Why Do Sellers (Usually) Prefer Auctions?

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Abstract

We compare the most common methods for selling a company or other asset when participation is costly: a simple simultaneous auction, and a sequential process in which potential buyers decide in turn whether or not to enter the bidding. The sequential process is always more efficient. But pre-emptive bids transfer surplus from the seller to buyers. Because the auction is more conducive to entry - precisely because of its inefficiency - it *usually* generates higher expected revenue. We also discuss the effects of lock-ups, matching rights, break fees (as in takeover battles), entry subsidies, etc. [94 words]

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Why do sellers like auctions? The simple answer is that auctions involve bidders competing simultaneously, and this benefits sellers.

But when entry is costly, auctions are inefficient. Sequential mechanisms are more efficient, because later potential bidders can use the information from early bids in deciding whether to undertake costly search and entry. If a seller can capture enough of the efficiency gain from sequential entry, it is more profitable for it to deal with just one bidder at a time, letting the bidders respond to the *potential* competition from potential buyers, with the entry costs for *actual* competition being incurred only when the early bidders turn out to have low values.[1]

Nevertheless, it seems to be widely believed that, as a practical matter, any additional rents created by real-world sequential mechanisms — and more — usually accrue to buyers. In a poll of private equity firms, while about 90 percent said they preferred to avoid auctions when acting as buyers, about 80 percent said they prefer to use auctions when acting as sellers.[2]

Bruce Wasserstein, the most successful U.S. mergers and acquisitions banker of the past 30 years, writes "It can be helpful to think of the range of possibilities in terms of two types — the classic...auction and the negotiated sale...A wide-ranging auction generally maximizes value...Naturally, sophisticated bidders will do their best to circumvent the auction format." (Wasserstein, 2001.) In a negotiated sale a seller commonly agrees to a price with a single bidder while retaining the right to talk subsequently to other bidders according to a "go shop" agreement that gives it a limited amount of time to find a better offer; the agreement does not usually permit the seller to subsidize a new bidder to compete, and the original bidder retains the right to beat any new bidder's offers. While some sellers, especially in the "middle market", purposely choose to negotiate sequentially with one buyer at a time, it is more often buyers who attempt to avoid auctions, often by making by a pre-emptive "jump bid".[3]

The paper is *not* primarily about "optimal" mechanisms. In our model, if the seller has enough information and commitment power to compute and charge optimal entrance fees

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[1] Nihat Aktas, Eric de Bodt, and Richard Roll (2007) and Audra Boone and Harold Mulherin (2008) emphasize the value of potential competition in obtaining a good offer from a buyer.

[2] "Auction Process Roundtable", *Mergers and Acquisitions,* December 2006, pp. 31-32.

Warren Buffet famously states each year in his annual report under "Acquisition Criteria": "[W]e don't want to waste our time ...*We don't participate in auctions.*" The italics are in the original.

Similarly, several large petrochemical companies are known for trying to maintain reputations for refusing to enter auctions either for buying businesses or for supplying products, at the same time as they sell businesses using auctions and procure supplies through "reverse" e-commerce auctions wherever possible.

[3] "Middle market" transactions are typically defined as those between $50 million and $500 million. See also Wasserstein (2001), and also see Boone and Mulherin (2007a,b) for a detailed breakdown of actual sales methods.

and subsidies, it can run an efficient sequential procedure that extracts all buyers' surplus.[4] But, although we will discuss optimal mechanisms for well-informed, powerful, sellers in sections VI and VII, real-world sellers' abilities seem more limited. (Conversely, if buyers who arrive in sequence can make credible take-it-or-leave-it offers they can extract all the surplus, but buyers' powers are also usually more restricted.)

This paper therefore focuses on comparing the two dominant methods for selling public companies - a simple "plain vanilla" simultaneous auction and an equally simple model of a sequential sales mechanism - when the seller has realistically-limited power and information. Similar alternative ways of selling are observed for many other assets.[5] We will show why the auction format usually - but not always - maximizes value for the seller and why sophisticated bidders will usually prefer to subvert an auction by making pre-emptive jump bids when they can.

In both processes we model, an unknown number of potential bidders make entry decisions sequentially, before learning their values. In an *auction*, no credible bidding is possible until all entry decisions have been taken. In a *sequential mechanism*, potential bidders arrive in turn. Each one observes the current price and bidding history and decides whether to pay the entry cost to learn its value. If it does, and if it succeeds in outbidding any current incumbent (who can respond by raising its own bid), it can also make any additional jump bid it wishes to attempt to deter further entry. In both processes, we assume the seller does not have the power or credibility to commit to a take-it-or-leave-it minimum (reservation) price above a buyer's minimum possible value.

Of course, there are many variants of these two processes, and exactly which extensive-form game is the best representation of any specific real-life sales process is often open to debate. So we also examine the sensitivity of our results to simple modifications of our basic simultaneous and sequential processes.

Our central result is that the straightforward, level-playing-field competition that an auction creates is *usually* more profitable for a seller than a sequential process, even though the sequential mechanism is *always* more efficient in expectation (as measured by the winner's expected value less expected aggregate entry costs).

The sequential process is more efficient because although it attracts fewer bidders in

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[4]Our model (in which buyers have no information about their values before entering) is similar to a special case of Jacques Cremer, Yossi Spiegel, and Charles Zheng (forthcoming), though our model, unlike theirs, assumes the number of potential bidders is unknown. See also John Riley and Richard Zeckhauser (1983), Preston McAfee and John McMillan (1987, 1988), Roberto Burguet (1996), Egil Kjerstad and Steinar Vagstad (2000), Flavio Menezes and Paulo Monteiro (2000), Paul Milgrom (2004), and Cremer, Spiegel, and Zheng (2007) and for discussions of optimal mechanisms in related contexts. See Paul Klemperer (1999, 2004), Vijay Krishna (2002), Milgrom (2004), and Menezes and Monteiro (2005) for syntheses of a large fraction of the auction literature.

[5]Obvious examples are fine art and housing – see the conclusion.

expectation, it attracts more bidders when those bidders are most valuable - the existence or absence of early bids informs subsequent entry decisions and attracts additional bidders when the early ones turn out to be weak. But buyers' ability to make pre-emptive jump bids, which inefficiently deter too many potential rivals from entering, harms the seller.[6]

We identify four factors that may cause the expected revenue between the auction and the sequential mechanism to differ. The first three all unambiguously favor the auction. Two of these factors are fairly straightforward. First, even in the most favorable circumstances, a sequential process could only be superior if the queue of potential bidders is sufficiently longer than the number that would compete in an auction. Second, in a sequential mechanism bidders who deter entry choose a price where the expected distribution of winning values is such that an additional entrant would expect to earn zero. By contrast, except in a knife-edge break-even case, bidders deterred from an auction face a distribution of winning values that make entry strictly unprofitable.

These two factors would be nullified with an infinite stream of potential bidders, and when parameters are such that the expected profits of the marginal bidder who does not enter the auction is exactly zero. The third factor is therefore crucial: the value of the winning bidder is generally less dispersed in the sequential process, because that process is more likely to attract one high-value bidder but will never attract more than one. But entrants prefer more dispersion in the value they have to beat, because dispersion makes the entrant's option to buy more valuable – this is just the standard consumer-theory result that consumers prefer more random prices. Therefore, the expected value of the top bidder in the auction must be higher than in the sequential mechanism to deter entry.

Thus, contrary to our usual instinct that auctions are profitable because they are efficient, *it is precisely the inefficiency of the auction - that entry into it is relatively ill-informed and therefore leads to a more random outcome - that makes it more profitable for the seller.*

These three factors all speak to the superiority of the auction in generating entry, and each factor provides a separate reason why the expected *value* of the winner in the auction must exceed the expected value of the winner in the sequential mechanism. But the seller ultimately cares about the expected price paid, and the comparison of expected prices is a somewhat different question. The fourth factor, then, involves the gap between expected

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[6]Michael Fishman (1988) considered a model with two potential bidders, both of whom would have participated in an auction. In this case, a pre-emptive offer could deter entry by the second bidder but a sequential mechanism would not increase the number of bidders.

David Hirshleifer and Ivan Png (1989), Kent Daniel and Hirshleifer (1998), and Robert Easley and Rafael Tenorio (2004) emphasised jump bidding in the context of bidding costs (rather than costs of entry). Among the large theoretical literature on endogenous entry into auctions, see McAfee and McMillan (1987), Ronald Harstad (1990, 2003), McAfee, Dan Vincent, Mike Williams and Melanie Williams Havens (1993), Burguet and Joszef Sakovics (1996), Menezes and Monteiro (2000), Kate Larson and Tuomas Sandholm (2001), and the references therein.

winning values and expected revenues, and this can work either for or against auctions.

However, we show that for typical demand specifications a higher expected winner's value directly implies higher expected revenue. More generally, the fourth factor (whatever its sign) is only significant when the third factor (which always favours the auction) is large. So an auction is generally better for a seller.

Why do bidders capture more than 100 percent of the extra rents created by the sequential mechanism? We will see that pre-emptive bidding is crucial: jump-bidding allows buyers to choose partial-pooling deterrence equilibria which over-deter entry relative to the social optimum. Although the information that bidders signal makes the sequential mechanism more efficient than the auction, the over-deterrence transfers enough rents to the bidders that sellers generally prefer auctions even when there is an infinite stream of potential buyers. With fewer potential buyers even efficient signalling can hurt sellers, because there is less potential for (the absence of) signalling to induce additional entry when early bidders are weak, so bidders earn some scarcity rents in a sequential mechanism even when there are more of them than would enter an auction.

With enough potential bidders, a sequential mechanism in which jump-bidding is prohibited may be desirable from the seller's viewpoint, because this retains the efficiency advantages of a sequential process while curtailing over-deterrence. But in markets such as that for corporate takeovers, it is generally the buyers who name prices, and pre-emptive jump bids are common.

Part of the appeal of the auction is that the competition it creates allows the seller to do well independent of any knowledge of bidder values or any ability to exploit that knowledge. Arming either buyers or sellers with greater commitment power and the knowledge necessary to use it will modify the equilibrium in the sequential mechanism. If the seller can opportunistically subsidize entry it can increase expected revenue – mainly through the *threat* to subsidize. The seller's ability to subsidize future bidders both prompts stronger types of current bidders to make higher deterring offers, and prevents some of the socially excessive deterrence – so in many cases subsidies never actually need to be paid. But the sequential mechanism with subsidies can only be more profitable than an auction when the seller *both* has sufficient information and power to set the subsidies optimally *and* has enough potential bidders. And if a sequential *buyer* can make plausible threats to withdraw if a seller seeks additional bids, then the advantage of the auction becomes larger.

In sum, the auction looks more profitable than practical versions of the sequential mechanism: absent extreme power on either side, sellers will generally prefer auctions and buyers will generally prefer sequential mechanisms.

The efficiency implications of our model should be viewed with care. While the model suggests the sequential mechanism is more efficient than the auction, this may reflect the

model's narrow view of efficiency. First, because the sequential mechanism gives more surplus to the earliest bidders, potential entrants may dissipate their surplus in a race to the front of the queue. In this case, net social surplus will equal seller revenue. And second, even if bidders' rents are not dissipated in this way, mechanisms that give sellers a greater share of the value of the asset sold provide greater incentives to create valuable assets. So since the auction usually raises more revenue, it may well be more socially efficient overall.

We begin (in section I) with a model of a random number of symmetric risk-neutral potential bidders with search costs of finding out their (private) values of, for example, a company for sale.[7] Section II solves for equilibrium behavior in the auction and in the sequential mechanism. We show that auctions are less efficient (section III) but likely to raise more revenue (section IV) than sequential bidding, and explain why. While it is possible to design examples where bidders prefer auctions, they typically prefer sequential processes (section V). In section VI we show that our contrast between simple simultaneous and sequential mechanisms is robust to natural variants of these processes, although the sequential mechanism becomes more attractive to sellers if buyers can be prevented from jump bidding. We also show that our results extend straightforwardly to simple common-value settings. In section VII we consider how bidding subsidies and other simple tactics such as lock-ups, break-up fees, and matching rights can be used by sellers to improve their expected revenue from the sequential mechanism. Section VIII concludes.

I The Model

We compare two mechanisms for selling an asset for which there is a queue of risk-neutral potential buyers. The probability that at least j potential bidders exist, given the existence of at least $j - 1$, is ρ_j, $0 \leq \rho_j \leq 1$. We write $\{\rho_j\}_{j=1}^{\infty}$ for $\rho_1, \rho_2, \rho_3, \dots$. Each buyer decides in turn whether or not to pay a cost c to enter the sales mechanism. If it enters, it then immediately learns its own private value, which is drawn independently from the distribution $F(v)$ with a continuous density $f(v)$. Without any loss of generality, assume $F(\underline{v}) = 0$, $F(\overline{v}) = 1$, $\underline{v}$ equals the seller's value, and $\overline{v} \leqslant \infty$. All this is common knowledge among the buyers (the seller needs no knowledge of $F(\cdot)$, c, or $\{\rho_j\}_{j=1}^{\infty}$).

