

# Why Do People Pay Taxes? Prospect Theory Versus Expected Utility Theory

Sanjit Dhami\*      Ali al-Nowaihi†

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## Abstract

*Given actual probabilities of audit and penalty rates observed in the real world, tax evasion should be an extremely attractive gamble to an expected utility maximizer. However, in practice, one observes too much compliance relative to the predictions of expected utility. This paper considers an alternative theoretical model that is based on Kahneman and Tversky's cumulative prospect theory. The model predicts empirically plausible magnitudes of tax evasion despite low audit probabilities and penalty rates. An increase in the tax rate leads to an increase in the amount evaded- a result, which is in broad agreement with the evidence, but is contrary to the prediction made by expected utility theory. Furthermore, we show that the optimal tax rates predicted by prospect theory, in the presence of tax evasion behaviour, are consistent with actual tax rates.*

*Keywords: Reference Dependence, Loss Aversion, Decision Weights, Prospect Theory, Expected Utility Theory, Tax Evasion, Optimal taxation.*

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\*Department of Economics, University of Leicester, University Road, Leicester. LE1 7RH, UK. Phone: +44-116-2522086. Fax: +44-116-2522908. E-mail: [Sanjit.Dhami@le.ac.uk](mailto:Sanjit.Dhami@le.ac.uk).

†Department of Economics, University of Leicester, University Road, Leicester. LE1 7RH, UK. Phone: +44-116-2522898. Fax: +44-116-2522908. E-mail: [aa10@le.ac.uk](mailto:aa10@le.ac.uk).

## 1. Introduction

Why do people pay taxes? Theoretical work, largely based on expected utility theory (EUT), has struggled to cope with the stylized facts. The tax evasion puzzle goes something like this. The audit probability,  $p$ , is extremely low, with realistic magnitudes ranging from  $p = 0.01$  to  $p = 0.03$ , while the penalty rate,  $\lambda$ , that is paid in addition to repayment of the evaded tax liabilities, ranges from 0.5 to 2.0; see for example Alm et. al. (1992), Andreoni et. al. (1998) and Bernasconi (1998). The seminal applications of EUT to the tax evasion problem, by Allingham and Sandmo (1972) and Yitzhaki (1974), show that the taxpayer will choose to evade taxes if the expected return on evading the tax,  $1 - p - p\lambda$ , is positive. Using observed values of  $p$  and  $\lambda$  the expected return on tax evasion is between 91 and 98.5 percent. This contradicts the empirical evidence in two ways.

1. With a positive expected return to tax evasion, EUT predicts that all taxpayers should hide some (but, typically, not all) of their income. Experimental evidence suggests that at least 20 percent of tax payers fully declare all their income, even when  $p = 0$ ; see, for example, Alm et. al. (1992). Furthermore, those who decide to evade, appear to hide all their income. More accurately, tax payers, if they decide to evade, hide some sources of income completely while fully declaring other sources; see, for example Pudney et. al (2000).
2. To square the predicted extent of tax evasion under expected utility with the evidence, tax payers should be risk averse to an absurd degree; see, for example, Skinner and Slemrod (1985) and Alm et al. (1992).

But these are not the only paradoxes arising from the application of EUT to tax evasion.

3. Yitzhaki (1974) showed that under the reasonable assumption of decreasing absolute risk aversion, an increase in the tax rate leads to a decrease in tax evasion. Experimental, econometric and survey evidence generally rejects this result<sup>1</sup>.
4. Obligatory advanced tax payments should not influence the taxpayer's evasion decision under EUT. However, empirical and experimental evidence show that obligatory advanced tax payments do reduce tax evasion; see, for example, Yaniv (1999).

The Allingham-Sandmo-Yitzhaki model has been extended in a number of directions to include endogenous income, dynamic effects, social norms, tax avoidance and uncertainty in the outcome of audits, among others; see, for example, Andreoni et. al. (1998).

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<sup>1</sup>See, for example, the references cited in Andreoni et. al. (1998) and in al-Nowaihi and Pyle (2000). Feinstein (1991) is a notable exception that lends empirical support to Yitzhaki's result.

While these extensions are, of course, very worthwhile, as long as EUT is retained the above paradoxes remain. Therefore, much recent work has investigated several non-EUT frameworks.

Several alternatives using non-EUT models have been proposed to explain the tax evasion puzzle. For instance, the stigma associated with being caught evading taxes might make taxpayers reluctant to evade, despite the existence of a favorable gamble; see for example Gordon (1989) and al-Nowaihi and Pyle (2000). Bernasconi (1998) imposes a technical restriction on preferences called “orders of risk aversion” based on earlier work by Segal and Spivak (1990)<sup>2</sup>. It turns out that for realistic  $(p, \lambda)$  pairs, if risk aversion is of order 2 then tax evasion will take place, but if it is of order 1 it might not. However, not all technical restrictions on preferences are founded on independent empirical evidence, and, hence, it is not easy to evaluate their plausibility for specific contexts and for actual behavior; see for example Starmer (2000).

On the other hand, preference restrictions can be based on a substantial body of independent experimental evidence, as for instance, in the “prospect theory” alternative to EUT given by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). We apply prospect theory to the tax evasion puzzle using the latter version of prospect theory expounded in Tversky and Kahneman (1992). There are two basic foundations of prospect theory, both supported by a large body of experimental evidence.

1. Unlike EUT, where the carrier of utility is final wealth, the carriers of utility in prospect theory are gains and losses relative to some “reference point”. Experimental evidence indicates that, relative to the reference point, individuals evaluate gains and losses, differently. In particular, losses loom larger than gains: a phenomenon that in prospect theory is termed as “loss aversion”. Denote by  $x$  the payoff, relative to the reference point, in some state of the world and by  $v(x)$  the corresponding utility in that state. Loss aversion implies that the disutility arising from a loss of  $x$  pounds is larger in magnitude than the utility arising from the gain of  $x$  pounds i.e.  $v(x) < -v(-x)$ ,  $x > 0$ . Furthermore,  $v(x)$  is concave for gains, and convex for losses relative to the reference point. These points are illustrated in Figure 1.1.
2. The second foundation of prospect theory is that, unlike EUT, probabilities enter non-linearly into the evaluation of a lottery. Consistent with experimental evidence, individuals use an inverted  $S$  shaped cumulative weighting function which overweights low probabilities and underweights high probabilities. Figure 1.2 shows a weighting function due to Prelec (1998) (this function has the form  $w(\mathbf{p}) = \exp[-(-\ln \mathbf{p})^\alpha]$ ; details can be read in subsection 2.6 below)

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<sup>2</sup>For a certain class of gambles, orders of risk aversion imposes restrictions on the first and second derivatives of the risk premium term with respect to the tax rate.

