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Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings

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Abstract The goal of this article is to present a sketch of what, following the German social theorist Arnold Gehlen, may be termed "sensuous cognition." The starting point of this alternative approach to classical mental-oriented views of cognition is a multimodal "material" conception of thinking. The very texture of thinking, it is suggested, cannot be reduced to that of impalpable ideas; it is instead made up of speech, gestures, and our actual actions with cultural artifacts (signs, objects, etc.). As illustrated through an example from a Grade 10 mathematics lesson, thinking does not occur solely *in* the head but also *in* and *through* a sophisticated semiotic coordination of speech, body, gestures, symbols and tools.

Keywords Cognition · Gestures · Graphs · Objectification · Multimodality · Mathematical meaning · Semiotics

The first sign of language is usually a gesture
C. Kenneally, 2007, p. 133

Right after the presentation of a research report on gestures and mathematics, someone seated in the row in front of me whispered to his neighbour: "I really don't see the connection with mathematics and learning!"

Skepticism about the role of gestures in the learning and teaching of mathematics is rooted, I want to suggest, in a historical tradition that, implicitly or explicitly, and to different extents, most of us share about cognition. In this long-standing tradition, it is assumed that thinking is a pure mental activity – something *immaterial*, independent of the body, occurring *in* the head. According to this viewpoint, it naturally follows that gestures in particular and the body in general do not to have a relevant cognitive import. If something can enlighten us about the intricacies of students' mathematical thinking and the depth of their knowledge, it is not the gestures that they make or the actions that they carry out but the symbols that they write. Tests, quizzes and other contemporary forms of

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knowledge assessment are based on this belief, shaped by rationalist and idealist epistemologies over the course of centuries.

My point here is not to diminish the cognitive role of the written. It is rather an invitation to entertain the idea that mathematical cognition is not only mediated by written symbols, but that it is also mediated, in a *genuine sense,* by actions, gestures and other types of signs.

Although contemporary theories in mathematics education may not be incompatible with the cognitive potential of gestures and kinesthetic actions, the problem of the role of gestures in learning can hardly be solved by merely admitting that gestures and the body have "something" to do with cognition. We still have to determine *how* gestures relate to learning and thinking. Now, in order to accomplish this, we need to specify the cognitive role of the body and the senses. And, of course, this is no small endeavour. Piaget, for instance, set down the basis of an epistemology that sought to theorize the role of the senses in concept formation. Drawing on Kant – who attempted a reconciliation between empiricist and rationalist trends – Piaget emphasized the important role of sensory–motor actions. However, although cognitively important, for Piaget the sensory–motor realm appeared as an ephemeral route towards abstract thinking: if body and artifacts played an epistemological role, it was indeed only that of highlighting the logical structures that supposedly underlay all acts of knowing. As a result, Piaget's epistemology – undoubtedly one of the most influential epistemological theories of the twentieth century – excluded gestures and the body from the analysis of abstract thinking – and so too did the educational theories related to it.

The goal of this article is not to provide a detailed account of mathematical cognition and the place of gestures therein. The goal is much more modest. It is to present a rough sketch of what, following Gehlen (1988), I want to term "sensuous cognition." This alternative sketch to mental-oriented views of cognition is the purpose of Section 2, which is preceded by a brief overview of current discussions about gestures (Section 1). In Section 3, I deal with the problem of learning and discuss the role of gestures through a classroom example. In the course of the article, I will argue that gestures, considered in isolation, have a very limited cognitive scope and that the cognitive possibilities of gestures can only be understood in the broader context of the interplay of the various sensuous aspects of cognition as they unfold against the background of social praxes.

1 Gestures: why and what for?

During the nineteenth century, in keeping with evolutionary ideas, some anthropologists claimed that gestures were a kind of precursor to language. Frank Cushing, in his studies on the Zuñi language, went on to suggest that "single terms or monophrastic words of many sorts have been single-hand made, and sentence–words or holophrastic terms have as often been double-hand or gesture made" (Cushing 1892, p. 291). Indeed, the names for numbers and other names in language reflected, according to Cushing's analysis, an underlying gestural origin. The concepts behind such terms were, as he called them, *manual concepts*.

In the first part of the twentieth century, studies on gestures experienced a kind of decline until a new impetus arose, in part as the result of attempts at teaching chimpanzees to use a kind of Sign Language, as well as the spread of a renewed interest in understanding the origins of language (Deacon 1997; Kendon 2004; Savage-Rumbaugh and Lewin 1994; Tomasello and Call 1997).

Despite a significant and growing contemporary interest in gestures, many unresolved problems remain. For example, the exact role of gestures is still a vigorously debated problem among contemporary psychologists and linguists.[1] Some researchers consider gestures as facilitators of verbal expression (Freedman 1977). Proponents of this view contend that people gesture in the course of their search to achieve a more adequate verbal expression. As Goldin-Meadow, Nusbaum, Kelly, & Wagner (2001) suggest, "gesturing may prime a speaker's access to a temporarily inaccessible lexical item and thus facilitate the processing of speech" (p. 521). For others, gestures and speech are part of the same cognitive source. Gestures can reveal aspects of the mental contents of the speaker to us. McNeil, who has been one of the promoters of this idea, suggests that, through gestures, "people unwittingly display their inner thoughts and ways of understanding events of the world" (McNeill 1992, p. 12). Gestures, according to this approach, are a kind of window to accessing thinking (see also Alibali, Bassok, Solomon, Syc, & Goldin-Meadow 1999; Garber & Goldin-Meadow 2002). Kita (2000) presents a different perspective. He suggests that gestures and speech are not part of the same cognitive source and argues that gestures derive from virtual actions made by the speaker in a virtual space of acting on the objects of discourse.