In the *Auction* mechanism, players cannot make any credible bid commitments until after entry stops. So when potential buyers make their entry decisions they observe only how many bidders have entered thus far. When entry has stopped, there is a standard English auction in which the bidder with the highest value wins and pays the value of the second highest.

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[7]Equivalently, symmetric risk-neutral potential suppliers can make costly investments which yield (private) costs of selling to a procurer.

Our *Sequential* mechanism is one that is commonly assumed for the sale of a business (see, e.g., Fishman (1988) and the subsequent literature based on his model). The first bidder simply chooses an initial bid. Any subsequent entrant may enter an English auction with the incumbent high bidder, and the winner of that competition may then, if it wishes, make a jump bid before finding out whether any subsequent potential entrant(s) actually exist. Bidders cannot lower bids that they have already made. Potential entrants observe all previous bids at the time they make their entry decisions, and when entry stops the current high bidder wins the asset at the price it bid.

We consider the perfect Nash equilibrium of the auction, which is unique except for knife-edge cases with two equilibria when the last bidder is indifferent to entering; in this case we take the worse case (the last bidder does not enter) for the auction.

We also consider all the perfect Nash equilibria of the sequential mechanism in which any buyer with a value greater than or equal to some constant cutoff value v^* makes a high enough preemptive bid that all subsequent entry is deterred.[8] However, we focus most attention on the unique perfect sequential equilibrium (as is standard in contexts like this one – see below).[9] We refer to this equilibrium deterring value of the sequential mechanism as V_S. We will refer to the minimum value that a bidder would need to have to deter if its value were known by potential entrants as V_K.[10]

To avoid trivialities, we assume there is at least one potential bidder and maybe more ($\rho_1 = 1$ and $\rho_2 > 0$), and that at least two bidders would be attracted into an auction if they exist. The seller can demand a minimum price of $\underline{v}$ in both mechanisms but no higher. There is no discounting.

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[8]We will note later that there are other equilibria that are not (perfect) "symmetric cutoff" equilibria, but they do not seem very plausible.

[9]In a perfect sequential equilibrium, no bidder wishes to deviate from the equilibrium if a potential entrant who observes an out-of-equilibrium price would assume, if possible, that the bidder's type is among some set K (a subset of $[\underline{v}, \overline{v}]$) such that (i) all types in K would benefit from the deviation *if* it was then inferred that the bidder's type was in K, and (ii) all types not in K would prefer not to deviate given the aforementioned inference.

This refinement is also variously called "credible" or "neologism-proof" or "F-G-P" after its developers (Sanford J. Grossman and Motty Perry (1986) and Joseph Farrell (1993)) – though Farrell's definition is very slightly different, this is unimportant here; see also Drew Fudenberg and Jean Tirole's (1991) standard text. Technically, we use the natural extension of the refinement to infinite strategy spaces; also a bit more than perfect sequential is needed to select a unique equilibrium when $\rho_j = 1 \ \forall j$, and in that case we just use the limit of the (unique) perfect sequential equilibria as $\rho_j \to 1 \ \forall j$ (this is also the unique equilibrium that Farrell's definition chooses $\forall \{\rho_j\}_{j=1}^{\infty}$, because he requires all types in K would *strictly* benefit in part (i) above).

[10]The "S" of V_S signifies not only that it refers to the *S*equential mechanism, but also that V_S is this mechanism's revenue - or *V*alue to the *S*eller - in the important special case $\rho_j = 1 \ \forall j$. The "K" of V_K signifies that it corresponds to bidders' values being *K*nown.

Notation

We write $v_i(k)$ for the actual i^{th} highest value among k bidders, and n^* for the maximum number of entrants into the auction. We write $S(v)$ for the expected surplus of a bidder who competes in an ascending auction against a second bidder, when their values are drawn independently from $F(\cdot)$, conditional on the *second* bidder's value *exceeding* v.[11]

Finally, we write $MR(v) \equiv [v - \frac{1-F(v)}{f(v)}]$; using expected marginal revenues ($EMRs$), together with the standard Facts 1 and 2 below, is not needed for our proofs, but assists intuition and greatly simplifies calculations.[12]

Fact 1 [Revenue Equivalence Theorem] (Roger Myerson (1981), Riley and William Samuelson (1981).) *Consider **any** sales process that awards a good to one of a set of risk-neutral bidders, each of whom has a privately-known value for the good independently drawn from a distribution $F(v)$ with a continuous density $f(v)$; $F(\underline{v}) = 0$, $F(\overline{v}) = 1$. If the sales process gives zero surplus to any bidder with value $\underline{v}$, then the seller's expected revenue from each bidder is that bidder's EMR conditional on it winning, times its probability of winning;*

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[11]We never need to compute $S(v)$ explicitly. It is also cumbersome to calculate it directly. However, it is easily calculated using Fact 1 below as the difference between the value of the bidder's gross winnings and its expected payments, that is (see note 13), $\int_{x=\underline{v}}^{\overline{v}} \{x - MR(x)\} f(x) P_1(x) dx$, in which $MR(x) = x - \frac{1-F(x)}{f(x)}$, and $P_1(x)$ is the probability with which the first bidder wins conditional on his value being x, that is, $\frac{F(x)-F(v)}{1-F(v)}$ for $x \geq v$. So $S(v) = \frac{1}{1-F(v)} \int_{x=v}^{\overline{v}} [1 - F(x)][F(x) - F(v)] dx$.

[12]Thus if one graphs a "demand curve" with v on the price axis, and $1 - F(v)$ on the quantity axis, then $MR(v)$ is just the corresponding marginal revenue curve - a bidder whose value v is drawn from $F(v)$ would on average buy $1 - F(v)$ units at a take-it-or-leave-it price v, so has expected demand $1 - F(v)$.

We make *no* assumptions on the $MR(v)$ function.

The mathematics of marginal revenue for auctions was introduced by Roger Myerson (1981); he coined the terms "virtual valuation" and "virtual utility" in Myerson (1984). Jeremy Bulow and John Roberts (1989) first used the nomenclature "marginal revenue", and explained the connection with ordinary monopoly theory. This connection, and "MR analysis", were extended to a broader set of assumptions by Bulow and Klemperer (1996). MR analysis simplifies the analysis in several ways.

First (when, as in our model, bidders' signals are independent) bidtakers' and bidders' surpluses depend only on the expected identity of the winner and not on prices, or on auction form, or on which bidders actually participate: see Fact 1.

Second, it is often easier to work with the winner's marginal revenue than the price, since the price often depends upon *other* bidders' values, including in an ascending auction and in our sequential mechanism: see, e.g., the discussion of factor (b) below Lemma 5. Doing so also often avoids performing multiple integrations by parts: see, e.g., the calculations of bidders' surpluses in notes 11 and 15, which both use the fact that $MR(x)f(x)$ has a particularly easy form.

Third, crucially, working with marginal revenues permits state-by-state dominance arguments that do not apply to state-by-state revenues. This is *exactly* as in ordinary monopoly theory (because, of course, the marginal revenue of an additional unit sold accounts for both the direct effect on revenue (the price paid) *and* the indirect effect of the revenue lost from the lower price received for other units); in particular, a monopolist can raise the contribution to profits (price *less* cost) of some units by setting a higher or lower price than the optimum, but it will weakly lower the contribution to marginal revenue *less* marginal cost of *every* unit sold, since the optimum is set at $MR = MC$, so there is state-by-state dominance in MRs, but not in prices. See, e.g., the discussion of Lemma 4. See Bulow and Klemperer (1996) for another application to auction theory of state-by-state dominance in MRs where there is no dominance in prices or revenues.

the seller's total expected revenue equals the EMR of the winning bidder.[13]

This Fact will be useful both in calculating expected seller revenue, and in determining how much any potential bidder would be willing to pay to enter a sales process.

Fact 2 *The EMR of all bidders above any value $\widehat{v}$ equals $\widehat{v}$.*[14]

II   Equilibria of the Mechanisms

A   Equilibrium of the Auction

The auction treats all entrants symmetrically, independent of their order of entry, so it is immediate that all of the first n^* potential bidders enter, if they exist, and there is no subsequent entry, and

Lemma 1 *The maximum number of entrants into the auction, n^*, is the largest integer, n, for which the expected surplus from being one of n bidders in the auction strictly exceeds c.*[15]

We do not need to compute the expected revenue explicitly, but it is easy to do so: it is the sum of expected revenue conditional on there being precisely j bidders times the probability that precisely j bidders exist, for all $j \in \{1, 2, ...n^*{-}1\}$, *plus* the expected revenue when the maximum possible number of bidders, n^*, enters times the probability that at least n^* potential bidders exist. So, using Fact 1 separately for each case

Remark 1 *The Auction's expected revenue*

$$= \sum_{j=1}^{n^*-1} \prod_{k=1}^{j} \rho_k (1{-}\rho_{j+1}) \int_{x=\underline{v}}^{\overline{v}} j f(x) F^{j-1}(x) MR(x) dx + \prod_{k=1}^{n^*} \rho_k \int_{x=\underline{v}}^{\overline{v}} n^* f(x) F^{n^*-1}(x) MR(x) dx.$$

Fact 1 also implies:

Remark 2 *The Auction's expected revenue would be unaffected if we assumed any other standard auction, such as a first-price sealed-bid auction.*

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[13]That is, the expected payments of buyer j are $\int_{x=\underline{v}}^{\overline{v}} MR(x) f(x) P_j(x) dx$ where $P_j(x)$ is the probability of buyer j being awarded the good conditional on his value being x (so also j's expected surplus depends only on $P_j(x)$). The seller's expected revenue equals $\int_{x=\underline{v}}^{\overline{v}} MR(x) P(x) dx$, in which $P(x) \equiv \sum_j f(x) P_j(x)$ is the probability density with which the good is awarded to any bidder with value x.

[14]That is, $\frac{1}{1-F(\widehat{v})} \int_{x=\widehat{v}}^{\overline{v}} (x - \frac{1-F(x)}{f(x)}) f(x) dx = \widehat{v}$. This is mathematically equivalent to the fact that for a monopolist the integral of marginal revenue equals price times quantity and so the average marginal revenue of the buyers must equal the price.

[15]The winner's value will be $v_1(n)$ and in our (English) auction the price will be $v_2(n)$, so per-bidder expected surplus equals $\frac{1}{n}(E\{v_1(n)\} - E\{v_2(n)\})$. This is most easily computed using Fact 1 that the expected auction revenue ($E\{v_2(n)\}$) equals the EMR of the winner. Since the probability density of the highest value among n is $nf(x)F^{n-1}(x)$, per-bidder expected surplus $= \frac{1}{n} \int_{x=\underline{v}}^{\overline{v}} nf(x) F^{n-1}(x) \{x - MR(x)\} dx = \int_{x=\underline{v}}^{\overline{v}} [1 - F(x)] F^{n-1}(x) dx$.

B Equilibrium of the Sequential Mechanism

In the equilibrium of the sequential mechanism, the first entrant makes a jump bid that is high enough to deter subsequent entry if its value exceeds some cutoff value, v^*, and otherwise bids the minimum price $\underline{v}$ – just as Fishman originally showed in his two-bidder version of this model. Any subsequent entrant whose value exceeds the current high bid competes with the current incumbent, and the price then rises until the lower-value of these two bidders quits at the point at which the price reaches its value. If it was the current incumbent who quit, the new entrant will then jump bid to the higher price that deters subsequent entry if its value exceeds the cutoff value, v^*, but will otherwise not raise the price further prior to any further entry. (If it was the entrant that quit, there is also no jump bid prior to further entry, in equilibrium, because the current incumbent's value must be below v^*.) If there is no jump bid, another potential bidder then enters, if one exists.

The deterrence value, v^*, cannot be lower than the value V_S that satisfies $S(V_S) = c$ (because if it were, a prospective entrant would profit from entering against a deterring bid - entering and winning would deter all future entry, and the gross surplus from competing against an incumbent whose value is v^* or higher is by definition $S(v^*)$, and clearly $S(v^*) > c$ if $v^* < V_S$). But an incumbent whose value is as low as V_S can deter entry only if it is pooled with types with higher values – if its value was known to be as low as V_S, entry against it would be known to be profitable. So while $v^* = V_S$ is a perfect Nash equilibrium, there are other equilibria in which higher-value incumbents are separated.

The perfect constant-cutoff Nash equilibrium in which the fewest types deter entry is the one in which the only types who deter are those who would deter if their actual values were known (namely those whose values at least equal the value V_K which satisfies $\int_{x=V_K}^{\overline{v}}[x - V_K]f(x)dx - c = 0$). In these equilibria a bidder with value V_S finds it too costly to pool with higher-value incumbents, and instead accommodates entry.[16]

However, as in Fishman's model, only the $v^* = V_S$ equilibrium is perfect sequential. In other equilibria, bidders with values above V_S could gain from deviating to a bid that signals a value of (just) V_S or more if the potential entrant were then to infer the equilibrium deterring value is actually only V_S, *and* all other types of bidders would lose from making such a bid however the potential entrant were to respond;[17] so the logic of perfect-sequential

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[16] Equilibria with v^* even higher than V_K are ruled out by our assumption of a constant deterring value, and also fail not only the intuitive criterion, but also the weaker "test of dominated messages", even in any single period - see proof of Lemma 2 in Appendix. (There is also a plethora of perfect *non-constant*-cutoff equilibria, but it is straightforward using MR analysis that the constant V_K and constant V_S equilibria define the upper and lower bounds in expected revenue among all these that pass the "test of dominated messages"; and there are even *non*-cutoff equilibria, for example, entry could be deterred only by some particular set of prices exceeding the price that would correspond to value V_S, but these seem particularly implausible.)

