Why illuminant direction is fundamental to texture analysis

M.J. Chantler

Department of Computing and Electrical Engineering
Heriot-Watt University
Riccarton
Edinburgh EH14 4AS
Tel. 031 451 3352 (or 031 451 3328)
Fax. 031 451 3327
e-mail mjc@cee.hw.ac.uk

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Why illuminant direction is fundamental to texture analysis

M.J. Chantler

Department of Computing and Electrical Engineering, Heriot-Watt University

Abstract : This paper shows that directed illumination used in the image acquisition process can act as a *directional filter* of three-dimensional texture. An image model of texture is presented which, given the illuminant vector, may used to predict the directional characteristics of image texture. Simulations and the results of laboratory experiments are presented that confirm the predicted directional filtering effects. The image model is used to predict the output of a directional texture measure : Laws' *L5E5* operator [1]. Empirical results using four samples of isotropic texture confirm that the operator's output is significantly affected by changes in the angle of tilt of the illuminant. They also show that the model provides a good basis for predicting the behaviour of such operators. Finally the effect of changes in illuminant tilt on the distributions of the operator for two isotropic textures are presented. These results show that considerable mis-classification would result in using an *L5E5* based classifier if the illuminant tilt angle were changed between training and classification sessions.

1. Introduction

Although the field of automated texture classification has seen many advances in the past twenty years, one associated area of research has received little attention. The effect of variation in illuminant direction on the classification process has rarely been discussed within the open literature. This is surprising, as it well known that the characteristics of lighting do affect the appearance of texture. For instance photographers will often use single point light sources positioned to provide directed illumination at low grazing angles if they wish to emphasise textural qualities [2] and there are many other circumstances in which the characteristics of the illumination are known to affect the appearance of texture. It is therefore all the more puzzling that its effect on automated texture classification has largely been ignored. A number of papers do describe "rotation invariant" texture classification schemes, but they *do not* consider the effect of the rotation of the illuminant direction [3-11]. They therefore implicitly assume that image texture is due solely to surface markings (rather than surface relief) or that the illuminant is omnidirectional ('flat'). However, in order to test their algorithms against a common standard, they have used what has become the *de facto* benchmark — a set of scanned images from

Brodatz's texture album [2]. Many of these textures clearly violate one or both of these assumptions.

This paper presents theory and empirical results which show that changes in the direction of the illuminant vector do significantly affect the characteristics of image texture. In particular the directionality of image texture is shown to be affected by variation in the illuminant's angle of tilt. This is important because many texture classification and segmentation schemes use directional feature measures. One such measure - Laws' *L5E5* operator - is examined in detail and is shown to be significantly affected by changes in tilt angle.

Although the effect of illuminant changes on the appearance of three-dimensional texture presented here may seem obvious, it is believed that this is the first time that such effects have been investigated and reported from the perspective of automated texture classification and segmentation.

2. An image model of three-dimensional texture

This section presents a model of the image of an illuminated three-dimensional texture. It is based on theory developed by Kube and Pentland [12] and further developed by Chantler [13 & 14]. The theory, which is given in the appendix, develops an expression for the spectrum of image texture in terms of the surface texture's spectrum and the illuminant vector. It assumes :

(i) a Lambertian surface (i.e. perfectly diffuse reflection),

(ii) an orthogonal camera model,

(iii) a constant illuminant vector over the scene,

(iv) a viewer-centred co-ordinate system, in which the reference plane of the surface is perpendicular to the viewing direction,

(v) that shadowing and occlusion are not significant,

(vi) that slope angles are low, and

(vii) that image texture is solely due to surface height variation.

The Lambertian reflectance model is linearised, expressed in terms of partial derivatives, and applied to a frequency domain representation of the surface texture — resulting in the following expression for the Fourier transform of the image :

$$F_I(\omega,\theta) = -i\omega F_H(\omega,\theta).\cos(\theta-\tau).\sin\sigma \qquad (1)$$

where

$F_H(\omega,\theta)$ is the Fourier transform of the height map of the surface,

ω is the angular frequency of the Fourier component,

θ is its direction w.r.t. the *x*-axis,

i represents a 90° phase change,

σ and τ are the slant and tilt angles of the illuminant vector **L** as defined in Figure 1.

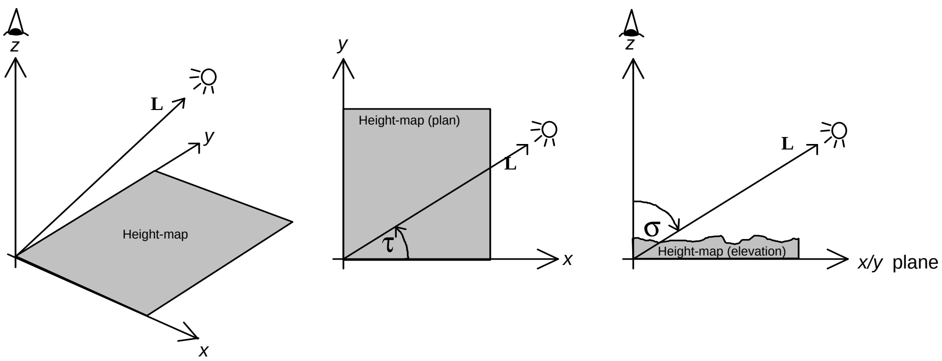


Figure 1 - Definition of axis and illumination angles

From (1) it is clear that the theory predicts that image texture is a function of the height-map of the physical texture *and* the slant (σ) and tilt (τ) of the illuminant. Most importantly it predicts that the directionality of image texture is not only a function of surface relief *but is also a function of the illuminant's angle of tilt τ.* The influence of each of these factors may be more easily identified if the model is divided into three components :