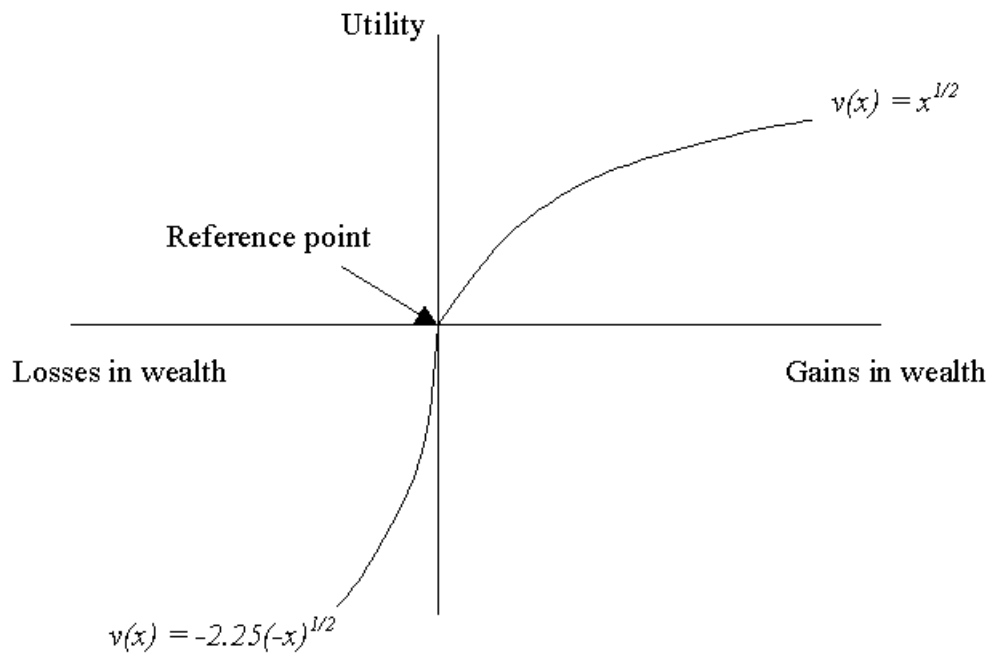


Figure 1.1: Preferences Under Prospect Theory

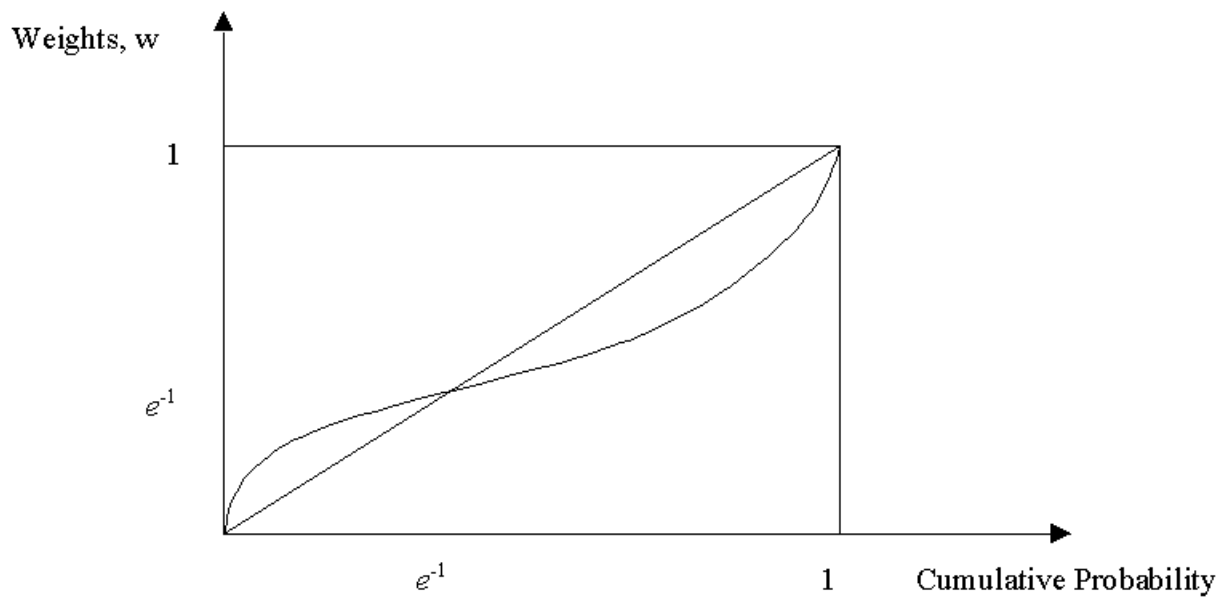


Figure 1.2: The Weighting Function Under Prospect Theory

Prospect theory has been successfully used to explain a range of puzzles in economics, such as the disposition effect, asymmetric price elasticities, elasticities of labour supply that are inconsistent with standard models of labour supply and the excess sensitivity of consumption to income; see, for example, Camerer (2000). An important application of prospect theory is to the equity-premium puzzle in finance, which is similar in spirit to the tax evasion puzzle in that both can be formulated in terms of a similar portfolio choice problem. The equity premium puzzle asks the following question. Why are annual real rates of return on stocks, relative to bonds, about 6 percent higher since 1926 when the standard deviation for the two investments equals 0.2 and 0.0 respectively? Using EUT, it is difficult to make sense of this apparently high premium on stocks, unless individuals have coefficients of relative risk aversion in excess of 30, which is unrealistic<sup>3</sup>. Benartzi and Thaler (1995) successfully use prospect theory to explain the equity premium puzzle based on loss averse agents who are also myopic in the sense of having limited planning horizons. An obvious parallel with tax evasion is to imagine evasion and compliance as two investments and ask why people invest disproportionately large amounts in the latter when returns on the former are so high?

List (2004) concentrates on testing one aspect of prospect theory, namely, ‘reference dependence’ also known as the ‘endowment effect’. He concludes (p. 622) “the data for ordinary consumers provides strong support in favor of prospect theory”. However, he also concludes that for dealers (p. 622-23) and experienced consumers (p. 624) “neoclassical theory predicts reasonably well”. In our view, however, List’s evidence can bear a different interpretation. Namely, that traders and experienced consumers reframe their problem so that their reference point becomes (say) expected profit or expected revenue, rather than an endowment of a particular good. To summarise, while List’s work strongly supports prospect theory for ordinary consumers, it does not, necessarily, reject prospect theory for dealers and experience consumers.

Prospect theory has also been used to explain the tax evasion puzzle. The experimental studies of Alm et al. (1992) suggest that one possible explanation for why people pay taxes might potentially be based on prospect theory. Individuals might be using a non-linear transformation of probabilities to overweight the probability of a tax audit, which provides for an obvious deterrent to tax evasion activity, however, they do not attempt to formalize their suggestion. It is generally agreed that the most satisfactory approach of transforming probabilities is that of rank dependent expected utility; see, for example, Starmer (2000). Rank dependent expected utility (RDEU) explains a number of paradoxical results of EUT, e.g. the “Allais paradox”, but Eide (2001) shows that the paradoxical comparative

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<sup>3</sup>Mankiw and Zeldes (1991) calculate that a coefficient of relative risk aversion of 30 implies that the certainty equivalent of the prospect - win 50,000 or 100,000 each with probability 1/2 is only 51,209. This is an absurdly high degree of risk aversion.

static results of the Allingham-Sandmo-Yitzhaki model carry over to RDEU.

Yaniv (1999) applies the idea of loss aversion in prospect theory to the tax evasion problem, however, his underlying framework is very similar to expected utility theory. In particular, he uses linear weighting in probabilities while prospect theory uses a non-linear inverted  $S$  shaped cumulative weighting function. Furthermore, his use of “choice heuristics” to combine common terms in income arising in different states of nature is at odds with the treatment of prospect theory in Tversky and Kahneman (1992).

Our paper is closest in spirit to Bernasconi and Zanardi (2002). Bernasconi and Zanardi (2002) choose an arbitrary reference point that does not necessarily depend on the tax rate. Hence, it is possible that in both states of the world (i.e. ‘income is audited’ and ‘income is not audited’), relative to some fixed reference point, either the tax payer is in the domain of gains or in the domain of losses or in both domains. It is, however, counterintuitive that the reference point of the taxpayer, when the tax evasion decision is made, should not depend on the legal tax liabilities<sup>4</sup>. Thus, for instance, if the tax payer is always in the domain of gains, then the analysis is exactly as under expected utility and the tax evasion puzzle remains outstanding. It is only in the other two complementary cases i.e. when the taxpayer is always in the domain of losses or in both domains in different states of the world, that prospect theory has any bite in their analysis. Furthermore, they do not use cumulative transformations of probability and so it is likely that the optimal choice could be stochastically dominated by other options; see for instance, Starmer (2000).

Stigma from the detection of tax evasion seems to be recognized as an important factor in the tax evasion literature; see, for example, Slemrod and Skinner (1985). In related contexts, Besley and Coate (1992) also argue for the importance of stigma as a feature of actual behavior. Hence, we integrate stigma from tax evasion within prospect theory. Including stigma does not change the comparative static results of our model, but it allows us to predict the level of aggregate tax evasion.

We broadly ask the following questions. Is it possible to resolve the tax evasion puzzles, (1)-(4) above, using prospect theory? What role does stigma play in partitioning the population into those who pay and those who evade taxes? Furthermore, is the prospect theory framework amenable to optimal tax analysis? and, if yes, are the magnitudes of optimal tax rates predicted by using a prospect theory approach realistic<sup>5</sup>?

The results are as follows. Empirically relevant combinations of  $p, \lambda$  lead to realistic patterns of tax evasion when one applies cumulative prospect theory to the tax evasion decision. On the other hand, the predictions of EUT are substantially at odds with the

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<sup>4</sup>For instance, the old and often quoted adage in public finance- “an old tax is no tax”- seems to have an obvious implication that the existing level of tax liabilities conditions ones post-tax reference income.

<sup>5</sup>To the best of our knowledge, this is the first time that this question has been considered within a prospect theory framework.

evidence. Under prospect theory, an increase in the tax rate leads to an increase in the amount evaded, which is in agreement with the bulk of the evidence. This is in direct contrast to the predictions made by EUT. Finally, the optimal tax rates predicted by prospect theory, in the presence of tax evasion behavior, are consistent with actual tax rates.

Convexity of the value function for losses is the most important element of prospect theory for the explanation of the tax evasion puzzle. However, the other elements, loss aversion and non-linear weighting of (cumulative) probabilities, as well as stigma, all play their part in arriving at a good empirical fit. It is important to note that we do not select our parameter values to get a good fit. Quite the contrary, the parameters of our model come from independent experimental evidence, unrelated to the specific problem of tax evasion. Thus, our paper, on the one hand, argues that prospect theory can provide a satisfactory explanation of tax evasion, and, on the other hand, also argues that the phenomenon of tax evasion provides independent confirmation of prospect theory.

The plan of the paper is as follows. Section 2 describes the basic model and provides a brief overview of the method of prospect theory. Section 3 solves for the equilibrium tax evasion under EUT and prospect theory; it also provides a diagrammatic interpretation of the results and reports a quite comprehensive set of comparative static results. Section 4 examines the predictions of expected utility and prospect theory with respect to the audit probabilities and the penalty rates. Section 5 derives the optimal tax rate under prospect theory. Section 6 examines alternative reference incomes. Finally, Section 7 concludes. All proofs are collected in the appendix.