[17] In fact, a perfect sequential equilibrium requires only that all other types would lose from making such a

equilibrium is that the potential entrant should make such an inference and a bid that signals a value of at least V_S therefore does deter entry. A large literature argues that the unique perfect-sequential equilibrium is the only reasonable one in games like this one,[18] and we therefore focus primarily on this, $v^* = V_S$, equilibrium.

At any time, the (lowest) *bid* that deters further entry (i.e., the deterring price at that time) is such that a bidder with value v^* is just indifferent about deviating to accommodating entry. That is, the bidder with value v^* is indifferent between making a deterring bid and deviating to a strategy of never jump bidding but always bidding the lowest price possible to beat any challenger until entry closes, or until the price exceeds v^*.[19] Summarizing (see Appendix for the details of the proof):

Lemma 2 *There exists a perfect Nash equilibrium of the Sequential mechanism in which any buyer with a value greater than or equal to v^* makes a bid that deters all subsequent entry if and only if $v^* \in [V_S, V_K]$*

$$\text{where } V_S \text{ satisfies} \quad S(V_S) = c, \tag{1}$$

$$\text{and } V_K \text{ satisfies} \quad \int_{x=V_K}^{\overline{v}} [x - V_K] f(x) dx - c = 0. \tag{2}$$

In the unique perfect sequential equilibrium, $v^* = V_S$.

Example If $F(v) = v$ for $v \in [0,1]$ and ρ is constant ($\rho_j = \rho \leq 1 \ \forall j > 1$), any new entrant whose value is v^* or more deters entry by jump bidding to the price $v^* - \frac{1-\rho}{\rho} \log\left(\frac{1-\rho p}{1-\rho v^*}\right)$ ($> p$), after defeating the existing incumbent at price p. (For the first entrant, $p = \underline{v}$ ($= 0$

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bid *if* the potential entrant makes the inference that they are types above V_S (see note 9), so the refinement is (even) more compelling than usual in our context.

[18] Fishman (1988, 1989) restricts attention to this equilibrium in his two-potential-bidders version of our model, contending that only this one is "credible". Andrei Shleifer and Robert Vishny (1986) claim "The case for the minimum bid equilibrium is compelling.... There is no basis for [any alternative] belief, since it is common knowledge that all types would like to [pay] the lowest possible price." And Mark Bagnoli, Roger Gordon, and Barton Lipman (1989) say "[We do not] clutter the text with details [of equilibria that are not perfect sequential]." Other authors who take similar views and pays scant attention to alternatives include Dan Bernhardt and David Scoones (1993), Kevin McCardle and S. Viswanathan (1994), and Yeon-Koo Che and Tracy Lewis (forthcoming), among others. Riley's (2001) authoritative survey of the signalling literature makes a strong general argument for focusing on the perfect sequential equilibrium. Finally, Andrew McLennan's (1985) argument for "justifiable" equilibrium, and note 17, provide additional arguments that this is the only natural equilibrium in our context.

A possible argument against the perfect sequential equilibrium is that the seller prefers the $v^* = V_K$ equilibrium, and could perhaps in effect achieve it by slightly changing the game to one in which the only reasonable equilibrium is $v^* = V_K$. We therefore discuss this possibility in section VII.

[19] If a bidder with value v^* is just indifferent about deviating, a bidder with a lower (higher) value would strictly gain (lose) by deviating, since higher-value types prefer strategies with higher probabilities of winning (and deviating strictly reduces the bidder's probability of winning). Not deviating therefore signals a value $\geq v^*$ and so successfully deters entry if $v^* \geq V_S$.

here)). Expected revenue equals $\frac{v^*}{\rho}\left[1 + \frac{(1-\rho)^2}{(1-\rho v^*)}\right] + \frac{2(1-\rho)}{\rho^2}\ln(1-\rho v^*)$.[20] In the unique perfect sequential equilibrium, $v^* = V_S = 1 - \sqrt{6c}$. (Calculations are in the Appendix.)

We do not need to compute expected revenue explicitly, but it is straightforward to do so using Fact 1 - the expected revenue equals the EMR of the winner - since this only requires knowing the identity of the winner. (Because the deterring price generally changes over the course of the game - since it depends on the value of the previous high bidder, and on the sequence of subsequent ρ_j's - calculating revenue by computing the actual price contingent on any sequence of bidders would be extremely messy.):

Remark 3 *The Sequential mechanism's expected revenue*

$$= \sum_{j=1}^{\infty}\prod_{k=1}^{j}\rho_k(1-\rho_{j+1})\int_{x=\underline{v}}^{v^*}jf(x)F^{j-1}(x)MR(x)dx + \sum_{j=1}^{\infty}\prod_{k=1}^{j}\rho_k F^{j-1}(v^*)(1-F(v^*))v^*.$$

(The first term sums (over j) the probability that the game will end with exactly j bidders, all of whom have values less than v^*, times the winner's EMR conditional on this event; this term may be negative. The second term is the probability that the winner's value exceeds v^*, times v^*, since (using Fact 2) the winner's EMR equals v^* conditional on this event.)

Because Fact 1 means expected revenue depends only on the identity of the winner, assumptions that affect neither v^* nor the fact that the highest actual entrant wins, are unimportant. In particular:

Remark 4 *Expected revenues (and each potential buyer's expected surplus) would be unaffected by bidders knowing the number of potential entrants who actually exist in advance of bidding (assuming it is common knowledge that they know this), and/or by the timing of entrants' jump-bidding (whether it is before, or during, or subsequent to competing with a current incumbent, or at all of these times).*[21]

When there are an infinite number of potential bidders, the lowest deterring type must bid its full value, v^*, to deter, since it must be indifferent about not deterring and if it did not deter it would then surely face entry by someone with at least as high a value and so earn zero surplus. So the deterring *price* always equals v^* in this case, and therefore (as can be confirmed directly using Remark 3):

Remark 5 *If $\rho_j = 1 \ \forall j$, expected revenue $= v^*$.*

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[20] The deterring *price* is increasing in both p and ρ, and equals the deterring *value*, v^*, both in the limit as $p \to v^*$ and in the limit as $\rho \to 1$; as $\rho \to 0$, the deterring price equals price, p; as $\rho \to 0$ and 1, expected revenue equals 0 and v^*, respectively – see Remark 5.

[21] Actual prices depend upon these assumptions, of course. See the Example in the Appendix for an illustration. For the result about buyers' expected surpluses see note 13.

III   Efficiency

It is a standard result that the amount of entry into a private-value ascending auction is efficient (see Richard Engelbrecht-Wiggans (1993)). So, since in both our mechanisms the last potential entrant makes its entry decision based on the possibility of having to compete in an ascending action with the strongest actual entrant, it is straightforward that all the entry decisions are socially efficient *given* the available information (details are in the Appendix):

Lemma 3 *Conditional on the information available to a potential entrant, it makes the socially correct decision about whether to enter in either of the sales mechanisms.*

Note that in the sequential mechanism the higher is v^*, the more efficient is the equilibrium. A social planner who could observe each bidder's value upon entry would choose entry until the highest value was such that the expected contribution of a new bidder was less than or equal to zero, that is, until a bidder was found with a value greater than or equal to V_K, since $\int_{x=V_K}^{\overline{v}} [x - V_K]f(x)dx - c = 0$. So the V_K equilibrium is socially optimal.

In other equilibria, entrants who find out their values are in the range $[v^*, V_K)$ pool with those above V_K and inefficiently prevent the further entry that would occur if their values were publicly observed. However, all these equilibria are more efficient than the auction, because they all provide prospective bidders with better information than the auction does. (A potential sequential bidder knows whether or not there is an incumbent bidder with a value in excess of v^*, and would never revise its entry decision if it were also told the number of previous entrants; a potential auction bidder, by contrast, knows only the number of other entrants, and might revise its decision if it were told whether or not there was an incumbent with a value of more than v^*.) Therefore, since all the entry decisions are socially efficient *given the available information*, the entry decision in the sequential mechanism is more efficient (in expectation) whenever the entry decisions in the two mechanisms differ.[22] Both mechanisms select efficiently among all bidders who actually enter, so:

Proposition 1 *The Sequential mechanism is more efficient than the Auction.*

Since when there is an infinite number of potential bidders the deterring *price* always equals v^* (see the discussion leading to Remark 5), the expected profit of any potential entrant, conditional on there being no previous bidder above the cutoff, is $\int_{x=v^*}^{\overline{v}} [x - v^*]f(x)dx - c$ in this case, and (2) then implies that bidders all earn zero expected profits in the V_K

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[22]Note when $v^* = V_S$ a potential entrant is just indifferent to entering, and so entry is a matter of indifference for efficiency as well. However, the entry decision is then strictly more efficient than in the auction when bidders cease entering the auction even though none have yet bid V_S.

equilibrium. So the V_K equilibrium is not only efficient but fully extractive in the "infinite potential bidder" case.

IV Revenue

The auction usually beats the sequential mechanism on revenue. We begin by showing (details in the Appendix) that:

Lemma 4 *For any given value of v*, if the auction is more profitable in expectation than the sequential mechanism for some sequence of $\{\rho_j\}_{j=1}^{\infty}$, then the auction is more profitable in expectation if any ρ_j is reduced.*

This result is *not* obvious in terms of actual *prices*, because the sequential mechanism encourages bidders to signal with high bids in early stages so, for example, conditional on only one entrant actually existing ex-post, the sequential mechanism earns more revenue on average, ex-post, than the auction (which then earns the minimum possible price). And even if the number of potential bidders actually existing is known in advance (and is more than one), the sequential mechanism's realised *price* may be higher or lower than the auction's. However (though we do not require EMR's for the proof), the intuition *is* obvious using EMR's, since conditional on n^* or fewer bidders actually existing, the auction *always* achieves a higher EMR,[23] while additional bidders beyond n^* clearly have no effect on the auction but always (weakly) raise the sequential mechanism's EMR.

We can now show that the auction is superior in expected revenue to the sequential mechanism under likely conditions:

Proposition 2 *The Auction is more profitable in expectation than the Sequential mechanism if (i) the equilibrium of the latter is its unique perfect sequential equilibrium and $S(\cdot)$ is convex (as it is for economists' most commonly used distributions of demand) and/or if (ii) $\prod_{j=3}^{n^*+1} \rho_j$ is sufficiently small (i.e., the probability of at least n^*+1 potential entrants existing, conditional on at least two existing, is sufficiently small).*

Proof: Part (i): by Lemma 4, if the result holds for $\rho_j = 1 \ \forall j$, it holds for all $\{\rho_j\}_{j=1}^{\infty} \leq 1$. Also, $S(V_S) = c \geq E\{S(v_2(n^*))\} \geq S\{E(v_2(n^*))\}$, in which the equality is the entry condition for the sequential mechanism (equation (1)), the first inequality is the entry condition for the auction (an $(n^*+1)^{st}$ potential entrant would not want to enter

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[23]This does *not* rely on any assumption about MRs. The auction selects the bidder with the highest value among its entrants, so (by Fact 2) its EMR equals the expected second-highest value which exceeds the actual MR of any other entrant.

the auction given that it would compete against a bidder whose value exceeds the second-highest of the n^* previous entrants' values, $v_2(n^*)$), and the second inequality is Jensen's inequality (if $S(\cdot)$ is convex). So, since $S'(\cdot) < 0$, we have $V_S \leq E(v_2(n^*))$. Since if $\rho_j = 1 \; \forall j$, the expected revenue from the auction $= E(v_2(n^*))$, and the revenue from the sequential mechanism $= v^*$ (see Remark 5), $v^* = V_S$ suffices for the result.

Part (ii): recall from Remark 4 that the sequential mechanism's (and the auction's) expected revenue would be unaffected if the actual number of potential entrants was revealed to everyone prior to any entry decisions. So consider this case. Since the auction is strictly more profitable than the sequential mechanism in expectation when there are at least two, but not more than n^*, potential entrants (see note 23, or Step 1 of the proof of Lemma 4 - the auction, but not the sequential mechanism, always sells to the highest-value potential entrant in this event), this dominates the possibly-greater expected profitability of the sequential mechanism when there are more than n^* potential entrants, if the relative probability of the latter event is sufficiently small. (When there is known to be just one potential entrant, both sales methods earn $\underline{v}$.) $\square$

There are three reasons why the auction is usually *strictly* more profitable than the sequential mechanism. These correspond to the facts that each of the first three inequalities in the proof of part (i) (namely $\{\rho_j\}_{j=1}^{\infty} \leq 1$, $c \geq E\{S(v_2(n^*))\}$, and $E\{S(v_2(n^*))\} \geq S\{E(v_2(n^*))\}$) are usually strict; they also correspond to the factors listed in the introduction.