Differences in perspective about the role of gestures in cognition have, to an important extent, to do with theoretical conceptions about thinking. Conceiving of thinking as something intrinsically mental easily leads to thinking of gestures as "windows" on inner thoughts or as conveyors of ideas that are already somewhere in the mind awaiting the proper material, namely, verbal expression.

The perspective that I am advocating here takes a different point of departure. Its starting point is a "material" or "textual" conception of thinking. The very texture of thinking, I want to suggest, cannot be reduced to that of impalpable mental ideas; it is also made up of speech and our actual actions with objects and all types of signs. Thinking, hence, does not occur solely *in* the head but *in* and *through* language, body and tools. As a result and from this perspective, gestures, as a type of bodily action, are not considered as a kind of window that illuminates the events occurring in a "black box" – they are not clues for interpreting mental states. They are rather *genuine constituents* of thinking.

In the next section, in tune with some anthropologists and cultural psychologists (Geertz 1973; Vygotsky 1986; Wertsch 1991), and drawing on the work of the social theorist Arnold Gehlen (1988), I present a sensuous conception of thinking that is at variance with the standard mental conception of it.

2 Toward a sensuous–cultural conception of cognition

In his seminal book, *Man: His Nature and Place in the World*, Gehlen (1988) suggested that humans, unlike other animals, have been denied a secure existence guided by instincts. Gehlen – Jürgen Habermas' teacher and one of the representatives of what came to be known in the twentieth century as *Anthropological Philosophy* – reasoned that while other animals are endowed with specific instincts and highly developed sensorial systems that they enjoy from birth, the human senses, in contrast, are highly unspecialized. And yet the human senses, Gehlen argued, are the species' very means of survival. In order to continue to exist, humans have to create shelters, stock food, plan and so on. In other words, humans

[1] See e.g. Goldin-Meadow (2003), Kendon (1981, 1993), Kita (2003), McNeill (2005, 2000). For a general overview of gestures and their potential in teaching and learning see Roth (2001).

have to act in the world through thought and imagination, which, for Gehlen, are versatile and sophisticated forms of sensuous action.

In Gehlen's account, the lack of specialization of the human senses is compensated for by a kind of plasticity and collaboration between them. Touch and sight, for instance, collaborate with each other. Through a tactile experience, I can feel the weight of an orange; through a perceptual one, I can have a sense of its relative chromatic characteristics. Later, I can feel its porous skin even if it is out of my actual tactile reach. Touch and sight collaborate at close distance. Sight and language, in contrast, collaborate at long distance. The intertwining of sensuous experiences leads to a special *detached intimacy* with the surrounding objects, whence the objectivity of the things in our world emerges. The crucial epistemic dimension of this sensuous cognition led Gehlen to assert that "What is to qualify as reality must fulfill two conditions: it must be established via two heterogeneous pathways and it must be reflected, even if only through language, in our ability to sense ourselves" (Gehlen 1988, p. 153).

In other words, what Gehlen is suggesting is that knowing can only be ensured through a multi-sensorial experience of the world and a kind of self-sensuous apprehension of things and us. This multi-sensory characteristic of cognition is not specific to humans; it is shared by other primates as well. There is, for instance, a very well-known tendency in primates to touch what they see. As primatologist Juan Carlos Gómez points out, "[apes'] visual representations are very soon coordinated with patterns of tactual–kinesthetic information arising from the same targets. This results in complex cross-modal or multisensory representations of objects" (Gómez 2004, p. 33). However, there seem to be limits to the collaboration between the senses of apes. Apes may recognize that a rope may have several characteristics that are important for reaching an object, e.g. length and transportability. However, as the experiments conducted by Köhler (1951) showed, chimpanzees attempted to reach a fruit placed relatively out of their reach, with the help of a wire or a rope. What the chimps did not realize is that to reach the fruit with a tool, the latter has to have a certain *rigidity*. Now *rigidity*, like weight, is a *tactile experience, not a visual one.* Thus, compared to the case of other primates, human sensorial organs *collaborate* together to a greater extent, so that what we perceive is endowed with a variety of sensuous coordinated characteristics.

Gehlen's account of cognition is a non-Rationalist one. And, it is far removed from Empiricism. Indeed, for Gehlen, the individual is not a passive receiver of sensations. The individual is an "acting being" through and through (Gehlen 1988, p. 16). But if action is at the core of Gehlen's conception of the mind, he does not allow us to succumb to the temptation to directly derive intellection from the senses. Referring to the concrete individuals of society, he says:

> I am not proposing to derive consciousness, imagination, and language from bodily processes ... Thought, representation, and imagination rest ... upon a broad base of 'sensorimotor' functions expressed through the hands, eyes and in language. It would, however, be an inexcusable simplification to attempt to attribute the former to the latter or to claim one evolved from the other. (Gehlen 1988, p. 12)

As Gehlen claims, the problem is much more complicated. The environment in which humans live is not a "natural" environment, but a symbolic one, transformed by previous human activity.