## 2. The Model

Each taxpayer has some exogenous taxable income  $W$  and can choose to declare some amount  $D \in [0, W]$ . The government levies a tax at some constant marginal rate  $t$  on declared income. An exogenous fraction  $p \in [0, 1]$  of the taxpayers are audited, subsequent to the filing of tax returns, and the audit reveals the true taxable income. If caught, the dishonest taxpayer must pay the outstanding tax liabilities  $t(W - D)$  and a penalty  $\lambda t(W - D)$  where  $\lambda > 0$  is the penalty rate. If evasion is discovered, the taxpayer also suffers some stigma  $s(W - D)$  where  $s$  is the stigma rate on evaded income<sup>6</sup>. There is a continuum of individuals with stigma rates  $s \in \left[ \underline{s}, \bar{s} \right]$  with density  $\phi(s)$  and distribution

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<sup>6</sup>As in Gordon (1989) and Besley and Coate (1992), such stigma enters linearly, as a monetary equivalent, into the payoff in that state of the world. Although, a natural interpretation of stigma might include factors such as the tug on one's conscience, and loss of face among family and community etc., other interpretations of stigma are possible. These might include, in an appropriately specified dynamic game, the effect on one's current and future earnings arising from the bad publicity arising from tax evasion.

function  $\Phi(s)$ . Denoting by  $Y_A$  and  $Y_{NA}$  respectively, the income of the taxpayer when he is audited and when he is not,

$$Y_{NA} = W - tD, \text{ and} \quad (2.1)$$

$$Y_A = W - tD - t(1 + \lambda)(W - D) - s(W - D). \quad (2.2)$$

### 2.1. Sequence of Moves

The tax authorities move first, making an announcement of  $p$  and  $\lambda$ . The taxpayer then makes the decision to either fully report his or her income or evade a fraction of it. Finally, the government audits a fraction  $p$  of the returns and dishonest taxpayers are required to give up a fraction  $(1 + \lambda)$  of their unreported income.

### 2.2. The Decision Problem Under Expected Utility Theory

Denote the set of outcomes, possibly income levels, in a gamble as  $Y_1, \dots, Y_n$ . Under EUT, the carrier of utility is the final wealth level and not, unlike prospect theory, wealth relative to some reference income level. In order to facilitate comparison with the results under prospect theory, the utility of any outcome is defined as follows.

$$v(Y_i) = \begin{cases} Y_i^\beta & \text{if } Y_i \geq 0 \\ 0 & \text{if } Y_i < 0 \end{cases} \quad (2.3)$$

In (2.3) the parameter  $\beta \in (0, 1)$ <sup>7</sup>. Given  $p$  and  $\lambda$  announced by the tax authorities, the taxpayer chooses declared income  $D \in [0, W]$  in order to maximize expected utility, which, using 2.1 and 2.2, is given as

$$E[U] = \begin{cases} (1 - p)[Y_{NA}]^\beta + p[Y_A]^\beta & \text{if } Y_A \geq 0 \\ (1 - p)[Y_{NA}]^\beta & \text{if } Y_A < 0 \end{cases}. \quad (2.4)$$

### 2.3. Utility of an Outcome Under Prospect Theory

Denote the reference income of the taxpayer as  $R$  and the income relative to the reference point as  $X_i = Y_i - R$ ,  $i = 1, \dots, n$ . As in Kahneman and Tversky (1979) and Tversky and Kahneman (1992), the utility associated with any outcome  $X_i$  is defined as  $v(X_i)$ , and

$$v(X_i) = \begin{cases} X_i^\beta & \text{if } X_i \geq 0 \\ -\theta(-X_i)^\beta & \text{if } X_i < 0 \end{cases} \quad (2.5)$$

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<sup>7</sup>From (2.2) it is possible that  $Y_A < 0$ . The simplest way to incorporate  $Y_A < 0$  into EUT is to define  $v(Y_A) = 0$ , as in (2.3). Other possibilities are  $v(Y_A) = -v(-Y_A)$  and  $v(Y_A) = -\infty$ . None of our results depend on which specification we use for  $v(Y_A)$  when  $Y_A < 0$ .



where  $\beta \in [0, 1]$  and  $\theta > 1$  are preference parameters. Based on experimental evidence Kahneman and Tversky (1979) and Tversky and Kahneman (1992) suggests that  $\beta \cong 0.88$  while  $\theta \cong 2.25$ .

#### 2.4. The Reference Point Under Prospect Theory

Although prospect theory does not provide sufficient guidance to determine the reference point in each possible situation, in several cases there is an obvious candidate for a reference point. We take the legal after-tax income,  $(1 - t)W$ , as the reference point in this paper using the old adage in public finance that “an old tax is no tax”<sup>8</sup>.

#### 2.5. The Decision Problem Under Prospect Theory

A brief sketch of the generic formulation under prospect theory is given below; details can be seen in Tversky and Kahneman (1992). There are two possible states of the world in the model, defined by  $\Lambda$  such that

$$\Lambda = \{\text{Audit}, \text{No Audit}\}.$$

The “Audit” state occurs with probability  $p$  and the “No Audit” state occurs with probability  $1 - p$ . Relative to the reference income, in the “Audit” state the taxpayer’s income is  $X_A = Y_A - (1 - t)W = -[\lambda t + s](W - D) \leq 0$ , while in the “No Audit” state it is  $X_{NA} = Y_{NA} - (1 - t)W = t(W - D) \geq 0$ . Substituting (2.1) and (2.2) into these expressions we get

$$X_{NA} = t(W - D), \text{ and} \tag{2.6}$$

$$X_A = -[\lambda t + s](W - D). \tag{2.7}$$

Define the set  $X$  as the set of monetary consequences, hence,

$$X = \{-[\lambda t + s](W - D), 0, t(W - D)\}$$

The “neutral outcome” 0 is also included in  $X$  and all other outcomes are measured as gains or losses relative to it. Hence, the taxpayer is in the domain of ‘gains’ in the “No Audit” state and the magnitude of the gains is the amount of the tax evaded. In the complementary “No Audit” state the taxpayer is in the domain of losses; the magnitude of the loss equals the penalty on the evaded tax plus the cost of stigma.

Consider the following prospect, denoted by  $f$ .

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<sup>8</sup>The implications of a reference income other than  $(1 - t)W$  are taken up in Section 6.

$$f = \{-[\lambda t + s](W - D), p; t(W - D), 1 - p\}$$

In this literature,  $f^+$  and  $f^-$  respectively denote the positive and negative parts of the prospect<sup>9</sup>. To each possible prospect  $f$ , prospect theory assigns a number  $V(f)$  such that prospect  $f$  is preferred to prospect  $g$  if and only if  $V(f) > V(g)$ . The “value equation” in prospect theory, which the individual tries to maximize, is very similar in form to the ‘rank expected utility models’ and is given by

$$V(f) = \pi_i^+ v(t(W - D)) + \pi_i^- v(-[\lambda t + s](W - D)), \quad (2.8)$$

The payoff arising from any individual outcome,  $v(X_i)$ ,  $X_i \in X$ , is defined in (2.5). Unlike EUT, the outcomes are not simply weighted by objective probabilities but rather they are weighted by some “decision weights”  $\pi_i$  which are non-linear transformations of objective probabilities. Furthermore, the positive and negative parts of the prospect are generally separately weighted by different weighting functions denoted by  $\pi^+$  and  $\pi^-$  respectively.

The decision weights are transformations of cumulative probabilities<sup>10</sup>, as in rank dependent models of expected utility. In particular, at the highest (respectively lowest) value of the outcome in the prospect  $f^+$  (respectively  $f^-$ ),  $\pi_i^+ = w^+(1 - p)$  (respectively  $\pi_i^- = w^-(p)$ ), where  $w^+$  and  $w^-$  are “weighting functions” that apply respectively to the positive and the negative parts of the prospect; these are formally defined below. Hence, in the special case of there being only one outcome each in the domain of gains and losses, as is the case in the generic tax evasion problem, the decision weights based on cumulative probabilities are simply equal to weights based on objective probabilities. Using the definition of  $v(X_i)$  in 2.5, this allows the expression 2.8 to be written as

$$V(f) = w^+(1 - p) [t(W - D)]^\beta + w^-(p)(-\theta) [(\lambda t + s)(W - D)]^\beta \quad (2.9)$$

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<sup>9</sup>In the tax evasion case,  $f^+ = \{0, p; t(W - D), 1 - p\}$  and  $f^- = \{[\lambda t + s](W - D), p; 0, 1 - p\}$ , hence,  $f = f^+ - f^-$ . Notice that the neutral outcome is included in each of the prospects because it is neither positive nor negative. The distinction between  $f^+$  and  $f^-$  is made because the probability weighting functions for the negative and positive parts of the prospect are typically distinct; see Tversky and Kahneman (1992) for the details.

<sup>10</sup>In the more general case suppose that there are  $n$  possible outcomes  $X_1, \dots, X_n$  in the domain of gains with associated objective probabilities  $p_1, \dots, p_n$ . Then the decision weight  $\pi_i^+(X_i) = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n)$  while  $\pi_n^+(X_n) = w^+(p_n)$ . The function  $w : [0, 1] \rightarrow [0, 1]$  is a weighting function that transforms objective probabilities into subjective weights. In the domain of cumulative probabilities, Tversky and Kahneman (1992) show that the shape of the weighting function consistent with experimental evidence is an inverted  $S$  shape.

Denoting the  $m$  possible outcomes in the domain of losses  $X_{-m}, \dots, X_{-1}$  with associated probabilities  $p_{-m}, \dots, p_{-1}$  one defines  $\pi_i^-(X_i) = w^-(p_{-m} + \dots + p_i) - w^-(p_{-m} + \dots + p_{i-1})$  with  $\pi_{-m}^-(X_{-m}) = w^-(p_{-m})$ . The decision weights for the tax evasion case in the paper now follow from noting that  $n = m = 1$ .

