as the collateral casualties increase, the sensitivity of the population to these casualties necessarily ebbs due to population constraints; that is, $\theta'(C)$, the derivative of the recruitment function w.r.t. the casualty

Figure 2. Case Lose in (a), and case Contain in (b).



Note. The insurgency wins in the dark-grey region, and the government contains the insurgency in the light-grey region.

Figure 3. Two variations of case Lose/Contain.



Note. The insurgency wins in the dark-grey region, and the government contains the insurgency in the light-grey region.

rate C, converges as C grows (e.g., $\theta(\cdot)$ sigmoid, logarithmic, or linear). If, as one would expect, the intelligence function μ levels off as G increases, then $i(G)$ approaches a constant level (appendix); see Figures 2 and 3. Under some extremely favorable conditions for the government, e.g., when μ sharply increases for small values of I, this constant level of $i(G)$ may be close to zero, meaning that the insurgency can be weakened to the point that it becomes insignificant.

3. Results

For any intelligence and recruitment functions satisfying the assumptions made in §2, the insurgency-counterinsurgency dynamics must fall into one of three scenarios, labeled Lose, Contain, and Contain/Lose. In Lose, the intelligence-gathering capabilities of the government are so poor, the recruitment to the insurgency is so high, or the government reinforcement rate is so low that for any initial conditions the insurgency always defeats the government. In Contain, the government reinforcement rate is relatively high, and therefore it can contain the insurgency and keep it at a constant level that, depending on the intelligence and recruitment functions, may be very low. Lastly, Lose/Contain is the scenario that, in the authors' view, most closely matches reality. In this scenario the government can either contain the insurgency or lose, depending on the model parameters, on the intelligence function μ, and on the insurgency recruitment function θ.

The scenario Lose occurs when $i(G) > \beta/\alpha$ for all $G \geqslant 0$. In this case the insurgency wins regardless of the initial conditions and the government forces attrition rate. Consider the three regions labeled A, B, and C in Figure 2(a). In region A both the government forces and the insurgency grow over time. In region B the insurgency still grows and reaches a size large enough to cause severe attrition to the government forces, which ultimately get beaten. In region C both forces decrease in size. If the insurgency force is initially large, it can effectively attack the government force. However, the large signature of the insurgency also enables the government force to effectively combat the insurgents. In this case the government loses while the insurgency force weakens. On the other hand, if the insurgency force is initially low, the government efforts cause significant collateral damage, bolstering the insurgency to the point that the government debilitates and eventually collapses. Therefore, regardless of the insurgents' initial force level, when the intelligence-gathering capabilities of the government grow slowly with the insurgents' strength, or the resentment generated by civilian casualties caused by government actions is not negligible, or the government reinforcement rate β is too low, the government is bound to lose, even with very favorable initial force ratios or attrition coefficients.

The scenario Contain arises if $\beta/\alpha \geqslant i(G)$ for all $G \geqslant 0$. This case is illustrated in Figure 2(b). Similarly to Lose, in region A both the government forces and the insurgency grow over time. In region B the government forces grow and the insurgency weakens. Because $i(G)$ is always bounded away from zero, the engagement reaches a point where the government controls the insurgency at a certain constant level. In region C, where both sides decrease in size, two cases are possible. In the first case the solution crosses into region B, leading eventually to controlling the insurgency (light-grey region). In the second case, marked by a dark-grey region, the insurgency wins. We show in the appendix that the boundary of the dark-grey region, in which the insurgency wins, is well approximated by

$$I > \begin{cases} \dfrac{\beta}{\alpha} + G\sqrt{\dfrac{\gamma}{\alpha}\left[(1+\theta'(0))\dfrac{\beta}{\alpha P} - \theta'(0)\right]} \\ \qquad \text{if } \dfrac{\beta}{\alpha P} > \dfrac{\theta'(0)}{1+\theta'(0)}, \\ \beta/\alpha, \quad \text{otherwise.} \end{cases} \tag{7}$$