First, there is never an infinite stream of potential bidders in practice (i.e., $\rho_j < 1$, for some[24] $j > 2$). Since the sequential mechanism only has an advantage when a long tail of bidders actually enters, reducing the likely number of potential bidders reduces its chance of beating the auction (recall Lemma 4).

Second, the jump-bids in the sequential mechanism are "fine-tuned" to make entry just barely unattractive to an additional bidder, while the expected profit of the marginal bidder who does not enter the auction is generally strictly negative (i.e., $c > E\{S(v_2(n^*))\}$).

Third, surplus from entry conditional on the value of the second-highest bidder is likely to be convex (i.e., $E\{S(v_2(n^*))\} > S\{E(v_2(n^*))\}$). So since the value of the second-highest bidder is random, its expected value (which equals the auction's expected revenue) has to be higher to deter entry than if this value were fixed; by contrast, the sequential mechanism's cutoff value (which determines the marginal entrant's surplus from that mechanism) is fixed.

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[24] We need $j > 2$ so that there is positive probability that at least 2, but not an infinity of, potential entrants exist (see the proof of Proposition 2 part (ii)).

Why does the auction's randomness usually *make it more profitable?*

The reason the auction's randomness usually makes it more profitable, that is, the reason $S(v)$ is usually convex, is best understood by recognising that two separate factors determine how the condition that the marginal prospective entrant be unwilling to enter the mechanisms affects the mechanisms' relative profitabilities. These factors are (a) the standard consumer-theory result that the prospective entrant's surplus if it were to enter either mechanism would be convex in the *value, v*, of the bidder who wins assuming the prospective entrant stays out (the prospective entrant can be thought of as having the option to buy at the price v, which option becomes more valuable as the distribution of v becomes more dispersed); and (b) the relationship between the *values* of the winners of the mechanisms and the *prices* they pay.

Both factors (a) and (b) depend upon the fact that even if there were an infinite number of potential bidders, and the *expected* values of the winners of the two mechanisms were the same, the *dispersion* of values faced by a new entrant into the auction would be greater than the dispersion of values faced by a new entrant into the sequential mechanism. More precisely:

Lemma 5 *If the expected value of the winner of the auction with $n \geq 2$ bidders equals that of the winner of a sequential mechanism with cutoff value v^* and an infinite number of potential bidders, the distribution of the sequential-mechanism-winner's value second-order stochastically dominates that of the auction-winner's value in the sense of Michael Rothschild and Joseph Stiglitz (1970).*

To understand this, see Figure 1 which pictures the case of uniform $F(\cdot)$. In this case, the sequential-mechanism-winner's value is drawn from a constant density truncated below v^*, while the auction-winner's value is drawn from the increasing function $F^{n^*}(\cdot)$. [INSERT FIGURE 1 HERE] So the truncated density must be the higher at v^*, since both densities integrate to 1. It must therefore also be the lower at $\overline{v}$ if both densities have the same mean. So the densities cross just twice, and the truncated distribution is therefore less risky. Considering a general $F(\cdot)$ simply distorts this picture without affecting the "two-crossings" property (see proof of Lemma in Appendix).[25]

Factor (a), then, is that, conditional on the two processes having the same expected winning value, a new entrant would find the auction more attractive because its distribution of winning values is more dispersed. Equating the attractiveness of the two mechanisms by increasing the expected value of the winner of the auction would require adding more

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[25] For intuition, observe that the sequential mechanism-winner's value is drawn from $F(\cdot)$ truncated at the fixed value v^*, while the auction-winner's value can be thought of as drawn from $F(\cdot)$ truncated at the *random* value $v_2(n^*)$.

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bidders, which would raise the expected second-highest value and so the expected revenue in the auction. Alternatively, reducing the expected value of the winner of the sequential mechanism would require reducing its cutoff value, and therefore its expected revenue. So factor (a) always favours the auction.

Factor (b) determines whether a greater dispersion of winning *values* leads to greater or lower *prices*, conditional on any given expected winning value. Since the expected revenue of any mechanism is the expected MR of the winning bidder, greater dispersion favours (hurts) the auction if MR is convex (concave) in v. For demand functions commonly used by economists, including linear, exponential, and constant elasticity, $MR(v)$ is affine,[26] so factor (b) is irrelevant. Furthermore, factor (b) can only matter much if winners have very dispersed valuations. But more dispersed winners' valuations also increase the importance of factor (a) which unambiguously benefits the auction.

In short, all four factors (the first two inequalities of the proof of part (i) of proposition 2, and both the factors that determine the third inequality of the proof) go in the same direction if $MR(v)$ is convex, and this is therefore a *very* easily sufficient condition for expected revenue to be greater in the auction than in the sequential mechanism.

A more formal way to understand why the auction's randomness usually makes it more profitable is to use directly the microeconomic theory result that any consumer's welfare is *always* convex in the prices she faces. So the expected surplus, call it $T(v)$, of a bidder who competes against another bidder with a value of *exactly* v is *always* convex. Of course, the auction's relative profitability depends upon the convexity of $S(v)$, that is, the expected surplus of a bidder who competes against another bidder with a value *at least* v. But since $S(v) = E\{T(w) \mid w \geq v\}$ and $v = E\{MR(w) \mid w \geq v\}$ (this is Fact 2), we can show that $S(v)$ is a convex transformation of v if $T(v)$ is a convex transformation of $MR(v)$, i.e., if $MR(v)$ is a convex transformation of $-T(v)$. But because $T(v)$ is convex, $-T(v)$ is concave, so it *easily* suffices that $MR(v)$ is convex. That is (for proof see Appendix):

Lemma 6 $S(v)$ *is convex in* v *if* $MR(v)$ *is convex in* v.

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[26]For all these cases, the ratio of the slope of MR to the slope of (inverse) demand (v) is constant (i.e., $\frac{d^2}{dv^2}MR(v) = 0$ everywhere): for uniformly-distributed signals ($F(v) = \frac{v-\underline{v}}{\overline{v}-\underline{v}}$), which generate linear demand, the ratio of the slope of MR to the slope of demand $= \frac{1}{2}$; for constant-elasticity distributed signals ($F(v) = 1 - (\frac{v}{\underline{v}})^\eta$, $\eta < -1$), the ratio $= \frac{\eta}{\eta+1}$; and for exponentially-distributed signals ($F(v) = 1 - e^{-\lambda(v-\underline{v})}$), which generate log-linear demand, the ratio $= 1$.

The condition $\frac{d^2}{dv^2}MR(v) = 0$ is satisfied by any member of the generalised Pareto distribution.

Note that convexity of $MR(v)$ in v (i.e., $\frac{d^2}{dv^2}MR(v) \geq 0$) is *not* the same as convexity of the marginal revenue curve as it is usually drawn (which corresponds to convexity in "quantity" ($1-F(v)$), rather than "price" (v)). MR convexity is equivalent to concavity in the inverse hazard rate (or $2(h'(v))^2 \leq h(v)h''(v)$, in which $h(v) \equiv \frac{f(v)}{1-F(v)} = $ the hazard rate). It is not hard to show that if expected demand is convex (i.e., $f'(\cdot) \leq 0$), a sufficient condition for $S(\cdot)$ to be convex is $h'(\cdot) \geq 0$.

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Examples of relative profitabilities of mechanisms

As an example, consider bidders with valuations uniformly distributed on $[0, 1]$, that is $F(v) = v$ for $v \in [0, 1]$. This corresponds to bidders forming a linear demand curve $p = 1 - q$, in the limiting case where there are m such bidders each demanding $1/m$ units at their values, as $m \to \infty$. If $c = .06$, the auction would attract up to $n^* = 3$ bidders and yield expected revenue of .50 if there are at least three potential bidders. A fourth bidder would have an expected loss of exactly .01 and so would not enter.

To have the same expected winner's value, even with an infinite supply of potential bidders, the sequential mechanism's cutoff, v^*, would need to be .50 - see Figure 1. But to reduce a new entrant's expected profits to $-.01$, a bidder in the sequential mechanism would have to credibly signal only that its value exceeded $1 - \sqrt{.3} \approx .452$ (because $S(1 - \sqrt{.3}) = .05$, so if $c = .06$ a new bidder would have an expected loss of .01). So roughly $.50 - .45 = .05$ of the extra revenue of the auction over that from the sequential mechanism is attributable purely to "convexity of surplus": although the auction yields an expected winner's value of .75, the sequential mechanism only needs an expected winner's value of .726 to reduce an entrant's net profits to -.01, because its winner's value is less dispersed. Because $MR(v) = 2v - 1$, with MR neither concave nor convex, the difference in the expected value of the winner (here, between .75 and .726) translates linearly into twice that difference in expected revenues.

If bidders pool on jump bids that make a new entrant's expected profits 0 instead of $-.01$, then the cutoff value is lowered to .40 (because $S(.4) = .06 = c$, so $V_S = .4$), extending the auction's advantage by another .05. Finally, if there were only three potential bidders (so $\rho_j = 1$ for $j \leq 3$ and zero otherwise), the "finite supply of bidders" reduces the sequential mechanism's expected revenue by .05 more, to about .35 while leaving the auction's revenue unchanged at .50. This example is described in detail in the Appendix.

Of course, by choosing c so that an additional entrant is barely deterred in an auction (so $c = .05$ here) and $\rho_j = 1 \ \forall j$ the last two effects are nullified. But even then, for the seller to prefer the sequential mechanism, the lower dispersion of the sequential mechanism's high value (worth the difference between .50 and .45 in the above example) must be compensated by concavity in the MR function. However, MR concavity only matters to the extent that the winning values in the auction and sequential mechanism differ. And the greater these differences, caused by the auction's greater dispersion, the greater the auction's advantage in expected winning value must be, making it that much harder for any concavity to help the sequential mechanism enough to catch up — even ignoring the other two factors.

Nevertheless, it is possible to find examples in which the sequential mechanism does win – in particular by choosing a distribution with a marginal revenue function that is essentially flat at higher values (so that the higher expected v_1 does little for the auction) and that

conveniently becomes sharply lower just below V_S (so that the auction is disproportionately punished for low outcomes)[27] — and then combining this with a large stream of potential bidders and a carefully chosen c. For such an example, assume bidders are equally likely to have the values 0, $\frac{8}{11}$, and 1 (or a continuous approximation to this three point distribution) and $\rho_j = 1 \ \forall j$. The expected revenue from the sequential mechanism then exceeds the expected revenue from the auction if (but only if) $c \in (\frac{80}{1782}, \frac{81}{1782})$, in which case c is low enough that $V_S = \frac{8}{11}$ and yet high enough that only 3 bidders enter the auction, so the greater chance that revenue will be 0 in the auction matters more than the possibility that the auction's revenue will be 1.[28]

If $v^* = V_K$ the sequential mechanism is both socially optimal and fully extractive, as we have seen in section III, and therefore it *must* beat the auction in expectation if (but only if) the number of potential bidders is sufficiently greater than n^*.

In short, there is a reasonable presumption that the expected price is higher in the auction than in the sequential mechanism.

V   Bidders' Surplus

Since the seller's profits are usually higher in the auction, but social welfare is lower, bidders' surplus must usually be lower also. We now show that not only does this result usually hold for total bidders' surplus, but that it is even more likely to hold for the surplus of the first bidder to enter. So the casual-empirical observation that the first bidder to enter will generally seek to pre-empt an auction is borne out. We begin by showing (see Appendix for details):

Lemma 7 *More bidders enter the Auction, in expectation, than the Sequential mechanism if (i) the equilibrium of the latter is its unique perfect sequential equilibrium and/or if (ii) $\prod_{j=3}^{n^*+1} \rho_j$ is sufficiently small.*

The intuition is that the sequential mechanism sorts bidders better, in that entry stops as soon as one bidder with a high enough value to deter future entrants is found, while

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[27]The standard statistical distribution, $F(v)$, that comes closest to these desiderata is the Normal distribution, for which it is *just* barely possible for the sequential mechanism to win. But it can do so only if $c \in (0.2821\sigma, 0.2823\sigma)$ (in which σ is the standard deviation - we let $\underline{v} = -\infty$). Furthermore, the maximum amount by which the sequential mechanism's expected profits can exceed the auction's, for any values of the parameters c, μ, σ, and $\{\rho_j\}$, is 0.001σ, whereas if c is just below $.2821\sigma$ the auction's expected revenue is greater by over 500 times as much, even if $\rho_j = 1 \ \forall j$. A more compelling example requires accentuating the "hump" — as we do in the example below by inserting a large central atom.