> [T]he perceivable world around us is undeniably the result of human activity. From a purely visual point of view, this world is highly symbolic, that is, a realm of *suggested experiences* which conveys to us the nature and usefulness of the objects around us. Exposure to an unchecked flood of impressions presents man, even at the earliest age,

> with the problem of coping with it, of relieving himself of this burden, of taking action against the world sensuously impinging upon him. This action consists of *communicative*, manipulative activities involving experiencing objects and then setting them aside and these activities have no immediate value for gratifying drives. The world is thereby 'processed' in communicative, relieved movements; its open abundance is experienced, 'realized', and subsequently absorbed. (Gehlen 1988, p. 31)

Now, the experience Gehlen is talking about is not something carried out by a solitary being. From the outset, it is directed toward someone else: "Experience is not a 'lonely' process, because an action directed toward a 'you' forms the basic structure of all psychical processes. Any thing can assume the role of this 'you', if we involve it in our experience." (Gehlen 1988, p. 153)

Although language could be considered "an extension of a system of deeply rooted interrelationships of movement and sensation" (Gehlen 1988, p. 11), it could hardly be reduced to it. Language – which from a phylogenetic viewpoint could only have emerged at the group level – arises ontogenetically from sensation *and practical communication*. It is hence both subjective *and* social.

In a similar vein, neurocognitivist Bruce Wexler has recently argued that

> language is not a property of the human brain but rather of human society and culture. If all living people were rendered permanently speechless and illiterate, their offspring and succeeding generation would be unable to speak despite having normal brains, and language, this most distinctive of all human characteristics, would be lost to the human species. (2006, p. 121)

Within this perspective, gestures can be considered as a part of individuals' sensuous attempts at dealing with abstract cultural ideas. Gestures may be seen as part of one of the sensuous modes – the tactile mode – which is demonstrated in the efforts at conceptually grasping something. My interest, however, is not in gestures in general, but in gestures as they appear in teaching and learning settings.

In the next section, I discuss some excerpts from a mathematics lesson and intertwine practical and theoretical considerations that frame the cognitive role of gestures in the learning and teaching of mathematics. To anticipate some of the claims that I will be making, I will present the problem of learning as a problem deeply related to the grasping of cultural conceptual entities and to the concomitant problem of the students' consciousness development. I will argue in favour of a multisemiotic view of consciousness and emphasize the role of gestures within the multilayered socio-cultural structure of movements and actions through which we objectify the world.

3 Making sense of graphs: mathematical and physical origins

The excerpt that I will discuss in this section comes from a longitudinal research program whose goal was to investigate the role of body, tools and symbols in the formation of abstract mathematical concepts. Previous analyses that we conducted in 2002 in a Grade 11 class (16–17-year-old students; number of students n=25) suggested that the students had significant difficulties in distinguishing between the phenomenological description of a physical event and its description in a Cartesian graph. Although the students did not experience difficulties in plotting points on a time–space Cartesian graph, a qualitative general description of the physical event was very often difficult to achieve, in particular

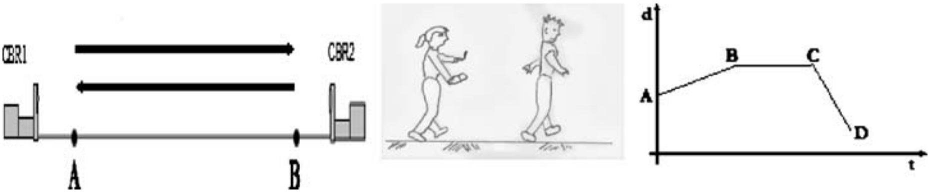


Fig. 1 To the *left*, the diagram of the car experiment. The car starts from A, moves to B, stops for 2 s and comes back to A. The *middle* and *right parts* show an illustration of the problem to be discussed in this paper: Pierre and Marthe's walk and its graph

because of confusions between the physical and the mathematical origins. The students, indeed, repeatedly tended to represent the physical beginning of, e.g., a certain motion with the mathematical origin (0, 0) of the Cartesian Graph (Radford, Cerulli, Demers, & Guzmán 2004; Radford, Demers, Guzmán, & Cerulli 2003). In a 2004–2007 longitudinal research program, we followed a Grade 10 class as it moved through Grades 11 and 12 and worked with the teachers to design teaching situations that could help the students reflect about physical events in a mathematical way.

In one of the Grade 10 activities, the students conducted an experiment in which a car moved at a constant speed from point A to point B, stopped at B for 2 s and then went back to point A at the same constant speed. Two probes – Calculator-Based Rangers© or CBRs, placed as shown in Fig. 1 (left) – were connected to one TI-83+ calculator each. Two students activated the CBRs at the precise moment in which a third student activated the remote controlled car. Before showing the students the time–distance graphs produced by each one of the CBR-Calculator systems, the students were required to draw the graphs and justify them.[2]

This activity – which aimed at making the students aware of the distinction between the physical origin of an event and the mathematical origin of its representation – was followed by other activities of increasing difficulty. In one of these activities, which will be discussed here, the students dealt with a problem that introduced some elements of relative motions. In this problem, two children, Pierre and Marthe, walked in a straight line, the latter pointing a CBR at the former (see Fig. 1, middle). The graph showed the relationship between the elapsed time (horizontal axis) and the distance between the children (vertical axis) as measured by the CBR (see Fig. 1, right).