Comparing the value equation in (2.8) or (2.9) with the expression for expected utility in (2.4), aside from the difference that the carriers of value in prospect theory are gains and losses relative to some reference point rather than final wealth levels, the two main differences are the following. First, one uses decision weights in prospect theory to aggregate outcomes while one uses direct objective probabilities under EUT. Second, there is loss aversion (arising through the term  $-\theta$ ) in prospect theory while there is none under EUT.

## 2.6. The Weighting Function in Prospect Theory

Empirical evidence is widely consistent with an inverted  $S$  shaped form for the weighting function; see for example Kahneman and Tversky (1979), Tversky and Kahneman (1992) and Prelec (1998). Denoting by  $\mathbf{p}$  the cumulative probability, Prelec (1998) derives the following weighting function (see figure 1.2),

$$w(\mathbf{p}) = w^+(\mathbf{p}) = w^-(\mathbf{p}) = \exp [ -(-\ln \mathbf{p})^\alpha ], \quad (2.10)$$

where  $0 < \alpha \leq 1$ . There are several advantages in using this weighting function, relative to the others suggested in the literature<sup>11</sup>. First, it has an inverted  $S$  shape which is consistent with experimental evidence. Second, it is based on axiomatic foundations. Third, it has the same form for gains and losses. For these reasons, we choose to use this weighting function in this paper. Several other weighting functions have been considered in the literature, that are based on transformations of individual (and not cumulative) probabilities, however, as is well known, in decision weighted models, these lead to the violation of first order stochastic dominance and for this reason these are not attractive<sup>12</sup>; see, for example, Starmer (2000).

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<sup>11</sup>Tversky and Kahneman (1992) suggest the following weighting function, based on experimental evidence, in the domain of gains and losses respectively

$$w^+(\mathbf{p}) = \mathbf{p}^\gamma \{ \mathbf{p}^\gamma + (1 - \mathbf{p})^\gamma \}^{-1/\gamma}; \quad \gamma \cong 0.61 \quad (2.11)$$

$$w^-(\mathbf{p}) = \mathbf{p}^\delta \{ \mathbf{p}^\delta + (1 - \mathbf{p})^\delta \}^{-1/\delta}; \quad \delta \cong 0.69 \quad (2.12)$$

<sup>12</sup>An example is the function proposed by Camerer and Ho (1994),  $f(p) = 1 - (1-p)^\gamma / \{ p^\gamma + (1-p)^\gamma \}^{1/\gamma}$  with  $f(0.7) \cong 0.7$ . For an application of this sort of weighting function to an essentially expected utility framework of tax evasion, see Bernasconi (1998).

### 3. The Tax Evasion Decision

This section derives the optimal tax evasion decision under expected utility and prospect theory, analytically, as well as diagrammatically.

#### 3.1. The Tax Evasion Decision Under Expected Utility

Differentiating (2.4) in the region  $Y_A > 0$ , the first order condition to the tax evasion problem is given by

$$\frac{\partial E[U]}{\partial D} = -t\beta(1-p)[Y_{NA}]^{\beta-1} + p\beta[t\lambda + s][Y_A]^{\beta-1} \leq 0, \quad D \frac{\partial E[U]}{\partial D} = 0; \quad D \geq 0 \quad (3.1)$$

where  $Y_A$  and  $Y_{NA}$  are defined in (2.1) and (2.2) respectively. The second order condition is given by

$$\frac{\partial^2 E[U]}{\partial D^2} = -t^2\beta(1-\beta)(1-p)[Y_{NA}]^{\beta-2} - \beta(1-\beta)p[t\lambda + s]^2[Y_A]^{\beta-2} < 0. \quad (3.2)$$

From 3.2 it follows that the necessary and sufficient condition for the maximization of expected utility in the region  $Y_A > 0$ , is

$$\frac{\partial E[U]}{\partial D} \geq 0 \quad \text{for } D = W \quad (3.3)$$

$$\frac{\partial E[U]}{\partial D} = 0 \quad \text{for } 0 < D < W \quad (3.4)$$

$$\frac{\partial E[U]}{\partial D} \leq 0 \quad \text{for } D = 0 \quad (3.5)$$

If the taxpayer does not evade ( $D = W$ ) then  $Y_A = W - tD > 0$  and, hence, (3.1), (3.2), and the first row of (3.3) all apply. In particular,  $\frac{\partial E[U]}{\partial D} \geq 0$  gives  $-t(1-p) + p[t\lambda + s] \geq 0$  and, hence,

$$\lambda(p) \geq \frac{1-p}{p} - \frac{s}{t} \quad (3.6)$$

In  $(\lambda, p)$  space, (3.6) gives the set of  $(\lambda, p)$  points such that a taxpayer with stigma  $s \in \left[ \underline{s}, \bar{s} \right]$  reports the full amount of income. At all point below this set, the taxpayer chooses to evade some strictly positive fraction of income. Using (3.6), in the benchmark case of  $s = 0$ , the taxpayer will evade some fraction of income when  $1 - p - p\lambda > 0$ , as claimed in the introduction.

Solving (3.4) for an interior solution to the declared income, one obtains

$$D = W \left[ 1 - (1 - t) \left[ (t\lambda + s) + t \left( \frac{(t\lambda + s)p}{t(1 - p)} \right)^{\frac{1}{1-\beta}} \right]^{-1} \right]. \quad (3.7)$$

Implicit differentiation of the first order condition (3.4), or direct differentiation of (3.7) gives rise to all the well known results under EUT, in particular, that the declared income is increasing in  $p$  and  $\lambda$ . Since stigma  $s$  plays a role that is identical to  $\lambda$ , it is easy to check that declared income is also increasing in  $s$ . It is also straightforward to derive the result in Yitzhaki (1974), namely that the declared income increases with the tax rate if preferences exhibit decreasing absolute risk aversion<sup>13</sup>. These results are summarized, without proof, in Proposition 1 below.

**Proposition 1** : *Under EUT, tax evasion is decreasing in  $p$ ,  $\lambda$ , and  $s$ . If preferences exhibit decreasing absolute risk aversion, then tax evasion is decreasing in the tax rate.*

### 3.2. The Tax Evasion Decision Under Prospect Theory

The taxpayer maximizes the value equation in (2.9) by an appropriate choice of  $D$ , given the weighting function in (2.10), which weights gains and losses symmetrically; rewriting (2.9)

$$V(f) = (W - D)^\beta h(\lambda, p, s, t, \theta, \alpha, \beta), \quad (3.8)$$

where

$$h = h(\lambda, p, s, t, \theta, \alpha, \beta) = t^\beta \left( \exp[-(-\ln(1 - p))^\alpha] - \theta \exp[-(-\ln p)^\alpha] \left( \lambda + \frac{s}{t} \right)^\beta \right) \geq 0 \quad (3.9)$$

Since the optimization problem stated in (3.8) is monotonic in  $D$ , the solution is given by

$$\begin{cases} D = 0 & \text{if } h > 0 \\ D \in [0, W] & \text{if } h = 0 \\ D = W & \text{if } h < 0 \end{cases} \quad (3.10)$$

The solution to the tax evasion problem under prospect theory is a bang-bang solution<sup>14</sup>. The solution is shown in a diagram below that also clarifies the intuition behind the result.

<sup>13</sup>These results are widely available and so no derivations are presented; see for example Myles (1995).

<sup>14</sup>By using other reference incomes, which are not as obvious and plausible as the legal after-tax income,  $(1 - t)W$ , one can generate interior solutions; this is shown in Section 6. An advantage of the bang-bang solution, however, is that it is very amenable to analysis and also descriptive of several forms of tax evasion which take the form of hiding certain activities completely from the tax authorities; see, for example, Pudney et. al. (2000).