$$F_s(\omega,\theta) = -i\omega F_H(\omega,\theta) \qquad (2)$$

$$F_\tau(\omega,\theta) = \cos(\theta-\tau) \qquad (3)$$

$$F_\tau(\omega,\theta) = \sin\sigma \qquad (4)$$

They are referred to here as the surface height function, the illuminant tilt angle response, and the illuminant slant angle response, respectively. It is the tilt angle response (3) which is of most significance here. It predicts that image acquisition using directed illumination acts as a *directional filter* of texture, that is, it predicts that the components of a texture in the same direction (θ) as the tilt angle of the illumination (τ) will be accentuated compared with those components at right angles to τ. This is unfortunate as the majority of feature sets used in classification and segmentation exploit the directional characteristics of image texture.

3. The response of image texture to changes in illuminant tilt

The above theory has important implications for the classification and segmentation of images of three-dimensional texture. However, many assumptions were made during its derivation. In particular the average slope angles were assumed to be low to allow the Lambertian reflectance model to be linearised. Simulation was used to investigate the effect of increasing slope angles. Lambert's cosine rule was used to render height-maps of isotropic fractal surfaces. The surfaces were synthesised with a range of height variances in order to provide a range of average slope angles. All surfaces were generated using Fourier filtering [15] with a power roll off factor $\beta = 3.5$. FFTs (fast Fourier transforms) of the resulting images showed that the image texture was directional. Polar plots where generated from these FFTs and are shown Figure 2.

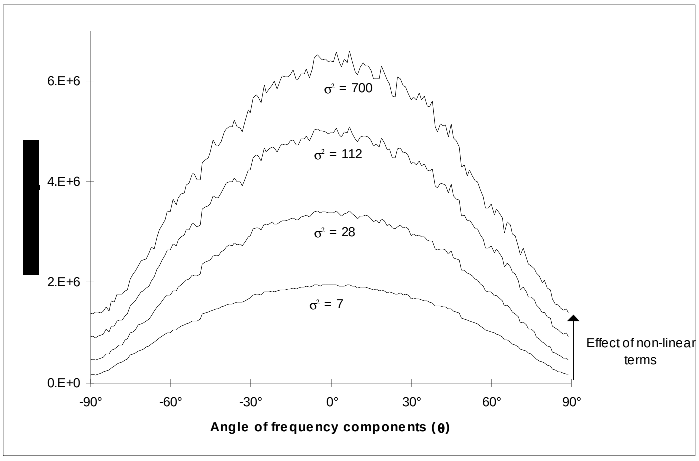


Figure 2- Polar plots of magnitude spectra of images of four synthetic surfaces of varying surface variance

In these plots each point on a graph is the sum of the magnitudes of the Fourier components in that direction (θ), i.e. each point is a function of the energy of the image texture in that direction. These graphs show that the directional characteristics of an image of a surface with low surface variance ($\sigma^2 = 7$) are of the form $\cos(\theta - \tau)$ as predicted. However, the plots of surfaces with higher variances show the effect of the non-linear terms that were neglected in the theory (see appendix). They suggest that the directional characteristics would be more accurately modelled using a *raised* cosine. Thus the *tilt response component* should be of the form :

$$F_\tau(\omega, \theta) = m_\tau \cos(\theta - \tau) + b_\tau \qquad (5)$$

Unfortunately the parameters m_τ and b_τ are a function of both the surface variance *and* the degree of shadowing and hence must be determined empirically for each texture. Further detail on these effects are available in [13]. The main point is, however, that the simulations support the proposition that image capture using directed illumination can act as a directional filter of texture, all be it in the modified form of (5).

3.1 Laboratory experiment - four physical textures

A set of experiments using real textures was performed to further investigate the effects that changes in illuminant tilt have on image texture. Four textures were chosen that were isotropic in appearance - so as to minimise their contributions to the directionality of the image textures. The textures were sprayed matte white to eliminate any albedo texture and to provide an approximately Lambertian reflectance characteristic. The texture samples were mounted perpendicularly to the camera's line of sight at a distance of 3.3m, and illumination was provided by a 500W lamp 1.6m from the subject. The tilt angle of the illumination was varied in 10° steps while all other parameters were kept constant. Sample images of the textures are shown in Figure 3.