The students were required to explain Marthe and Pierre's walk, knowing that their CBR-Calculator system produced the graph shown in the far right of Fig. 1. In our lessons, the students spend substantial periods of time working together in small groups of three or four. At certain points, the teacher (who interacts continuously with the different groups during the small group-work phase) conducts general discussions allowing the students to expose, compare and contest their different solutions. To collect data, we use four or five video cameras, each filming one small group of students.

In the interpretation of the graph, the students usually assumed that Marthe was not moving, contrary to the statement of the problem. The problem was then transformed into a problem for which the students had a solution.[3] In one of the small groups of three students

[2] Another paper dealing with the study of motion with the CBR is Arzarello and Robutti (2004).

[3] Many years ago, Fischbein called attention to the fact that, often, students transform a new problem into a different one in order to fit it into a familiar conceptual model for which they have a solution at hand (Fischbein 1989).

in which the class was divided – a group formed by Rita, Bobby and Monique – the students' interpretation was that Pierre walked (segment AB); then he stopped (segment BC); then he went back (segment CD).[4]

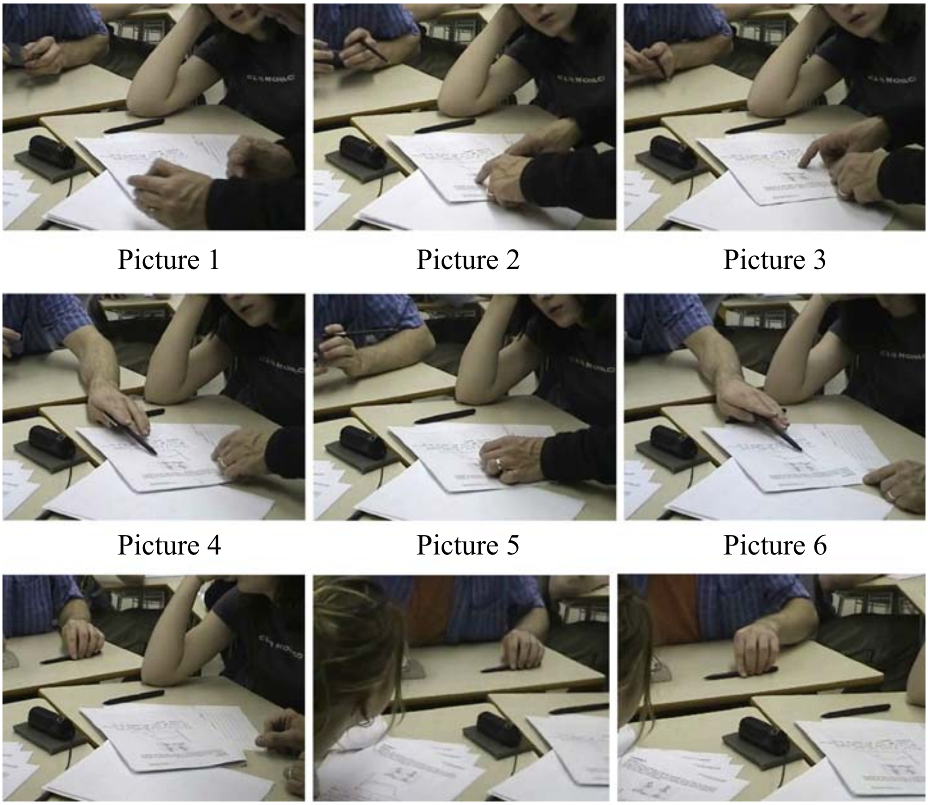
The students were not really sure about their interpretation. There was a sense that Marthe and the CBR were not accounted for in the graph. Furthermore, it was not clear whether or not the segment CD represented a return or a change in direction. The students could not reach a consensus, so, since the teacher was busy discussing with another small group of students, they called me. I was videotaping Bobby's group and, aided by a headphone, I had been able to follow their discussion. As I approached the group, Rita, pointing to the segment CD, asked whether or not "they walk in a different direction." As in the previous passage, the next passage includes many gestures whose cognitive role will be discussed below (in the transcript, the numbers in brackets refer to the elapsed seconds; *Pn* refers to picture numbers in Fig. 2, and LR to me).