As β/α decreases, the slope of the line determined by Equation (7) approaches zero, to the point that the dark-grey region covers the entire area determined by $I > \beta/\alpha$. Arguably, Contain is not very likely because it presumes unrealistically high government reinforcement rate. If $\theta(C) = \theta C$ for C small, then $\beta \geqslant \alpha i(0) = \alpha P\theta/(1+\theta)$. Based on Iraq data (O'Hanlon and Cambell 2007), $P = 27$ M, and reasonable estimates for α and θ are 0.01 and 10, respectively. These values lead to a lower bound of 270,000 on the rate of reinforcement—more than one-half the current total government plus coalition forces in Iraq—for scenario Contain to result.

The scenario Lose/Contain occurs when $i(G)$ intersects with $I = \beta/\alpha$. This situation is illustrated in Figure 3. In Figure 3(a), $i(G)$ is nonincreasing, and in Figure 3(b), $i(G)$ is not monotone.

In the case of Figure 3(a), points in regions A or C contained in the dark-grey area lead to region B, where the insurgency wins. The light-grey area of regions A or C lead to region D, where the government controls the insurgency. It is shown in the appendix that the dark-grey area in Figure 3(a) is approximately bounded by the line

$$I = \frac{\beta}{\alpha} + (G - G^*)\frac{-a_{22} + \sqrt{a_{22}^2 - 4\alpha a_{21}}}{2\alpha}, \tag{8}$$

where a_{21} and a_{22} are positive constants, and $i(G^*) = \beta/\alpha$; see the appendix for more details.

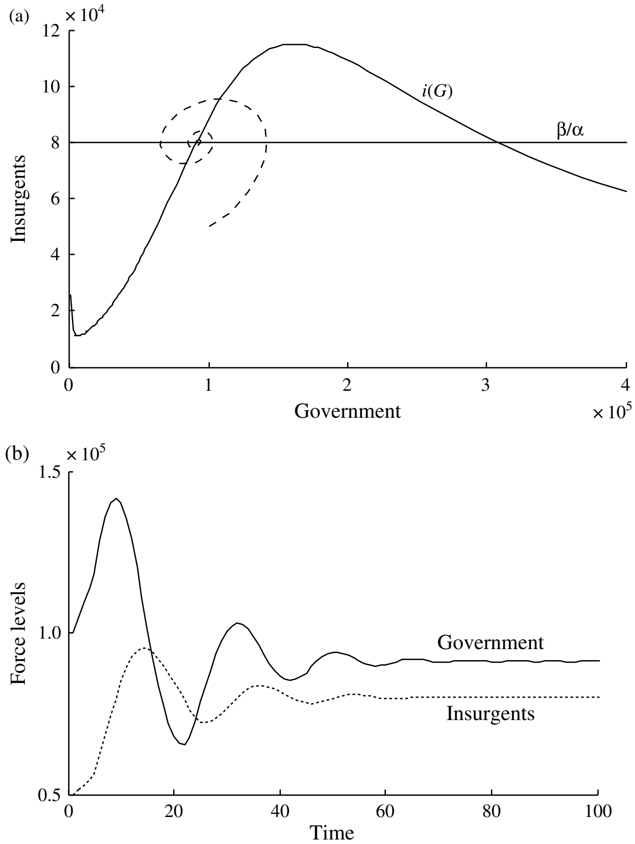
Concerning the situation of Figure 3(b), the government's strength decreases when $I > \beta/\alpha$, and increases when $I < \beta/\alpha$. When $I > i(G)$, the insurgency weakens, whereas the insurgency gains strength when $I < i(G)$. Points in the light-grey area lead to containment of the insurgency, whereas points in the dark-grey area cause the insurgency to win, except for the light-grey ellipsoid, where the points converge possibly in oscillating manner to the stalemate point $(G, I) = (\hat{G}, \beta/\alpha)$. This oscillating convergence to a stalemate is due to the exponential recruitment of insurgents that results from the killing of innocent victims in the vicinity of the deadlock point, and is rationalized as follows: When I is small and G large, poor intelligence results in many collateral casualties in the population, thus strengthening I ($\uparrow$ arrow from region E to region C). Thusly strengthened, the insurgents generate effective attrition to the government, making G decrease ($\leftarrow$ arrow from C to B). The increase in I (and thus μ) also enables the government to target the insurgents more accurately, causing I to decrease ($\downarrow$ arrow from B to A). Finally, the reduced number of insurgents enables the government to regain its strength ($\rightarrow$ arrow from A to E). In the appendix we show that closed orbits, where G and I oscillate without ever spiraling toward $(\hat{G}, \beta/\alpha)$, are not possible—there are only two feasible outcomes: Either the government loses, or it contains the insurgency. In mathematical parlance, the stalemate point $(\hat{G}, \beta/\alpha)$ in Figure 3(b) is an *attractor*, and the point $(G^*, \beta/\alpha)$ is a *saddlepoint*. Figures 4(a) and 4(b) depict the transient dynamics of G and I that lead to that point. Figure 4(a) is the phase-plane portrait, whereas Figure 4(b) presents the transient behavior of the two sides.