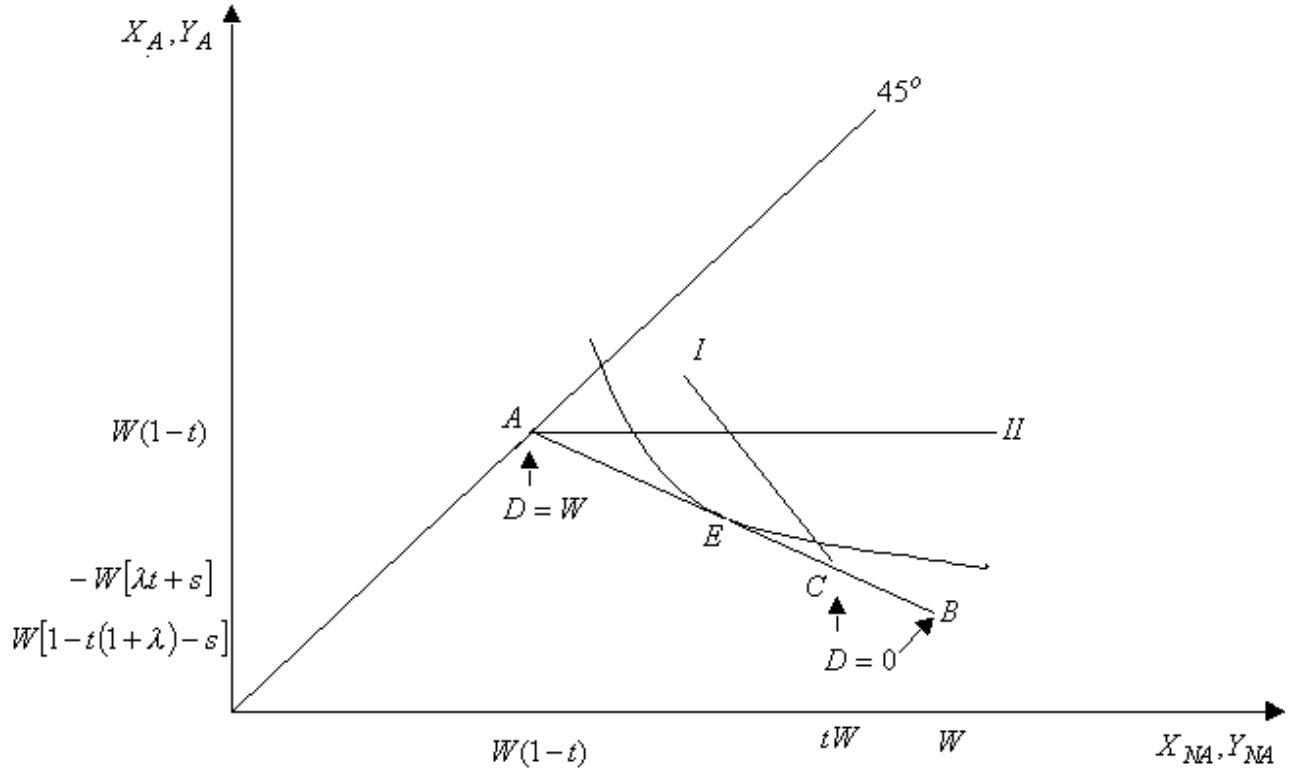


Figure 3.1: Tax evasion under expected utility and prospect theory

### 3.3. A Diagrammatic Representation of the Solution in State Space

It is instructive to compare the solutions under EUT and prospect theory in the space of incomes in the two states of the world, “Audit” and “No Audit”; this is done in Figure 3.1. Under EUT, the carriers of wealth are final wealth levels, so the origin corresponds to  $(0, 0)$ . Under prospect theory, however, the carriers of utility are gains and losses relative to the reference point,  $W(1-t)$ , hence, the origin is at  $(W(1-t), W(1-t))$ , which corresponds to point  $A$  in the figure. Therefore, the diagram for expected utility is drawn in  $(Y_A, Y_{NA})$  space with origin at  $(0, 0)$  while the diagram for prospect theory is drawn in  $(X_A, X_{NA})$  space with origin at  $[W(1-t), W(1-t)]$ .

#### 3.3.1. Tax Evasion Under Expected Utility (Diagrammatic Representation)

In Figure 3.1, point  $A$  corresponds to the case where the taxpayer declares all income so  $D = W$ . Using (2.1) and (2.2), it can be checked that the coordinates of this point are  $W(1-t), W(1-t)$ . Point  $B$  at the other extreme, corresponds to the case where  $D = 0$ ; again using (2.1) and (2.2), it can be checked that the coordinates of this point

are  $W, W [1 - t(1 + \lambda) - s]$ . Hence, the slope of the budget constraint  $AB$ , any point of which reflects a tax evasion decision  $D \in [0, W]$ , can be checked to equal  $-(\lambda + \frac{s}{t})$ . Using (2.4), the slope of an indifference curve under EUT is given by  $\frac{dY_A}{dY_{NA}} = \frac{-(1-p)}{p} \frac{v'(Y_{NA})}{v'(Y_A)}$ . Along a ray of  $45^\circ$  from the origin where  $Y_A = Y_{NA}$  the slope of the indifference curve is  $\frac{dY_A}{dY_{NA}} = \frac{-(1-p)}{p}$ .

Whether the individual evades any tax or not depends on whether, at point  $A$ , the indifference curve is steeper or flatter than the budget constraint. The diagram shows the case where the former is true so that at the equilibrium point for the taxpayer under EUT,  $E$ , some strictly positive tax evasion takes place.

The comparative static results with respect to  $s$  and  $\lambda$  are now straightforward. An increase in  $s$  or  $\lambda$  lowers point  $B$  and so the budget constraint swivels down while being anchored at point  $A$  but the slope of the indifference curve along the  $45^\circ$  line is unchanged. Hence, by making the budget constraint steeper, an increase in  $s$  or in  $\lambda$  reduces evasion by pushing the equilibrium point  $E$  in the north-west direction.

Yitzhaki (1974) showed that, within an EUT framework, an increase in the tax rate on the other hand, lead to an increase in the declared income under decreasing absolute risk aversion. The reason is as follows. Because the penalty paid on undeclared income and the tax rate are proportional to each other, a change in the tax rate results in no substitution effect, only an income effect. An increase in the tax rate makes the taxpayer poorer and, hence, more risk averse. So, under DARA, she declares more of her income.

### 3.3.2. Tax Evasion Under Prospect Theory (Diagrammatic Representation)

Under prospect theory, by using (2.6) and (2.7), it is easy to check that the decision  $D = W$  corresponds to  $[X_A, X_{NA}] = [W(1 - t), W(1 - t)]$  which is the origin in  $X_A, X_{NA}$  space and is denoted by the point  $A$ . Again, using (2.6) and (2.7) check that the tax evasion decision  $D = 0$ , corresponds to the coordinates  $tW, -(\lambda t + s)W$  which in the diagram is shown as point  $C$ . The slope of the budget constraint,  $AC$ , under prospect theory, then equals  $-(\lambda + \frac{s}{t})$ , which is identical to the slope of the budget constraint,  $AB$ , under expected utility. Hence, the budget constraint  $AC$  lies on top of  $AB$  but is shorter.

Using (2.9), the slope of the indifference curve is given by  $\frac{dX_A}{dX_{NA}} = \frac{-w(1-p)}{w(p)} \frac{v'(X_{NA})}{v'(X_A)}$ . Substituting the values for  $X_A$  and  $X_{NA}$  one can check that  $\frac{dX_A}{dX_{NA}} = \frac{-w(1-p)}{w(p)} \left(\frac{1}{\theta}\right) \left(\lambda + \frac{s}{t}\right)^{1-\beta}$ . Thus, the slope of the indifference curve is independent of  $X_{NA}$  and so the indifference curves are linear which leads to corner, bang-bang, solutions. If the indifference curves are relatively flat (see the indifference curve marked  $II$ ) then the taxpayer does not evade any taxes while in the complementary case (see the indifference curve marked  $I$ ) the taxpayer does not declare any income.

To fix ideas, it is useful to think of individuals undertaking several activities, some of which cannot be evaded, while there are others in which evasion takes the form of the individual completely hiding the activity for tax purposes. Hence, tax evasion takes the form of either complete evasion of the evadable activities or complete reporting of the non-evadable activities<sup>15</sup>.

Comparing the slopes of the budget constraint and the indifference curves under prospect theory, the following is immediate

$$\left\{ \begin{array}{lll} \text{Case : I} & D = 0 & \text{if } \left(\frac{1}{\theta}\right) \frac{w(1-p)}{w(p)} > \left(\lambda + \frac{s}{t}\right)^\beta \\ \text{Case : II} & D \in [0, W] & \text{if } \left(\frac{1}{\theta}\right) \frac{w(1-p)}{w(p)} = \left(\lambda + \frac{s}{t}\right)^\beta \\ \text{Case : III} & D = W & \text{if } \left(\frac{1}{\theta}\right) \frac{w(1-p)}{w(p)} < \left(\lambda + \frac{s}{t}\right)^\beta \end{array} \right. \quad (3.11)$$

The diagrammatic equilibrium derived in (3.11) is identical to the analytical equilibrium in (3.10). It is clear from the inequality given in Case III of (3.11), that in prospect theory, the following factors are conducive to eliminating tax evasion, so that declared and actual incomes equal (i.e.  $D = W$ ).

1. Low tax rates,  $t$ .
2. High levels of stigma,  $s$ .
3. High penalties for tax evasion,  $\lambda$ .
4. High levels of loss aversion,  $\theta$ .
5. Overweighting of the probability of a loss,  $p$ .

The comparative static properties of the tax evasion model under prospect theory are formally summarized below.

**Proposition 2** : *Ceteris-paribus*: (1)  $\exists \lambda = \lambda_c \geq 0$ , such that, when  $\lambda > \lambda_c$ , the taxpayer declares the full amount of income, while in the complementary case, income is evaded. (2)  $\exists p = p_c \in [0, 1]$ , such that, when  $p > p_c$ , the taxpayer declares the full amount of income, while in the complementary case, tax is evaded. (3)  $\exists s = s_c \in \left[ \underline{s}, \bar{s} \right]$ , such that, when  $s > s_c$ , the taxpayer declares the full amount of income, while in the complementary case, tax is evaded.

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<sup>15</sup>The empirical evidence presented in Pudney et. al. (2000) is suggestive of this form of tax evasion. For instance, they find that there is no evidence of a disincentive impact of extra punishment on those who decide to evade, which would, for instance, be the case if one is at a corner solution.



Comparing Propositions 1 and 2 it turns out that EUT and prospect theory both predict that tax evasion decreases with an increase in  $p$ ,  $\lambda$ , and  $s$ . However, the main difference in the two approaches is in the predicted effect of the tax rate on the amount evaded. EUT predicts, as in Yitzhaki (1974), that (with decreasing absolute risk aversion) individuals evade less income as the tax rate increases while prospect theory predicts the more factual result that tax evasion increases with an increase in the tax rate. This is formally shown in Proposition 3 below.