"beans1" : a tray of butter beans | *"chips1" : a tray of gravel chips* | *"stones1" a tray of beach pebbles* | *"rock1" : a small piece of conglomerate*

Figure 3-Sample images of the test textures : top row captured at $\tau = 0°$ (i.e. the light source to the right images); bottom row captured at $\tau = 90°$ (light source above images)

The variation in the images' directional characteristics caused by a change of illuminant tilt angle from 0° to 90° is discernible but not obvious. FFTs of the images showed that they were clearly directional and that the directional distribution of energy rotated with the illuminant tilt angle — the maxima always coinciding with the tilt angle as predicted by equation (3). Sample FFTs of images of *rock1* are shown in Figure 4.

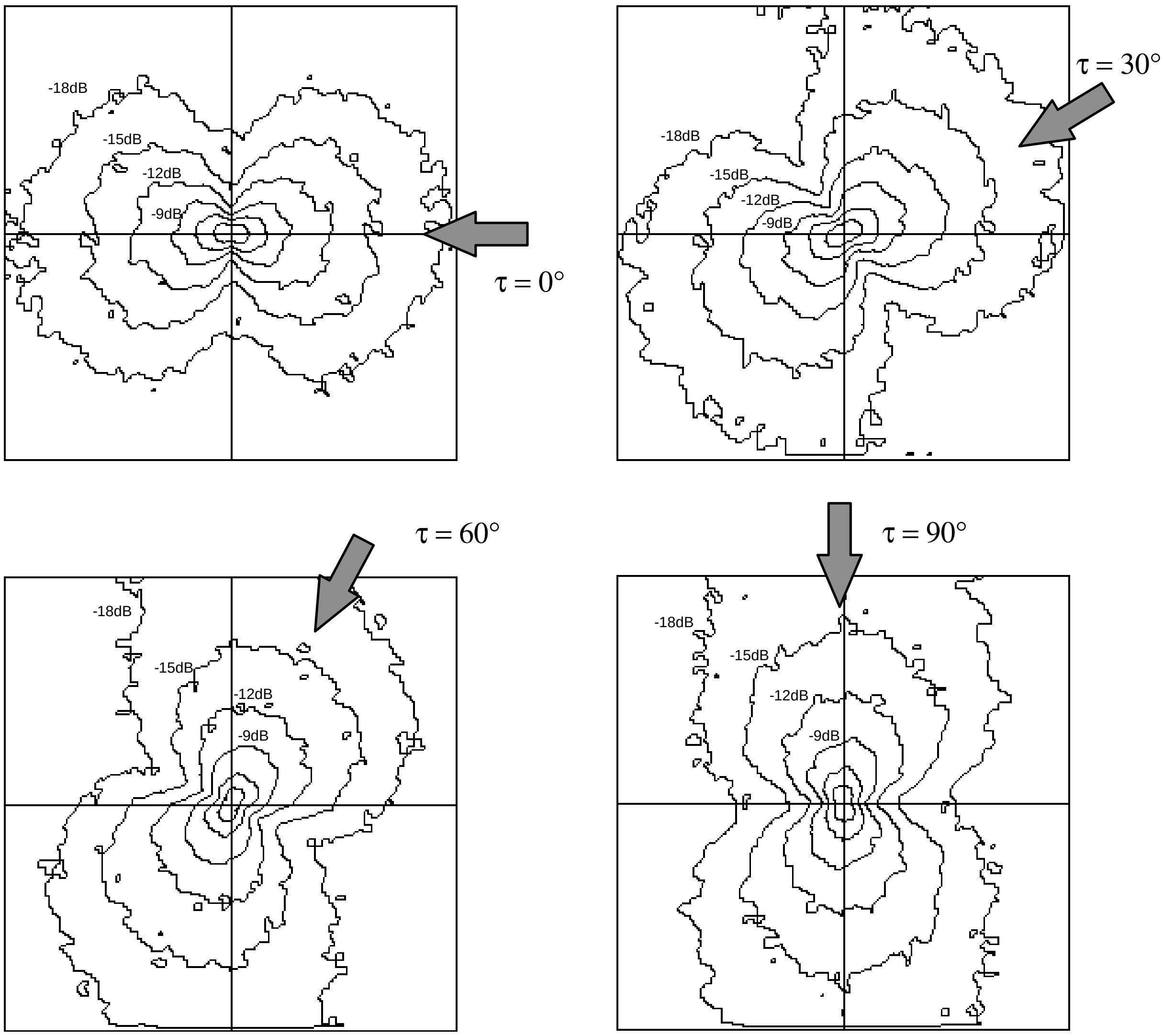


Figure 4 - Effect of illuminant tilt angle on two-dimensional FFT plots of images of texture *rock1*
Notes : (a) arrows indicate direction of illumination, and
(b) FFTs shown as log contour plots of magnitudes with d.c. at the centre.