4. Discussion and Conclusions

The three scenarios lead to several observations. First, the government always loses if there is no reinforcement to its force because the intelligence capabilities of the government degrade with the attrition of its force, causing many innocent casualties and indirectly strengthening the insurgency, which eventually takes over.

Second, there may exist more than one stalemate scenario—one in the decreasing part of $i(G)$, and another in the increasing part of $i(G)$; see Figure 3(b). A stalemate in the part where $i(G)$ decreases may necessitate a large government force. However, it is a safe stalemate for the government because it is robust; if the reinforcement

Figure 4. Phase-plane portrait of the stalemate point in (a), and transient dynamics in (b).



rate is insufficient, it would take longer for a large government force to be defeated before the β can be corrected. In our model the reinforcement rate β is fixed, which leads to unbounded value of G in a containment scenario. We do not expect that this situation could occur because once the insurgency approaches its stable contained level, one of two things may occur: Either the insurgency, realizing it cannot grow, settles with the government and the insurgency situation ends, or the government reduces its reinforcement rate such that a stalemate situation is reached where G remains constant too. When the population is very sensitive to innocent casualties, and as a result the recruitment to the insurgency accelerates, the government and insurgents may approach a stalemate in an oscillating manner. The opposite is also true: If there is oscillating convergence toward a stalemate, the population must be very responsive to unintentional civilian casualties. If the government recruitment rate β decreases, then the stalemate force levels of both G and I decrease (see Figure 3(b)), resulting in less violence and a smaller government force. The downside of a lower stalemate point is that a smaller government force has a higher risk of losing if the combat situation changes, or if the parameters of the insurgency are poorly estimated.

Last, recall that our model represents a best-case situation from the government perspective. Under the reasonable assumptions regarding the behavior of the intelligence function μ, we conclude that the government cannot completely eradicate the insurgency by force alone. If the government can gather significant accurate intelligence when the insurgency is very small, it can reduce the insurgency to a small manageable size. Finally, "soft" actions such as reconstruction, civil support, and effective propaganda may positively affect the population support for the government and thus improve intelligence (increase the value of μ) obtained from human sources. Such actions can only improve the prospects of defeating the insurgency.

Appendix

This section brings theoretical support to several results stated without proof in the main body of the paper.