Episode 1:

1. LR: (0.00) Here she moves (*pointing to Marthe*).... So they're both walking (1.72) (*gesture with two hands*; see *P1*) Yes? They're walking, they're both walking (6.43) (*gestures with both index fingers on the drawing*, see *P2*). So, what happened here? (8.48) (*pointing gesture along segment AB*, see *P3*).
2. Rita: (11.47) Ummm...
3. Bobby: (*Talking about point B, which is the point indicated by the stopped gesture*) (11.77) They're both stopped (13.50).
4. Monique: (13.83) No, she ... she remained still (15.57).
5. Bobby: (17.14) But he, the distance (*pointing to segment BC, P4*) (18.80) between them does not increase (21.13).
6. LR: (22.27) Here (*pointing to AB*, *P5*), does the distance increase? (23.88)
7. Bobby: (*responding immediately*) (24.20) Yes... Yes! (25.34)... (25.91) Oh! So, he (26.46), he walks faster (27.79), he walks... (*P6* (28.84) *pointing at point A and moving his pen slowly along the segment AB*).
8. Rita: (28.95) They stop (inaudible).
9. Bobby: (31.08) Ok, look (32.32) ... (36.68) Ok, hey, (37.75) I got it (37.92). (40.42) She... (42.02) That's him, this is her. (*He uses his pen and his protractor to simulate the motion on the desk*) (44.06) He walks more quickly (45.68), so it does this (*the distance between the pen and the protractor increases*) (*P7*), and (47.67) there's a bigger distance (49.00). (*He points on the graph*) (49.58) After, they both stop (*P8*) (*he points to segment BC on the graph*), after (51.45) she starts running faster than him (54.11). So... (*the students see that the distance between the pen and the protractor decreases, P9*).
10. Monique: (57.14) Ok...

When addressing Rita's question, I made two different gestures with both hands. I wanted to emphasize the idea that Pierre and Marthe were walking in a straight line, but also that they were *both* moving. The first two-hand gesture was an enactment of the walk made in the air (*P1*). The second one, at 6.43 (*P2*), was also an enactment but this time with two fingers and starting from the drawing of Pierre and Marthe on the activity sheet. I took

[4] As Rita summarized their ideas, "... this (*the graph*) is what he did (*pointing to Pierre*). So, he walked normally, he stopped for like, so many seconds, and then he went back...".



Picture 1 Picture 2 Picture 3

Picture 4 Picture 5 Picture 6

Picture 7 Picture 8 Picture 9

Fig. 2 Gestures and artifact-mediated actions in the student's attempt at understanding the graph

advantage of my discussion with the group and, at 8.48 (*P3*), I encouraged a reinterpretation of segment AB, given that the fact that Pierre and Marthe were *both* moving had been mentioned. A very long pause of 3.29 s followed, until Bobby, focusing on point B, suggested that they were stopped. The statement was still about both Pierre and Marthe but not in a relational way. Bobby's utterance still dealt with two unrelated movements, where a horizontal line is synonymous of rest (as in the car experiment). So far, the possibility of relative motion could only be considered in the simplest case, where no motion was going on. In line 5, 3.64 s later, Bobby elaborated further on the idea, and, pointing to segment BC, argued that in this part of the graph the distance did not increase, (*P4*). In line 6, I took advantage of Bobby's use of the term "increase" to ask the question about segment AB again, positioning one finger at each end of the segment (*P5*). Then, things happened very quickly.

Indeed, in line 7 Bobby answered affirmatively, as if referring to something trivial, and then there was a lapse of 1.14 s. The second "Yes!" was followed by the expression "Oh!" which revealed the attainment of a crucial awareness. Bobby pointed to Pierre and, translating the increase of distance into speeds, introduced a *comparative* expression – "he walks faster" – that put Pierre's walk into a qualitative relationship with Marthe. Right after, so as to understand better, he repeated the sentence but it was left incomplete ("he walks...," line 7), or rather the sentence was completed by a careful and long gesture from A to B that took the place of the previous linguistic comparative (*P6*). Bobby's "sentence" was in fact one made up of words and gestures, one through which he attended *perceptually* and *kinaesthetically* to segment AB and – we may venture – could *feel* the distance increase.

3.1 Objectification

In the first 29 s – roughly up to Picture 6 – Bobby underwent a process of *objectification*, i.e. a social process over the course of which something started making sense (Radford 2002, 2003) – here, a highly condensed cultural mathematical meaning presented through a graph.

Now, because mathematical meanings are general, they cannot be noticed through mere observation. Had Bobby been able to perceptually go point by point through each one of the infinity of points constituting a graph, he still would not have been able to derive, from such scrutiny, the graph's mathematical meaning. In other words, what we see in the sensual plane cannot fully account for its conceptual character. Objectifying something is an active process of sense-making that takes place in the interplay of sensuous activity and its continuous revised interpretation.

Although still partial and tenuous, Bobby's grasping of the mathematical meaning of the graph – its *objectification* – was accomplished through spoken words, gestures, bodily actions, artifacts (the pen, etc.) and mathematical signs. In short, it was accomplished through what I have called elsewhere *semiotic means of objectification* (Radford 2003). Within this theoretical approach (Radford 2006a, 2008), the pedagogical import of Bobby's gestures and bodily actions is related to his attempts at making the mathematical meanings' diaphanous corporeality palpable through personal senses.