Figure 4's plots clearly show that the illuminant tilt has a considerable impact on the directionality of images of the texture sample *rock1* and that changes in tilt may cause any classifier using directional characteristics considerable problems. Plots of the other textures gave similar results. From the contour plots of Figure 4 it is difficult to assess how well the predicted tilt response (5) can be used to represent the distributions. Hence Figure 5 contains polar plots of the FFTs. They are of images of the four textures captured at $\tau = 0°$ and are shown together with the best fit[1] raised cosine functions : $y = m_\tau \cos(\theta - \tau) + b_\tau$.

[1] The parameters of the raised cosine were calculated using least squares linear regression.

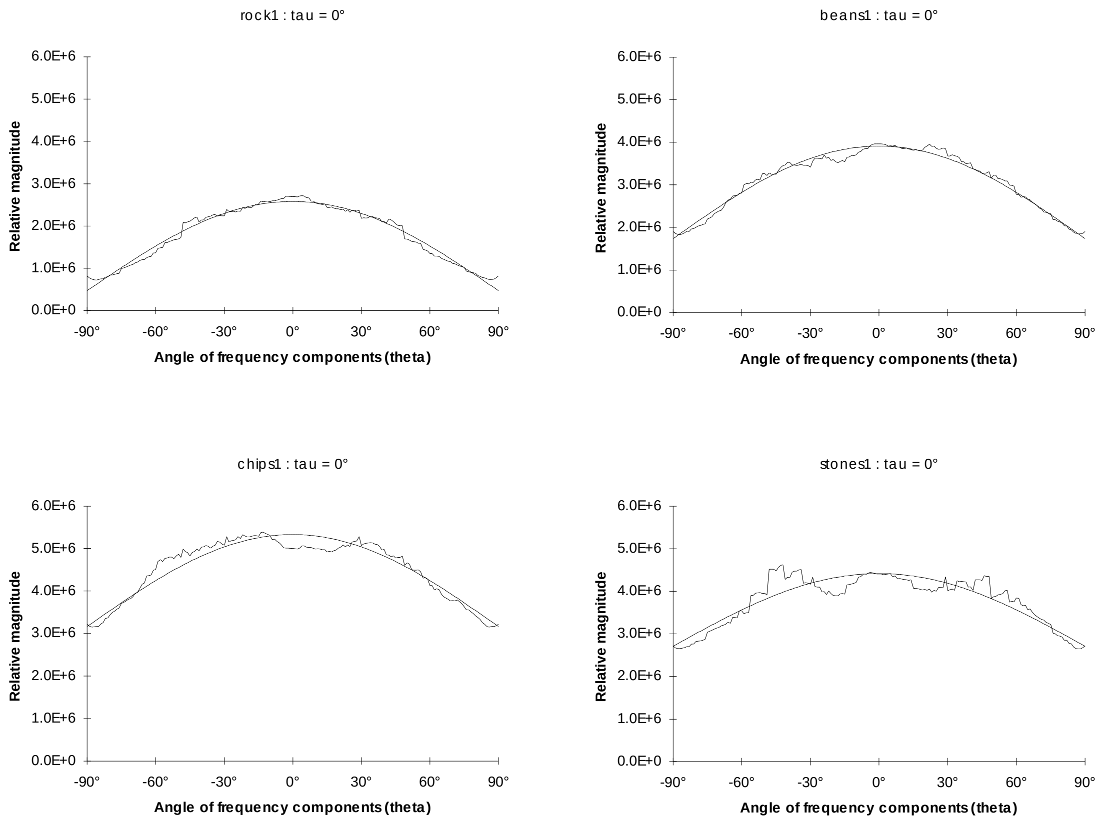


Figure 5- Polar plots and best fit raised cosines for the four test textures (images capture at $\tau = 0°$).

The polar plots above confirm that all four image textures exhibit distinct directional characteristics which resemble a raised cosine. These empirical results therefore

(i) confirm that image texture directionality is not only a function of surface relief but is also dependent upon illuminant tilt, and

(ii) support the $\cos(\theta - \tau)$ relationship between illuminant tilt and image texture, but show that it may be more accurately modelled by adding a term to account for the raised cosine effect, that is, it confirms that the tilt component would be better represented by equation (5).

These phenomena complicate the task of texture classification; as many texture measures used in such schemes exploit directional characteristics. Hence the next section examines the effect that changes in illuminant tilt angle have on the output of one texture measure which is popular in the literature.

4. The effect of variations in illuminant tilt angle on Laws' L5E5 texture measure

The tasks of texture classification and segmentation both require a means of quantifying textural characteristics so that the similarities between texture samples or regions may be measured. Such textural characteristics are normally computed using a set of feature measures [16 & 17]. This section uses both theory and laboratory experiment to investigate the response of *one* feature measure to changes in illuminant tilt, the purpose being two-fold : the first is to show convincingly that changes in illuminant direction do significantly affect the output of a directional texture measure, and the second is to show that the theory presented above can be used to predict the behaviour of such an operator. Note that the single Laws' feature is used here for illustrative purposes only — in practice a combination of complementary features would be used for classification.