A. The Function $i(G)$ Levels Off as G Grows

We have $i'(G)=0$ if and only if the marginal increase in insurgency recruitment equals the marginal increase in attrition, that is,

$$\frac{d\theta(C)}{dG}=\frac{d(\gamma G(\mu(G,I)+(1-\mu(G,I))I/P))}{dG},$$

where (recall) $C=\gamma G(1-\mu(G,I))(1-I/P)$, and (G,I) are such that

$$\gamma G\big(\mu(G,I)+(1-\mu(G,I))I/P\big)$$
$$=\theta\big(\gamma G(1-\mu(G,I))(1-I/P)\big). \tag{9}$$

Using the chain rule and elementary manipulations, the above becomes

$$\big(1-\mu(G,I)-G\partial\mu(G,I)/\partial G\big)$$
$$\cdot(1-I/P)(1+\theta'(C))-1=0, \tag{10}$$

where $\theta'=d\theta/dC$. Therefore, if the left-hand side (LHS) of Equation (10) converges to zero as G grows, we have that $i(G)$ levels off.

Suppose that $G\partial\mu(G,I)/\partial G\to 0$. If $\theta'(C)\to 0$, Equation (9) causes I to decrease as G increases. The same also is true about μ for G sufficiently large. Therefore, the LHS of Equation (10) converges to zero as G grows. If $\theta'(C)\to\eta>0$, then Equation (9) becomes $\mu(G,I)+(1-\mu(G,I))I/P=\eta(1-\mu(G,I))(1-I/P)+o(G)$, where $o(G)$ denotes a function $h(G)$ such that $h(G)/G\to 0$ as $G\to\infty$. Therefore, the LHS of Equation (10) converges to zero as G grows, with η in place of $\theta'(C)$.

B. The Case of Constant μ and Linear θ

Consider the case where the level of intelligence μ is constant throughout and the recruitment function $\theta(C)=\theta C$

is linear. If $\mu \leqslant \theta/(1+\theta)$, then $i(G) = P(1-((1+\theta)\cdot(1-\mu))^{-1})$ for all Gs, and we have three possibilities: (i) $\beta/\alpha < P(1-((1-\mu)(1+\theta))^{-1})$, so that we fall in case Lose; (ii) $\beta/\alpha > P(1-((1-\mu)(1+\theta))^{-1})$, which leads to case Contain; and (iii) $\beta/\alpha = P(1-((1-\mu)(1+\theta))^{-1})$ gives rise to a stalemate, where both G and I approach a constant positive force level. When $\mu > \theta/(1+\theta)$, a linearization around $(0, \beta/\alpha)$ produces the Jacobian

$$\begin{pmatrix} 0 & -\alpha \\ \gamma((1+\theta)(1-\mu)(1-(\beta/\alpha)/P)-1) & 0 \end{pmatrix},$$

with eigenvalues

$$\pm\big(\alpha\gamma\big(1-(1+\theta)(1-\mu)(1-(\beta/\alpha)/P)\big)\big)^{1/2}.$$

The eigenvector associated with the negative eigenvalue is $V^{(-)} = (1, v^{(-)})$, where $v^{(-)} = ((\gamma/\alpha)(1-(1+\theta)(1-\mu)\cdot(1-(\beta/\alpha)/P)))^{1/2}$. Therefore, I wins if $I_0 > \beta/\alpha + G_0((\gamma/\alpha)(\mu-\theta(1-\mu)) + \gamma\beta(1+\theta)(1-\mu)/P\alpha^2)^{1/2}$ in a vicinity of $(G, I) = (0, \beta/\alpha)$.

C. Dark-Grey/Light-Grey Region Boundary in Case Contain

A linearization around $(0, \beta/\alpha)$ leads to the Jacobian

$$\begin{pmatrix} 0 & -\alpha \\ \gamma(\theta'(0)-(1+\theta'(0))(\beta/\alpha)/P) & 0 \end{pmatrix},$$