[28]To calculate the continuous distribution analogue, note that a deterred entrant expects zero profits from entry if $v^* = V_S$, so a buyer with value $\frac{8}{11}$ enters with probability $\frac{11c}{1-11c}$. The expected value of the winner in the auction, $.892$, comfortably exceeds the expected value of the winner in the sequential mechanism (between $.864$ and $.865$ depending on c).

the auction attracts a random number of high-value bidders and therefore on average needs more bidders to achieve the same degree of entry deterrence. We now have (see Appendix):

Lemma 8 *The first bidder obtains a smaller fraction of the total bidder surplus, in expectation, in the Auction than in the Sequential mechanism if (i) the equilibrium of the latter is its unique perfect sequential equilibrium and/or if (ii) $\prod_{j=3}^{n^*+1} \rho_j$ is sufficiently small.*

The intuition is that not only are there more bidders on average among whom to split the surplus in the auction (see the previous Lemma), but also the surplus is tilted towards the first bidder in the sequential mechanism, because that bidder sometimes deters others in that mechanism, while all of the first n^* bidders always enter the auction if they exist. It follows straightforwardly from these lemmas and Propositions 1 and 2 that:

Proposition 3 *The Auction yields lower total bidder surplus, lower surplus per participating bidder, and lower surplus of the first entrant, in expectation, than the Sequential mechanism if (i) the equilibrium of the latter is its unique perfect sequential equilibrium, and $S(\cdot)$ is convex and/or if (ii) $\prod_{j=3}^{n^*+1} \rho_j$ is sufficiently small.*

If both conditions (i) and (ii) fail, and the sequential mechanism has $v^*= V_K$ when $\rho_j = 1 \; \forall j$, the auction is more attractive to all bidders. It is also possible to find examples in which all bidders prefer the auction with $v^*= V_S$ when $\rho_j = 1 \; \forall j$ (and $S(\cdot)$ is not convex), but parameters must be chosen very carefully – such examples are clearly even more restricted than those for which the seller prefers the sequential mechanism.[29]

In short, bidders, especially those "at the front of the queue" (who are the most influential), mostly prefer sequential mechanisms; our model supports the advice that bidders should avoid auctions if possible.

VI   Robustness

There are many variants of our simple canonical models of auctions and sequential sales in reality. And which extensive form best represents any given real-world process is often unclear. So how robust are our results?

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[29]The example we gave in the previous section, for which the seller preferred the sequential mechanism, is also one in which the auction is the most attractive mechanism for bidders in total surplus, surplus per participating bidder, and surplus for the first entrant. (If $\rho_j = 1 \; \forall j$, the first bidder's expected surplus is $\frac{28}{297} - c$ and $\frac{27}{297} - c$ in the auction and the sequential mechanism, respectively.) Of course, the set of $\{\rho_j\}_{j=1}^{\infty}$ for which these statements hold is even smaller than the set for which the seller prefers the sequential mechanism.

A   Common Values

We have focused upon an independent private value model. However, the simplest common value version of our model gives essentially the same results:

To make things simple, assume exactly N potential bidders exist, that is, $\rho_j = 1$ for $j \leq N$; $\rho_{N+1} = 0$; and our existing model, except interpreting the jth bidder's draw from $F(v)$ as its *signal*, not its value; instead, all bidders share a common value which equals the sum of all N potential bidders' signals. For this subsection alone, we assume that the hazard rate $f(v)/(1 - F(v))$ is weakly increasing in v.[30] It is straightforward that in the symmetric equilibrium of an ascending auction with n bidders the winner's actual surplus equals the difference between the two highest signals, so the entry condition for the auction, and the computation of n^*, is exactly as before. Furthermore (1), that is, $S(v^*) = c$, defines a perfect sequential equilibrium of the sequential mechanism, as before, if c is not too large.[31] The equilibrium parallels that of the independent private value case; as before, deterring bids are determined so that a bidder with signal v^* is indifferent between deterring and not deterring.

So it is easy to see that our result about profitability (Proposition 2) holds exactly as before. Furthermore, all that matters for efficiency is the expected number of bidders, so Lemma 7 suffices for the result that the sequential mechanism is the more efficient sales process (Proposition 1), and our result about bidders' preferences (Proposition 3) also follows.

Calculation of equilibrium (and our results) remains straightforward when there is a random number of bidders in a "sum-of-signals" model, so long as the existence of an additional bidder always weakly increases the expected value of the asset, regardless of the bidder's signal. This would be true if, for example, the common value was the sum of the signals of all those potential bidders who exist and either $\underline{v} \geq \rho_j E(v)\ \forall j$, or ρ_j is (weakly) increasing and $\underline{v} \geq 0$. (If a new entrant appearing and paying for a signal could reduce the asset's expected value, the entrant might be unwilling to top the current bid even if it's signal exceeded the incumbent's; so the sequential mechanism's winner might not be the bidder with the highest signal, and the incumbent's signal would no longer be a sufficient statistic for determining whether further entry would be deterred.)

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[30]Bulow and Klemperer (2002) explain the importance of this assumption in common-value models, and provide additional information about their solution.

[31]We require that $S(v^*) = c$ yields $v^* \geq E(v)$. Otherwise, there are complications akin to those for $\rho_j \in (0, 1)$ that we discuss later in this subsection.

B   Simultaneous (Random) Entry into the Auction

No important result is affected if potential bidders make simultaneous, instead of sequential, entry decisions into the auction. This is true whether we assume as in, e.g., Dan Levin and James L. Smith (1994), that potential bidders know the actual number of potential entrants when they make their entry decisions, or we assume potential bidders only know the distribution of the number of other potential entrants when they make their decisions.

The logic of Proposition 2 holds exactly as before: because the second-highest value in the auction is random, the auction price must be higher than the sequential mechanism's price to deter entry, if $S(\cdot)$ is convex.[32] And since the random-entry auction is less efficient than the auction of our basic model (bidders have less information when they make their entry decisions), the auction remains less efficient than the sequential mechanism (Proposition 1).

C   Soft-closing Auctions

A seller may lack the ability or desire to commit to closing an auction if none of the initial entrants bids high enough. Assuming they have the ability to jump bid prior to any further entry, then if none of the initial bidders signals a value of at least v^*, an additional bidder or group of bidders (if any exist) will enter in a second round, and so on, until either there are no more potential entrants, or at least one bidder has a value exceeding v^* and offers a deterring price.

It is easy to check that the expected revenue from such a "soft-closing" auction may be either higher or lower than in our basic auction, depending on parameters.[33] Furthermore, this auction is *always* weakly more profitable and less efficient, in expectation, than the sequential mechanism: when only one bidder enters the "soft-closing" auction at a time, the mechanisms are identical, and when more than one bidder enters at a time, the auction has lower efficiency, but higher expected revenue, because it will attract at least as many if not more bidders than the sequential mechanism, and it may attract more than one bidder

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[32]The proof is essentially unchanged if we simply replace $v_2(n^*)$ by the actual second-highest value of whatever (random) number of bidders actually enter. (The only difference is that when no bidders actually enter a random-entry auction, an additional bidder would only pay $\underline{v}$ (instead of a higher price) but this reinforces the result.)

The random-entry auction may be either more or less profitable than the auction of our basic model.

David Reiley (2005) shows internet auctions are better modeled by random than by deterministic entry.

[33]For example, assume that there are an infinite number of potential bidders, each with values uniform on $[0, 1]$. Let c be close to $2/27$. Then the basic auction will attract three bidders and have expected revenue of $1/2$. The sequential mechanism would yield $V_S = 1/3$. If c is just over $2/27$, the hybrid auction will attract two bidders at a time until at least one is willing to pay $1/3$ or more, and expected revenue is $4/9$. If c is just under $2/27$ the hybrid auction will attract three bidders at a time until at least one is willing to pay $1/3$ or more and expected revenue is $7/13$.

with a value in excess of V_S.[34]

D   Sequential Mechanisms Without Jump Bidding

One assumption that *does* importantly affect our profitability comparisons is that entrants into the sequential mechanism can make jump bids to scare off subsequent entry.

The assumption is unimportant for efficiency. The greater expected efficiency of the sequential mechanism relative to the auction is due entirely to the information revealed by bidders competing until there is a single survivor prior to any further entry; the extra information prospective entrants learn by observing pre-emptive jump bids (or their absence), contributes nothing additional:

Proposition 4 *The expected efficiency of the unique perfect sequential equilibrium of the Sequential mechanism would be unaffected if jump bids were not possible.*

Proof: Absent jump bidding, potential entrants' information about the incumbent's value is only that it exceeds the value of the second-highest-value entrant to date, which the competition between the bidders allows it to observe. So entry will continue after the entry of the first bidder with a value above V_S. However, entry that takes place while there is just one bidder whose value exceeds V_S is on average neutral for efficiency, since revealing only that the incumbent's value exceeds V_S would leave potential entrants indifferent about entering, and potential entrants make the socially correct decisions given their information - see Lemma 3. Conditional on there having been two entrants whose values exceed V_S, so that the higher of these two bidders' values must be drawn from a distribution of $F(\cdot)$ truncated somewhere above V_S, further entry is socially undesirable. But as soon as two such bidders have entered, the price will anyway be driven up to V_S or higher, thus deterring all further entry even without jump bidding.□

But although jump bids are unimportant for total surplus, they play a crucial role in transferring it from the sellers to the buyers. When there are no rents to be earned from bidder scarcity ($\rho_j = 1 \ \forall j$) bidder profits are *entirely* due to the ability to make jump

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[34]The mechanisms yield different prices only when more than one bidder enters simultaneously in the last round of the auction. Since no bidders prior to this round have values exceeding V_S, the logic of Step 1 of the proof of Lemma 4 applies to this group of bidders. (As usual, the sequential mechanism's *price* in some states exceeds the auction's price. Also as usual, Facts 1 and 2 provide a quicker proof: the mechanisms have different winners only when at least two auction entrants' values exceed V_S. The auction winner's value then exceeds at least one other value above V_S, so its EMR exceeds V_S, while the sequential mechanism's winner's EMR equals V_S in this case. So the auction wins.)

Another possible assumption is that bidders in the soft-closing auction cannot jump-bid prior to further entry. In this case, too, the new procedure may be more or less profitable in expectation than the basic auction, but is always more profitable in expectation than the (basic) sequential mechanism.

bids: absent jump bidding, a potential entrant knows only that the current incumbent's value exceeds the actual value of the second-highest-value entrant to date. So entry will continue until the value of the second-highest-value entrant is at least V_S, at which point the competition between the two highest-value entrants will demonstrate that at least one has a value above V_S, and there will be no subsequent entry.[35] So, with an infinite supply of potential entrants, each new entrant knows it will win only if and when it beats one bidder whose value exceeds V_S. So it expects to earn zero profits (by equation (1)), that is:

Proposition 5 *If jump bids were not possible in the Sequential mechanism, and there is an infinite supply of potential entrants, all bidders would earn zero surplus.*

Because the "no jump bidding" mechanism would be both fully extractive (when $\rho_j = 1 \; \forall j$) and as efficient as the perfect sequential equilibrium of our basic sequential mechanism, which is in turn more efficient than the auction, it must yield higher expected revenue than the auction in this case. (Of course, the auction remains more profitable if sufficiently few potential entrants are expected – Part (ii) of Proposition 2 holds exactly as before.)

The reason jump bidding is so profitable for buyers is that it allows them to partially pool; many buyers who would not be able to deter entry if their values were known are able to do so by pooling with higher-value buyers. The perfect sequential equilibrium maximizes the size of the pool. By doing so, it also minimizes the cost of signaling to those bidders who would anyway be able to deter. From the seller's viewpoint, the cost of this strategic signalling outweighs the efficiency advantages of the sequential mechanism.

Observe that a special case of our model is equivalent to one in which all potential purchasers are initially present, if $F(\cdot)$ has sufficient dispersion that no one will ever bid without first paying the cost, c, to learn its value, and if the "auction mechanism" (but not the sequential mechanism) prohibits any learning of values after the bidding has begun. So our results here closely parallel those of Olivier Compte and Philippe Jehiel (2007) who model a fixed number of bidders, some of whom know their values for an object, while others are initially uninformed. They find that if there are sufficiently many bidders, then allowing uninformed bidders to purchase information about their actual values during a continuous ascending auction is more profitable than permitting them to buy such information only prior to the auction - in other words, in this context in which jump bids are not permitted, a "sequential" process is more profitable than a simultaneous auction. Leonardo Rezende (2005) and Eric Rasmusen (2006) obtain similar results.

So if the number of potential entrants is sufficiently large, running a sequential process at the same time as prohibiting jump bidding might be a seller's best option - if this is

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[35]The expected number of entrants in the sequential mechanism without jump bids is therefore twice what it would be if jump bidding were possible (if $\rho_j = 1 \; \forall j$).

practical to implement.