4.1 The Laws' L5E5 operator

Laws developed a set of simple feature measures for texture classification and segmentation [1]. They comprise a set of two-dimensional filters (also referred to simply as 'masks') each of which is coupled to a *mean square* or *sum of absolutes* macro statistic. The macro statistic provides a measure of the energy in the pass band of the filters within a moving window. The most popular filters used are 5 x 5 and in common with all of Laws' masks are derived from the following non-recursive filters :

L3 = (1,2,1) - Level detection,

E3 = (-1,0,1) - Edge detection, and

S3 = (-1,2,-1) - Spot detection.

Only the response of the L5E5 operator will be presented here, it is obtained from the following convolution :

$$L5E5 = E3 * L3 * L3^T * L3^T$$

and its mask is shown in Figure 6(a). As it comprises four *separable* filters its frequency response is simply derived :

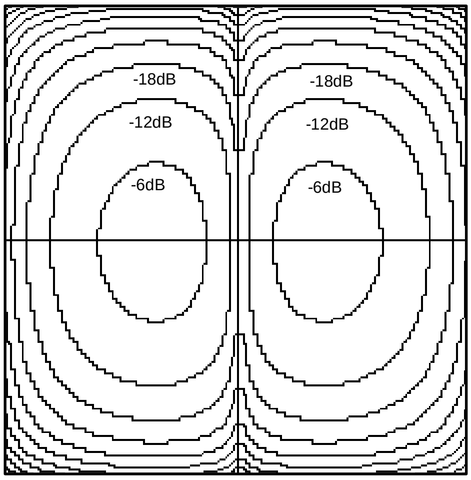
$$\begin{aligned} \left|H_{L5E5}(\omega_1,\omega_2)\right| &= \left|H_{E3}(\omega_1).H_{L5}(\omega_1).H_{L5}(\omega_2).H_{L5}(\omega_2)\right| \\ &= 2\sin\omega_1.2\left(1+\cos\omega_1\right).2(1+\cos\omega_2).2(1+\cos\omega_2) \end{aligned} \qquad (6)$$

where ω_1 and ω_2 are angular frequencies in *x* and *y* directions respectively.

The *L5E5*'s two-dimensional frequency, Figure 6(b), shows that it is a directional texture measure being sensitive to components around θ = 0°. The *complete L5E5* texture measure, as used here, consists of the mask shown above followed by a 31 x 31 mean square macro statistic. Thus the mean output of this operator may be predicted using the model of image texture presented above (1) together with the frequency response of the *L5E5* mask (6). As only variations due to changes in the illuminant's tilt are of interest, it is assumed that the illuminant's slant does not vary, and the contribution of the corresponding component in the image model is a

-1	-2	0	2	1
-4	-8	0	8	4
-6	-12	0	12	6
-4	-8	0	8	4
-1	-2	0	2	1

(a)



(b)

Figure 6 - The Laws' L5E5 mask (a) and its frequency response (b)

constant k_σ. Thus the model presented in equation (1) reduces to :

$$F_I(\omega,\theta) = F_s(\omega,\theta).F_\tau(\omega,\theta).k_\sigma \tag{7}$$

Now if the height maps of the test textures are assumed to be isotropic and fractal then the magnitude of the surface response component may be represented by :

$$\left|F_s(\omega,\theta)\right| = k_\beta \omega^{-\beta_I/2} \tag{8}$$

The parameters k_β and β_I may be estimated by obtaining the gradient and y-intercept of the best-fit straight line to the average log-log radial plots of the magnitude spectra of the image. Hence the image texture magnitude spectra of the four samples may be modelled by combining (5), (7) and (8) to give :

$$\left|F_I(\omega,\theta)\right| = k_\beta \omega^{-\beta_I/2}\left(m'_\tau \cos(\theta-\tau) + b'_\tau\right).k_\sigma$$ (9)[2]

Now if only relative magnitudes are required k_σ may be eliminated and all remaining parameters estimated for each of the test textures as described previously. Thus the output of the first stage of Laws' *L5E5* operator is

$$\begin{aligned}\left|Y_{L5E5}(\omega,\theta)\right| &= \left|H_{L5E5}(\omega,\theta)\right|\left|F_I(\omega,\theta)\right| \\ &= \sin(\omega\cos\theta)\{1+\cos(\omega\cos\theta)\}\{1+\cos(\omega\sin\theta)\}^2.k_\beta\omega^{-\beta/2}\{m'_\tau\cos(\theta-\tau)+b'_\tau\}.k_\sigma\end{aligned} \tag{10}$$

[2] Note the primes added to symbols m_τ and b_τ indicate that these parameters have been normalised to avoid including a factor related to the total power of the texture twice.

where :

$\omega\cos\theta = \omega_1$, $\omega\sin\theta = \omega_2$, and

$|Y_{L5E5}(\omega,\theta)|$ is the two-dimensional magnitude spectrum of the output of *L5E5*.