In the second half of the previous episode, which occupies line 9, Bobby continued the process of objectification. Through the terms "look" and "hey," he tried to get his group-mates' attention and, with the help of a pen and a protractor, simulated Pierre and Marthe's walk. In *P7* he moved both objects, making sure that the protractor (signifying Pierre) went faster than the pen (signifying Marthe), while accompanying the coordinated action with a phrase that emphasized Pierre ("He..."), what Pierre was accomplishing ("...walks...") and how he was accomplishing it ("... more quickly"). The linguistic description qualified the action and filled it up with theoretical content. In return, the action made an increasing space between the pen and the protractor *perceptually* visible (*P8*), and it could now be linguistically qualified as a "bigger" space between the objects, achieving a relational interpretation of segment AB. Right after *P8*, Bobby left both objects on the table and showed on the graph, through an indexical gesture, that the distance remained the same. In *P9*, both objects continued to move in the same direction, but the pen went faster than the protractor. Again, the action was intertwined with a sentence ("She starts running faster than him") and, as in the previous cases, through indexical gestures, the action was related to the corresponding segments on the graph.

3.2 Thinking or explaining?

One way of interpreting the previous passage is to assert that, at instant 24.20, Bobby suddenly came to understand the meaning of the graphic. All of a sudden, something clicked in his mind and everything became clear. Bobby would have experienced what is traditionally called the "Aha! Moment." From that instant onward, Bobby was not really thinking, but rather explaining to others the outcome of his fortunate enlightening moment. As a result, what the analysis shows is the role of gestures in explaining and not in thinking.

Although interesting, this interpretation rests on an objectionable and over-simplified dichotomy between thinking and explaining. There is, of course, an obvious way in which this dichotomy and its ensuing interpretation can be dismissed: it can be claimed that all explanation involves thinking. But there is a less obvious consideration that makes the dichotomy even more problematic. It is this second consideration that is of interest for me here. I want to argue that Bobby did not think all these details *in* his head. He did not plan out the whole process ahead of time. He was watching these details unfold at the same time as his group-mates. He was thinking through his own actions and words. In *seeing–feeling* the former and *listening* to the latter, he was gaining a better mathematical understanding of the problem.

Of course, this interpretation does not mean that he did not have certain ideas before he started using the pen and protractor in his bodily actions. At 31.08 (the beginning of line 9), he certainly had an idea. Indeed, a prosodic analysis conducted with Praat – a voice dedicated software (http://www.praat.com) – revealed that speech intensity was at its peak when he uttered the expression “Oh!” (line 7). This expression was indeed pronounced at 81 dB, whereas Bobby’s speech intensity was elsewhere around 71–78 dB. He certainly “saw” something in the elapsed moment from 24.20 to 25.34 s, i.e. between the first “Yes” and the second “Yes!” in line 7. However, the initial idea was only the tip of the iceberg. In order to reach a solid understanding of the idea, Bobby had to reach a deeper layer of consciousness – a layer where, through multimodal activity, the details were worked out, articulated and tested. Through this multimodal activity, he was gaining a deeper level of consciousness of the cultural meaning of the graph, a level that might be characterized by the passage from one form of intuitive sentience to a more fully fledged form of reflective consciousness.

This is why, from the perspective here advocated, the previous passage is not an example of an individual thinking process that ended in a happy “Aha! Moment,” followed by a generous social act of explaining to others. To really understand the meaning of the graph, Bobby had to go through a *second process of objectification* (a process of about 23 s [from 31.08 to 54.11] in Episode 1, line 9). In the course of this second process of objectification, Bobby’s explanations, his gestures and bodily actions were addressed to the others *as much as to himself.* To become conscious of something abstract is, generally speaking, a long process. To account for this process is, in fact and to a large extent, what objectification is about.

3.3 Semiotic nodes

A third step towards a deeper level of objectification occurred right after the previous episode. Rita and Monique saw and heard Bobby’s intricate motor–tactile–linguistic (in short, multimodal) argument. But they did not reach the same level of objectification. Bobby, then, decided to start the argument of line 9 over again (in the following transcript *Pn* refers to the picture number in Fig. 3):

Episode 2:

1. Bobby: (00.00) Ok, if ... if she... if that’s the CBR, and this is the girl (4.19) (*he shows the pen*), and this is the guy (5.34) (*he shows the protractor*), they (*P1*) both start to walk (*he moves the objects*) but he walks faster (9.31) (*P2*)