E Buyers' Bargaining Power

Buyers may be able to improve their position, and reduce expected revenues in the sequential mechanism, if they can build a reputation for exiting if an offer they make is not accepted. In the extreme case where sequential buyers can all fully commit to making take-it-or-leave-it offers, and $\rho_j = 0$ for at least one (perhaps large) j, the unique subgame-perfect equilibrium is for the first bidder to make an offer of $\underline{v}$ with the seller accepting. (The game is trivial to solve recursively.) This mechanism obviously minimizes seller surplus and, if $v - MR(v)$ is weakly decreasing in v (that is, the hazard rate $f(v)/(1 - F(v))$ is weakly increasing in v), maximizes buyers' expected surplus.[36]

The more general principle is that if there is a high enough chance that a buyer will leave, then the seller may be forced to accept a price that would not deter future bidders, because of the risk of losing the current bidder. The increase in buyer's surplus from the lower price plus the efficiency loss from reduced entry causes a double hit to the seller.

If it is easier to credibly threaten to withdraw if a seller pursues new entrants in a sequential mechanism than to withdraw in the middle of bidding in an auction, this is another reason for sellers to prefer auctions.

F Other Assumptions

Remarks 2 and 4 illustrate that several assumptions about the extensive forms of the mechanisms are unimportant. Of course, other assumptions might matter more.[37] But the bottom

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[36]If $v - MR(v)$ is increasing in v, bidders who cannot resell may, for example, depending on bidding costs, do better in expectation, in aggregate, if the seller sets a reserve price above $\underline{v}$.

[37]For example, we have not considered (ex-ante) asymmetries among bidders. Both simple ascending auctions, and simple sequential procedures can perform very badly in this context. A first-price auction (or an Anglo-Dutch auction, see Klemperer (2002)) may perform better, but the premium on the seller having the power to implement more complex procedures may be large.

Also important is that a simple auction is perhaps more easily undermined, than a sequential process, by collusion, but the "soft closing" auction may mitigate this problem. (McAfee et al (1993) discusses collusion in a two-bidder version of our sequential mechanism.)

For the sale of corporate assets a preference for speed, discretion, or a long-term buyer (such as Buffett), or a financial buyer likely to retain current management, or a management buyout (where the management will prefer a low sales price) can make a sequential process preferable to some sellers.

Our model also ignores bidding costs (in Daniel and Hirshleifer (1998)'s early work on sequential models of takeovers (without entry costs) these make a simultaneous auction more efficient, as well as more profitable, than a sequential process), any other technological costs of choosing one selling mechanism over another (see, e.g., Ruqu Wang (1993, 1995, 1998) and Michael Arnold and Steven Lippman (1995)); legal issues (see, e.g., Peter Cramton and Alan Schwartz (1991), Ronald Gilson and Bernard Black (1995)); seller credibility (see, e.g., Guiseppe Lopomo (2000), David McAdams and Michael Schwarz (2007)); etc.

line is that varying our assumptions seems more likely to reinforce our results than to undermine them. The main caveat is that if the seller has the power to prevent jump bidding, *and* there are likely to be many bidders, the sequential mechanism looks very attractive to it. On the other hand, if buyers have any countervailing power – in particular, the ability to credibly withdraw if additional buyers are sought – then the auction is especially attractive to sellers.

VII   Promoting Entry

Sequential sales perform poorly for sellers, despite their relative efficiency, because auctions attract more entrants when they matter most. We now explore whether, or when, tactics to promote entry can enable sellers to capture greater revenues from a sequential mechanism.

A   Cash Subsidies

If the seller knows $F(\cdot), c$, and $\{\rho_j\}_{j=1}^{\infty}$ *and* can pre-commit to a set of rules *and* the full common knowledge assumptions obtain for buyers, it can assure efficient entry and extract all surplus by using a sequence of entry fees and subsidies that give the buyers no expected surplus.[38] This seems unrealistic. But it is natural to ask whether there is any simpler modification of the basic sequential mechanism that yields an outcome corresponding to the seller's "favorite" equilibrium ($v^*= V_K$), while satisfying the perfect sequential equilibrium refinement that is usually imposed?

The answer is "yes", *provided* the seller also knows $F(\cdot), c$, and $\{\rho_j\}_{j=1}^{\infty}$. If the seller has the ability to partially subsidize (but not tax) new entry, this raises the signal (and therefore the price) needed to deter entry. In particular, if the size of the possible subsidy is chosen correctly, incumbent bidders with values between V_S and V_K are prevented from being able to pool with those above V_K. Even better, when the weaker types are separated from the stronger ones an unsubsidized entrant will compete against them. So while the *threat* of the

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[38]The simplest efficient mechanism commits to a sequence of reserve prices for successive buyers, chosen so that any buyer with value V_K is just willing to pay its reserve price rather than face additional entry. If entry ceases, a simple ascending auction is held between all those who have entered. To make the mechanism fully-extractive requires charging each successive entrant an entrance fee equal to his expected surplus from entry. See section 3.3 of Cremer, Spiegel, and Zheng (forthcoming), who also show that fully extractive efficient mechanisms can be implemented in considerably more general models than ours. (Cremer, Spiegel, and Zheng also generalise to allow for time discounting, so it can be efficient to have multiple entrants simultaneously, and generalise to bidders with different entry costs and with values drawn from different and non-independent distributions, so the optimal mechanism of an all-powerful buyer is then considerably more complex.)

If the seller can commit to subsidise future entrants, it can extract all surplus without needing to set reserve prices, as we now show.

subsidy is necessary, the subsidy never actually needs to be paid! The outcome is therefore exactly the same as the V_K equilibrium we discussed earlier, and is therefore more profitable than the auction for $\rho_j = 1 \; \forall j$. That is,

Proposition 6 *Consider the Sequential mechanism with the additional feature that after any buyer enters and bids, the seller has the ability but not the obligation to offer to subsidise up to $c - S(V_K)$ of the next potential entrant's cost of finding out its value. The unique perfect sequential equilibrium of this mechanism is more profitable in expectation than the Auction for a set of sequences of $\{\rho_j\}_{j=1}^{\infty}$ that includes $\rho_j = 1 \; \forall j$. Buyers with values below V_K cannot deter entry, but no subsidies are paid in equilibrium.*

Proof: If, when the current high bid is p, a single final buyer enters, the joint benefit to the new bidder and the seller is $v_j - p$ if $v_j > p$, and 0 otherwise, where v_j is the new bidder's actual value, regardless of whether the incumbent outbids the entrant. So from (2) the seller would like to induce (at least) one additional entrant if $p < V_K$. Therefore, since the seller can if it wishes subsidise any potential bidder so that that bidder's total entry costs are no more than $S(V_K)$, a *price* below V_K would only deter entry if it signalled that the deterring incumbent's value was at least V_K. Obviously, no bidder with a value below V_K will be willing to set a price above V_K, so it follows that no bidder with a value below V_K will deter entry.

As in Lemma 2, in the unique perfect sequential equilibrium, the pool of deterring bidders is as large as possible, so all those with values above V_K *will* deter. Clearly no subsidy is then required to induce entry if the potential entrant faces only bidders who have not deterred (since an unsubsidized entrant earns at least zero in expectation if it will be able to buy at a price no more than V_K), and no feasible subsidy will induce entry once a deterring bid has been made. Therefore, no subsidies are actually paid in this equilibrium.

Furthermore, the discussion at the end of section III then implies the equilibrium is both efficient and fully extractive when $\rho_j = 1 \; \forall j$, and so more profitable than the auction in this case.□

Clearly the modified sequential mechanism will be more profitable than the auction for sequences of $\{\rho_j\}_{j=1}^{\infty}$ that are "close enough" to $\rho_j = 1 \; \forall j$, but the argument of Proposition 2 part (ii) shows that for "sufficiently small" $\{\rho_j\}_{j=1}^{\infty}$ the auction remains more profitable.[39]

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[39]Another way to encourage entry is to prohibit incumbent bidders from raising their bids in response to it. Entry will then continue until the price V_K is bid. If $\rho_j = 1 \; \forall j$, this is socially efficient and leaves no expected surplus for bidders, so achieves the maximum expected revenue obtainable from any mechanism. Aaron Edlin (2002) discusses regulatory regimes based on this principle. However, it may be hard to credibly commit to sticking to this strategy. It is also dangerous if there are likely to be few potential entrants: in this case, the fact that such a high *price* (V_K) is required to deter entry means bidders may instead gamble

B Lock-ups, Break-up Fees, and Matching Rights

The previous subsection shows that entry subsidies can make the sequential mechanism both more efficient and more profitable than the auction. By making entry subsidies larger still, the sequential mechanism can often be made even more profitable, by further raising the signal required to deter entry.

However, if the seller has the ability to offer larger subsidies than $c - S(V_K)$, then subsidies must sometimes actually be paid – even in circumstances in which a subsidy would not have had to be paid if large subsidies were not possible – because bidders who fear *future* high subsidies to attract entry against them may need to be paid to be persuaded to enter today.[40] Furthermore, the gains to the seller from reducing buyer surplus are reduced by the inefficiency of the incremental entry induced by these larger subsidies.

The seller could do better than simply offering large subsidies if it could pre-commit to avoiding future inefficient subsidies. In particular, it could benefit from a provision that prohibited it from paying any further subsidies to additional bidders if the current entrant signalled a value of at least V_K.

Such a combination of a subsidy to a current entrant and a promise to limit future subsidies parallels the common practice of sellers of public companies of offering "lock-ups" that include a break-up fee (equivalent to cash paid out of corporate assets) and "matching rights" that prohibit subsidies to future bidders in return for a sufficiently high bid.[41]

So our analysis suggests a role for lock-up-like combinations of current subsidies and promises about future subsidies, but the details will depend not only on the bargaining power of the seller against any new entrant, but also on the specification of the bargaining protocol between the seller and the current incumbent (since bargaining between these actors can lead to even more efficient outcomes). This is beyond the scope of the current paper.

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on low-ball bids being enough to win.

This subsection's result that the modified sequential mechanism is more profitable in expectation than the auction for a set of $\{\rho_j\}_{j=1}^{\infty}$ that includes $\rho_j = 1 \ \forall j$, applies to *any* perfect Nash equilibrium, not just to the unique perfect sequential equilibrium.

[40]In particular, when large subsidies are feasible a first bidder with a value above V_K may prefer to make the minimum allowable bid and risk entry if ρ_2 is small, rather than bid V_K to deter entry (because a lower bid that simply signals a *value* of V_K may not – by contrast with the low-subsidy case – be enough to deter entry); the second bidder may then require a subsidy since it may have to face future highly-subsidized entry (it will certainly require a subsidy if $\rho_j = 1 \ \forall \ j > 2$). The need to sometimes actually pay a subsidy means that in some examples the ability to offer subsidies above $c - S(V_K)$ does not help the seller in expectation.

[41]A break-up fee guarantees the bidder a fixed sum in return for making its offer, payable if the deal fails to be completed – typically because of a topping offer by another bidder – and are typically accompanied by "matching rights". Since the break-up fee is a sunk cost once agreed to, it only distorts an auction if the recipient makes an initial offer above its value in order to receive it. See Ian Ayres (1990). Che and Lewis (2007) show the merits of breakup fees in Fishman (1988)'s model. (Sometimes buyers will offer sellers break-up fees, for example, as protection against a deal collapsing because of antitrust problems. Break-up fees are limited to 1% of the value of the bid in the UK, but can be much higher in the US.) Related issues are discussed by Michael Rothkopf, Harstad and Yuhong Fu (2003).

However, we are sceptical of the broad use of subsidies in practice. Subsidies can easily be abused by a seller who wishes to bias the process. It is not uncommon to see the board of a target company granting subsidies (with matching rights) to an initial bidder that has made a deal with management, effectively deterring other bidders.[42] And potential first bidders may be deterred by the possibility of the board paying subsidies to a management-led counter-bid or other favored alternative bidder.[43]

Even putting aside both this moral hazard problem, and the problem of attracting non-genuine bidders who pocket subsidies without investigating serious bids,[44] the tactics considered here depend sensitively on the seller's information. The basic sequential mechanism and the auction can be implemented by a seller who knows nothing about the distribution of bidders' values, or bidders' perception of that distribution, or their cost of entry. But the tactics here require knowledge of all these things. And with endogenous entry the implementation of an "optimal mechanism" is fraught with danger: there is a small difference between a tactic that extracts all surplus and one that discourages all entry. So while the ability to offer a limited special deal to a "white knight" involving paying its fees, or giving it cheap options or a lock-up, can increase expected revenue, this depends upon the seller being a revenue-maximiser with sufficient informational resources[45] and commitment powers to fine–tune its use of these tactics.

In addition, of course, even with modifications, the sequential mechanism beats the auction only if there are "enough" potential bidders. All told, there is good justification for the U.S. legal system's strong encouragement of auctions for the sale of public companies (see Cramton and Schwartz (1991)).

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[42]Paying break-up fees to initial bidders is especially suspect. Consider, for example, the recent $33 billion buyout of hospital owner HCA, which gave the group that included management a $300 million break-up fee and matching rights in return for an offer that exceeded the pre-takeover valuation by about $3 billion. Since the management group was probably also better informed than any potential rival, it is no surprise that investment bankers were then unable to find a competing offer.

[43]For example, if $\rho_j = 1 \ \forall j$ and the first potential bidder perceives any risk that future subsidies will be larger than optimal, it will never enter, since it would be forced to either bid above V_K or to suffer subsidised competition.