**Proposition 3** : *Ceteris-paribus*,  $\exists t = t_c \in [0, 1]$  such that the individual does not evade taxes if  $t < t_c$  but evades taxes if  $t > t_c$ .

The explanation behind Proposition 3 is as follows. Because the penalty paid on undeclared income,  $t\lambda(W - D)$ , is proportional to the tax rate,  $t$ , a change in the tax rate has no substitution effect, just an income effect. An increase in the tax rate makes the tax payer poorer. Hence, under decreasing absolute risk aversion, EUT predicts a tax payer would become more risk averse, and so evade less. By contrast, under prospect theory the value function is convex for losses. So, if the reference point is such that an audited taxpayer finds himself in the domain of losses (as in this paper) then an increase in the tax rate causes him to be poorer and, hence, more risk seeking. So he evades more.

Under prospect theory, attitude to risk is determined, not just by the shape of the value function, but is also influenced by loss aversion and non-linear transformation of probabilities. These issues are explored in Propositions 4 and 5 below.

**Proposition 4** : *Ceteris-paribus*,  $\exists \theta = \theta_c \in \mathbb{R}_+$  such that for all  $\theta > \theta_c$  the taxpayer does not evade any tax, while for all  $\theta < \theta_c$  a strictly positive level of evasion arises.

From Proposition 4 we see that an increase in the loss aversion parameter reduces tax evasion. In this case, the loss in income (relative to the reference point) in the event of an audit looms even larger than before, reducing the amount of tax evasion.

**Proposition 5** : (1) Let  $R(\alpha) = \frac{w(1-p)}{w(p)}$ , where the weighting function  $w(p) = \exp[-(-\ln p)^\alpha]$  and the parameter  $\alpha \in [0, 1]$  (see (2.10)). Then  $R(0) = 1$  and  $R(1) = \frac{1-p}{p}$ . (2) Suppose  $p < \frac{1}{2}$ . Then  $R(\alpha)$  is strictly increasing in  $\alpha$ , and (a) If  $\theta \left(\lambda + \frac{s}{t}\right)^\beta \leq 1$  then the taxpayer evades for all  $\alpha$ . (b) If  $\theta \left(\lambda + \frac{s}{t}\right)^\beta \geq \frac{1-p}{p}$  then the taxpayer does not evade for any  $\alpha$ . (c) If  $1 < \theta \left(\lambda + \frac{s}{t}\right)^\beta < \frac{1-p}{p}$ , then  $\exists \alpha_c \in [0, 1]$ , given by  $R(\alpha_c) = \theta \left(\lambda + \frac{s}{t}\right)^\beta$ , such that the taxpayer does not evade for all  $\alpha < \alpha_c$  and evades for  $\alpha > \alpha_c$ . (3) Suppose  $p > \frac{1}{2}$ . Then  $R(\alpha)$  is strictly decreasing in  $\alpha$ , and (a) If  $\theta \left(\lambda + \frac{s}{t}\right)^\beta \leq \frac{1-p}{p}$  then the taxpayer evades for all  $\alpha$ . (b) If  $\theta \left(\lambda + \frac{s}{t}\right)^\beta \geq 1$  then the taxpayer does not evade for any  $\alpha$ . (c) If  $\frac{1-p}{p} < \theta \left(\lambda + \frac{s}{t}\right)^\beta < 1$ , then  $\exists \alpha_c \in [0, 1]$ , given by  $R(\alpha_c) = \theta \left(\lambda + \frac{s}{t}\right)^\beta$ , such that the taxpayer evades for all  $\alpha < \alpha_c$  and does not evade for  $\alpha > \alpha_c$ .

From Proposition 4(2), we see that, if  $p < \frac{1}{2}$  then a reduction in  $\alpha$  reduces tax evasion. This is because the overweighting of a low probability of an audit increases. The reverse holds if  $p > \frac{1}{2}$ , because the underweighting of a high probability of an audit increases.

#### 4. Model Calibration: Audit Probabilities, Penalty Rates And Tax Evasion

What accounts for the high degree of tax compliance when actual  $(\lambda, p)$  combinations are such that  $1 - p - p\lambda > 0$ ? To answer this question, it is useful to start with the following question. Given reasonable magnitudes of tax evasion in the population, what combinations of  $(\lambda, p)$  are required to sustain that level of evasion. Since the weighting function in prospect theory is highly non-linear, analytical answers are not possible. Hence, the analysis below provides some simulation results in a calibrated model.

From estimates in Tversky and Kahneman (1992) we know that (approximately)  $\beta = 0.88$  and  $\theta = 2.25$ . We use a tax rate of 30 percent i.e.  $t = 0.3$ . From Bernasconi (1998) we know that in actual practice 30 to 60 percent of taxpayers report their incomes correctly. Since these figures would include those who unintentionally report incorrectly we use the more conservative estimate that only about 30 percent evade taxes. We assume that the stigma rate is distributed uniformly over the unit interval i.e.  $s \in [0, 1]$  so that, in monetary terms, stigma costs do not exceed the magnitude of the income evaded. From Proposition 2, we know that all individuals characterised by  $s > s_c$  do not evade taxes. Hence, the stylized fact that approximately 30 percent of taxpayers evade taxes corresponds to  $s_c = 0.3$ <sup>16</sup>.

The question that we ask then is the following. Given realistic magnitudes of tax evasion, i.e. 30 percent, how close are the predicted  $\lambda, p$  values under EUT and prospect theory, respectively, to their actual values?

Under expected utility then, using 3.6, the locus of  $(\lambda, p)$  combinations that need to be consistent with the actual data is given by

$$\lambda^{EU}(p) = \frac{1-p}{p} - \frac{0.3}{0.3} \quad (4.1)$$

Using 3.11, and the form of the weighting function in (2.10), a similar locus of  $(\lambda, p)$  combinations under prospect theory is given by

$$\lambda^{PT}(p) = \left(\frac{1}{2.25}\right)^{\frac{1}{0.88}} \Gamma(p)^{\frac{1}{0.88}} - \frac{0.3}{0.3}, \quad (4.2)$$

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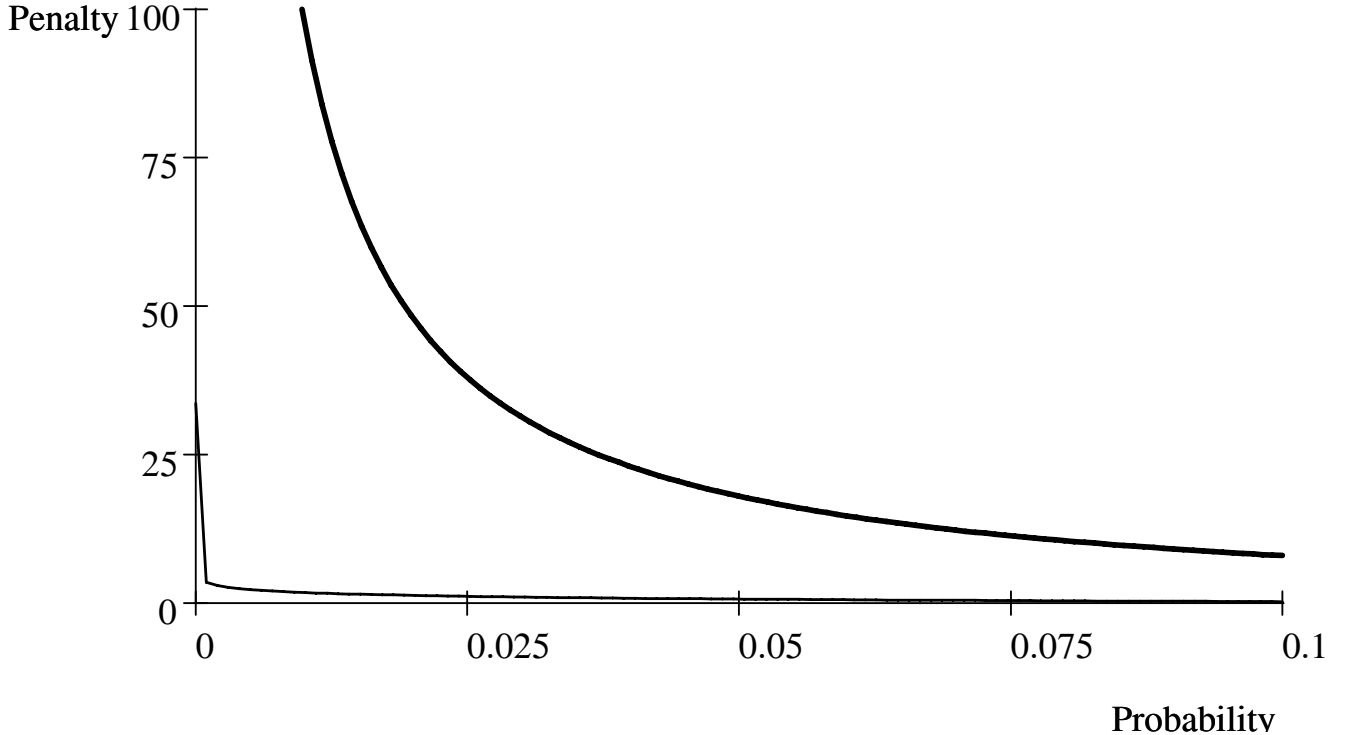
<sup>16</sup>The results are fairly robust to values of  $s$  other than 0.3 used in the calibration exercise.

where

$$\Gamma(p) = \exp [(-\ln p)^{0.35} - (-\ln(1 - p))^{0.35}]$$

We have substituted the value of  $\alpha = 0.35$  in  $\Gamma(p)$ ; this does not affect the qualitative comparison between the EUT and Prospect theory approaches.