The second stage of the operator provides an estimate of the "power" of the filtered image texture and hence the integral of the PSD (Power Spectral Density) assuming that the process under consideration is at least wide-sense stationary. The PSD may in turn be obtained from the magnitude of the Fourier transform of the output of the filter. Hence the mean output of the L5E5 operator will be :

$$\begin{aligned}\overline{y^2_{L5E5}} &= \frac{1}{4\pi^2}\int_{\omega=0}^{\infty}\int_{\theta=0}^{2\pi}|Y_{L5E5}(\omega,\theta)|^2\,d\theta d\omega \\ &= \frac{1}{4\pi^2}\int_{\omega=0}^{\infty}\int_{\theta=0}^{2\pi}\left[\sin(\omega\cos\theta)\{1+\cos(\omega\cos\theta)\}\{1+\cos(\omega\sin\theta)\}^2.k_\beta\omega^{-\beta/2}\{m'_\tau\cos(\theta-\tau)+b'_\tau\}.k_\sigma\right]^2 d\theta d\omega\end{aligned} \qquad (11)$$

The solution of the above integral for the general case is not trivial. However, it may be estimated numerically when the values of the parameters are known. Hence the four parameters (m'_τ, b'_τ, k_β, and β_I) were calculated for each of the four isotropic test textures. The integral (11) was evaluated for each of these four sets of parameter values for nineteen angles of illuminant tilt (0° to 180° in 10° steps). The results are shown in Figure 7.

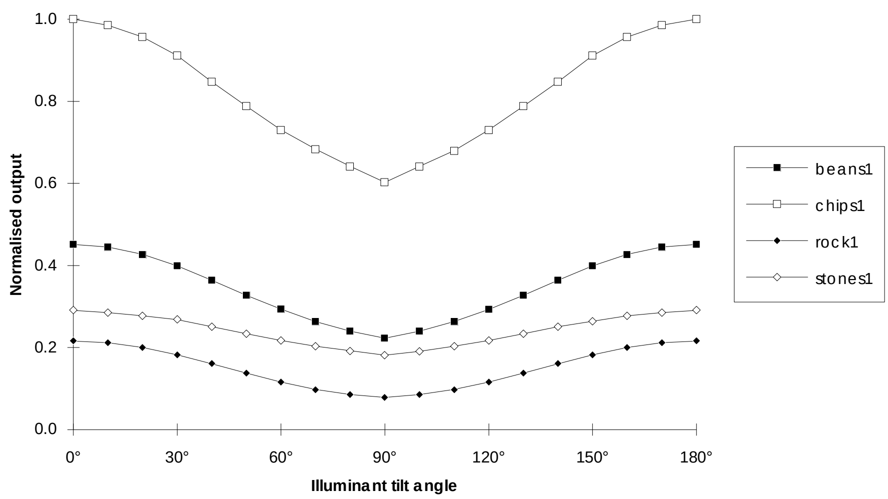


Figure 7- Predicted effect of tilt angle variation on L5E5 output[MJC3]

Thus the above theory predicts that the *L5E5* feature measure is affected by changes in the illuminant's angle of tilt. For comparison Figure 8 shows the equivalent results obtained by processing images of the textures with the feature measure itself — an *L5E5* mask coupled to a 31 x 31 *mean square* macro statistic.

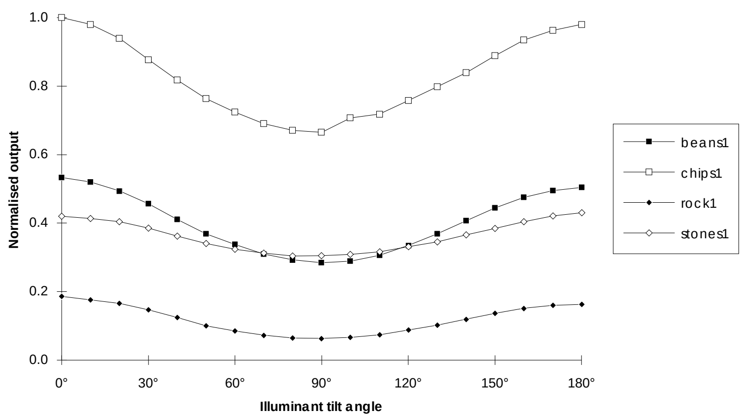


Figure 8 - Observed effect of tilt angle variation on L5E5 mean output

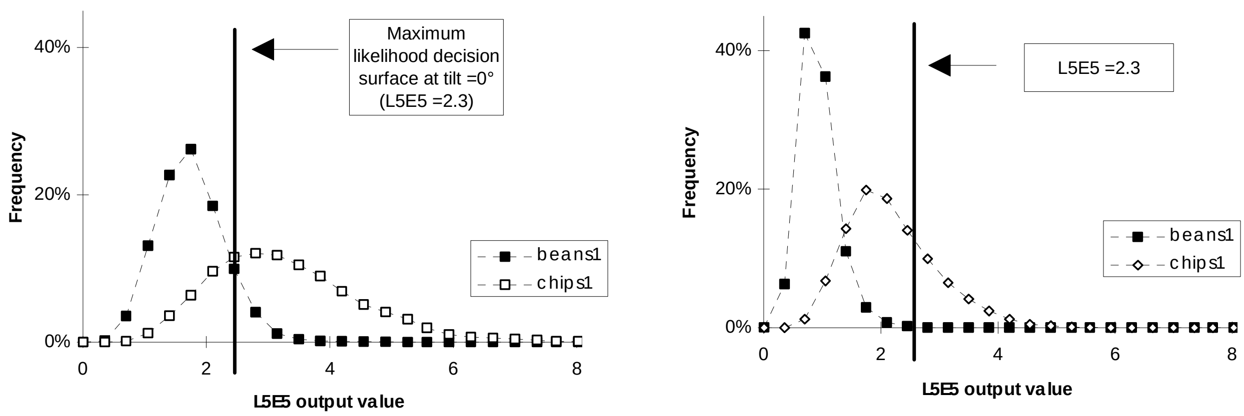
The two figures above show that :