A further concern arises if management can pay subsidies in shares. Giving shares is cheaper than paying cash, because the recipient values them more than the seller – either the recipient sells out at the sale price that would have resulted anyway, or the recipient is the final winner and so values them more than this, and because it gives the recipient a toe-hold advantage (see Bulow, Ming Huang and Klemperer (1999)). So management has a temptation to offer a subsidy in shares even against a bid of V_K – but no bidder will then be prepared to make an original bid of V_K.

[44]Giving break-up fees and matching rights only in return for a "good" offer mitigate the second problem but without eliminating it (it is usually not hard for a winner to find an excuse to withdraw) and at the cost of possibly introducing inefficiency (see note 41).

[45]But if the seller is thought to have too *much* information, for example, more information than the bidders about the probability of further potential entrants, then the current potential bidder may be reluctant to deal with the seller.

VIII Conclusion

Much academic and practitioner opinion argues that sellers benefit from the "level playing field" competition of a simultaneous auction. For example, Bulow and Klemperer (1996) show under mild conditions that even a simple ascending auction with no reserve price, is more profitable than the best possible process that can be conducted with one fewer bidder. But models such as this assume no entry costs, so an auction is also most efficient. If participating in the sales process is costly, the case for auctions is less compelling, because sequential mechanisms are more efficient.

Nevertheless, as a practical matter, auctions generally work best for sellers. As a weapon for negotiating a price, the threat of refusing an offer and waiting for more competitors is less effective than actually inviting the competitors straight in - even though the latter strategy is more socially costly. Buyers, on the other hand, usually gain (in expectation) by disrupting an auction process and forcing sellers to deal with them sequentially. Why?

The auction has three clear advantages for the seller: first, it requires only a limited number of potential bidders to achieve its maximum expected revenue; the sequential mechanism's advantage in being able to consider more buyers if earlier offers are too low is only relevant if the number of potential bidders exceeds the number that would participate in an auction. Second, buyers in a sequential mechanism can make jump bids that just deter entry and so increase their expected profits at the the seller's expense. Third, the greater dispersion in the winner's value in the auction means its expected value must also be higher – and although it does not follow that the price it pays is necessarily higher, the greater entry means it usually will be.

So sellers only rarely benefit from the efficiencies created by sequential search.

The main exception is that if it is difficult for buyers to make credible public jump bids, a simple sequential mechanism may be more profitable than an auction *if* there are also sufficiently many potential buyers and the buyers have sufficiently high costs of collecting enough information to bid. This might help explain why sequential sales procedures are common for residential housing, where search costs, inspection fees, etc., may be high and where (in many jurisdictions) it is hard to commit to a firm offer. For more valuable properties and commercial real estate, search costs are a smaller fraction of values, the number of bidders may be smaller, and their abilities to make credible bids may be greater – consistent with the greater prevalence of auctions for these transactions.[46]

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[46]Our model abstracts from several important issues in the sales of both companies (see note 37) and houses (in particular, there are flows of buyers and sellers into and out of the housing market so issues of timing are especially important).

Similarly, the reasons some art dealers argue that auctions can be less profitable than discretely showing a significant work of art to potential buyers one at a time, negotiating with whoever shows interest, probably

In takeover battles, however, the set of potential buyers is usually quite limited, and jump bids are routine, consistent with the evidence that sophisticated value-maximising sellers usually prefer auctions, while buyers more often prefer to pre-empt them if they can.

With credible jump bidding, unless the number of potential bidders is likely to be large *and* the seller has the information and the ability to enforce contracts including lock-ups, matching rights, and entry subsidies, etc., the auction is more profitable on average except under delicate conditions on demand curvature, bidding costs, and the supply of potential bidders; for the seller to prefer the sequential mechanism requires several stars to be aligned — and for the seller to realize that they are so. If the likely number of potential bidders is small, or if the seller has only limited information about valuations, entry costs, or the number of bidders, or needs to standardize procedures across many sales, as with some government entities, a simultaneous auction is the seller's best bet: it should structure the sales process (for example, by timing the release of critical information - see Wasserstein (2001)) so that bidders are unable or unwilling to make early bids. The preference for auctions should be particularly strong in the sale of public companies where jump bids can be made public and where, because of concerns about moral hazard, one might wish to restrict management in its ability to discriminate between bidders.

Taking a broader perspective, this paper and Bulow and Klemperer (1996) both show the power of competition. Bulow and Klemperer (1996) showed that the extra competition provided by a single extra bidder dominates the extra value of any clever negotiating strategies or regulatory schemes that might be designed to extract more rents from competing firms. The current paper shows that, except in rare cases, *potential* competition is not a good substitute for *actual* competition: the straightforward, level-playing-field competition that a simple auction creates is usually more profitable for a seller than a sequential procedure that sometimes attract more bidders, but prevents direct, simultaneous competition among all participants on equal terms.

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lie outside our model. The publicity of an auction can create stigma (since sales are often associated with "the three D's" - divorce, (financial) distress, and death), attract unwelcome attention from tax authorities (so many private sales take place in Monaco or Liechtenstein), and undermine the common practice of commissioning a copy of the art sold to hang in its place (after selling the original in a different continent). (There may also be an agency problem: it is hard for a dealer to justify high fees if he merely consigns clients' assets to auction.)

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Appendix A. Examples of the Equilibrium of the Sequential Mechanism

Let bidders' values be drawn from the uniform distribution $F(v) = v$ for $v \in [0, 1]$. Then (1) $\implies \int_{x=V_S}^{\overline{v}} [1-x][x-V_S]/[1-V_S]dx = c$ (using note 11's formula for $S(v)$) $\implies V_S = 1 - \sqrt{6c}$. That is, a potential entrant would be just indifferent to entering against an incumbent with a value in the range $[1 - \sqrt{6c}, 1]$ (the probability of defeating such an incumbent will be $\frac{1}{2}\sqrt{6c}$, and the expected profit conditional on doing so will be $\frac{1}{3}\sqrt{6c}$, so the net benefit from entering would be $\frac{1}{2}\sqrt{6c} \cdot \frac{1}{3}\sqrt{6c} - c = 0$).

Let $c = .06$, so $V_S = .4$. So – starting from any point of the game, and independent of past history or future $\{\rho_j\}$ – a bidder must signal that it has a value of at least .4 to deter further entry. It can do this by making a bid that, if its value were .4, would give it the same surplus if it successfully deterred entry as if it did not jump bid and accommodated further entry.

3-bidder Example

If there are exactly 3 bidders ($\rho_j = 1$ for $j \leq 3$, and zero otherwise), a first bidder with the deterring value of .4 who chose *not* to deter would win with probability $(.4)^2$, and earn $\frac{1}{3}(.4)$ conditional on winning, yielding an expected profit of .0213. So if the first bidder's value is .4 or more it will jump bid to $.4 - .0213 = .3787$ and there will be no further entry.

If the first bidder's value is less than .4, it will bid $\underline{v} = 0$, the second bidder will then enter, and the two bidders will raise the price continuously until the lower of their two actual values is reached. Call this price p. If it was the second entrant that quit, the third bidder then enters (it remains unprofitable for the first bidder to deter) and if the third bidder's value exceeds p, the price continues to rise until the lower of the two remaining bidders' values is reached.

If, instead, it was the incumbent bidder (that is, the first entrant) that quit at p, the new incumbent has the opportunity to jump bid to deter the final potential entrant. If the new incumbent had the minimum deterring value, .4, but did not jump bid, its expected profit from competing with the third bidder would be $p(.4 - p)$ (from the case in which the third value is below p) *plus* $\frac{1}{2}(.4 - p)^2$ (from the case in which the third value is between p and .4) equals $.08 - p^2/2$. So if the second bidder's value exceeds .4, it makes a jump bid from p to $.32 + p^2/2 \ (> p)$ immediately after defeating the first bidder at the price p, and there is then no further entry. If its value is below .4 it does not jump bid, the third bidder then enters and, as before, if the third bidder's value exceeds p the price then rises again until the lower of the two remaining bidders' values is reached.

To compute expected revenue, it is easiest to use MR analysis:[47] with probability $.4^3$ all

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[47] To compute expected revenue directly, note that with probability .6 the first bidder deters at price .3787.

three bidders have values between 0 and .4 and so MRs uniformly distributed between -1 and -.2 (since "demand" is linear when values are uniformly distributed). In this case, the winner will be the highest-value bidder, and have the highest of these three MRs which is, on average, $-.4$. With the remaining probability at least one bidder will have a value above .4, and the first of these bidders in the queue will win yielding an expected MR of .4 (using Fact 2). So total expected revenue equals $(.4^3)(-.4) + (1 - .4^3)(.4) = .3488$. (It is easy to check that the auction's expected revenue equals .5).

Alternative Timing of Jump Bidding

As noted in Remark 4, each bidder's expected surplus would be unaffected by the timing of entrants' jump-bidding. If, for example, the first bidder bid 0, a second bidder with the minimum deterring value of .4 would expect surplus $.4(.4/3)$ by not deterring (it would have the highest signal with probability .4, and would then face two bidders with values uniformly distributed below .4). So if the second bidder jumps immediately to $.4. - .4(.4/3) = .3467$, it will scare off both the incumbent (the first bidder) and the subsequent potential entrant and obtain the same surplus. (An alternative assumption is that the incumbent would not, in equilibrium, immediately be scared off if its value exceeds the jump bid. In this case, the second bidder jumps to a price of .3425 and is then bid up (to an average of .3713) when the first bidder's value exceeds .3425, and the third bidder again does not enter.) Expected revenues are, of course, unaffected.

Constant-ρ Example

If instead $\rho_j = \rho \leq 1 \; \forall j \geq 2$, a new entrant with the deterring value of v^* who defeated an existing incumbent at price p but chose *not* to deter would win if no higher-value bidder entered, and would then pay the maximum of p and the highest among subsequent entrants' values. The probability that no subsequent entrant's value would exceed any amount $x \leq v^*$ is $(1 - \rho)[1 + \rho x + \rho^2 x^2 ...] = \left(\frac{1-\rho}{1-\rho x} \right)$, so the probability density of the highest among subsequent entrants' values equalling x would be $\frac{\partial}{\partial x} \left(\frac{1-\rho}{1-\rho x} \right) = \frac{\rho(1-\rho)}{(1-\rho x)^2}$ for $p \leq x \leq v^*$, and this entrant's expected surplus would therefore be $\left[(v^* - p) \left(\frac{1-\rho}{1-\rho p} \right) + \int_{x=p}^{v^*} (v^* - x) \frac{\rho(1-\rho)}{(1-\rho x)^2} dx \right] = \frac{1-\rho}{\rho} \ln \left(\frac{1-\rho p}{1-\rho v^*} \right)$. So the deterring bid of any entrant whose value is v^* or more, and who de-

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With probability $.6(.4) = .24$ the second bidder will deter the third bidder after beating the first bidder at price p uniformly distributed between 0 and .4 and the deterring price is $.32 + p^2/2$, so the average price in this case equals $\int_{p=0}^{.4} (.32 + p^2/2) dp = .3467$. With probability $.6(.4)^2 = .096$ neither of the first two bidders will deter and the third bidder's value is at least .4, so it will win at a price equal to the higher of the first two bidders' values, which is on average $(2/3).4 = .2667$. Finally, with probability $(.4)^3 = .064$ all three bidders have values below .4 and the expected price will be the expected second-highest of these three values, that is, $(1/2).4 = .2$. So total expected revenue equals $.6(.3787) + .24(.3467) + .096(.2667) + .064(.2) = .3488$.

feats the existing incumbent at price p ($p = \underline{v} = 0$ for the first entrant) is $v^* - \frac{1-\rho}{\rho} \ln\left(\frac{1-\rho p}{1-\rho v^*}\right)$.

To calculate expected revenue, we find the expected MR of the winning bidder, as for the 3-bidder example above.[48] The probability that there exists no bidder with a value above any $x \leq v^*$ is $1 - \frac{1-x}{1-\rho x}$. So with probability $\frac{1-v^*}{1-\rho v^*}$ there will be a bidder with a value of at least v^*, and so an expected MR of v^*. With probability density $\frac{\partial}{\partial x}\left(1 - \frac{1-x}{1-\rho x}\right) = \frac{1-\rho}{(1-\rho x)^2}$ the winner's value is $x < v^*$, and $MR(x) = 2x - 1$, so the expected revenue from the mechanism is $v^* \frac{1-v^*}{1-\rho v^*} + \int_{x=0}^{v^*}(2x-1)\frac{1-\rho}{(1-\rho x)^2}dx = \frac{v^*}{\rho}\left[1 + \frac{(1-\rho)^2}{(1-\rho v^*)}\right] + \frac{2(1-\rho)}{\rho^2}\ln(1 - \rho v^*)$.

In the perfect sequential equilibrium, $v^* = V_S = 1 - \sqrt{6c}$ and, for $c = .06$, $v^* = V_S = .4$ (as in the 3-bidder example above). If, for example, $\rho = .8$ the expected revenue $\approx .29$. (It is not hard to check the auction's expected revenue $\approx .37$.)