In the diagram below, the horizontal axis represents the audit probability  $p$  and the vertical axis represents the audit penalty,  $\lambda$ . Since most realistic audit probabilities are in the range 0.01 to 0.03 the horizontal axis is shown up to a maximum value of 0.10. The upper, thicker, curve plots the  $(\lambda, p)$  locus for expected utility in (4.1) and the lower, thinner, curve plots the  $(\lambda, p)$  locus for prospect theory in (4.2). It is clear that the  $(\lambda, p)$  locus for EUT is everywhere significantly above that for prospect theory<sup>17</sup>.



Combinations of  $\lambda, p$  consistent with 30% tax evasion under EUT and PT. The numerical magnitudes of  $(\lambda, p)$  that correspond to the two loci in the figure are shown in the table below.

**Table I: Comparison of  $\lambda$  Under Expected Utility and Prospect Theory**

$p$	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050
$\lambda^{EU}(p)$	197.67	97.67	64.33	47.67	37.67	31.00	26.23	22.67	19.89	17.67
$\lambda^{PT}(p)$	1.55	1.21	1.00	0.86	0.75	0.66	0.58	0.52	0.46	0.41

$\lambda^{EU}(p)$  and  $\lambda^{PT}(p)$  are the penalty rates consistent with a given  $p$ , and a tax evasion rate of 30 percent, under EUT and Prospect theory, respectively. For most realistic audit

<sup>17</sup>Note that the vertical axis gives the penalty rate. Hence, a penalty rate of 50, for instance, means that for each pound of tax evaded, the penalty is 50 pounds.

probabilities i.e.  $p \in [0.01, 0.03]$  the estimate of the penalty rate ranges from 0.66 to 1.21 for Prospect Theory. This is consistent with actual values for the audit probabilities which range from 0.5 in USA to 2.0 in Italy; see Bernasconi (1998) for these figures. On the other hand, EUT predicts a penalty rate that is on average about 100 times larger in this range! The above is summarised, without proof, by the following Proposition.

**Proposition 6** : *For realistic magnitudes of tax evasion (approximately 30 percent) and audit probabilities (1 to 3 percent) the penalty rate predicted by Prospect theory is 0.66 to 1.21 while that predicted by EUT is 31 to 98. The penalty rate predicted by Prospect theory is in agreement with observed rates while that predicted by EUT is about 100 times larger.*

This provides strong vindication for the choice of prospect theory in explaining actual parameters of policy choice relevant for the tax evasion problem. It is remarkable that the predicted magnitudes of  $(\lambda, p)$  are so close to the actually observed values when one considers that the parameter values used for the calibration exercise were obtained from independent experimental evidence applied to generic situations of risk.

## 5. Optimal Choice of the Tax Rate

Tax authorities typically have limited choice over both  $p$  and  $\lambda$ . The constitutions in most Western democracies require that comparable offenses must be punished in a comparable manner, hence, the penalty rates levied by the tax authorities are severely constrained by penalties for similar cases of fraud and negligence. Although, in principle, the tax authorities have more flexibility in determining the audit probability, significant changes in  $p$  are fairly resource intensive and typically the tax authorities face fairly stringent resource constraints in this endeavour. Hence, the audit probabilities in actual practice tend to lie between 0.01 and 0.03 for most countries. However, there is a somewhat greater element in the choice of a tax rate, an issue which this section turns to, below.

In actual practice, the objective function of a government over the tax rate can be fairly complex and it can involve difficult trade-offs between efficiency and equity as well as complicated political economy considerations. The exposition below, however, follows an old lineage in tax policy, in postulating that the government acts like a revenue maximizing leviathan, whose sole objective is to maximize total tax revenues. It turns out that even in this fairly conservative and limited framework, for plausible parameter values, one gets realistic magnitudes of optimal tax rates using prospect theory.

Using (3.11) one can find cutoff values of stigma  $s^*$ , such that all individuals with  $s > s^*$  choose to declare their (full) income, while all individuals with  $s < s^*$  choose to fully evade their income. It can be checked, using (3.11), that

$$s^* = t\Psi; \text{ where } \Psi = \left(\frac{1}{\theta}\right)^{\frac{1}{\beta}} [\exp((-\ln p)^\alpha - (-\ln(1-p))^\alpha)]^{\frac{1}{\beta}} - \lambda. \quad (5.1)$$

Hence, total tax revenue is given by

$$T = (tW) [1 - \Phi(s^*)]$$

where  $\Phi(s)$  is the distribution function of stigma and  $\phi(s)$  is the corresponding density.

The government's objective is to choose its optimal tax rate  $t^*$  in order to maximize the tax revenue  $T$ . The first order condition is

$$\frac{\partial T}{\partial t} = [1 - \Phi(s^*)] - t\phi(s^*)\frac{\partial s^*}{\partial t} \leq 0; \quad t \geq 0 \quad (5.2)$$

The second order condition is given by

$$\frac{\partial^2 T}{\partial t^2} = -\Psi [2\phi(s^*) - t\Phi''(s^*)].$$

Hence, a sufficient condition for the second order condition to hold is that the distribution function be (weakly) convex i.e.  $\Phi''(s^*) \geq 0$ . The comparative static properties of the optimal tax rate are summarized in the following proposition.

**Proposition 7 :** *If  $\Phi''(s^*) \geq 0$  and  $\phi(s)$  is log concave then the optimal tax rate  $t^*$  is increasing in  $\lambda$ ,  $p$ ,  $\theta$  and  $\beta$ .*

Several commonly used density functions, such as the Gaussian, exponential, logistic, chi-squared, Laplace and the uniform are log concave; for a more complete list see Bagnoli and Bergstrom (1989).

The intuition behind Proposition 7 can be understood by rewriting the first derivative of  $T$ , stated in (5.2) and evaluated at the optimal tax choice  $t^*$ , as follows:

$$\frac{\partial T}{\partial t} = [1 - \Phi(s^*)] \left(1 - \left(\frac{\phi(s^*)}{1 - \Phi(s^*)}\right) t^* \frac{\partial s^*}{\partial t}\right) \quad (5.3)$$

Consider, for instance, the comparative static affect of an increase in  $\lambda$ . From Proposition 2 we know that  $s^*$  is decreasing in  $\lambda$  and so  $[1 - \Phi(s^*)]$  increases. Log concavity of  $\phi(s)$  ensures that the likelihood ratio  $\frac{\phi(s^*)}{1 - \Phi(s^*)}$  falls when  $s^*$  falls.  $\frac{\partial s^*}{\partial t}$  is independent of  $\lambda$  and using the envelope theorem, we can ignore the indirect effects of  $\lambda$  on the optimal tax choice  $t^*$ . Hence, an increase in  $\lambda$  increases the marginal benefit of increasing  $t$ , and so,  $t^*$  must increase. The intuition behind the other comparative static effects can be similarly constructed.

## 5.1. Optimal tax rates in a calibrated model.

**Example 1** : A particularly simple and tractable case of a log concave distribution is the uniform so that  $s \sim U \left[ \underset{-}{s}, \bar{s} \right]$ . Defining  $\Delta s = \bar{s} - s$ , the first order condition (5.2) can be written as

$$\frac{\partial T}{\partial t} = \left[ 1 - \frac{s^* - s}{\Delta s} \right] - \frac{t\Psi}{\Delta s} \leq 0; \quad t \geq 0. \quad (5.4)$$

For an interior solution, one can solve (5.4) to get the following simple formula for the optimal tax rate

$$t^* = \frac{\bar{s}}{2\Psi} \quad (5.5)$$

To calibrate the model, we use the values of  $\theta$  and  $\beta$  suggested by Tversky and Kahneman (1992) i.e.  $\theta = 2.25$ ,  $\beta = 0.88$ . For audit probability and the penalty rate we use conservative values suggested by Bernasconi (1998) i.e.  $p = 0.03$ ,  $\lambda = 2.0$ . Finally, for the domain of the uniform distribution we use  $s \sim U [0, 1.5]$ , which is also likely to be a fairly conservative figure. Substituting these calibrated values in (5.5) one gets the optimal tax rate to be 33.33 percent which is not unrepresentative of Western democracies. What is reassuring about the optimal tax analysis is that a relatively conservative formulation generates reasonably realistic estimates of the optimal tax. It is also suggestive of the further use of prospect theory in a wider range of optimal tax models.