a) variation of illuminant tilt does affect the *L5E5* operator's output, and

b) that the image model presented above does allow the form of the operator's response to be predicted, indicating that this theory may be useful for the development of an illuminant tilt-compensation scheme.

The behaviours of feature means are obviously important; however, they do not provide sufficient information to allow the full effects on classification and segmentation to be assessed. A small variation in mean due to change in illuminant tilt may be very significant for distributions of large variance but insignificant for those of small variance. Hence what is required is the behaviour of the distributions.

Figure 9 shows the distributions of *L5E5* for two of the test textures captured at two angles of illuminant tilt (τ = 0° and 90°).



(a) τ = 0° *(b) τ = 90°*

Figure 9 - Effect of tilt variation on L5E5 distributions[MJC4]

Assuming equal prior probabilities the maximum likelihood classifier trained under an illuminant tilt of 0° would have a decision surface at L5E5 = 2.3 - see Figure 9(a).

Figure 9(b) shows the result of changing the tilt to 90° : the mean of *chips1* is now clearly to the left of L5E5 = 2.3. Thus if a classifier were trained at $\tau = 0°$ then the majority of the class *chips1* would be incorrectly classified at an illuminant tilt of $\tau = 90°$. Thus changes in the illuminant's tilt have been shown to significantly affect the output of Laws' *L5E5* operator. Experiments with the other directional texture measures (Laws, co-occurrence and fractal dimension based) gave similar results. Experiments with omnidirectional texture measures showed that they may also be affected by changes in illuminant tilt when used with directional surface textures [13].

5. Conclusions

The main conclusions concern a) the directional filtering effect caused by directed illumination and b) the model of image texture presented above.

This paper has shown that the use of directed illumination during image capture can act as a directional filter of texture, and that a texture's directional characteristics are therefore not just a function of surface relief but are also affected by the illuminant's angle of tilt. The illumination conditions are therefore of fundamental importance to the analysis of images of three-dimensional texture. This is particularly so for classification and segmentation schemes that use directional feature measures. Changes in illumination direction over the scene due for instance to close proximity point lighting, or changes in illumination direction that occur between training and classification sessions, could both cause considerable problems for such systems — as evidenced by the effect of variation of tilt angle on the *L5E5* operator. These points seem obvious but to the author's knowledge have not been addressed before within the open literature on automated texture analysis.

The second conclusion concerns the model of image texture presented above. Although it is based on a linearised version of Lambert's law and hence extremely simple, and although many assumptions were made in its derivation, it has been very successfully used to predict the directional filtering effect. It has also been used, all be it in a modified form, to predict the behaviour of Laws' *L5E5* feature measure. In both these cases the predictions have compared well with empirical results. Thus the model provides a means of predicting the behaviours of image textures and feature measures in response to changes in illumination, and could therefore be used as the basis for tilt compensation schemes or a illuminant tilt estimation methods.