where we use the fact that μ equals zero along either axis. Therefore, the eigenvalues are $\pm(\alpha\gamma(\theta'(0)-(1+\theta'(0))(\beta/\alpha)/P))^{1/2}$. If $(\beta/\alpha)/P > \theta'(0)/(1+\theta'(0))$, the solutions approach $(0, \beta/\alpha)$ along the direction of the eigenvector $V^{(-)} = (1, v^{(-)})$ associated with the negative eigenvalue, where $v^{(-)} = ((\gamma/\alpha)(\theta'(0)-(1+\theta'(0))\cdot(\beta/\alpha)/P))^{1/2}$. Therefore, I wins if $(I-\beta/\alpha)/G > v^{(-)}$ in a neighborhood of $(0, \beta/\alpha)$, which is Equation (7). Moreover, $v^{(-)} \to 0$ as $(\beta/\alpha)/P - \theta'(0)/(1+\theta'(0)) \to 0$. Thus, when $(\beta/\alpha)/P \leqslant \theta'(0)/(1+\theta'(0))$, the insurgents win if $I > \beta/\alpha$ in the vicinity of $(0, \beta/\alpha)$.

D. Dark-Grey/Light-Grey Region Boundary in Case Lose/Contain of Figure 3(a)

We linearize around $(G^*, \beta/\alpha)$, which produces the Jacobian

$$\begin{pmatrix} 0 & -\alpha \\ a_{21} & a_{22} \end{pmatrix},$$

where

$$a_{21} = -\gamma(1+\theta'(C))\left[G^*\frac{\partial\mu(G^*,\beta/\alpha)}{\partial G}\left(1-\frac{\beta/\alpha}{P}\right) + \mu(G^*,\beta/\alpha) + (1-\mu(G^*,\beta/\alpha))(\beta/\alpha)/P\right] + \gamma\theta'(C)$$

and

$$a_{22} = -\gamma G^*(1+\theta'(C))\left(\frac{\partial\mu(G^*,\beta/\alpha)}{\partial I}\left(1-\frac{\beta/\alpha}{P}\right) + \frac{1-\mu(G^*,\beta/\alpha)}{P}\right).$$

Leaving aside the trivial case where $\mu(G^*,\beta/\alpha)=1$, we have $a_{22}<0$ and, because $i'(G^*)<0$, $a_{21}<0$. The eigenvalues are $(a_{22}\pm(a_{22}^2-4\alpha a_{21})^{1/2})/2$, meaning that $(G^*,\alpha/\beta)$ is a saddlepoint. The eigenvector associated with the negative eigenvalue is $V^{(-)}=(1,v^{(-)})$, where $v^{(-)}=(-a_{22}+(a_{22}^2-4\alpha a_{21})^{1/2})/(2\alpha)$. Therefore, in a neighborhood of $(G^*,\beta/\alpha)$, I wins if (G,I) lie to the left of the line with slope $v^{(-)}$ that passes through $(G^*,\beta/\alpha)$. An elementary algebraic manipulation shows that solutions in this region satisfy $(I-\beta/\alpha)/(G-G^*)>v^{(-)}$, in agreement with Equation (8).

E. Properties of the Equilibrium Point $(\hat{G},\beta/\alpha)$ in Figure 3(b)

A linearization around $(\hat{G},\beta/\alpha)$ generates a Jacobian

$$\begin{pmatrix} 0 & -\alpha \\ b_{21} & b_{22} \end{pmatrix},$$

where b_{21} and b_{22} are defined like a_{21} and a_{22}, respectively, with $\hat{G}$ in place of G^*. We have $b_{22}<0$, and $b_{21}>0$ because $i'(\hat{G})>0$. Therefore, the eigenvalues are both negative real if $b_{22}^2-4\alpha b_{21}>0$, and complex with negative real part otherwise. In either case, the equilibrium point $(\hat{G},\beta/\alpha)$ is asymptotically stable. Moreover, the Dulac function $(1,(-\gamma G(\mu+(1-\mu)I/P)+\theta(\gamma G(1-\mu)\cdot(1-I/P)))^{-2})$ together with the Dulac-Bendixon theorem (Perko 2001) show that the system $\dot{G}=-\alpha I+\beta$ and $\dot{I}=-\gamma G(\mu+(1-\mu)I/P)+\theta(\gamma G(1-\mu)(1-I/P))$ admits no periodic orbits on R_+^2. This justifies our assertion that the only possible outcomes in this scenario are Lose and Contain.

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