## 6. Convex Indifference Curves Under Prospect Theory and alternative reference incomes

Suppose now that reference income equals  $\nu(1-t)W$  where  $\nu > 0$ ; the text assumes that  $\nu = 1$ . Define by  $I_A$  and  $I_{NA}$  respectively, the income of the taxpayer, relative to the reference point, when the taxpayer is and is not audited. It is easy to check that

$$I_A = W [(1-t)(1-\nu) - \lambda t - s] - D [\lambda t + s]$$

$$I_{NA} = W [1 - \nu(1-t)] - tD$$

In order to define the budget constraint, notice that at  $D = 0$ ,  $I_A = W [(1-t)(1-\nu) - \lambda t - s]$  and  $I_{NA} = W [1 - \nu(1-t)]$  while at  $D = W$ ,  $I_A = I_{NA} = W [(1-t)(1-\nu)]$ . From this it is easy to compute the slope of the budget constraint in  $I_A, I_{NA}$  space to be  $-\left[\lambda + \frac{s}{t}\right]$ , which is identical to the one in the text with  $\nu = 1$ , hence, movements of the reference

point do not affect the slope of the budget constraint. Therefore, in terms of Figure 3.1, one proceeds as before but by defining a new origin (corresponding to a new point  $A$ ) with coordinates given by  $\{W(1-t)(1-\nu), W(1-t)(1-\nu)\}$ .

Also check that the slope of the indifference curve in  $I_A, I_{NA}$  space is now given by:

$$\frac{dI_A}{dI_{NA}} = \frac{-w(1-p)}{\theta w(p)} \left[ \frac{-I_A}{I_{NA}} \right]^{1-\beta} < 0$$

The sign follows because  $I_A$  is a loss relative to the reference point. Differentiating again, it is easy to show that  $\frac{d^2 I_A}{dI_{NA}^2} \geq 0$ . In terms of Figure 3.1, one now gets smooth, convex downward sloping indifference curves even in the prospect theory case. This should typically give an interior solution to the tax evasion decision compared to the bang-bang solution one gets under linear indifference curves.

## 7. Conclusions

Why should people pay taxes when, given relatively low audit probabilities and penalties, evasion should be extremely attractive, at least to expected utility maximizers? This is puzzling, given “too much observed compliance” relative to the predictions of expected utility theory. This paper considers an alternative theoretical model that is based on cumulative prospect theory. Prospect theory characterizes individuals as loss averse with respect to some reference income. Furthermore, these individuals overweight small probabilities while underweighting large ones.

Results in the paper show that despite the existence of low audit probabilities and penalty rates in actual practice, the magnitude of tax evasion predicted by prospect theory is consistent with the data. Individuals are also predicted to respond to an increase in the tax rate by increasing the amount evaded. This accords with the bulk of the evidence, but contrasts with the converse prediction made by expected utility theory. Finally, we show that the optimal tax rates predicted by prospect theory, in the presence of tax evasion behavior, are consistent with actual tax rates.

Prospect theory was developed to explain actual choice under generic situations of risk. There is a large body of evidence supporting prospect theory, see for example, Kahnemann and Tversky (2000) and Camerer, Loewenstein and Rabin (2004).

In applying prospect theory, we do not ‘fix’ the parameter values of the model to make it fit the evidence on tax evasion. Instead, we use the parameters of human choice that are revealed from independent experimental evidence. Prospect theory is able to explain the tax evasion puzzles. Its predictions about the magnitudes of optimal income taxes in the presence of tax evasion are indicative of actual magnitudes. Hence, the paper concludes that the behavior of tax payers provides strong support for prospect theory.

## 8. Appendix

*Proof of Proposition 2*

Using 3.10, the definition of  $h(\cdot)$  in (3.9) and by defining  $\lambda_c = \left(\frac{w(1-p)}{\theta w(p)}\right)^{1/\beta} - \frac{s}{t}$ , and  $s_c = t \left\{ \left(\frac{w(1-p)}{\theta w(p)}\right)^{1/\beta} - \lambda \right\}$  it follows that when  $\lambda < \lambda_c$ ,  $h > 0$  and so  $D = 0$ . Conversely when  $\lambda > \lambda_c$ ,  $h < 0$  and so  $D = W$ . The same argument applies to the inequalities  $s < s_c$  and  $s > s_c$ .

Since  $w(p)$  is continuous and increasing in  $p$ , hence, the function  $\frac{w(1-p)}{w(p)}$  is continuous and decreasing in  $p$  over the range  $[0, \infty]$ . From (3.11) the three cases of  $D = 0$ ,  $D = [0, W]$  and  $D = W$  are distinguished by the relative magnitudes of  $\frac{w(1-p)}{w(p)}$  and  $\theta \left(\lambda + \frac{s}{t}\right)^\beta$ . Since  $\frac{w(1-p)}{w(p)}$  is continuous over  $[0, \infty]$ , there exists some  $p = p_c$  such that for all  $p < p_c$ ,  $\frac{w(1-p)}{w(p)} > \theta \left(\lambda + \frac{s}{t}\right)^\beta$  and so  $D = 0$  while at  $p = p_c$ ,  $\frac{w(1-p)}{w(p)} = \theta \left(\lambda + \frac{s}{t}\right)^\beta$  and so  $D = [0, W]$ . Finally, for  $p > p_c$ ,  $\frac{w(1-p)}{w(p)} < \theta \left(\lambda + \frac{s}{t}\right)^\beta$  and so  $D = W$ . ■

*Proof of Proposition 3*

Using 3.10, and the definition of  $h(\cdot)$  in (3.9) and by defining  $t_c = s \left[ \left(\frac{w(1-p)}{\theta w(p)}\right)^{1/\beta} - \lambda \right]^{-1} \geq 0$  it follows that when  $t = t_c$ ,  $h = 0$ . When  $t < t_c$ ,  $h < 0$  and so  $D = W$ , whereas if  $t > t_c$ ,  $h > 0$  and so  $D = 0$ . ■

*Proof of Proposition 4*

Using 3.10, and the definition of  $h(\cdot)$  in (3.9) and by defining  $\theta_c = \left(\frac{w(1-p)}{\theta w(p)}\right) \left(\lambda + \frac{s}{t}\right)^{-\beta} \geq 0$  it follows that when  $\theta = \theta_c$ ,  $h = 0$ . When  $\theta > \theta_c$ ,  $h < 0$  and so  $D = W$ , whereas if  $\theta < \theta_c$ ,  $h > 0$  and so  $D = 0$ . ■

*Proof of Proposition 5*

Since  $R(\alpha) = \frac{w(1-p)}{w(p)}$ , where  $w(p) = \exp[-(-\ln p)^\alpha]$  and  $\alpha \in [0, 1]$  a straightforward calculation gives  $R(0) = 1$ ,  $R(1) = \frac{1-p}{p}$  and that  $R(\alpha)$  is strictly increasing for  $p < \frac{1}{2}$  and strictly increasing for  $p > \frac{1}{2}$ . This establishes (1). (2) and (3) then follow as simply corollaries, using (3.11). ■

*Proof of Proposition 7*

Implicitly differentiate 5.2 successively with respect to  $\lambda$ ,  $p$ ,  $\theta$  and  $\beta$ . The generic derivative is

$$\frac{\partial t^*}{\partial x} = t^* \left( -\frac{\partial^2 T}{\partial t^2} \right)^{-1} \left[ \left( \frac{\phi(s^*)}{1 - \Phi(s^*)} \right)^2 \frac{\partial}{\partial s} \left[ \frac{\phi(s^*)}{1 - \Phi(s^*)} \right] + 1 \right] \left( -\frac{\partial \Psi}{\partial x} \right), \quad (8.1)$$

where  $x = \lambda, p, \theta$  and  $\beta$  and  $\Psi$  is defined in (5.1). The condition  $\Phi''(s^*) \geq 0$  ensures that the second order condition holds while log concavity of  $\phi(s)$  ensures that  $\frac{\partial}{\partial s} \left[ \frac{\phi(s^*)}{1 - \Phi(s^*)} \right] \geq 0$ ,



hence, the sign of  $\frac{\partial t^*}{\partial x} = -\text{sign}(\frac{\partial \Psi}{\partial x})$ . For  $x = \lambda, p, \theta$  and  $\beta$  and using (5.1) check that the following hold.

$$\frac{\partial \Psi}{\partial \lambda} = -t^* \leq 0$$

$$\frac{\partial \Psi}{\partial \theta} = t^* \left( \frac{-1}{\beta} \right) \theta^{-\beta-1} [\exp((-\ln p)^\alpha - (-\ln(1-p))^\alpha)]^{\frac{1}{\beta}} \leq 0$$

$$\frac{\partial \Psi}{\partial \beta} = \left( \frac{-t^*}{\beta^2} \right) \frac{\log \left[ [\exp((-\ln p)^\alpha - (-\ln(1-p))^\alpha)]^{\frac{1}{\beta}} \right]}{[\exp((-\ln p)^\alpha - (-\ln(1-p))^\alpha)]^{\frac{-1}{\beta}}} \leq 0$$

$$\frac{\partial \Psi}{\partial p} = \left( \frac{-\alpha t^*}{\beta \theta} \right) \frac{\left[ \frac{(-\ln p)^{\alpha-1}}{p} + \frac{(-\ln(1-p))^{\alpha-1}}{(1-p)} \right]}{[\exp((-\ln p)^\alpha - (-\ln(1-p))^\alpha)]^{1-\frac{1}{\beta}}} \leq 0$$

The signing of these derivatives uses the fact that  $\exp((-\ln p)^\alpha - (-\ln(1-p))^\alpha) \geq 0$ , which is a necessary condition for  $s^* \in \text{int} \left[ \underset{-}{s}, \bar{s} \right]$  (see 5.1); this follows for any of the parameter values used in this paper. The statement in the proposition now follows by using (8.1) in conjunction with the sign of the derivatives of  $\Psi$  with respect to the various parameters. ■

## 9. References

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