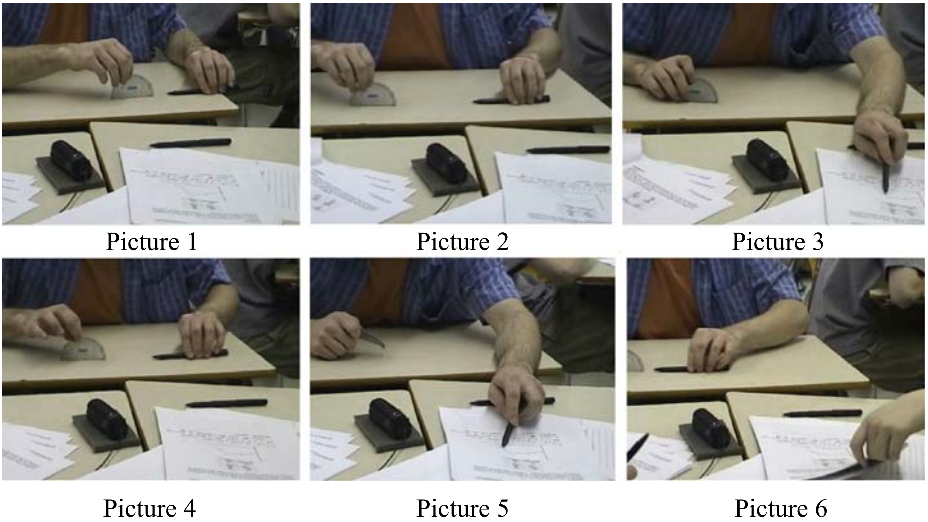


Fig. 3 Bobby's second process of objectification

2. Rita: (9.41) Yes...
3. Bobby: (9.76) After they stop for a bit (11.44) (*gesturing*; *P3*) (12.61) and then...
4. Rita: (13.14) She continues to walk (14.42).
5. Bobby: (13.44) (*interrupting*) Once they start again, (15.10) she's going to start walking faster (17.38) (*P4*). (17.68) Because the distance of the CBR decreases (21.63) (*gesturing*; *P5*). (22.72) Between him and her, the distance decreases, so... (25.85) it's going to do this (26.7) (*moving the pen and the protractor, P6*).

This time, the actions were better tuned and refined. Indeed, in the last part of Episode 2 (more specifically, from 44.06 to 54.11 s), the amount of time spent doing actions was 7.93 s (78% of the episode from 44.06 to 54.11 s). In Episode 3 (from 0.00 to 26.7), this amount was 5.73 s (i.e. 21.4% of that episode). Here, Bobby spent most of his time talking and gesturing. From 44.06 to 54.11 in Episode 2, he made several indexical gestures that amounted to a total of 2.58 s; in Episode 3, he gestured for a total of 5.58 s.

In a previous paper, to highlight the multimodal nature of processes of objectification (processes which, as we saw earlier, resort to different sensuous channels), my collaborators and I introduced the concept of *Semiotic Nodes*. Semiotic nodes, we suggested, are pieces of the students' semiotic activity where action, gesture and word work together to achieve knowledge objectification (Radford et al. 2003). The last part of Episode 1, discussed above, and Episode 2 are examples of semiotic nodes.

The previous analysis of time spent gesturing and carrying out actions sheds some light on the dynamic nature of semiotic nodes. The analysis suggests that, as objectification progresses, there is a shift in the *configuration* of the semiotic means of objectification in semiotic nodes: actions become shorter and gestures and language become more relevant. The centre of gravity of the semiotic nodes *moves*.

One last example will help to illustrate this idea better. After Episode 2, the students continued discussing Bobby's argument. Talking rather to herself, Rita said: "They walk together ..." and accompanied her utterance with a complex gesture made with two fingers which replaced Bobby's previous sequence of actions and gestures (see Fig. 4, *P1*). In *P2* she said: "This is Marthe (*moving her index finger and moving it towards her thumb which*

Fig. 4 Bobby repeating Rita's gestures with theoretical enriched content

represented Pierre), and she continues walking...." In P3, Bobby gestured back, making a gesture similar to Rita's and said: "because the distance increases...," referring to segment AB.

Here, the semiotic node has become further contracted; fewer actions and gestures are required. The centre of gravity leans toward language.

Although I cannot go further in the analysis, it is worth noting that later on, when time and space became quantified, the students were required to find the algebraic formulas of the segments and interpret the coefficients in physical terms. They tested their interpretation by carrying out the experiment with the CBR and the calculator, and general classroom discussions brought out the possibility of different solutions. New signs came to enrich the semiotic nodes and a deeper understanding of the mathematical meanings was reached.

4 Synthesis and concluding remarks

In this article, I dealt with the role of gestures and bodily actions in the learning of mathematics. The article is in fact an invitation to see gestures through a fresh lens and consider the idea that gestures and the body may indeed be important sources, not only of early pre-linguistic conceptual formation (as in Piaget's genetic epistemology), but of abstract thinking as well.

As I pointed out from the outset, the idea of paying serious attention to gestures is not to diminish the role of the written in the teaching and learning of mathematics. Neither it is to assert that "pure" mental thinking is a chimerical presumption or that *all* thinking involves gestures. As one of the reviewers put the matter, "a proficient mathematician could handle the question on Pierre and Marthe without gestures by pure thinking." In a parenthetical note, the same reviewer added: "I could [handle the question] without any gestures or tools." And I certainly could too. However, as we saw in the episode discussed here, in order to make sense of the graph, many students who are in the process of learning cannot. Instead, they resort to an amazing number of gestures and actions. The problem was therefore to account for this impressive gestural activity.

Now, the insertion of the body into an account of cognition in general and mathematics cognition in particular could hardly succeed without revisiting the idea of cognition itself (Arzarello 2006a, b; Lakoff and Johnson 1999; Lakoff and Núñez 2000; Nemirovsky 2003; Núñez, Edwards, & Matos 1999; Radford, Bardini, Sabena, Diallo, & Simbagoye 2005; Seitz 2000). There are some theoretical problems that need to be tackled in order to overcome critics like Eagleton who see – in the claims of embodied perspectives – futile attempts that end up being no more than "the return in a more sophisticated register of the old organicism" or even a token of "the post-modern cult of pleasure" and love of the concrete (Eagleton 1998, p. 158).