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References

1. K.I. Laws, "Textured image segmentation", Ph.D. thesis, Dept. Electrical Engineering, University of Southern California, Jan. 1980.
2. P. Brodatz, "*Textures : a photographic album for artists and designers*", Dover, New York, 1966.
3. L.S. Davis, "Polarograms : a new tool for image texture analysis", Pattern Recognition, V13, No. 3, 1981, pp219-223.
4. C. Sun & W.G. Wee, "Neighbouring Grey Level Dependence Matrix for Texture Classification", Computer Vision, Graphics & Image Processing, V23, 1983, pp341-352.
5. R.L. Kashyap & A. Khotanzad, "A Model-Based Method for Rotation Invariant Texture Classification", IEEE Transactions on Pattern Analysis and Machine Intelligence, V8, July 1986, pp472-481.
6. G. Eichmann & T. Kasparis, "Topologically Invariant Texture Descriptors", Computer Vision, Graphics & Image Processing, V41, 1988, pp267-281.
7. F.S. Cohen, Z. Fan & M.A. Patel, "Classification of Rotated and Scaled textured Images Using Gaussian Markov Field Models", IEEE Transactions on Pattern Analysis and Machine Intelligence, V13, , pp192-202.
8. S.J Clarke, "The analysis and synthesis of texture in side scan sonar data", Ph.D. thesis, Heriot-Watt University, May 92.
9. Y. Choe & R.L. Kashyap, "Modelling, Estimation, and Pattern Analysis of Random Texture on 3-D Surfaces", Rpt. TR-EE-91-4, Purdue University, Jan. 91.
10. D. Chetverikov, “Experiments in rotation-invariant texture discrimination using anisotropy features”, Proceedings of the 6th Int. Conf. on Pattern Recognition, Munich, 1982, pp1071-1073.
11. D. Chetverikov, “GLDH based analysis of texture anisotropy and symmetry”, Proceedings of the 12th Int. Conf. on Pattern Recognition, Jerusalem, 1994, pp444-448.
12. P. Kube, P. & A. Pentland, "On the imaging of fractal surfaces", IEEE Transactions on Pattern Analysis and Machine Intelligence, V10, No. 5, pp704-707, Sept. 1988
13. M.J. Chantler, "The effect of variation in illuminant direction on texture classification", Ph.D. thesis, Heriot-Watt University, August 94.
14. M.J. Chantler, G.T. Russell, L.M. Linnett, "Illumination : a directional filter of texture ?", Proceedings of the British Machine Vision Conference BMVC94, York, September 1994, pp449-458.
15. D. Saupe, "Algorithms for random fractals" in *The science of fractal images*, H-O Peitgen & D Saupe (eds.), Springer Verlag, 1988, pp71-113.
16. J.M.H. du Buf, M. Kardan & M. Spann, "Texture feature performance for image segmentation", Pattern Recognition,

V23, No. 3/4, 1990, pp291-309.

17. L. Van Gool, P. Dewaele & A. Oosterlinck, "Texture analysis anno 1983", Computer Vision, Graphics & Image Processing, V29, 1985, pp336-357

Appendix A

This appendix presents the theory used to develop the image model of three-dimensional texture.

The normalised image intensity $I(x,y)$ of the surface $V_H(x,y)$ is

$$I(x,y) = \mathbf{n} \cdot \mathbf{L} = \frac{-p\cos\tau\sin\sigma - q\sin\tau\sin\sigma + \cos\sigma}{\sqrt{p^2 + q^2 + 1}} \qquad (12)$$

where

$\mathbf{n}$ = the unit vector normal to the surface at the point (x,y)

$p = \dfrac{\partial V_H}{\partial x} \qquad q = \dfrac{\partial V_H}{\partial y}$

$V_H(x,y)$ is the height-map of the surface

$\mathbf{L} =$ (cosτ.sinσ, sinτ.sinσ, cosσ) is the unit vector towards the light source

τ and σ are the illuminant vector's *tilt* and *slant* angles as defined below.

Now in a departure from [12] and without loss of generality, choose a new axis (x',y',z) which is rotated τ about the z axis such that the projection of **L** onto the x-y plane will be parallel to the x' axis. In this new axis system the expression for intensity simplifies to

$$I(x,y) = \mathbf{n} \cdot \mathbf{L} = \frac{-r\sin\sigma + \cos\sigma}{\sqrt{r^2 + t^2 + 1}} \qquad (13)$$

where

$r = \dfrac{\partial V_H}{\partial x'}$, and $t = \dfrac{\partial V_H}{\partial y'}$

Taking the MacLaurin expansion yields

$$I(x,y) = (-r\sin\sigma + \cos\sigma)\left[1 - \frac{(r^2+t^2)}{2!} + \frac{9(r^4+t^4)}{4!}\ldots\ldots\right] \tag{14}$$

Now if the surface slope angle is less than 15°, then $r^2, t^2 << 1$; and the quadratic and higher order terms may be neglected. Note that the error introduced by this approximation for a slope angle of 15° is 3.5%. With this approximation (14) becomes

$$I(x,y) = (-r\sin\sigma + \cos\sigma) \tag{15}$$

which is simply the mean, plus a linear contribution of the surface gradient, measured in the direction of the illuminant's tilt angle.

Now the partial derivative operator $\frac{\partial}{\partial x'}$ is a *linear* operator [13], and in the frequency domain may be represented by :

$$\mathcal{F}\left[\frac{\partial V_H}{\partial x'}\right] = i\omega\cos(\theta - \tau)F_H(\omega,\theta) \tag{16}$$

where

ω is the angular frequency of the Fourier component

θ is its direction w.r.t. the *x*-axis

$\mathcal{F}[g(x,y)]$ is the two-dimensional Fourier transform of *g(x,y)*, and

$F_H(\omega,\theta) = \mathcal{F}[V_H(x,y)]$

Now, from (15)

$$I(x,y) = -\frac{\partial V_H}{\partial x'}\sin\sigma + \cos\sigma \tag{17}$$

Hence if the mean is ignored, the Fourier transform of the image intensity is :

$$\begin{aligned} F_I(\omega,\theta) &= \mathcal{F}\left[-\frac{\partial V_H}{\partial x'}\sin\sigma\right] \\ &= [-i\omega F_H(\omega,\theta)][\cos(\theta - \tau)][\sin\sigma] \end{aligned} \tag{18}$$