Appendix B. Proofs of Lemmas

Proof of Lemma 2: It is straightforward from the discussion in the text that it is a perfect Nash equilibrium for any entrant, immediately after entry, to jump bid to the price that deters any future entry, if and only if it has a value $\geq v^*$, for some $v^* \geq V_S$. Furthermore, there cannot be a constant deterring "cut-off" value $> V_K$, because if two bidders had values $> V_K$ but $< v^*$ then a further new entrant would not be deterred and would face a minimum price $> V_K$. But its expected profits would be less than the profits from facing a fixed price of V_K, which from (2) are zero, which is a contradiction.[49] So $v^* \in [V_S, V_K]$.

Now consider any sequential equilibrium in which, at some stage, some bidders with values $\geq V_S$ do not deter; and at that stage let $\widetilde{p}$ be the bid which type V_S would be just indifferent about jumping to *if* such a jump bid did deter future entry. Then at this stage of such an equilibrium (i) all types with values $\geq V_S$ would deviate from their equilibrium strategy to bidding $\widetilde{p}$ *if* doing so did deter (since types that were anyway deterring would thereby deter at a lower price, and other types above V_S gain more than type V_S gains by deterring and therefore strictly gain) *and* (ii) no other type would deviate from its equilibrium strategy to bidding $\widetilde{p}$ even if doing so did successfully deter all further entry. So a type of bidder with value $\geq V_S$ failing to deter cannot, at any stage, be part of a perfect sequential equilibrium. The unique perfect sequential equilibrium therefore has $v^* = V_S$.[50]

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[48] As before, it is possible, but more cumbersome, to compute expected revenue directly - see note 47.

[49] Technically, this argument requires a positive probability of at least three potential entrants existing. However, any equilibrium in which the "cut-off value" is above V_K in *any* period not only fails the "intuitive criterion", but is not even robust to the "test of dominated messages" (see In-Koo Cho and David Kreps (1987), and Kreps, 1990, p.436): making the deterring bid corresponding to a "cut-off value" of V_K is strictly dominated for all types with values below V_K (however any potential entrant would respond, all these types would do better not to jump bid), so making this bid must be interpreted as signalling a value $\geq V_K$, and will therefore prevent further entry.

[50] Strictly, if $\rho_j = 1 \ \forall j$ this argument proves only that this is the unique neologism-proof equilibrium. It does not quite prove it is the unique perfect-sequential equilibrium, because a bidder with value below V_S is

Proof of Lemma 3: In both mechanisms, the direct private and social cost of entering is the same, that is, c.

In the auction the private and social contribution of each bidder is the same, namely zero for everyone other than the winner and the difference between the top two values for the winner. Since the expected contribution is therefore clearly also decreasing in the number of entrants, the correct number enter.

In the sequential mechanism, if no bidder with a value of v^* or more has yet entered, the social contribution from one more entry is at least $\int_{x=v^*}^{\overline{v}}(x - v^*)f(x)dx \geq c$ so entry is socially efficient. Also, since the entrant will be able to deter entry through a bid no greater than v^*, it is also profitable to enter, even ignoring potential profits from winning with a bid, or even a value, below v^*. If a bidder with a value of v^* or more has already entered, it is inefficient and unprofitable for a new bidder to enter even ignoring the possibility of others entering later: the expected private and social contributions from entry would be $S(v^*) \leq c$ (from Lemma 2, $S(V_S) = c$ and $v^* \geq V_S$), so the private and social choices are again identical.

Proof of Lemma 4:[51] From Remark 4, neither sales process's expected revenue would be affected if the actual number of potential entrants were revealed to everyone prior to any entry decisions. So consider this case. The result therefore follows if we can show that "if the auction is more profitable in expectation when there are exactly $k > 2$ potential bidders, then it is more profitable in expectation when there are exactly $k - 1$ potential bidders". (If there are two potential bidders, the reduction of one bidder trivially leaves both mechanisms with the same expected revenue, $\underline{v}$.) We will prove this in two steps: 1. $k \leq n^*$. 2. $k > n^*$.

Step 1. $k \leq n^*$. To show this we demonstrate that with n^* or fewer bidders the auction is always more profitable in expectation:[52] if no bidders' value exceeds v^*, the two sales processes yield the same price. If instead at least one potential bidder's value exceeds v^*, consider the first such bidder. The deterring price it sets is such if its value were v^* it would be indifferent between deterring at this price, and not deterring. But a bidder with value v^* is indifferent between not deterring, and paying a price equal to the minimum of v^* and the highest among all subsequent potential bidders' values. So the deterring price equals the

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weakly willing to bid V_S if $\rho_j = 1\ \forall j$, which spoils the exact Grossman-Perry argument (but not Farrell's). However, if bidders prefer no bid to a surely-losing one (e.g., there are small bidding costs) or of course if $\rho_j < 1$ for *any j, all* equilibria other than the constant-cutoff equilibrium with $v^* = V_S$ fail Grossman-Perry's exact refinement, as well as Farrell's.

[51] The working paper version of this paper gives a shorter proof using "MR analysis" following the intuition given in the main text.

[52] Note this would be immediate in Myerson (1981)'s "regular case" - which corresponds to assuming downward sloping MRs - because the auction is then the optimal method of sale to a set of potential bidders who will all enter.

expectation of the minimum of v^* and the highest among all subsequent potential bidders' values. But the auction would achieve the second highest among the current bidder's and all subsequent potential bidders' values, which is obviously higher in expectation.

Step 2. $k > n^*$. Reducing the number of bidders leaves the auction unaffected (since only n^* enter anyway), but (weakly) lowers the price in the sequential mechanism, since if any of the first $k - 2$ would deter they would do so at a lower price than if a total of k bidders were expected since the deterring price equals the expectation of the minimum of v^* and the highest among all subsequent potential bidders' values (as shown in the previous step); the result is obvious if none of the first $k - 2$ would deter.

Proof of Lemma 5: The winner of the sequential mechanism is the first bidder with a value exceeding v^*. So for $v > v^*$ the ratio of the density of the auction-winner's value to that of the sequential-mechanism-winner's value is $[nF(\cdot)^{n-1}f(\cdot)]/[\frac{f(\cdot)}{1-F(v^*)}]$, which is increasing. So the former density is first higher (for $v < v^*$), then lower (it obviously cannot always be higher), and finally higher again (it cannot stay lower since the distributions have the same mean), than the latter density, which implies second-order stochastic dominance.

Proof of Lemma 6: Recall $S(v)$ is the expected surplus of a bidder who competes in an ascending auction against a second bidder, when their values are drawn independently from $F(\cdot)$, conditional on the second bidder's value exceeding v. Clearly $S(v)$ is decreasing in v, so it is convex in v if, $\forall$ constants a, b, with $b < 0$, whenever the expression $S(v) - (a + bv)$ changes sign twice, it is first positive, then negative, then positive again.

Furthermore, since $S(v) = E\{T(w) \mid w \geq v\}$, in which $T(v)$ is the expected surplus of a bidder who competes against another bidder with a value of exactly v, and $v = E\{MR(w) \mid w \geq v\}$ (this is Fact 2) and, of course, $a = E\{a \mid w \geq v\}$, we have

$$S(v) - (a + bv) = E\{[T(w) - a - bMR(w)] \mid w \geq v\}$$
$$= \int_{w=\underline{v}}^{\overline{v}}[T(w) - a - bMR(w)]\left[\frac{f(w)}{1-F(v)}I_{[w \geq v]}\right]dw,$$

in which $I_{[w \geq v]}$ is the indicator function.

We know from standard consumer theory that $T(v)$ is convex and decreasing in v, so it is straightforward that if $MR(v)$ is convex in v, then whenever, with $b < 0$, the expression $[T(w) - a - bMR(w)]$ changes sign twice, it is first positive, then negative, then positive again. But it follows from Karlin (1968) that since $\left[\frac{f(w)}{1-F(v)}I_{[w \geq v]}\right]$ is known to be *Totally Positive* of order 3, the sign pattern is carried over[53] to $\int_{w=\underline{v}}^{\overline{v}}[T(w)-a-bMR(w)]K(v,w)dw$, as required.

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[53] This is the variation diminishing property of totally positive kernels developed in Karlin (1968). Jewitt (1987) gives a concise summary of the theory, and application to economics. We are very grateful to Ian Jewitt for helping us with this proof.

Proof of Lemma 7: Part (i): for $\rho_j = 1 \ \forall j$, assume for contradiction that the sequential mechanism has as many bidders as the auction in expectation. Then the distribution of highest values in the sequential mechanism must stochastically dominate the distribution in the auction, since entrants into the sequential mechanism win if and only if their values are v^* or more, while auction participants sometimes lose with values above v^* and sometimes win with values below v^*. So since an additional bidder's expected profits from entering the sequential mechanism are zero when $v^* = V_S$, its expected profits from entering the auction are positive, which is a contradiction. So the auction attracts more bidders in expectation.

Now, $\forall \{\rho_j\}_{j=1}^{\infty}$, the auction has more expected bidders attributable to potential bidder k, for $2 \leq k \leq n^*$ (since all these would participate in the auction and not all would participate in the sequential mechanism) and the sequential mechanism has more expected bidders attributable to potential bidder k, for any $k > n^*$. So since the auction has more expected bidders for $\rho_j = 1 \ \forall j$, it also has more expected bidders if we reduce (by lowering ρ_k) the number of expected bidders attributable to all potential bidders from the kth on.

Part (ii) is straightforward.

Proof of Lemma 8: Recall that expected surplus for all buyers and the seller would be unaffected if the number of potential entrants, k, were revealed in advance of the first bidder's entrance (Remark 4). So consider the mechanisms with k revealed in advance. Observe that *conditional on actually entering* a potential entrant's expected surplus from the sequential mechanism is independent of its position in the queue (if its value is less than v^* it wins whenever its value exceeds all other $k - 1$ signals; otherwise it wins with probability 1, and must pay the same that it would expect to pay if its value were Vs and it did not deter). So its expected share of surplus from the sequential mechanism equals the fraction of expected actual entrants that it represents, just as in the auction.

Part (ii) of the Lemma now follows when the probability of more than n^* entrants is sufficiently small. For part (i), there are clearly fewer expected entrants in the sequential mechanism than the auction if $k \leq n^*$ (because all potential entrants then enter the auction) and, since the expected number of entrants in the sequential mechanism is less than n^* (since it is less than n^* even if $\rho_j = 1 \ \forall j$, if $v^* = V_S$, by Lemma 7), the same is true if $k > n^*$. So because the first bidder always enters, it expects a larger share of the buyers' surplus in the sequential mechanism than in the auction for any k, and therefore also on average.
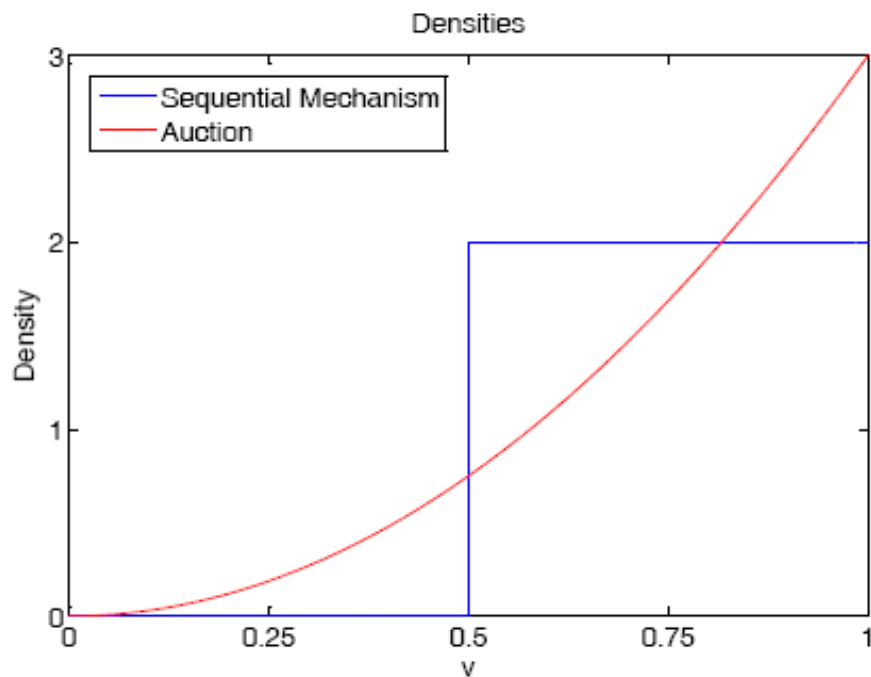
Figure 1:

Densities of winner's value for an Auction and Sequential Mechanism with identical expected winning bids and identical expected winner's values.

[Drawn for bidders' values uniformly distributed on $[0, 1]$;
for the Auction, $n^* = 3$, so expected winning bid $= \cdot 5$;
for the Sequential Mechanism we assume the same cutoff (winning) bid, $\cdot 5$,
so with an infinite number of potential entrants,
the expected winner's value $= \cdot 75$, in both cases.]