My aim in the preceding sections was not to solve the wide range of theoretical problems that a non-mental cognitive approach has to face, but to focus on one of the crucial concepts in mathematics education – the concept of thinking. Drawing on Gehlen's notion of multi-modal cognition, I was then led to argue that thinking is not something strictly mental. I advocated instead for a sensuous conception of thinking – one in which gestures and bodily actions are not the ephemeral symptoms announcing the imminent arrival of abstract thinking, but *genuine* constituents of it.

In this multi-modal view of thinking, the problem is not focused on gestures only. As a matter of fact, in normal learning settings, gestures, considered in isolation, have a very limited cognitive scope. The cognitive possibilities of gestures can only be understood in the broader context of the interplay between the various sensuous aspects of cognition as they unfold against the background of social praxes.

The praxis I am interested in is that of the mathematics classroom. Gestures, bodily actions and symbolic activity were therefore investigated accordingly. I focused on one excerpt from a teaching sequence whose goal was to create the conditions for the students' acquisition of historically constituted modes of theorizing and reflecting about motions. I dealt in particular with the students' understanding of the meaning of a time–distance Cartesian graph related to relative motions.

Within the educational perspective that I have been advocating – the theory of knowledge objectification (Radford 2006a, 2008) – one of the main pedagogical problems is to provide accounts of students' encounters with historically constituted mathematical meanings. Since meanings are general and hence beyond the realm of perceptual things, these encounters can neither be reduced to the empirical observation of concrete objects and signs, nor to the observation of simple problem-solving rituals advocated by direct teaching pedagogies. The problem is rather to offer the students the possibility of becoming as critically fluent as possible with cultural modes of thinking. The process of becoming critically fluent with historically constituted modes of thinking is thematized as social processes of *objectification*.[5] More specifically, by processes of objectification, I mean the active and creative social processes in the course of which students come to master, through practice, cultural mathematical forms of reflection and action while achieving, at the same time, the development of deeper layers of subjectivity and consciousness.

The analysis presented here aimed at evidencing the students' social processes of objectification. It focused on the crucial interrelated roles of perception, language, actions and gestures in the students' progressive awareness of the cultural logic behind sophisticated mathematical meanings. The analysis made evident the intricate configuration of the various semiotic systems in what we have termed *semiotic nodes*, i.e. those segments of the students' mathematical experience where objectification is achieved. Gestures and bodily actions, I suggested, are attempts at making palpable the diaphanous corporeality of mathematical meanings. The time-analysis of semiotic nodes intimated that, as objectification progresses, the configuration of the semiotic systems changes. The centre of gravity of the semiotic nodes *moves*. There is, indeed, a shift in the *configuration* of the semiotic means of objectification: actions become shorter and gestures and language become more relevant.

[5] To avoid misunderstandings and at the risk of repeating myself, I would like to emphasize that, in talking about *objectification*, I am *not* referring to conceptual objects of a transcendental ontological nature, but to human-made cultural objects (Radford 2002, 2004, 2006a, b).

The leaning of semiotic nodes towards language is not a surprise (for further empirical evidence, see e.g. Robutti 2006; Roth 2001). Language has an extraordinary capacity to shape scientific thinking – a point made by Vygotsky (1986), when he claimed that scientific thinking cannot come into existence without verbal thought. It might be worth recalling here the case of Emmanuelle Laborit, a girl who was born deaf and learned Sign Language when she was 7 years old. Before the age of 7, she tells us, there were no words or phrases in her mind: "just images" (Laborit 2003, p. 23, my translation). When she tries to think of her first memory, she says: "There are neither first nor last memory in my disorder. There are sensations. Eyes and a body to register sensation" (Laborit 2003, p. 27, my translation). Language, indeed, has a unique capacity for creating sustainable reference points with which to organize experience: "When I was taught 'yesterday' and 'tomorrow' in Sign Language and I understood their meaning, I was able to talk about them orally more easily, write about them more easily!" (Laborit 2003, p. 164). However, it would be an over-simplification to state that abstract thinking, as required in science and mathematics, is just the internalization of speech. In this respect, sensuous cognition seems a promising avenue through which to inquire about the problem of mathematical thinking from a more encompassing perspective. Indeed, one of the points made in this article is that the source of abstract mathematical thinking is to be found in the sophisticated linkage of language and the perceptual, auditory, tactile and kinesthetic sensorial channels. In the analysis of the classroom episode, it turned out, in fact, that gestures and bodily action contributed to the process of objectification by delineating the borders of perceptual attention (through, e.g., an indexical gesture), and by making available a kinesthetic meaning which expressed, in a motor–sense, the idea of increase, rest, decrease, etc.[6] In the course of the students' objectification, the *analogical* dimension of gestures and bodily actions (Lemke 2003; McNeill 2000) became qualified with cultural *synthetic* conceptual categories of language (e.g. "decrease") and, in a dialectic way, all together were fundamental in allowing the students to gain fluency in the targeted cultural mode of mathematical thinking.

Naturally, to better weigh the role of gestures and bodily actions in mathematics cognition, more detailed investigations are required. We need further investigations of the multi-modal cognitive dimension of mathematics cognition, through the development of theory and classroom analyses.

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[6] For further evidence see also Núñez (2007), Radford, Bardini, & Sabena (2007), and Sabena (in press).

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